18/05/2021
TUESDAY

ASSIGNMENT - 01

1AY18AU031.

(18AU62)

(1.) Explain three modes of heat trasfer.

Sol:- Basically there are 3 main modes of heat trasfer;

i) CONDUCTION:- It's a mode of heat trasfer in which energy is exchanged b/w the high temperature region to the low temp. region by kinetic motion / direct impact of molecules.
It can occur in solid, liquid and gas.

ii) CONVECTION:- It's a mode of heat trasfer in which energy is exchanged b/w the high temp region and low temp. region due to molecular motion & macroscopic motion of the fluid particles.
It occurs only in fluids.

iii) RADIATION:- It's a mode of heat trasfer in which heat trasfer takes place between two substances even without any medium through electromagnetic waves (quanta).
Radiation is max. if two substances exchanging heat are seperated by perfect vaccum.

(2.) State the laws governing 3 basic modes of heat trasfer.

Sol:- i) CONDUCTION → FOURIER LAW.

" The rate of heat flow by conduction in a given direction is propotional to the area normal to the direction of heat flow and to the gradient of temperature in that direction".

i.e $Q_x \propto A \cdot \frac{dT}{dx}$

$$\Rightarrow \boxed{Q_x = -KA\frac{dT}{dx}} \quad \text{--- W}$$

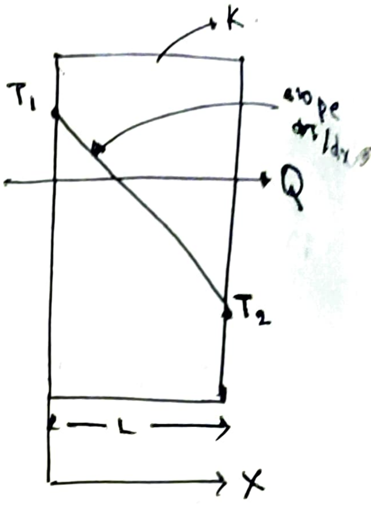


Fig :- Plane wall.

Also; $q_x = \frac{Q_x}{A} = -K\frac{dT}{dx}$ --- $\frac{W}{m^2}$

∵ Q_x = Rate of heat transfer in +ve 'x' direction.

$\frac{dT}{dx}$ = Temp. gradient in x-direction.

A = normal/cross sn area to the direction of heat flow

L = Thickness of the metal plate.

K = Thermal conductivity -- $\frac{W}{mK}$

q_x = heat flux -- W/m^2

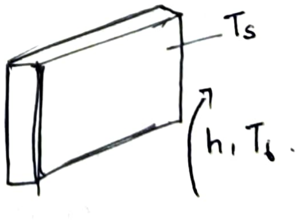
-ve = Temp ↓es in x-direction.

ii) CONVECTION → NEWTON'S LAW OF COOLING.

"The rate of heat transfer by convection between a surface and a fluid is directly proportional to the surface area of heat transfer and the temperature difference between them".

i.e $Q \propto A(T_s - T_f)$ $(T_s > T_f)$

$$\Rightarrow \boxed{Q = hA(T_s - T_f)} \quad \text{--- W}$$



Also; $q = \frac{Q}{A} = h(T_s - T_f)$ --- W/m^2

∵ T_s = surface temp --- °C.

T_f = fluid temp --- °C.

A = surface area --- m^2

Q = Rate of heat transfer --- W from surface of fluid

h = co-eff of heat transfer for convection

iii) RADIATION → STEFAN - BOLTZMANN LAW.

" Radiation energy emitted by a black body is propotional to the surface area and fourth power of it's absolute temperature".

i.e $Q \propto A T^4$

$\Rightarrow Q = \sigma A T^4$ --- W

also

$Q = \sigma \epsilon A T^4$ --- W

∵ Q = rate of heat (energy emitted in watts.

