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To solve linear programming using R studio, we need to install lpsolve package Install.packages("lpsolve")

PRACTICAL 1

GRAPHICAL METHOD USING R PROGRAMMING

R Program
#Find a geometrical interpretation and solution as well for the following LP problem
#Max z=3x1+5x2
#subject to constraints:
#x1+2x2<=2000
#x1+x2<=1500
#x2<=600
#x1,x2>=0
Load IpSolve
require(lpSolve)
Set the coefficients of the decision variables -> C of objective function
C <- c(3,5)
Create constraint martix B
A <- matrix(c(1, 2,
1, 1,
0, 1
), nrow=3, byrow=TRUE)
Right hand side for the constraints
B <- c(2000,1500,600)

```
# Direction of the constraints
constranints_direction <- c("<=", "<=", "<=")
# Create empty example plot
plot.new()
plot.window(xlim=c(0,2000), ylim=c(0,2000))
axis(1)
axis(2)
title(main="LPP using Graphical method")
title(xlab="X axis")
title(ylab="Y axis")
box()
# Draw one line
segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, x1 
# Find the optimal solution
optimum <- lp(direction="max",
                       objective.in = C,
                       const.mat = A,
                       const.dir = constranints_direction,
                       const.rhs = B,
                       all.int = T)
# Print status: 0 = success, 2 = no feasible solution
print(optimum$status)
# Display the optimum values for x1,x2
best sol <- optimum$solution
names(best_sol) <- c("x1", "x2")
print(best_sol)
```

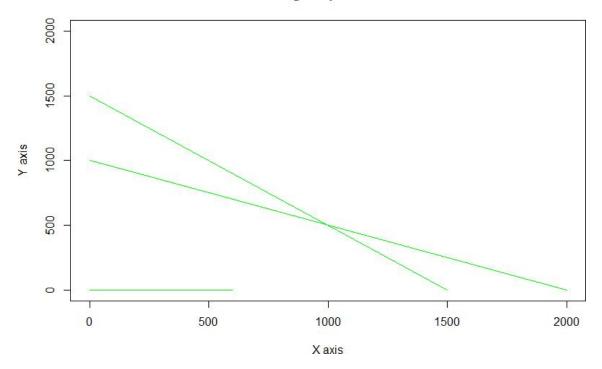
Check the value of objective function at optimal point print(paste("Total cost: ", optimum\$objval, sep=""))

OUTPUT:

```
[Workspace loaded from ~/.RData]
> # Right hand side for the constraints
> B <- c(2000,1500,600)
> # R Program
> # Load lpSolve
> require(lpSolve)
Loading required package: lpSolve
> ## Set the coefficients of the decision variables \rightarrow C
> C < -c(3,5)
> # Create constraint martix B
> A <- matrix(c(1, 2,
                 1, 1,
                 0, 1
+ ), nrow=3, byrow=TRUE)
> # Right hand side for the constraints
> B < -c(2000, 1500, 600)
> # Direction of the constraints
> constranints_direction <- c("<=", "<=", "<=")
> # Create empty example plot
> #plot(2000, 2000, col = "white", xlab = "", ylab = "")
> plot.new()
> plot.window(xlim=c(0,2000), ylim=c(0,2000))
> axis(1)
> axis(2)
> title(main="LPP using Graphical method")
> title(xlab="X axis")
> title(ylab="Y axis")
> box()
> # Draw one line
> segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
> segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
> segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, col = "green")
> # Find the optimal solution
> optimum <- lp(direction="max",</pre>
                objective.in = C_{\bullet}
                const.mat = A,
                const.dir = constranints direction,
                const.rhs = B,
                all.int = T)
```

```
> # Print status: 0 = success, 2 = no feasible solution
> print(optimum$status)
[1] 0
> # Display the optimum values for x1,x2
> best_sol <- optimum$solution
> names(best_sol) <- c("x1", "x2")
> print(best_sol)
    x1    x2
1000   500
> # Check the value of objective function at optimal point
> print(paste("Total cost: ", optimum$objval, sep=""))
[1] "Total cost: 5500"
```

