```python
'''Importing all necessary libraries'''

import torch
import torchvision
import torchvision.transforms as transforms
from torch.utils.data import DataLoader, TensorDataset, Subset,
random_split, ConcatDataset,SubsetRandomSampler
import torch.nn.functional as F
import matplotlib.pyplot as plt
import os
import torch.nn as nn
from torch.distributions.normal import Normal
import torch.optim as optim
import random
import numpy as np
import pandas as pd
from PIL import Image
from torchvision.utils import make_grid
from scipy.stats import norm
from sklearn.preprocessing import LabelEncoder
from sklearn.manifold import TSNE
```

Part 1

Dataset Preparation

```python
'''Importing the MNIST Dataest'''

transform = transforms.Compose([transforms.Pad(padding = 2),
                                transforms.ToTensor()
                               ])

train_dataset = torchvision.datasets.MNIST(
    root='./data',
    train=True,
    transform=transform,
    download=True
)

test_dataset = torchvision.datasets.MNIST(
    root='./data',
    train=False,
    transform=transform,
    download=True
)
```

```
Downloading http://yann.lecun.com/exdb/mnist/train-images-idx3-
ubyte.gz
Failed to download (trying next):
HTTP Error 404: Not Found

Downloading https://ossci-datasets.s3.amazonaws.com/mnist/train-
```

```
images-idx3-ubyte.gz
Downloading https://ossci-datasets.s3.amazonaws.com/mnist/train-
images-idx3-ubyte.gz to ./data/MNIST/raw/train-images-idx3-ubyte.gz

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./data/MNIST/raw

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ubyte.gz
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labels-idx1-ubyte.gz
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labels-idx1-ubyte.gz to ./data/MNIST/raw/train-labels-idx1-ubyte.gz

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./data/MNIST/raw

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ubyte.gz
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Downloading https://ossci-datasets.s3.amazonaws.com/mnist/t10k-
images-idx3-ubyte.gz
Downloading https://ossci-datasets.s3.amazonaws.com/mnist/t10k-
images-idx3-ubyte.gz to ./data/MNIST/raw/t10k-images-idx3-ubyte.gz

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./data/MNIST/raw

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ubyte.gz
Failed to download (trying next):
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Downloading https://ossci-datasets.s3.amazonaws.com/mnist/t10k-
labels-idx1-ubyte.gz
Downloading https://ossci-datasets.s3.amazonaws.com/mnist/t10k-
labels-idx1-ubyte.gz to ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz

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Extracting ./data/MNIST/raw/t10k-labels-idx1-ubyte.gz to
./data/MNIST/raw
```
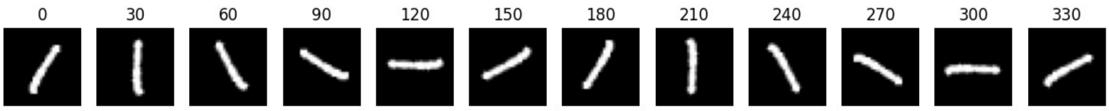

```
'''Due to limited computational budget, only using images labeled 1
and 2.
    Creating a new dataset, and plotting the images.'''

train_idx = [i for i, (img,label) in enumerate(train_dataset) if
label in [1,2]]
test_idx = [i for i, (img,label) in enumerate(test_dataset) if label
in [1,2]]

train_set = Subset(train_dataset, train_idx)
test_set = Subset(test_dataset, test_idx)

count = [0,0,0,0,0,0,0,0,0]
for img,label in train_set:
    count[label]+=1

print("Number of Images labeled 1: ",count[1])
print("Number of Images labeled 2: ",count[2])

fig, axes = plt.subplots(1, 12, figsize=(15, 2))
plt.title("Images in the Dataset")
for i in range(12):
    axes[i].imshow(train_set[i][0].squeeze(), cmap = 'gray')
    axes[i].axis('off')
plt.show()
```

```
Number of Images labeled 1:  6742
Number of Images labeled 2:  5958
```


```
'''Rotating each image in steps of 30 degrees. Thus creating a new
train and test dataset.
    Plotting an image labeled 1 in steps of 30 degrees.'''

angles = [0,30,60,90,120,150,180,210,240,270,300,330]

rotated_train_data_imgs = {theta: [] for theta in angles}
rotated_train_data_labels = []
to_pil = transforms.ToPILImage()
to_tensor = transforms.ToTensor()

for img,label in train_set:
    img_pil = to_pil(img)
    for theta in angles:
        temp_img = img_pil.rotate(theta,resample = Image.BICUBIC)
        rotated_train_data_imgs[theta].append(to_tensor(temp_img))
    rotated_train_data_labels.append(label)

rotated_test_data_imgs = {theta: [] for theta in angles}
```

```
rotated_test_data_labels = []

for img,label in test_set:
    img_pil = to_pil(img)
    for theta in angles:
        temp_img = img_pil.rotate(theta, resample = Image.BICUBIC)
        rotated_test_data_imgs[theta].append(to_tensor(temp_img))
    rotated_test_data_labels.append(label)

rotated_train_data_imgs = {theta:
torch.stack(rotated_train_data_imgs[theta]) for theta in angles}
rotated_test_data_imgs = {theta:
torch.stack(rotated_test_data_imgs[theta]) for theta in angles}

rotated_train_data_labels = torch.tensor(rotated_train_data_labels)
rotated_test_data_labels = torch.tensor(rotated_test_data_labels)


rot_train_dataset = {theta:
                     TensorDataset(rotated_train_data_imgs[theta],
                                   rotated_train_data_labels)
                     for theta in angles}

rot_test_dataset = {theta:
                     TensorDataset(rotated_test_data_imgs[theta],
                                   rotated_test_data_labels)
                     for theta in angles}


fig, axes = plt.subplots(1, 12, figsize=(15, 2))
for i in range(12):
    axes[i].imshow(rotated_train_data_imgs[i*30][0].squeeze(), cmap
= 'gray')
    axes[i].axis('off')
    axes[i].set_title(f"{i*30}")
# plt.title("Rotation of a sample image in steps of 30 degrees")
plt.show()


'''Splitting the train dataset into training and validation
dataset.'''

train_ratio = 0.8
val_ratio = 1 - train_ratio

num_train = int(len(rot_train_dataset[0]) * train_ratio)
num_val = len(rot_train_dataset[0]) - num_train
val_idx = random.sample(range(len(rot_train_dataset[0])), num_val)
train_idx = list(set(range(len(rot_train_dataset[0])))-set(val_idx))

rotated_train_dataset={theta: Subset(rot_train_dataset[theta],
train_idx) for theta in angles}
rotated_val_dataset={theta: Subset(rot_train_dataset[theta],
```

```
val_idx) for theta in angles}

print("Number of images in train dataest =
",len(rotated_train_dataset[0]))
print("Number of images in validation dataset =
",len(rotated_val_dataset[0]))
```




```
Number of images in train dataest =  10160
Number of images in validation dataset =  2540
```

```
'''Combining all the datasets correspoding to individual rotation
angles.'''

def combine_all_rotations(rot_dataset, shuffle):
    all_data = []
    for theta in angles:
        all_data.extend(list(rot_dataset[theta]))
    if shuffle:
        random.shuffle(all_data)
    data_tensors = torch.stack([x[0] for x in all_data])
    label_tensors = torch.tensor([x[1] for x in all_data])
    combined_dataset = TensorDataset(data_tensors, label_tensors)
    combined_dataloader = DataLoader(combined_dataset,
batch_size=batch_size, shuffle=shuffle)
    return combined_dataloader


batch_size = 64

train_loader = combine_all_rotations(rotated_train_dataset, True)
val_loader = combine_all_rotations(rotated_val_dataset, False)
test_loader = combine_all_rotations(rot_test_dataset, False)

fig, axes = plt.subplots(1, 12, figsize=(15, 2))
for i in range(12):
    image, _= train_loader.dataset[i]
    axes[i].imshow(image.squeeze(), cmap = 'gray')
    axes[i].axis('off')
plt.title("Sample images from the new dataset")
plt.show()
```