A = surface area of a black body -- m^2

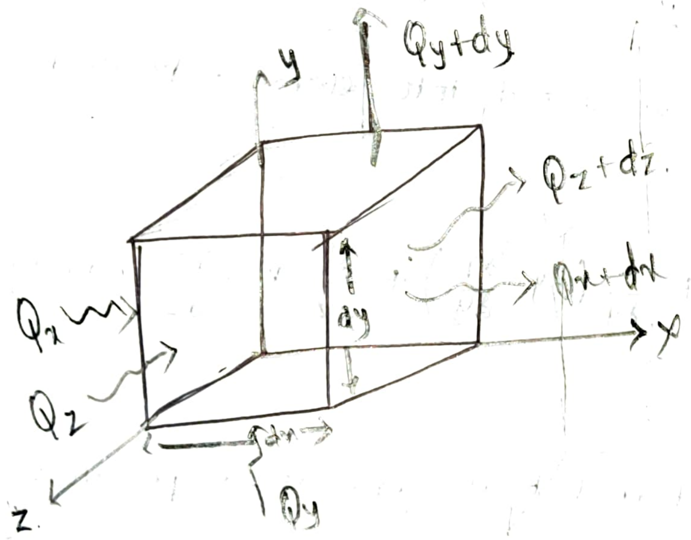
T = Abso. temp. of the body -- K (kelvin).

σ = S-B-const
= 5.67×10^{-8} W/m^2.

ϵ = emissivity.

(3) Derive general three dimensional heat conduction equation in cartesian co-ordinate.

sol:-



consider a small volume element in cartesian co-ordinates having side dx, dy, dz as shown.

The energy balance for this small volume element from the first law of thermodynamics.

Now;

WKT, from Fourier law; $Q = -KA\, dT/dx$.

$$\therefore \quad Q_x = -K_x \frac{\partial T}{\partial x} dy \cdot dz.$$

K_x = Thermal conductivity in x-direction.

The rate of heat flow out of the element in x-direction

$$Q_{x+dx} = Q_x + \frac{\partial (Q_x)}{\partial x} dx$$

$$= -K_x \frac{\partial T}{\partial x} dy\, dz - \frac{\partial}{\partial x}\left(K_x \cdot \frac{\partial T}{\partial x}\right) \cdot dx \cdot dy \cdot dz$$

Then,

net heat entering the element in x-direction is the diff b/w entering and leaving.

$$Q_x - Q_{x+dx} = \frac{\partial}{\partial x}\left(K_x \cdot \partial T/\partial x\right) dx \cdot dy \cdot dz.$$

1ly

$$Q_y - Q_{y+dy} = \frac{\partial}{\partial y}\left(K_y \cdot \partial T/\partial y\right) dx \cdot dy \cdot dz.$$

$$Q_z - Q_{z+dz} = \frac{\partial}{\partial z}\left(K_z \cdot \partial T/\partial z\right) dx \cdot dy \cdot dz.$$

Then net heat conducted into element $dx \cdot dy \cdot dz$ per unit time.

$$\hookrightarrow \left\{ \frac{\partial}{\partial x}\left(K_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \cdot \frac{\partial T}{\partial z}\right)\right\} dx \cdot dy \cdot dz. \longrightarrow (1)$$

<u>Now</u>;

Let q_g be the internal heat generation per unit time per unit volume,

$$\therefore \quad = q_g (dx \cdot dy \cdot dz) \longrightarrow (2)$$

<u>Now</u>;

The change in internal energy for the unit time;

= (mass of ele) (spfc heat) (change of temp. per unit time)

$$= (\rho)(dx \cdot dy \cdot dz) \times C_p \cdot \frac{\partial T}{dt}$$

$$= \rho C_p \frac{\partial T}{\partial t} dx \cdot dy \cdot dz \longrightarrow (3)$$

Also;

The work done by the element per unit time is very small cuz' the flow-work done by solids due to temp. change is negligible.

$$= 0 \longrightarrow (4)$$

Now substh;

$$(1) + (2) = (3) + (4)$$

$$\left\{ \frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z}\right) \right\} dx \cdot dy \cdot dz + q_g (dx \cdot dy \cdot dz)$$

$$= \rho c_p \frac{\partial T}{\partial t} \cdot dx \cdot dy \cdot dz + 0.$$

$\div$ B.S by $dx \cdot dy \cdot dz$.

$$\Rightarrow \frac{\partial}{\partial x}\left(k_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \cdot \frac{\partial T}{\partial z}\right) + q_g = \rho c_p \frac{\partial T}{\partial t}.$$

For isotropic materials;

$k_x = k_y = k_z = k$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}.$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$\because \alpha = \frac{k}{\rho C_p}$ ... thermal diffusivity.