LPP using Graphical method

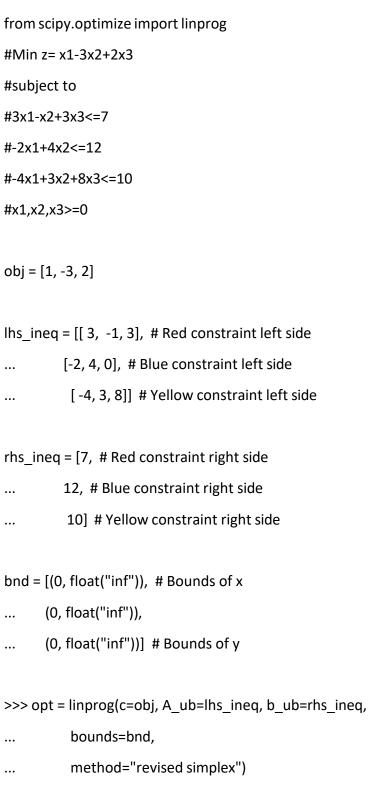


Simplex Method with 2 variables using Python

from scipy.optimize import linprog
#Max z=3x1+2x2
#subject to
#x1 + x2 <=4
#x1 - x2 <=2
#x1,x2>=0
obj = [-3, -2]
lhs_ineq = [[1, 1], # Red constraint left side [1, -1]] # Blue constraint left side
rhs_ineq = [4, # Red constraint right side
2] # Blue constraint right side
bnd = [(0, float("inf")), # Bounds of x
(0, float("inf"))] # Bounds of y
>>> opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
bounds=bnd,method="revised simplex")
>>> opt
opt.fun
opt.success
opt.x

```
;a Logout
; Jupyter simplex1with2variables Last Checkpoint: a day ago (unsaved changes)
                                                                                                                                                               Trusted Python 3 0
 File Edit View Insert Cell Kernel Widgets Help
Sonve following linear programming problem with two variables using simplex
                 method.
                 Max z=3x1+2x2
                 subject to
                 x1 + x2 <=4
                x1 - x2 <=2
                 X1,x2>=0
     In {1]: H fi:omcipy.optimize impoi:t linprog
     In [13], H ocj - [-3, -2]
    \begin{array}{lll} \text{In [14]:} & \textbf{H} & \text{lh.s\_ineq} = [(\ 1,\ 1], & \textit{I left sids of first constraint} \\ & (1,\ -1] \ \textbf{J} & \textit{I lt/ft sid!!!!=oi st!cond constraint} \end{array}
    In [15]: \mathbf{H} \operatorname{rh.s\_ineq} = [4, : right \ side \ constant \ of \ first \ constraint \ 2] : right \ side \ constant \ of \ first \ constraint
    In [16]: \mathbf{H} bnd= 1(0, float("inf")), \mathit{IBoundsotx} (0, float("inf"))] : \mathit{Boundsof} \mathit{y}
    In [17]: H >>> opt - linprog(e=cobj, A_ub=,lh.9_ineq, b_ub-rh.9_ineq, tound.9=bnd,method-"revi.9ed .9implex")
                     >>> opt
                      array ([], dtype=float64)
fun: -11.0
me9.sage: 'Optimization terminated succe9.sfully.'
nii.t: 2
!5lack: array([O., 0.])
         Out[17]:
                        status: 0
True
                                  array([3., 1.])
    In [18]: H opt.fun
        OUt[IS]: -11.0
```

Simplex Method with 3 variables using Python



>>> opt

;; JUpyter SimplexWith3Variables Last Checkpoint: 2 hours ago (autosa•,ed)

```
Trusted Python 3
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                                       Run 
C Code
    + :|< |',') | |-, + +
   In 127]: M from !5cipy.optimiz import linprog
   In {28]: H #Min 2= x1-3x2+2x3
                     obJ = 11, -3, 2]
   In 129]: M lh.9_ineq = I[ 3, -1, 3], : Red constraint lsft side (-2, 4, 0], I Bll le constraint left side [ -4, 3, 8]] i Yellov constraint left: side
   In JJO]: M rh!5_J.n q = 17, i Red constraint right side 12, i Blue constraint right side 10] f YslloJo'constraint right side
   In {33}: \mbox{\bf H} bnd = I {0, float ("inf")}, i Bottnds of x (0, float("inf")), (0, float ("l.nf"))] # Bounds of y
   \label{eq:linear} \begin{array}{lll} \text{In } \{34\}; & \textbf{M} >>> \text{opt= linprog}(e=\text{obJ}, \ A\_ub=\text{ln.5\_J.n q, b\_ub=\text{rh.5\_J.n q, bound.9=bnd, bound.9=bnd, method="revised simplex"}) \end{array}
                             array([], dtyp-e::float64) fun: -11.0
        0Ut[34]:
                        me5.5age: 'Optimization terminated .5ucce.5.5fully.' nit: 2
                         nit: 2
slack: array{[ 0., 0., 11.])
.5tatu.5: 0
                                    True
                                    array([4,, 5., 0,])
```

Simplex Method with Equality Constraints Using Python

```
from scipy.optimize import linprog
\#Max z=x+2y
#subject to
#2x+y<=20
#-4x+5y<=10
#-x+2y>=-2
\#-x+5y=15
\#x,y>=0
obj = [-1, -2]
lhs_ineq = [[ 2, 1], # Red constraint left side
        [-4, 5], # Blue constraint left side
         [1, -2]] # Yellow constraint left side
rhs_ineq = [20, # Red constraint right side
         10, # Blue constraint right side
         2] # Yellow constraint right side
lhs_eq = [[-1, 5]] # Green constraint left side
rhs_eq = [15]
                 # Green constraint right side
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
          A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
          method="revised simplex")
```