Latent Space Creation

```
'''Defining all the classes and functions necessary for VAE
training.'''
```

```python
device = ('cuda' if torch.cuda.is_available() else 'cpu')
lr = 0.0001
patience = 3
img_size= 32
channels = 1
embedding_dim = 16
epochs = 70
shape_before_flattening = (64,16,16)


output_dir='output'
os.makedirs('output', exist_ok = True)

training_dir=os.path.join(output_dir, 'training')
os.makedirs(training_dir, exist_ok = True)

weights_dir = os.path.join(output_dir, 'weights')
os.makedirs(weights_dir, exist_ok = True)
model_path = os.path.join(weights_dir, 'vae.pt')


def kl_loss(m, logvar):
    kld = -0.5*torch.sum(1+logvar - m.pow(2) - logvar.exp(), dim=1)
    return kld.mean()


def bce_loss(x_hat, x):
    return 1000*nn.BCELoss()(x_hat, x)


def vae_loss(y_pred, y_true):
    m, logvar, x_hat = y_pred
    return bce_loss(x_hat, y_true)+kl_loss(m, logvar)



class sampling(nn.Module):
    def forward(self, z_mean, z_log_var):
        batch, dim = z_mean.shape
        epsilon = Normal(0,1).sample((batch,dim)).to(z_mean.device)
        return z_mean+torch.exp(0.5*z_log_var)*epsilon


class Encoder(nn.Module):
    def __init__(self, img_size, embedding_dim):
        super(Encoder, self).__init__()
        self.conv1 = nn.Conv2d(in_channels = 1,
                               out_channels = 32,
                               kernel_size = 3,
                               stride = 2,
                               padding = 1)
        self.bn1 = nn.BatchNorm2d(32)
```

```
        self.conv2 = nn.Conv2d(in_channels = 32,
                               out_channels = 64,
                               kernel_size = 3,
                               stride = 1,
                               padding = 1)
        self.bn2 = nn.BatchNorm2d(64)
        self.conv3 = nn.Conv2d(in_channels = 64,
                               out_channels = 64,
                               kernel_size = 3,
                               stride = 1,
                               padding = 1)
        self.bn3 = nn.BatchNorm2d(64)
        self.flatten = nn.Flatten()
        self.fc1 = nn.Linear(64*16*16, 1024)
        self.fc_mean = nn.Linear(1024, embedding_dim)
        self.fc_logvar = nn.Linear(1024, embedding_dim)
        self.sampling = sampling()

    def forward(self, x):
        x = F.relu(self.bn1(self.conv1(x)))
        x = F.relu(self.bn2(self.conv2(x)))
        x=F.relu(self.bn3(self.conv3(x)))
        x= self.flatten(x)
        x=self.fc1(x)
        z_mean = self.fc_mean(x)
        z_logvar = self.fc_logvar(x)
        z=self.sampling(z_mean, z_logvar)
        return z_mean, z_logvar, z


class Decoder(nn.Module):
    def __init__(self, embedding_dim, shape_before_flattening):
        super(Decoder, self).__init__()
        self.fc1 = nn.Linear(embedding_dim, 1024)
        self.fc2 = nn.Linear(1024,
                        shape_before_flattening[0]
                        *shape_before_flattening[1]
                        *shape_before_flattening[2])
        self.reshape = lambda x:x.view(-1, *shape_before_flattening)
        self.deconv1 = nn.ConvTranspose2d(in_channels = 64,
                                          out_channels = 64,
                                            kernel_size = 3,
                                          stride = 1,
                                          padding = 1,
                                          output_padding = 0)
        self.deconv2 = nn.ConvTranspose2d(in_channels = 64,
                                          out_channels = 32,
                                          kernel_size = 3,
                                          stride = 1,
                                          padding = 1,
                                          output_padding = 0)
        self.deconv3 = nn.ConvTranspose2d(in_channels = 32,
                                          out_channels = 1,
                  kernel_size = 3,
```

```
                                            stride = 2,
                                            padding = 1,
                                            output_padding = 1)

    def forward(self, x):
        x = self.fc1(x)
        x= self.fc2(x)
        x = self.reshape(x)
        x = F.relu(self.deconv1(x))
        x = F.relu(self.deconv2(x))
        x = torch.sigmoid(self.deconv3(x))
        return x


class VAE(nn.Module):
    def __init__(self, encoder, decoder):
        super(VAE, self).__init__()
        self.encoder= encoder
        self.decoder = decoder

    def forward(self, x):
        z_mean, z_logvar, z = self.encoder(x)
        x_cap = self.decoder(z)
        return z_mean, z_logvar, x_cap


def validate(encoder, decoder, test_loader):
    encoder.eval()
    decoder.eval()
    r_loss_kl, r_loss_bce, r_loss_total = 0.0, 0.0,0.0
    num_batches = len(test_loader)
    with torch.no_grad():
        for batch_idx, (data, _) in enumerate(test_loader):
            data = data.to(device)
            encoded = encoder(data)
            decoded = decoder(encoded[2])
            loss_kl, loss_bce = kl_loss(encoded[0], encoded[1]) ,
bce_loss(decoded, data)
            r_loss_kl+=loss_kl
            r_loss_bce+=loss_bce
            r_loss_total += loss_kl+ loss_bce
    return (r_loss_kl/num_batches, r_loss_bce/num_batches,
r_loss_total/num_batches)

'''Training.'''

encoder = Encoder(img_size, embedding_dim).to(device)
decoder = Decoder(embedding_dim, shape_before_flattening).to(device)
vae = VAE(encoder, decoder).to(device)

optimizer = optim.AdamW(list(encoder.parameters()) +
list(decoder.parameters()),
                        lr,
```

```
                        weight_decay=1e-5)
scheduler = optim.lr_scheduler.ReduceLROnPlateau(optimizer,
                                                mode = "min",
                                                factor = 0.5,
                                                patience = patience)

best_val_loss = float("inf")
encoder.train()
decoder.train()


for epoch in range(epochs):
    train_loss = 0.0
    val_loss_kl, val_loss_bce, val_loss_total = 0.0,0.0,0.0
    running_loss = 0.0
    for batch_idx, (data, _) in enumerate(train_loader):
        data = data.to(device)
        optimizer.zero_grad()
        pred = vae(data)
        loss = vae_loss(pred, data)
        loss.backward()
        optimizer.step()
        running_loss += loss.item()
    train_loss += running_loss/len(train_loader)
    val_loss = validate(encoder, decoder, val_loader)
    val_loss_kl+=val_loss[0]
    val_loss_bce+=val_loss[1]
    val_loss_total += val_loss[2]
    print(f"epoch: {epoch} train loss = {train_loss:.4f} val loss kl
= {val_loss_kl:.4f} valloss bce = {val_loss_bce:.4f} val loss total=
{val_loss_total:.4f}")
    if(val_loss_total<best_val_loss):
        best_val_loss = val_loss_total
        torch.save({"vae":vae.state_dict()}, model_path)

    scheduler.step(val_loss_total)
```

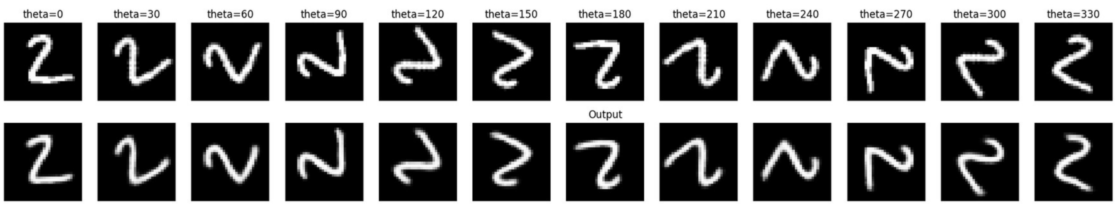
```
epoch: 0 train loss = 116.1134 val loss kl = 21.7237 valloss bce =
77.5487 val loss total= 99.2724
epoch: 1 train loss = 96.1418 val loss kl = 21.3747 valloss bce =
72.3218 val loss total= 93.6965
epoch: 2 train loss = 93.2484 val loss kl = 21.8136 valloss bce =
70.0885 val loss total= 91.9020
epoch: 3 train loss = 91.5650 val loss kl = 20.7411 valloss bce =
69.9045 val loss total= 90.6456
epoch: 4 train loss = 90.2980 val loss kl = 21.9437 valloss bce =
67.6679 val loss total= 89.6116
epoch: 5 train loss = 89.3986 val loss kl = 20.6957 valloss bce =
68.1720 val loss total= 88.8678
epoch: 6 train loss = 88.6298 val loss kl = 20.6144 valloss bce =
67.7189 val loss total= 88.3333
epoch: 7 train loss = 88.1106 val loss kl = 20.6480 valloss bce =
67.1165 val loss total= 87.7645
epoch: 8 train loss = 87.6243 val loss kl = 20.3612 valloss bce =
```

```
66.9033 val loss total= 87.2645
epoch: 9 train loss = 87.2407 val loss kl = 20.8734 valloss bce =
66.1236 val loss total= 86.9970
epoch: 10 train loss = 86.9601 val loss kl = 20.9174 valloss bce =
65.8791 val loss total= 86.7965
epoch: 11 train loss = 86.6494 val loss kl = 20.8121 valloss bce =
65.8846 val loss total= 86.6967
epoch: 12 train loss = 86.4231 val loss kl = 21.0929 valloss bce =
65.2490 val loss total= 86.3419
epoch: 13 train loss = 86.1669 val loss kl = 20.8362 valloss bce =
65.2172 val loss total= 86.0533
epoch: 14 train loss = 85.9674 val loss kl = 20.8503 valloss bce =
65.0671 val loss total= 85.9174
epoch: 15 train loss = 85.8267 val loss kl = 21.2121 valloss bce =
64.5810 val loss total= 85.7932
epoch: 16 train loss = 85.6445 val loss kl = 21.0318 valloss bce =
64.4872 val loss total= 85.5189
epoch: 17 train loss = 85.4893 val loss kl = 21.0323 valloss bce =
64.4368 val loss total= 85.4691
epoch: 18 train loss = 85.3652 val loss kl = 21.0183 valloss bce =
64.5080 val loss total= 85.5263
epoch: 19 train loss = 85.2275 val loss kl = 20.8464 valloss bce =
64.3385 val loss total= 85.1849
epoch: 20 train loss = 85.0929 val loss kl = 20.8621 valloss bce =
64.2328 val loss total= 85.0950
epoch: 21 train loss = 85.0113 val loss kl = 20.6340 valloss bce =
64.4477 val loss total= 85.0817
epoch: 22 train loss = 84.9035 val loss kl = 20.6908 valloss bce =
64.3600 val loss total= 85.0509
epoch: 23 train loss = 84.7745 val loss kl = 20.6354 valloss bce =
64.2518 val loss total= 84.8872
epoch: 24 train loss = 84.6842 val loss kl = 20.8638 valloss bce =
63.8649 val loss total= 84.7287
epoch: 25 train loss = 84.6025 val loss kl = 21.0588 valloss bce =
63.7322 val loss total= 84.7911
epoch: 26 train loss = 84.5080 val loss kl = 21.0353 valloss bce =
63.5725 val loss total= 84.6078
epoch: 27 train loss = 84.4120 val loss kl = 20.8158 valloss bce =
63.8385 val loss total= 84.6543
epoch: 28 train loss = 84.3607 val loss kl = 20.9313 valloss bce =
63.4472 val loss total= 84.3786
epoch: 29 train loss = 84.2703 val loss kl = 20.7234 valloss bce =
63.8422 val loss total= 84.5656
epoch: 30 train loss = 84.2150 val loss kl = 20.7905 valloss bce =
63.6582 val loss total= 84.4487
epoch: 31 train loss = 84.1527 val loss kl = 21.0305 valloss bce =
63.3502 val loss total= 84.3807
epoch: 32 train loss = 84.0841 val loss kl = 20.8152 valloss bce =
63.5279 val loss total= 84.3430
epoch: 33 train loss = 83.9892 val loss kl = 20.8771 valloss bce =
63.3802 val loss total= 84.2573
epoch: 34 train loss = 83.9706 val loss kl = 20.9474 valloss bce =
63.2385 val loss total= 84.1858
epoch: 35 train loss = 83.8729 val loss kl = 21.0030 valloss bce =
```

```
63.1889 val loss total= 84.1919
epoch: 36 train loss = 83.8708 val loss kl = 20.6011 valloss bce =
63.6119 val loss total= 84.2130
epoch: 37 train loss = 83.7703 val loss kl = 20.5103 valloss bce =
63.6374 val loss total= 84.1476
epoch: 38 train loss = 83.7553 val loss kl = 20.9811 valloss bce =
63.1420 val loss total= 84.1230
epoch: 39 train loss = 83.6783 val loss kl = 20.6823 valloss bce =
63.3115 val loss total= 83.9938
epoch: 40 train loss = 83.6401 val loss kl = 20.9163 valloss bce =
63.0644 val loss total= 83.9806
epoch: 41 train loss = 83.6216 val loss kl = 20.6977 valloss bce =
63.1485 val loss total= 83.8462
epoch: 42 train loss = 83.5744 val loss kl = 20.6147 valloss bce =
63.3116 val loss total= 83.9263
epoch: 43 train loss = 83.5197 val loss kl = 20.8140 valloss bce =
62.9767 val loss total= 83.7907
epoch: 44 train loss = 83.4958 val loss kl = 20.5626 valloss bce =
63.2158 val loss total= 83.7784
epoch: 45 train loss = 83.4483 val loss kl = 20.7847 valloss bce =
62.9660 val loss total= 83.7507
epoch: 46 train loss = 83.3894 val loss kl = 20.7375 valloss bce =
62.9576 val loss total= 83.6951
epoch: 47 train loss = 83.3654 val loss kl = 20.8656 valloss bce =
62.8897 val loss total= 83.7552
epoch: 48 train loss = 83.3588 val loss kl = 20.4180 valloss bce =
63.3602 val loss total= 83.7782
epoch: 49 train loss = 83.2938 val loss kl = 20.5081 valloss bce =
63.0667 val loss total= 83.5748
epoch: 50 train loss = 83.2443 val loss kl = 20.9435 valloss bce =
62.6185 val loss total= 83.5619
epoch: 51 train loss = 83.2198 val loss kl = 20.9932 valloss bce =
62.5498 val loss total= 83.5431
epoch: 52 train loss = 83.1730 val loss kl = 20.7395 valloss bce =
62.8871 val loss total= 83.6265
epoch: 53 train loss = 83.1540 val loss kl = 21.1079 valloss bce =
62.5840 val loss total= 83.6920
epoch: 54 train loss = 83.1307 val loss kl = 20.8149 valloss bce =
62.7144 val loss total= 83.5293
epoch: 55 train loss = 83.0780 val loss kl = 20.7437 valloss bce =
62.7914 val loss total= 83.5351
epoch: 56 train loss = 83.0745 val loss kl = 20.8455 valloss bce =
62.6493 val loss total= 83.4948
epoch: 57 train loss = 83.0392 val loss kl = 20.8614 valloss bce =
62.6519 val loss total= 83.5133
epoch: 58 train loss = 83.0235 val loss kl = 20.7159 valloss bce =
62.7860 val loss total= 83.5018
epoch: 59 train loss = 82.9934 val loss kl = 20.5919 valloss bce =
62.8557 val loss total= 83.4476
epoch: 60 train loss = 82.9500 val loss kl = 20.5740 valloss bce =
62.8993 val loss total= 83.4732
epoch: 61 train loss = 82.9483 val loss kl = 20.7973 valloss bce =
62.6179 val loss total= 83.4152
epoch: 62 train loss = 82.8969 val loss kl = 20.2087 valloss bce =
```