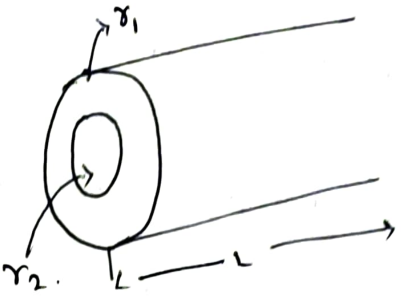
(4) Derive an expression for temperature distribution and rate of heat transfer for a hollow cylinder.

Soln:- consider a hollow cyclinder having

L = length.

r_1 = outer radius = r_0

r_2 = Inner radius = r_i



Assuming ;

steady state &

no heat generation.

Then we have;

$$\frac{1}{r}\frac{d}{dr}\left(kr \cdot \frac{dT}{dr}\right) = 0 \longrightarrow (1)$$

Assuming k = constant

$$\therefore (1) \Rightarrow \frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = 0.$$

$\int$ing.

$$r \cdot \frac{dT}{dr} = c_1 \Rightarrow \frac{dT}{dr} = \frac{c_1}{r} \rightarrow (2)$$

$$\int \text{ing } (2) \Rightarrow T(r) = c_1 \ln(r) + c_2. \longrightarrow (3)$$

As we have two constants c_1 and c_2 we require two boundary conditions.

at; $r = r_1$; $T = T_{s,1}$
$r = r_2$; $T = T_{s,2}$. $\rightarrow (4)$ [T_s = surface temp]

$\therefore$ (3) $\Rightarrow$ $T_{s,1} = c_1 \ln r_1 + c_2. \rightarrow (5)$ [(4) in (3)]

$T_{s,2} = c_1 \ln r_2 + c_2. \rightarrow (6)$

Now; subtracting (5) - (6).

$$\Rightarrow T_{s,1} - T_{s,2} = c_1 (\ln r_1 - \ln r_2)$$

$$= c_1 \ln (r_1/r_2)$$

$$\therefore c_1 = \frac{T_{s1} - T_{s2}}{\ln(r_1/r_2)}$$

from (6) ; $C_2 = T_{s2} - c_1 \ln r_2$

$\therefore \; T(r) = c_1 \ln(r) + T_{s2} - c_1 \ln(r_2)$

$= c_1 \ln(r/r_2) + T_{s2}$ [from c_1]

$$T(r) = \frac{T_{s1} - T_{s2}}{\ln(r_1/r_2)} \ln(r/r_2) + T_{s2} \longrightarrow (7)$$

Eq^n (7) is the temp. distribution.

NOW;

Heat Trasfer Rate;

$$q_r = -KA\frac{dT}{dr} = -K(2\pi r L)\frac{c_1}{r}$$ [from (2)] [A = Area of cyl]

$$\therefore \; q_r = -K 2\pi L c_1 \longrightarrow (8)$$

Eq^n (8) showing that it's not a f^n of "r", therefore heat trasfer rate will remain constant.

Also

Heat Flux;

$$q''_r = \frac{q_r}{A} = -K\frac{dT}{dr} = -K\frac{c_1}{r} \longrightarrow (9)$$

(9) It's a f^n of "r"; $\therefore$ cannot analyze heat trasfer through a cylinder.

(10) Define the three basic modes of heat trasfer & the governing laws with supporting relationship.

Sol^n:- combined Q' of (1) and (2). Refer.

6) Explain briefly the mechanism of conduction, convection and radiation of heat trasfer.

Soln:-

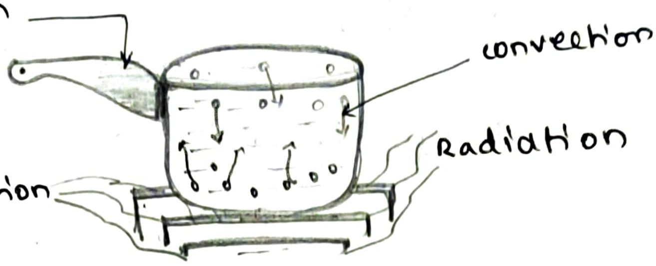


Fig:- water boiling on a gas in a container.

i) CONDUCTION:-

* It's a microscopic phenomenon.
* More energetic particles of a substance trasfer their energy to less energetic neighbours.
* can occur in solid, liquid & gas.
* Happens due to lattice vibrations in solids & collision of molecules and diffusion of molecules in liquids / Gases.
* In the above Fig. the conduction occurs from the metal container to its hand.
* Fourier law Governs this $[Q = -KA\, dT/dx \text{ -- } W]$.

ii) CONVECTION:-

* It's a macroscopic phenemon.
* Direction of heat course depends on the relative magnitudes of temperatures of fluid and the surface.
* The fluid particles themselves move & thus carry energy from high temp. level to a low temp. level.
* It has two types ↳ FREE (NATURAL) CONVECTION. ↳ FORCED CONVECTION.

