Linear Response Theory

Here the system is described by:

$$H = \underbrace{H_0}_{\text{Equilibrium Hamiltonian}} - \underbrace{A}_{\text{observable/operatorto which field couples}} \underbrace{F(t)}_{\text{external field}} \tag{1}$$

For F=0, the observable we are measuring is at its equilibrium value $\bar{B} = \bar{B_0} = tr(\rho_0 B) =< n|B|n > e^{-\frac{\beta E_n}{z}}$ not sure that this is right.This means that ρ_0 is a diagonal matrix, and I am not sure about the last line. Exponential is probably wrong.

We we add a perturbation in linear response theory we make several assumptions:

$$F(t)\ e^{i\omega t} e^{\eta t} \qquad \eta \to 0^+ \tag{2}$$

1. early times there is no perturbation
2. there is an adiabatic turning on of the perturbation
3. the time scale over which F changes is much slowler than the microscopic timescales/relaxation times/molecular collision times/local equilibrium. Implying
   $\rho(t) = \rho_0 - \Delta\rho(t)$

When we add a perturbation to the system in the form of F(t), the observable B is no longer time independent:

$$\bar{B}(t) = Tr(\rho(t)B) \tag{3}$$

$$\underbrace{\delta\bar{B}(t)}_{\text{response}} = \bar{B}(t) - \bar{B}_0 \tag{4}$$

Our goal is to get an expression for the response, thus we need an expression for $\rho(t)$

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H_t, \rho(t)] \tag{5}$$

Need to uinderstand where this comes from it is related to the Liouville operator

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H_0 - AF(t), \rho_0 + \Delta\rho(t)] \tag{6}$$

$$= \frac{1}{i\hbar}[H_0, \Delta\rho(t)] - \frac{1}{i\hbar}[A, \rho_0]F(t) + \text{(higher order terms)} \tag{7}$$

Note that since H_0 and ρ_0 commute their commutator becomes zero.

By convention: $H_0 * \hat{O} = [H_0, \hat{O}]$ we are changing pictures, but I do not know which picture we have changed to