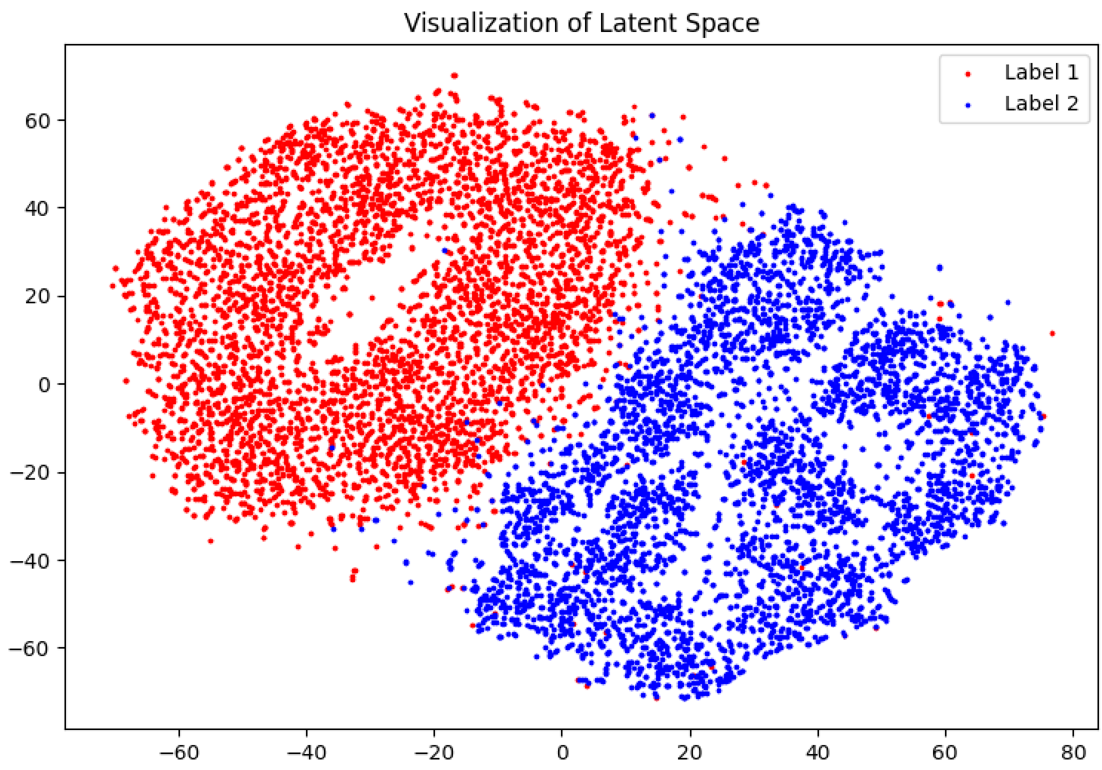
```
63.1748 val loss total= 83.3835
epoch: 63 train loss = 82.8696 val loss kl = 20.8835 valloss bce =
62.5377 val loss total= 83.4212
epoch: 64 train loss = 82.8485 val loss kl = 20.6887 valloss bce =
62.7026 val loss total= 83.3912
epoch: 65 train loss = 82.8372 val loss kl = 20.5825 valloss bce =
62.7495 val loss total= 83.3320
epoch: 66 train loss = 82.8070 val loss kl = 20.6564 valloss bce =
62.5380 val loss total= 83.1944
epoch: 67 train loss = 82.7727 val loss kl = 20.9650 valloss bce =
62.3681 val loss total= 83.3330
epoch: 68 train loss = 82.7589 val loss kl = 20.6929 valloss bce =
62.5665 val loss total= 83.2594
epoch: 69 train loss = 82.7392 val loss kl = 20.9446 valloss bce =
62.3855 val loss total= 83.3300
```

```
'''Visualizing the output of the Encoder and Decoder training.
    The top row is the input. It goes into the encoder and then the
decoder.
    The ouput by the decoder is plotted in the bottom row.'''

vae.eval()
fig, axes = plt.subplots(2, len(angles), figsize=(len(angles)*2, 4))

model = vae

for i, theta in enumerate(angles):
    input_tensor, _= rot_test_dataset[theta][0]
    input_tensor = input_tensor.to(device)
    # print(input_tensor.shape)
    with torch.no_grad():
        zm, zlogvar, zpred = model.encoder(input_tensor.reshape(1,
1, 32, 32))
        ypred = model.decoder(zpred)

    axes[0,i].imshow(input_tensor.cpu().squeeze(), cmap='gray')
    axes[0,i].set_title(f"theta={theta}")
    axes[0,i].axis("off")

    axes[1,i].imshow(ypred.cpu().squeeze(), cmap='gray')
    axes[1,6].set_title(f"Output")
    axes[1,i].axis("off")

plt.show()
```