* In the fig, convection occurs in the water molecules & the surface of the container.
* Newton's law of cooling governs this $[Q = hA(T_s - T_b) \ldots W]$

iii) RADIATION :-

* All bodies above temperature 0K emit radiation.
* When radiation falls on the body, it may be attenuated with a short distance from the surface, or get reflected from the surface or just pass through.
* In vaccum radiation propogates without any atteneuation.
* In the fig the radiation from the coils generate flame and thus boiling the water.
* Stefan-Boltzmann law Governs this

$$[Q = \sigma \epsilon A T^4 \ldots W]$$

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(12) Starting from fundamental, deduce general form of one dimensional heat conduction equation in cartesian co-ordinates and arrive at Fourier equation.

Sol: Consider a plane slab as shown.

L = thickness.

Temperature at two faces is constant & uniform.

i.e $T_0 = T_1$ at $x = 0$

$T = T_2$ at $x = L$.

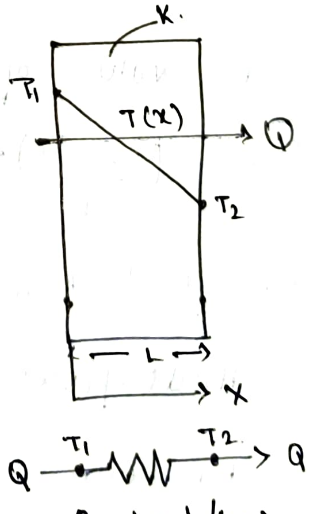


Fig 1: 3-D plane wall

Assumptions:-

↳ 1-D-conduction (i.e) $L \lll L$.

↳ Steady state conduction. (i.e) temp = const.

↳ No internal heat generation.

↳ Homogeneous material. (i.e) K = constant.

NOW;

the general diff. eq^n in cartesian co-ordinate;

$$\left[\frac{\partial}{\partial x}\left(K_x \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \cdot \frac{\partial T}{\partial z}\right)\right] + q_g = \rho C_p \frac{\partial T}{\partial t}.$$

Apply^n all assump^n the governing eq^n for plane slab is;

$$\frac{d}{dx}\left(K \cdot \frac{dT}{dx}\right) = 0.$$

i.e

$$\Rightarrow \frac{d^2T}{dx^2} = 0 \quad \Rightarrow \int ing \Rightarrow \int \frac{d^2T}{dx^2} = \int 0$$

$$\Rightarrow \frac{dT}{dx} = C_1 \quad \Rightarrow \int ing \Rightarrow \int \frac{dT}{dx} = \int C_1$$

$$\Rightarrow T = C_1 x + C_2 \rightarrow (1)$$

∵ C_1 and C_2 are constants to be determined.

Now the boundary conditions are)

i) At $x = 0$, $T = T_1$

$$T_1 = C_1 \times 0 + C_2 \quad \Rightarrow C_2 = T_1 \longrightarrow (2)$$

ii) At $x = L$, $T = T_2$

$$T_2 = C_1 L + C_2 \quad \Rightarrow C_1 = \frac{T_2 - T_1}{L} \longrightarrow (3)$$

Sub. the value of C_1 & C_2 in (1)

$$T = \left(\frac{T_2 - T_1}{L}\right)x + T_1 \longrightarrow (4)$$

(4) is called temp. distribution.

Now; Heat flow

WKT by Fourier Law; $Q = -KA\, dT/dx$ at $x = 0$.

from (4) $\Rightarrow$ $T = \left(\frac{T_2 - T_1}{L}\right)x + T_1$

$$\frac{dT}{dx}\Big|_{x=0} = \frac{T_2 - T_1}{L}$$

$$Q = -KA\left(\frac{T_2 - T_1}{L}\right)$$

$$\therefore \; Q = KA\left(\frac{T_1 - T_2}{L}\right)$$

further; $Q = \dfrac{T_1 - T_2}{\frac{L}{KA}} = \dfrac{\Delta T}{R_{cond^n}}$

(7) Write three dimension heat conduction equations in cartesian co-ordinate and explain the terms.