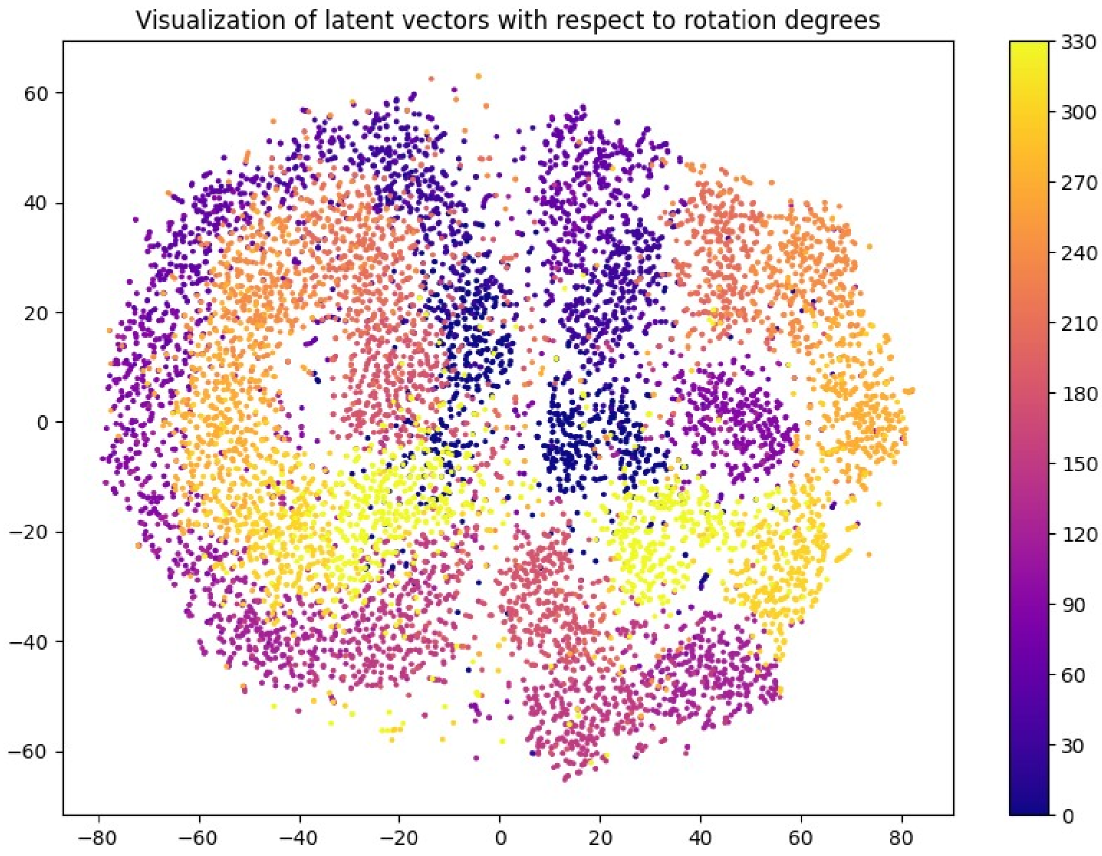

```
'''Latent Space Visualization.
    This plot only visualizes inputs with label 1 and 2.'''
```

```
latent_vectors = []
labels = []

for theta in angles:
    for batch_idx, (data, label) in
enumerate(rot_test_dataset[theta]):
        if batch_idx > 800:
            break
        label = label.item()
        data = data.to(device)
        with torch.no_grad():
            _, _, lvec = vae.encoder(data.reshape(1,1,32,32))
            latent_vectors.append(lvec.cpu().numpy())
            labels.append(label)

latent_vectors = np.concatenate(latent_vectors, axis=0)
labels = np.array(labels)
tsne = TSNE(n_components=2, perplexity=30, n_iter=1000,
random_state=56)
X_tsne = tsne.fit_transform(latent_vectors)
color_map = {1: 'red', 2: 'blue'}

plt.figure(figsize=(9, 6))
for label, color in color_map.items():
    mask = labels == label
    plt.scatter(X_tsne[mask, 0], X_tsne[mask, 1], color=color,
label=f'Label {label}', s=3)

plt.title("Visualization of Latent Space")
plt.legend()
plt.show()
```




```
'''Latent Space Visualization.
    This plot visualizes the rotation angles regardless of the image
label.'''

latent_vectors = []
labels = []

for theta in angles:
    for batch_idx, (data, label) in
enumerate(rot_test_dataset[theta]):
        if batch_idx > 800:
            break
        label = (label.item(), theta)
        data = data.to(device)
        with torch.no_grad():
            _, _, lvec = vae.encoder(data.reshape(1,1,32,32))
            latent_vectors.append(lvec.cpu().numpy())
            labels.append(label)

latent_vectors = np.concatenate(latent_vectors, axis=0)
labels = np.array(labels)
angle_labels = np.array([theta for _, theta in labels])

label_encoder = LabelEncoder()
encoded_labels = label_encoder.fit_transform(angle_labels)

reducer = TSNE(n_components=2, learning_rate=200, n_iter=1000,
random_state=56)
X_tsne = reducer.fit_transform(latent_vectors)
```

```python
plt.figure(figsize=(10, 7))
scatter = plt.scatter(X_tsne[:, 0], X_tsne[:, 1], c=angle_labels,
cmap='plasma', s=3)
cbar = plt.colorbar(scatter)

cbar.set_ticks(angles)
cbar.set_ticklabels([str(a) for a in angles])

plt.title("Visualization of latent vectors with respect to rotation
degrees")
plt.show()
```



Part 2

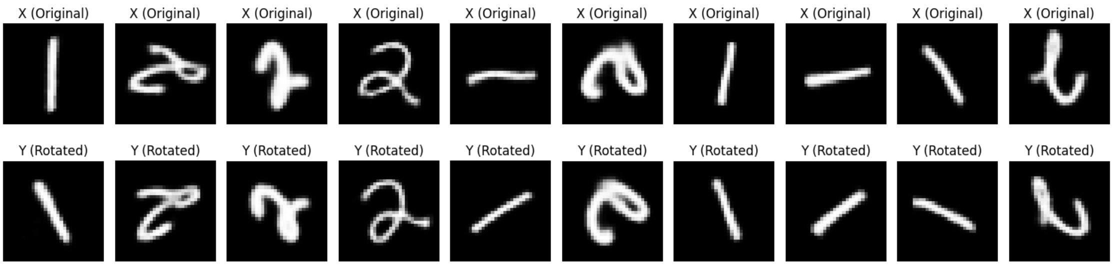
Supervised Symmetry Discovery

```python
'''As we want to learn latent vector rotation by 30 degrees, we will
need to create a new dataset.
    In the new dataset, each latent vector of an image is paired
with a latent vector
    which was generated by encoding the 30 degree rotated version of
the original image.
    This way, each l.vector has a target of its 30 degree rotated
version l.vector.
    We can see the plot to understand the dataset better.'''
```

```
def generate_new_loader(loader_dict, shuffle):
    X, y, y_temp = [], [], []
    count = 0
    len_of_data = len(loader_dict[0])
    num_samples = int(0.5*len_of_data)
    idx = random.sample(range(len_of_data), num_samples)
    for theta, loader in loader_dict.items():
        subset = Subset(loader, idx)
        new_loader = DataLoader(subset, batch_size=batch_size,
shuffle=False)
        for data in new_loader:
            inputs, _ = data
            inputs = inputs.to(next(vae.parameters()).device)
            _, _, vec = vae.encoder(inputs)
            if(inputs.shape[0]!=64):
                continue
            if(theta==0): count+=1
            X.append(vec.cpu().detach().numpy())
    X_new =[]
    for vec_x in X:
        tensor_vec_x = torch.Tensor(vec_x)
        X_new.append(tensor_vec_x)
    y_new = X_new[count:]+ X_new[:count]
    X_tensor = torch.cat(tuple(X_new), dim=0)
    y_tensor = torch.cat(tuple(y_new), dim=0)
    dataset = TensorDataset(X_tensor, y_tensor)
    return DataLoader(dataset, batch_size=batch_size,
shuffle=shuffle)


new_train_loader = generate_new_loader(rotated_train_dataset, True)
print("training set generated")
new_val_loader = generate_new_loader(rotated_val_dataset, True)
print("validation set generated")
new_test_loader = generate_new_loader(rot_test_dataset, False)
print("testing set generated")

X_sample, y_sample = next(iter(new_train_loader))
X_sample, y_sample = X_sample[:10], y_sample[:10]
X_sample = X_sample.to(device)
y_sample = y_sample.to(device)
X_decoded = vae.decoder(X_sample).cpu().detach().numpy()
y_decoded = vae.decoder(y_sample).cpu().detach().numpy()

fig, axes = plt.subplots(2, 10, figsize=(15, 4))

for i in range(10):
    axes[0, i].imshow(X_decoded[i].squeeze(), cmap="gray")
    axes[0, i].set_title("X (Original)")
    axes[0, i].axis("off")

    axes[1, i].imshow(y_decoded[i].squeeze(), cmap="gray")
    axes[1, i].set_title("Y (Rotated)")
```

```
    axes[1, i].axis("off")

plt.tight_layout()
plt.show()
```

```
training set generated
validation set generated
testing set generated
```




```
'''Defining all the classes and functions required for MLP
training.'''


lr_mlp = 0.01
patience_mlp = 3
epochs_mlp =75
mlp_model_path = os.path.join(weights_dir, 'mlp.pt')


class MLP(nn.Module):
    def __init__(self):
        super().__init__()
        self.fc1 = nn.Linear(embedding_dim, 64, bias = True)
        self.do1 = nn.Dropout(0.3)
        self.bc1 = nn.BatchNorm1d(64)
        self.relu1 = nn.ReLU()

        self. fc2 = nn.Linear(64, 128, bias = True)
        self.do2 = nn.Dropout(0.3)
        self.bc2 = nn.BatchNorm1d(128)
        self.relu2 = nn.ReLU()

        self.fc3 = nn.Linear(128, 64, bias = True)
        self.do3 = nn.Dropout(0.3)
        self.bc3 = nn.BatchNorm1d(64)
        self.relu3 = nn.ReLU()

        self.fc4 = nn.Linear(64, embedding_dim, bias = True)

        for m in self.modules():
            if isinstance(m,nn.Linear):
                nn.init.kaiming_normal_(m.weight)
                m.bias.data.zero_()

    def forward(self, x):
```

```python
        x = self.relu1(self.bc1(self.do1(self.fc1(x))))
        x = self.relu2(self.bc2(self.do2(self.fc2(x))))
        x = self.relu3(self.bc3(self.do3(self.fc3(x))))
        x = self.fc4(x)
        return x


def mse_loss(x_hat, x):
    return 50*nn.MSELoss()(x_hat, x)


def evaluate(model, test_loader):
    model.eval()
    loss= 0.0
    num_batches = len(test_loader)
    with torch.no_grad():
        for batch_idx, (data, label) in enumerate(test_loader):
            data = data.to(device)
            label = label.to(device)
            predicted = model(data)
            loss += mse_loss(predicted, label)
    return (loss/num_batches)

'''Training.'''


mlp = MLP().to(device)

optimizer = optim.AdamW(mlp.parameters(),
                        lr_mlp,
                        weight_decay=1e-5)
scheduler = optim.lr_scheduler.ReduceLROnPlateau(optimizer,
                                                 mode = "min",
                                                 factor = 0.5,
                                                 patience =
patience_mlp)

best_val_loss = float("inf")
mlp.train()
mlp = torch.nn.DataParallel(mlp)

for epoch in range(epochs_mlp):
    train_loss = 0.0
    val_loss = 0.0
    running_loss = 0.0
    for batch_idx, (data, label) in enumerate(new_train_loader):
        data = data.to(device)
        label = label.to(device)
        optimizer.zero_grad()
        pred = mlp(data)
        loss = mse_loss(pred, label)
        loss.backward()
        torch.nn.utils.clip_grad_norm_(mlp.parameters(),
```