Sol:- ↳ cartesian coordinate.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

$\alpha = \dfrac{K}{\rho C_p}$ = thermal diffusivity.

q_g = internal heat generation.

K = thermal conductivity.

↳ Polar coordinate.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

↳ spherical coordinate.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin\phi}\frac{\partial^2 T}{\partial \phi^2} + \frac{q_g}{K}$$

$$= \frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

(5) A wall is constructed of several layers. The first layer consists of brick ($K = 0.66 \, W/mK$), 25 cm thick, the second layer 2.5 cm thick mortar ($K = 0.7 \, W/mK$), the third layer 10 cm thick limestone ($K = 0.66 \, W/mK$) & outer layer of 1.25 cm thick plaster ($K = 0.7 \, W/mK$). The heat transfer co-eff on interior wall layers are $5.8 \, W/m^2K$ & $11.6 \, W/m^2K$ respt. Find;

a) overall heat transfer co-eff.

b) Overall thermal resistance per m^2.

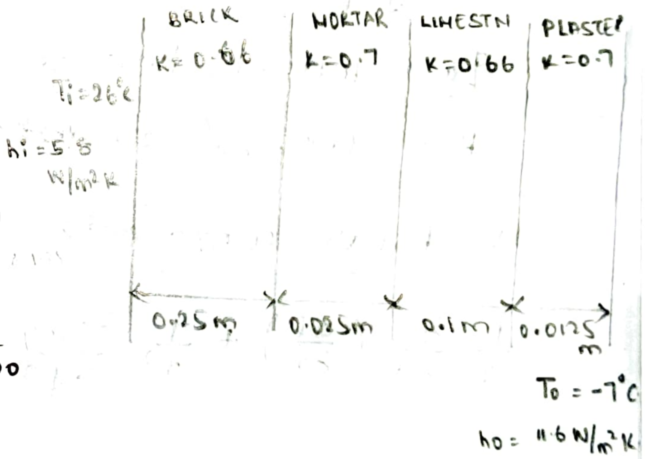
c) Rate of heat transfer per m^2, if the interior of the room is at 26°C while outer air is at −7°C.

d) Temp. at the junction between the morttrar and limestone.

Solⁿ:- Given;

$T_i = 26° + 273° = 299 \, K$

$T_o = -7° + 273 = 266 \, K.$



a) $U = ?$ (2-conv & 4-cond)

$$U = \frac{1}{\frac{1}{h_i} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{L_4}{K_4} + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{1}{5.8} + \frac{0.25}{0.66} + \frac{0.025}{0.7} + \frac{0.1}{0.66} + \frac{0.0125}{0.7} + \frac{1}{11.6}}$$

$\therefore U = 1.1869 \, W/m^2K.$

b) $R_t = ?$ considering $A =$ unity $= 1 \, m^2$.

$$\Sigma R_t = \frac{1}{h_i A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_o A}.$$

$= 1/5.8 + \frac{0.25}{0.66} + \frac{0.025}{0.7} + \frac{0.1}{0.66} + \frac{0.0125}{0.7} + 1/11.6$

$\therefore R_t = \underline{\underline{0.842}}\ K/W.$

c) $Q = ?$

$Q = \left(\frac{T_i - T_o}{R_t}\right) \Rightarrow \frac{299-266}{0.842}$

$\therefore Q = \underline{\underline{39.192\ KW}}.$

d) T_{jn} b/w Mortar & limestone = ?

#.

(8). A 240 mm steam main, 210m long is covered with 50mm of high temp. insulation ($K = 0.092 W/m°C$) & 40 mm low temp. insulation ($K = 0.062 W/m°C$) the inner & outer surface temp. is measured are 390°C & 40°C respt. Determine.

a) The total heat loss per hour.

b) The total heat loss per m^2 of outer surface.

c) Temp. b/w two layers of insulation. Neglecting condition through pipe material.

Soln :-

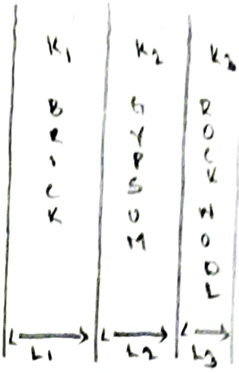
9. An exterior wall of house may be approximated by a 0.1 m layer of common brick ($K = 0.7\,W/m°C$) followed by point 0.04 m layer of gypsum plaster ($K = 0.48\,W/m°C$). What thickness of loosely packed rock wool insulation ($K = 0.065\,W/m°C$) should be added reduce the heat loss (or gain) through the wall by 80 %.

Sol:- Given; 3-layers;

BRICK ⇒ $L_1 = 0.1\,m$, $K_1 = 0.7\,W/m^\circ C$

GYPSUM ⇒ $L_2 = 0.04\,m$, $K_2 = 0.48\,W/m^\circ C$

$L_3 = ?$, $K_3 = 0.065\,W/m^\circ C$ of ROCKWOOL.



case i) Rock wool insulation not used;

$$Q_1 = \frac{A\,\Delta t}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}}.$$

$$\therefore Q_1 = \frac{A(\Delta t)}{1.32} \rightarrow (1)$$

case ii) Rock wool insulation used;

$$Q_2 = \frac{A(\Delta t)}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}} = \frac{A\,(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}}$$

$$\therefore Q_2 = \frac{A\,(\Delta t)}{1.32 + \frac{x}{0.065}} = \frac{A(\Delta t) \times 0.065}{0.0858 + x}. \rightarrow (2)$$

ALSO WKT;

$$Q_2 = (1 - 0.8)\,Q_1 = 0.2\,Q_1. \rightarrow (3)$$

$$\hookrightarrow \quad \frac{\cancel{A}\,(\cancel{\Delta t}) \times 0.065}{0.0858 + x} = \frac{0.2 \times \cancel{A}\,(\cancel{\Delta t})}{1.32}.$$

$$\Rightarrow \quad 0.0858 + x = 0.429$$

$$\therefore \quad x = 0.0588\,m \text{ or } 58.8\,mm = L_3.$$

∴ The thickness of the rock wool insulation should be 58.8 mm.

(11.) Air at 23°C blows over a hot plate made of carbon steel at 260°C. The plate is

0.6 m × 0.75 m and 2 cm thick. If 300W is lost from the plate surface by radiation, calculate the heat transfer & inside plate temperature. Assume convective heat transfer co-efficient as 25 W/m²°C & K for carbon steel as 43 W/m°C.

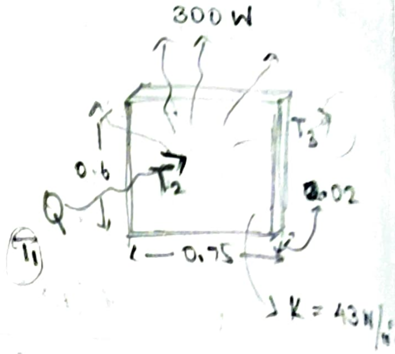


Fig:- C-S material

Soll- Given;

$A = 0.6 \times 0.75 = 0.45 m^2$

$t = 2 cm = 0.02 m$.

$Q_{Lost} = 300 W$ by radiation

$Q = ?$, $T_2 = ?$ (inside temp).

Let $T_1 = 23°C$ & $T_3 = 260°C$

$h = 25 W/m^2°C$, $K = 43 W/m°C$.

WKT; $Q = \frac{T_a - T_b}{R_{total}} = \frac{T_1 - T_3}{R_t}$.

$R_{tot} = R_{cond} + R_{conv}$

$= \frac{1}{hA} + \frac{L}{KA} = \frac{1}{25 \times 0.45} + \frac{0.75}{43 \times 0.45}$

$R_t = 0.127 °C/W$.

$\therefore Q = \frac{260 - 23}{0.127} = 1.866 KW - 300W$

$\therefore Q_1 = 1.56 KW$

To find T_2;

$Q_1 = Q_2$

$1.56 \times 10^3 W = \frac{T_2 - T_3}{R_t}$ ⇒ $1.56 \times 10^3 = \frac{T_2 - 260}{0.127}$

$\therefore T_2 = 458.12 °C$.

(13) An aircraft heat exchanger has a minimum wall temp. of 810K. The hot side and the cold side heat transfer co-eff. are $200\ W/m^2K$ and $400\ W/m^2K$ respt. Determine the maximum permissible unit thermal resistances per m^2 area of the metallic walls seperating the hot gas from the cold gas, if hot gas temp is 1200K and coolant temp. is the hot gas from the cold gas, if hot gas temp. is 1200K & coolant temp is 300K.

Soln :- Given;

$$R_{wall} = \frac{L}{KA} \text{ or } \frac{1}{hA} \quad \text{-- } \frac{K \text{ or } C}{W}.$$