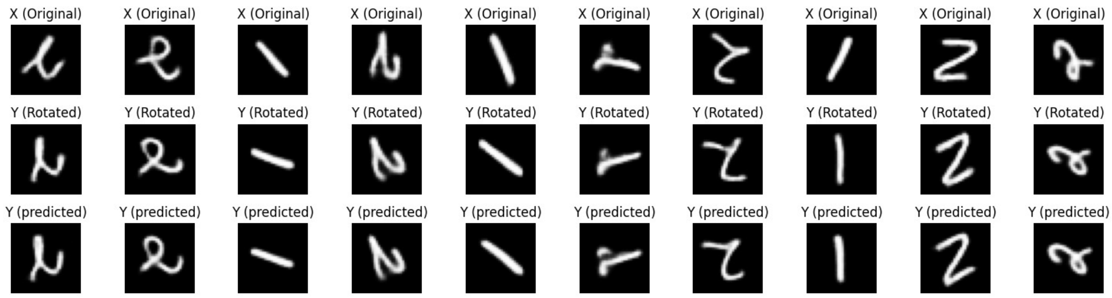
```
max_norm=1.0)
        optimizer.step()
        running_loss += loss.item()
    train_loss += running_loss/len(new_train_loader)
    val_loss += evaluate(mlp, new_val_loader)
    print(f"epoch: {epoch} train loss = {train_loss:.4f} val loss=
{val_loss:.4f}")
    if(val_loss<best_val_loss):
        best_val_loss = val_loss
        torch.save({"mlp":mlp.module.state_dict()}, mlp_model_path)

    scheduler.step(val_loss)
```

```
epoch: 0 train loss = 29.9880 val loss= 19.8742
epoch: 1 train loss = 17.4720 val loss= 16.8214
epoch: 2 train loss = 16.6675 val loss= 16.4569
epoch: 3 train loss = 16.4124 val loss= 16.3449
epoch: 4 train loss = 16.2353 val loss= 16.1315
epoch: 5 train loss = 16.1297 val loss= 16.2831
epoch: 6 train loss = 16.0605 val loss= 16.0285
epoch: 7 train loss = 15.9836 val loss= 16.1668
epoch: 8 train loss = 15.9377 val loss= 16.1064
epoch: 9 train loss = 15.8849 val loss= 15.9842
epoch: 10 train loss = 15.8532 val loss= 16.0052
epoch: 11 train loss = 15.8192 val loss= 15.9797
epoch: 12 train loss = 15.7824 val loss= 15.9199
epoch: 13 train loss = 15.7567 val loss= 16.0286
epoch: 14 train loss = 15.7165 val loss= 15.9921
epoch: 15 train loss = 15.7135 val loss= 15.8397
epoch: 16 train loss = 15.6917 val loss= 15.9900
epoch: 17 train loss = 15.6751 val loss= 15.9626
epoch: 18 train loss = 15.6568 val loss= 15.7513
epoch: 19 train loss = 15.6397 val loss= 15.8528
epoch: 20 train loss = 15.6259 val loss= 15.7367
epoch: 21 train loss = 15.6293 val loss= 15.7887
epoch: 22 train loss = 15.5983 val loss= 15.9373
epoch: 23 train loss = 15.5796 val loss= 16.0186
epoch: 24 train loss = 15.5691 val loss= 15.8027
epoch: 25 train loss = 15.2056 val loss= 15.5819
epoch: 26 train loss = 15.1720 val loss= 15.5077
epoch: 27 train loss = 15.1539 val loss= 15.4858
epoch: 28 train loss = 15.1467 val loss= 15.5081
epoch: 29 train loss = 15.1314 val loss= 15.5126
epoch: 30 train loss = 15.1275 val loss= 15.4733
epoch: 31 train loss = 15.1169 val loss= 15.4719
epoch: 32 train loss = 15.1098 val loss= 15.4786
epoch: 33 train loss = 15.1089 val loss= 15.4387
epoch: 34 train loss = 15.1012 val loss= 15.4693
epoch: 35 train loss = 15.0904 val loss= 15.5001
epoch: 36 train loss = 15.0878 val loss= 15.5154
epoch: 37 train loss = 15.0794 val loss= 15.4829
epoch: 38 train loss = 14.8757 val loss= 15.3420
epoch: 39 train loss = 14.8599 val loss= 15.3617
epoch: 40 train loss = 14.8440 val loss= 15.2975
```

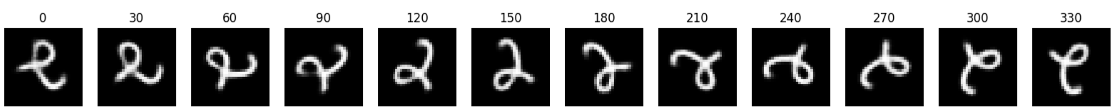
```
epoch: 41 train loss = 14.8446 val loss= 15.3044
epoch: 42 train loss = 14.8417 val loss= 15.3560
epoch: 43 train loss = 14.8398 val loss= 15.3275
epoch: 44 train loss = 14.8358 val loss= 15.3025
epoch: 45 train loss = 14.7160 val loss= 15.2653
epoch: 46 train loss = 14.7032 val loss= 15.2427
epoch: 47 train loss = 14.7031 val loss= 15.2614
epoch: 48 train loss = 14.6978 val loss= 15.2368
epoch: 49 train loss = 14.6956 val loss= 15.2670
epoch: 50 train loss = 14.6940 val loss= 15.2401
epoch: 51 train loss = 14.6884 val loss= 15.2527
epoch: 52 train loss = 14.6870 val loss= 15.2417
epoch: 53 train loss = 14.6244 val loss= 15.2306
epoch: 54 train loss = 14.6198 val loss= 15.2078
epoch: 55 train loss = 14.6179 val loss= 15.2268
epoch: 56 train loss = 14.6152 val loss= 15.2172
epoch: 57 train loss = 14.6139 val loss= 15.2134
epoch: 58 train loss = 14.6130 val loss= 15.2066
epoch: 59 train loss = 14.5784 val loss= 15.1944
epoch: 60 train loss = 14.5761 val loss= 15.1955
epoch: 61 train loss = 14.5742 val loss= 15.1943
epoch: 62 train loss = 14.5731 val loss= 15.1936
epoch: 63 train loss = 14.5723 val loss= 15.1907
epoch: 64 train loss = 14.5713 val loss= 15.2017
epoch: 65 train loss = 14.5709 val loss= 15.1885
epoch: 66 train loss = 14.5711 val loss= 15.1883
epoch: 67 train loss = 14.5687 val loss= 15.1956
epoch: 68 train loss = 14.5691 val loss= 15.1909
epoch: 69 train loss = 14.5676 val loss= 15.1942
epoch: 70 train loss = 14.5496 val loss= 15.1858
epoch: 71 train loss = 14.5480 val loss= 15.1840
epoch: 72 train loss = 14.5477 val loss= 15.1832
epoch: 73 train loss = 14.5470 val loss= 15.1881
epoch: 74 train loss = 14.5465 val loss= 15.1860
```

```python
'''Visualization of the result of the trained MLP.
    The original image, the rotated image, and the output of the MLP
by feeding in the original image are all plotted.'''

test_data = list(new_test_loader.dataset)

idx = random.sample(range(len(test_data)), 10)
X_sample, y_sample = zip(*[test_data[i] for i in idx])

X_sample = torch.stack(X_sample).to(device)
y_sample = torch.stack(y_sample).to(device)

X_decoded = vae.decoder(X_sample).cpu().detach().numpy()
y_decoded = vae.decoder(y_sample).cpu().detach().numpy()
y_predicted = vae.decoder(mlp(X_sample)).cpu().detach().numpy()

fig, axes = plt.subplots(3, 10, figsize=(15, 4))

for i in range(10):
```

```
    axes[0, i].imshow(X_decoded[i].squeeze(), cmap="gray")
    axes[0, i].set_title("X (Original)")
    axes[0, i].axis("off")

    axes[1, i].imshow(y_decoded[i].squeeze(), cmap="gray")
    axes[1, i].set_title("Y (Rotated)")
    axes[1, i].axis("off")

    axes[2,i].imshow(y_predicted[i].squeeze(), cmap = "gray")
    axes[2,i].set_title("Y (predicted)")
    axes[2, i].axis("off")

plt.tight_layout()
plt.show()
```




```
'''If we apply the MLP on an image 12 times, we can achieve a full rotation.
    This is a demonstration of the idea.'''


complete_rotation = []
l_vec = X_sample[1]
complete_rotation.append(vae.decoder(l_vec).cpu().detach().numpy())

for i in range(12):
    l_vec = mlp(l_vec.reshape(1,-1))

complete_rotation.append(vae.decoder(l_vec).cpu().detach().numpy())

fig, axes = plt.subplots(1, 12, figsize=(20, 4))

for i in range(12):
    axes[i].imshow(complete_rotation[i].squeeze(), cmap="gray")
    axes[i].set_title(f"{30*i}")
    axes[i].axis("off")

plt.show()
```



PART 3

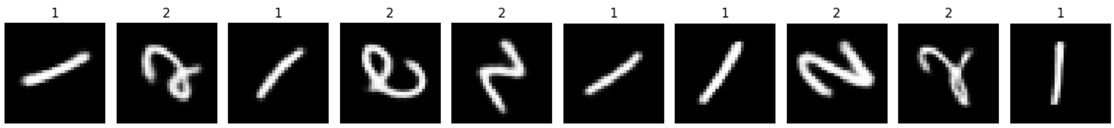
Unsupervised Symmetry Discovery

Induced Oracle:

```
'''For training the Induced Oracle, we will need to create a new dataset.
    Each image(including all the rotated versions) are paired with their original label(i.e. 1 or 2).
    Visualizing the new dataset with the images and their corresponding labels.'''


def generate_symmetry_loader(loader, shuffle):
    X, y= [], []

    for tensor, label in loader:
        tensor = tensor.reshape(tensor.shape[0],1, 32, 32)
        tensor = tensor.to(device)
        label = label.to(device)
        _, _, vec = vae.encoder(tensor)
        if(tensor.shape[0]!=64):
            continue
        X.append(vec.cpu().detach().numpy())
        y.append(label)
    X_new =[]
    for vec_x in X:
        tensor_vec_x = torch.Tensor(vec_x)
        X_new.append(tensor_vec_x)

    X_tensor = torch.cat(tuple(X_new), dim=0)
    y_tensor = torch.cat(y, dim=0)

    dataset = TensorDataset(X_tensor, y_tensor)
    return DataLoader(dataset, batch_size=batch_size, shuffle=shuffle)

sym_train_loader = generate_symmetry_loader(train_loader, True)
print("training set generated")
sym_val_loader = generate_symmetry_loader(val_loader, True)
print("validation set generated")
sym_test_loader = generate_symmetry_loader(test_loader, False)
print("testing set generated")

vae.eval()

X_sample, y_sample = next(iter(sym_train_loader))
X_sample, y_sample = X_sample[:10], y_sample[:10]

with torch.no_grad():
    X_sample = X_sample.to(device)
```

```python
    X_decoded = vae.decoder(X_sample).cpu()

fig, axes = plt.subplots(1, 10, figsize=(15, 4))

for i in range(10):
    axes[i].imshow(X_decoded[i].squeeze(), cmap="gray")
    axes[i].set_title(f"{y_sample[i]}")
    axes[i].axis("off")
plt.tight_layout()
plt.show()
```

```
training set generated
validation set generated
testing set generated
```




```python
'''Defining all the classes and functions required to train the
Induced Oracle.'''


lr_oracle = 0.001
patience_oracle = 3
epochs_oracle = 10
oracle_model_path = os.path.join(weights_dir, 'oracle.pt')


class Oracle(nn.Module):
    def __init__(self):
        super().__init__()
        self.fc1 = nn.Linear(embedding_dim, 64, bias = True)
        self.do1 = nn.Dropout(0.5)
        self.bc1 = nn.BatchNorm1d(64)
        self.relu1 = nn.ReLU()

        self. fc2 = nn.Linear(64, 128, bias = True)
        self.do2 = nn.Dropout(0.5)
        self.bc2 = nn.BatchNorm1d(128)
        self.relu2 = nn.ReLU()

        self.fc3 = nn.Linear(128, 32, bias = True)
        self.do3 = nn.Dropout(0.5)
        self.bc3 = nn.BatchNorm1d(32)
        self.relu3 = nn.ReLU()

        self.fc4 = nn.Linear(32, 2, bias = True)

        for m in self.modules():
            if isinstance(m,nn.Linear):
                nn.init.kaiming_normal_(m.weight)
                m.bias.data.zero_()
```

```python
    def forward(self, x):
        x = self.relu1(self.bc1(self.do1(self.fc1(x))))
        x = self.relu2(self.bc2(self.do2(self.fc2(x))))
        x = self.relu3(self.bc3(self.do3(self.fc3(x))))
        x = self.fc4(x)
        return x


def cross_entropy_loss(x_hat, x):
    x = x-1
    return 50* nn.CrossEntropyLoss()(x_hat, x)


def evaluate(model, test_loader):
    model.eval()
    loss= 0.0
    num_batches = len(test_loader)
    with torch.no_grad():
        for batch_idx, (data, label) in enumerate(test_loader):
            data = data.to(device)
            predicted = model(data)
            loss += cross_entropy_loss(predicted, label)
    return (loss/num_batches)

'''Training.'''


oracle = Oracle().to(device)

optimizer = optim.AdamW(oracle.parameters(),
                        lr_oracle,
                        weight_decay=1e-5)
scheduler = optim.lr_scheduler.ReduceLROnPlateau(optimizer,
                                                 mode = "min",
                                                 factor = 0.5,
                                                 patience =
patience_oracle)

best_val_loss = float("inf")
oracle.train()
oracle = torch.nn.DataParallel(oracle)

for epoch in range(epochs_oracle):
    train_loss = 0.0
    val_loss = 0.0
    running_loss = 0.0
    for batch_idx, (data, label) in enumerate(sym_train_loader):
        data = data.to(device)
        label = label.to(device)
        if label.dim() > 1:
            label = torch.argmax(label, dim=1)
```

```
        label = label.long()
        optimizer.zero_grad()
        pred = oracle(data)
        loss = cross_entropy_loss(pred, label)
        loss.backward()
        torch.nn.utils.clip_grad_norm_(oracle.parameters(),
max_norm=1.0)
        optimizer.step()
        running_loss += loss.item()
    train_loss += running_loss/len(sym_train_loader)
    val_loss += evaluate(oracle, sym_val_loader)
    print(f"epoch: {epoch} train loss = {train_loss:.4f} val loss=
{val_loss:.4f}")
    if(val_loss<best_val_loss):
        best_val_loss = val_loss
        torch.save({"oracle":oracle.module.state_dict()},
oracle_model_path)

    scheduler.step(val_loss)
```

```
epoch: 0 train loss = 7.8860 val loss= 1.9227
epoch: 1 train loss = 1.8218 val loss= 1.2789
epoch: 2 train loss = 1.4689 val loss= 1.1643
epoch: 3 train loss = 1.3215 val loss= 1.1063
epoch: 4 train loss = 1.1897 val loss= 1.1938
epoch: 5 train loss = 1.0788 val loss= 1.0845
epoch: 6 train loss = 1.0453 val loss= 1.2263
epoch: 7 train loss = 0.9473 val loss= 1.3585
epoch: 8 train loss = 0.8762 val loss= 1.3035
epoch: 9 train loss = 0.8495 val loss= 1.4080
```

```
'''Checking the accuracy on test dataset.'''

correct = 0
total = 0

for batch_idx, (data, label) in enumerate(sym_test_loader):
    pred_logit = oracle(data)
    pred = nn.Softmax(dim=1)(pred_logit)
    pred_labels = torch.argmax(pred, dim=1)
    label = label-1
    correct += (pred_labels == label).sum().item()
    total += label.shape[0]

accuracy_on_test = correct / total
print(f"Accuracy on test dataset: {100*accuracy_on_test:.4f}%")
```

```
Accuracy on test dataset: 99.6344%
```

Finding symmetries:

```
'''Defining all the classes and functions required for training.
    The model used for the generators is the same as the one used in
Supervised Symmetry Discovery.'''
```

```python
class G_model(nn.Module):
    def __init__(self):
        super().__init__()
        self.fc1 = nn.Linear(embedding_dim, 64, bias = True)
        self.do1 = nn.Dropout(0.5)
        self.bc1 = nn.BatchNorm1d(64)
        self.relu1 = nn.ReLU()

        self. fc2 = nn.Linear(64, 128, bias = True)
        self.do2 = nn.Dropout(0.5)
        self.bc2 = nn.BatchNorm1d(128)
        self.relu2 = nn.ReLU()

        self.fc3 = nn.Linear(128, 64, bias = True)
        self.do3 = nn.Dropout(0.5)
        self.bc3 = nn.BatchNorm1d(64)
        self.relu3 = nn.ReLU()

        self.fc4 = nn.Linear(64, embedding_dim, bias = True)

        for m in self.modules():
            if isinstance(m,nn.Linear):
                nn.init.kaiming_normal_(m.weight)
                m.bias.data.zero_()

    def forward(self, x):
        x = self.relu1(self.bc1(self.do1(self.fc1(x))))
        x = self.relu2(self.bc2(self.do2(self.fc2(x))))
        x = self.relu3(self.bc3(self.do3(self.fc3(x))))
        x = self.fc4(x)
        return x


def loss_inf(oracle_model, sym_model, epsilon, input, label):
    oracle.eval()
    with torch.no_grad():
        pred = oracle(input + epsilon*sym_model(input))
        logit = oracle(input)
        loss = torch.mean((pred-logit)**2) / epsilon**2
    # print(f"loss_inf = {loss}")
    return loss

def loss_norm(sym_model, input):
    norm = torch.norm(sym_model(input), dim=1, keepdim=True)
    mu = torch.mean(norm)
    num = input.shape[0]
    norm_loss = torch.mean((norm-1)**2)
    norm_loss += torch.mean((norm-mu)**2)
    # print(f"norm_loss={norm_loss}")
    return norm_loss
```

```python
def loss_ortho(sym_model_list, input):
    Ng = len(sym_model_list)
    total_loss = torch.tensor(0.0, device=input.device)
    for alpha in range(Ng):
        for beta in range(alpha + 1, Ng):
            g_alpha = sym_model_list[alpha]
            g_beta = sym_model_list[beta]
            dot_product = torch.einsum("bi,bi->b", g_alpha(input),
g_beta(input))
            total_loss += torch.mean(dot_product**2)
    # print(f"loss_ortho = {total_loss*300}")
    return total_loss*300

'''Another possible approach to closure loss is to minimize the out-
of-space components of the commutators with respect to the space of
generators,
    after flattening and Gram–Schmidt orthonormalization.'''

def gram_schmidt(vectors):
    basis = []
    for v in vectors:
        w = v.clone()
        for b in basis:
            proj_scalar = torch.sum(w * b, dim=1, keepdim=True)
            w = w - proj_scalar * b
        norm = torch.norm(w, dim=1, keepdim=True)
        w = w / (norm + 1e-8)
        basis.append(w)
    return torch.stack(basis, dim=1)

def loss_closure(sym_model_list, data):
    Ng = len(sym_model_list)
    batch_size = data.shape[0]
    for model in sym_model_list:
        if next(model.parameters()).device != data.device:
            model.to(data.device)
    outputs = [sym_model(data).flatten(start_dim=1) for sym_model in
sym_model_list]
    generators = torch.stack(outputs, dim=1)
    orthonormal_generators = gram_schmidt([generators[:, i, :] for i
in range(Ng)])
    total_loss = torch.tensor(0.0, device=data.device)
    for alpha in range(Ng):
        for beta in range(alpha + 1, Ng):
            C_alpha_beta = sym_model_list[alpha]
(sym_model_list[beta](data)) - sym_model_list[beta]
(sym_model_list[alpha](data))
            C_flattened = C_alpha_beta.flatten(start_dim=1)
            projections = []
            for i in range(Ng):
                basis_vec = orthonormal_generators[:, i, :]
                proj = torch.sum(C_flattened * basis_vec, dim=1,
keepdim=True) * basis_vec
                projections.append(proj)
```

```
            approx_C = sum(projections)
            error = torch.norm(C_flattened - approx_C, p=2)**2
            total_loss += error
    # print("closure_loss ",total_loss)
    return total_loss

def evaluate(test_loader, sym_model_list):
    model.eval()
    loss= 0.0
    num_batches = len(test_loader)
    with torch.no_grad():
        for batch_idx, (data, label) in enumerate(test_loader):
            data = data.to(device)
            loss += sum(loss_inf(oracle, sym_model, epsilon, data,
label) + loss_norm(sym_model, data) for sym_model in sym_model_list)
            loss +=  loss_closure(sym_model_list, data) +
loss_ortho(sym_model_list, data)
    return (loss/num_batches)

'''Training
    Only training 1 generator model.
    I was unable to obtain satisfactory results while training more
than 1 generator model.'''



lr_sym = 0.00003
Ng = 1
epochs_sym = 11
epsilon = 0.001

sym_model_path = [os.path.join(weights_dir, f'sym_model{i}.pt') for
i in range(Ng)]
sym_model_list = [G_model().to(device) for _ in range(Ng)]
optimizers = [
    optim.Adam(sym_model_list[i].parameters(), lr=lr_sym,
weight_decay=1e-5)
    for i in range(Ng)
]
schedulers = [
    optim.lr_scheduler.ReduceLROnPlateau(optimizers[i], mode="min",
factor=0.5, patience=patience_oracle)
    for i in range(Ng)
]

oracle.to(device)
oracle.eval()
for param in oracle.parameters():
    param.requires_grad = False

[sym_model.train() for sym_model in sym_model_list]
best_val_loss = float("inf")
```

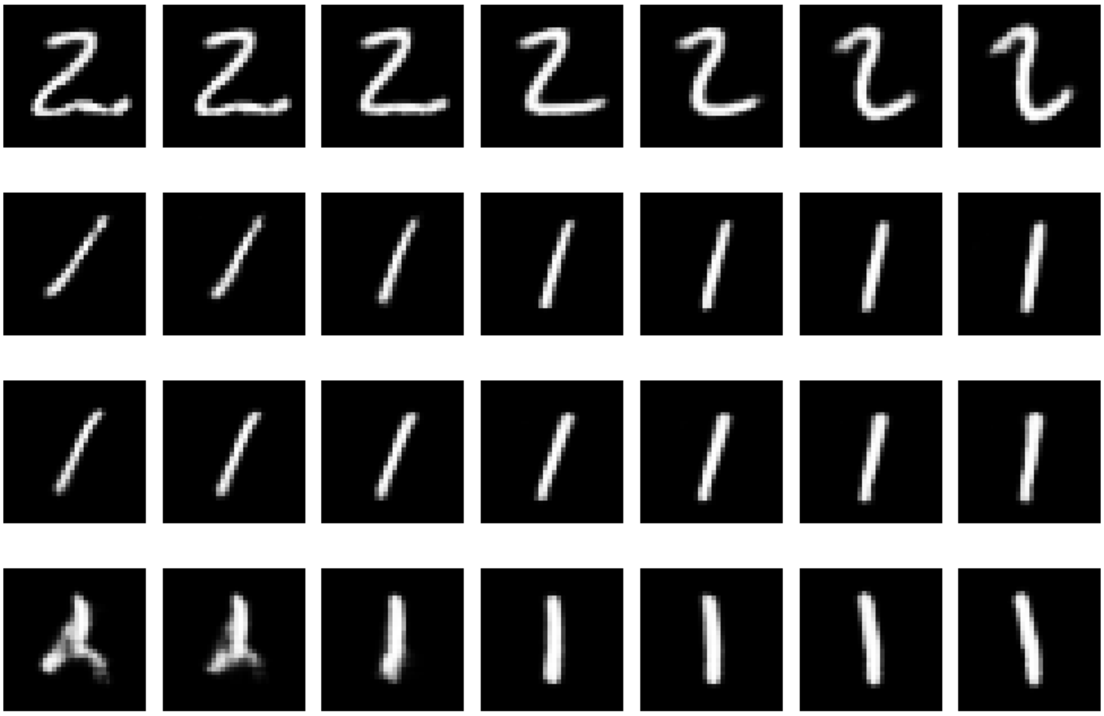
```
for epoch in range(epochs_sym):
    train_loss = 0.0
    running_loss = 0.0
    for batch_idx, (data, label) in enumerate(sym_train_loader):
        data = data.to(device)
        label = label.to(device).long()

        for i, sym_model in enumerate(sym_model_list):
            optimizers[i].zero_grad()
            loss = loss_inf(oracle, sym_model, epsilon, data, label)
+ loss_norm(sym_model, data)
            loss.backward()
            optimizers[i].step()
            running_loss += loss.item()

        if(Ng>1):
            for optimizer in optimizers:
                optimizer.zero_grad()
            loss_joint = loss_closure(sym_model_list, data) +
loss_ortho(sym_model_list, data)
            loss_joint.backward()
            for optimizer in optimizers:
                optimizer.step()
            running_loss += loss_joint.item()

    train_loss = running_loss / len(sym_train_loader)
    val_loss = evaluate(sym_val_loader, sym_model_list)
    print(f"Epoch {epoch} Train Loss: {train_loss:.4f} Val Loss:
{val_loss:.4f}")

    if val_loss < best_val_loss:
        best_val_loss = val_loss
        for i, sym_model in enumerate(sym_model_list):
            torch.save({"model_state_dict": sym_model.state_dict()},
sym_model_path[i])

    for scheduler in schedulers:
        scheduler.step(val_loss.item() if isinstance(val_loss,
torch.Tensor) else val_loss)
```

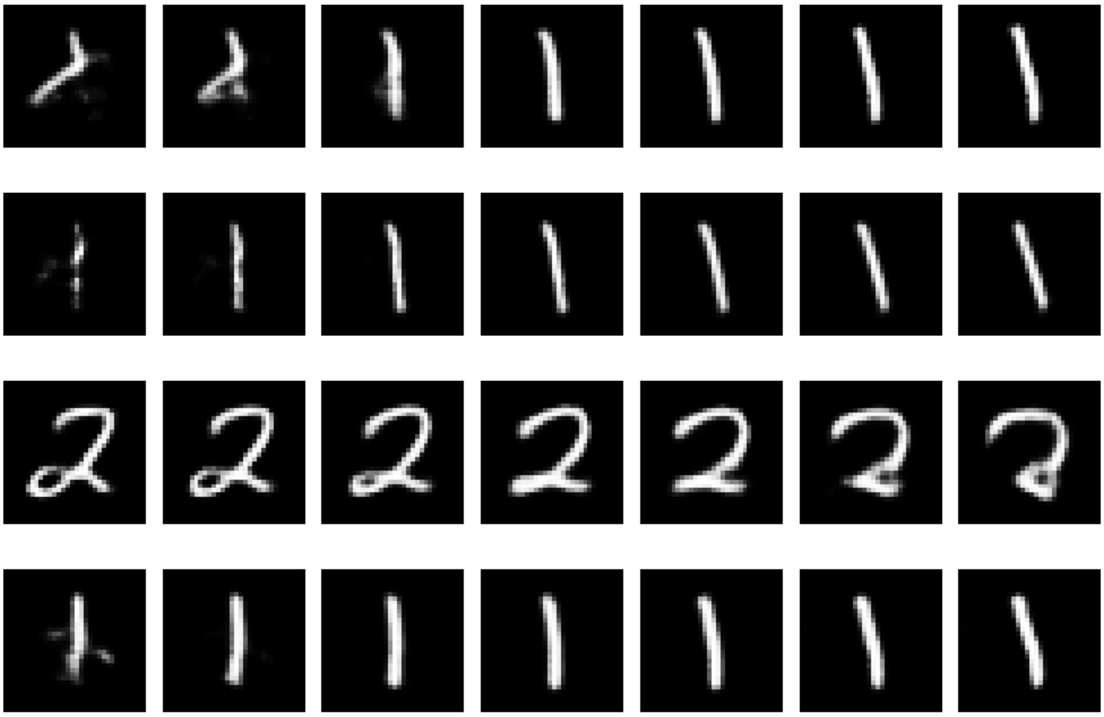
```
Epoch 0 Train Loss: 58.7272 Val Loss: 32.2490
Epoch 1 Train Loss: 24.7586 Val Loss: 20.3504
Epoch 2 Train Loss: 17.3095 Val Loss: 14.9750
Epoch 3 Train Loss: 12.6637 Val Loss: 10.9795
Epoch 4 Train Loss: 9.5604 Val Loss: 8.7722
Epoch 5 Train Loss: 8.1420 Val Loss: 7.6108
Epoch 6 Train Loss: 7.2858 Val Loss: 6.9603
Epoch 7 Train Loss: 6.4981 Val Loss: 6.3512
Epoch 8 Train Loss: 5.9172 Val Loss: 5.7803
Epoch 9 Train Loss: 5.3642 Val Loss: 5.1604
Epoch 10 Train Loss: 5.0005 Val Loss: 4.9476
```

```
'''Defining a function to clearly visualize the transformation by
our generator model.
```

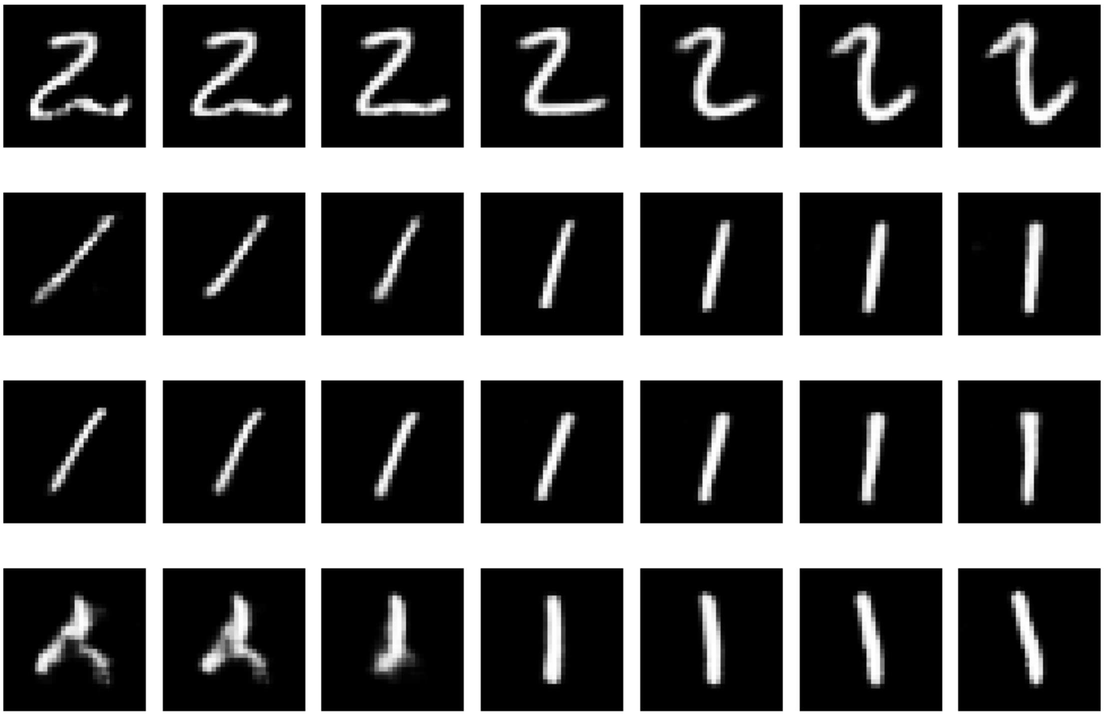
```
    The same method used in the paper has been implemented.
    The original image is situated in the centre and the
transformations are applied to the left and right in steps of a few
thousands.'''


def plot_the_result(its, Ng, sample_idx):
    X_sample, y_sample = next(iter(sym_test_loader))
    X_sample, y_sample = X_sample[sample_idx], y_sample[sample_idx]
    X_sample = X_sample.to(device)

    transformed_mat = []
    for sym_model in sym_model_list:
        sym_model.eval()
        trans_list = []
        X_sample_trans = X_sample.view(1, -1)
        trans_list.append(X_sample_trans)
        for iteration in range(its[-1]+1):
            X_sample_trans = X_sample_trans +
epsilon*sym_model(X_sample_trans)
            if(iteration in its):
                trans_list.append(X_sample_trans)
        X_sample_trans = X_sample.view(1,-1)
        for iteration in range(its[-1]+1):
            X_sample_trans = X_sample_trans -
epsilon*sym_model(X_sample_trans)
            if(iteration in its):
                trans_list.append(X_sample_trans)
        transformed_mat.append(trans_list)
    transformed_imgs = []
    for i in range(Ng):
        temp_list = []
        for j in range(7):
            img = vae.decoder(transformed_mat[i]
[j]).cpu().detach().numpy().squeeze()
            temp_list.append(img)
        transformed_imgs.append(temp_list)

    fig, axes = plt.subplots(Ng, 7, figsize=(10, 4))

    if(Ng==1):
        for i in range(Ng):
            img = transformed_imgs[i][0]
            axes[3].imshow(img, cmap="gray")
            axes[3].axis("off")
            for j in range(3):
                img = transformed_imgs[i][j+1]
                axes[j+4].imshow(img, cmap="gray")
                axes[j+4].axis("off")
            for j in range(3):
                img = transformed_imgs[i][j+4]
                axes[2-j].imshow(img, cmap="gray")
                axes[2-j].axis("off")
```

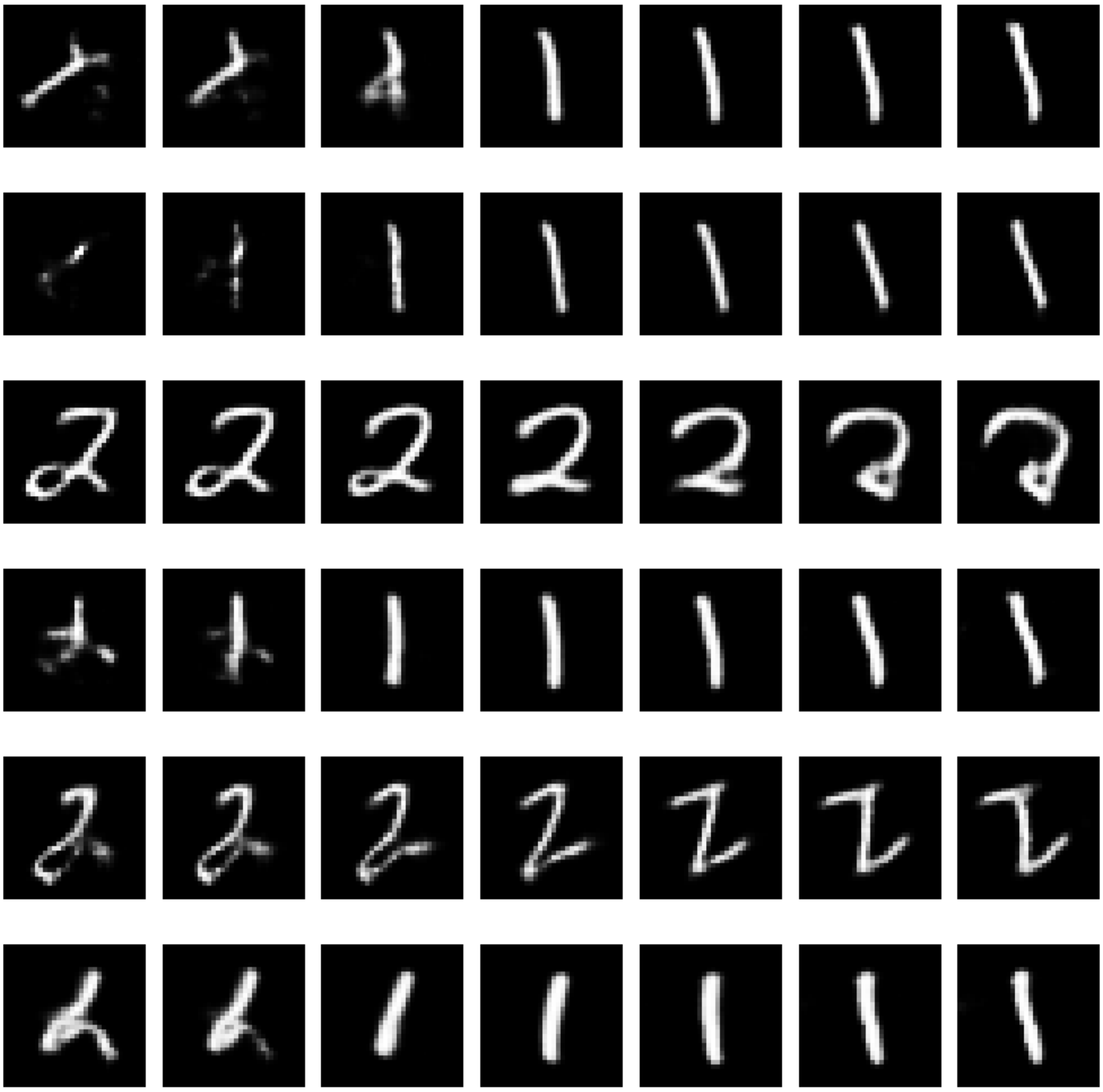
```
        plt.tight_layout()
        plt.show()

    elif(Ng>1):
        for i in range(Ng):
            img = transformed_imgs[i][0]
            axes[i][3].imshow(img, cmap="gray")
            axes[i][3].axis("off")
            for j in range(3):
                img = transformed_imgs[i][j+1]
                axes[i][j+4].imshow(img, cmap="gray")
                axes[i][j+4].axis("off")
            for j in range(3):
                img = transformed_imgs[i][j+4]
                axes[i][2-j].imshow(img, cmap="gray")
                axes[i][2-j].axis("off")

        plt.tight_layout()
        plt.show()

'''Plotting the transformation on 10 sample images from the test dataset in steps of 2000 iterations.
    We can see that some of the images are being rotated. But their rate of rotation could be improved.
    We can achieve this by increasing the steps from 2000 to 3000 or 4000.'''


for i in range(8):
    plot_the_result([2000,4000,6000],1,i)
```





```
'''Plotting the transformations of the same images in steps of
3000'''

for i in range(10):
    plot_the_result([3000,6000,9000],1,i)
```





```python
'''Plotting the transformations of images in steps of 4000.'''


for i in range(10):
    plot_the_result([4000,8000,12000],1,i)
```