method ="revised simplex" solves linear programming problem using two phase simplex method.

```
con: array([0.])
       fun: -16.8181818181817
 message: 'Optimization terminated successfully.'
       nit: 3
    slack: array([ 0. , 18.18181818, 3.36363636])
  status: 0
 success: True
          x: array([7.72727273, 4.54545455])
                                                                                                     Trusted | Python 3 O
 File Edit View Insert Cell Kernel Widgets Help
 ~
    In [1]: M from scipy.optimize import linprog
    In [2]: | #Max z=x+2y #subject to #2x+y<=20 #-4x+5y<=10
              #-4x+3y<=10
#-x+2y>=-2
#-x+5y=15
#x,y>=0
obj = [-1, -2]
    In [3]: | hasineq = [[ 2, 1], # Red constraint left side
... [-4, 5], # Blue constraint left side
... [ 1, -2]] # Yellow constraint left side
    In [5]: M lhs_eq = [[-1, 5]] # Green constraint left side
    In [6]: H rhs_eq = [15] # Green constraint right side
    In [8]: M opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
                    A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
method="revised simplex")
    In [9]: M opt
       Out[9]:
                con: array([0.])
fun: -16.818181818181817
               message: 'Optimization terminated successfully.'
    nit: 3
               slack: array([ 0. , 18.18181818, 3.36363636])
status: 0
success: True
                    x: array([7.72727273, 4.54545455])
```

BigM Simplex Method using Python

Solve Following linear programming problem using Big M Simplex method.

Min z=4x1+x2

subjected to:

$$3x1 + 4x2 >= 20$$

$$x1 + 5x2 >= 15$$

$$x1, x2 >= 0$$

from scipy.optimize import linprog

$$obj = [4, 1]$$

lhs_ineq = [[-3, -4], # left side of first constraint

... [-1, -5]] # right side of first constraint

rhs_ineq = [-20, # right side of first constraint

... -15] # right side of Second constraint

bnd = [(0, float("inf")), #Bounds of x1]

... (0, float("inf"))] # Bounds of x2

>>> opt = linprog(c=obj, A ub=lhs ineq, b ub=rhs ineq,

... bounds=bnd,method="interior-point")

>>> opt

method =" interior-point" solves linear programming problem using default simplex method.

Solve Following linear programming problem using Big M Simplex method.

RESOURCE ALLOCATION PROBLEM BY SIMPLEX METHOD

Use SciPy to solve the resource allocation problem stated as follows:

Max
$$z=20x1+12x2+40x3+25x4...$$
 (profit)

subjected to:

from scipy.optimize import linprog

obj = [-20, -12, -40, -25] #profit objective function

lhs_ineq = [[1, 1, 1, 1], # Manpower

... [3, 2, 1, 0], # Material A

... [0, 1, 2, 3]] # Material B

rhs_ineq = [50, # Manpower

... 100, # Material A

... 90] # Material B

opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,

... method="revised simplex")

Opt

Use SciPy to solve the resource allocation problem stated as follows:

```
Max z= 20x1 + 12x2 +40x3 + 25x4..... (profit)
           subjocled to:
               x_1 + x_2 + x_3 + x_4
                                        50-----
                                                      -----(manpower)
               3x1 + 2x2 + K3
                                      100 ----- (m.atei::ial A)
                       x.2 + 2x3
                                        90 -----(m.atei::ial 8)
                    x.1. x.2, x3, x.4 >-= 0
In [12]: M rrom :3cipy.optimize import linpi::og
In [13]: M obj = [-20, -12, -40, -25]
                                                    #prQfit objective function
In [14]: M lh:3 i.neq = [[1, 1, 1, 11, #l'.gnpover [3, 2, 1, 0], #l'ft.teric.l - [0, 1, 2, 311 # Materia.1 B
In [15]: M :i::h:3_ineq = 50, # ivr.,gnpover
                                  100, #- i'lateria.1 A
90] # i'la teria.1 B
```

```
\begin{array}{lll} \text{In} & \text{[161;} & \text{II} \text{ opt} & \text{-li.nprog(e=obj, A\_ub=lh:3\_l.neq, b\_ub=:t:h:3 ineq,} \\ & & \text{-method=":revi:3ed :3implex")} \end{array}
```

The inerdult tell:3 you that the maximal pirrofit is 3 1900 and coincer 3pend:3 to x1 = 5 and m = 45. I-e':3 not p?:Dfitable to principle the i3econd and fourth principle the given condition:3. You can draw i3eveiral interiner 3ting conclusion:3 below.

The thii::d preduct b:::ing:3 the lai::ge:3t pi::ofi t pe::: unit, :30the factor}; will produce it the mo:3"t.

The fir:3t :3lack i:3 O, whim mea.n:3 that the value:3 of the left and right :3ide:3 of the manpower (:fir:3t) on:3traint the :3ame. The factoi::ypi::oduce:3 50 unit:3pe::: day, a.nd tha-c':3 it:3 full capacity.

The :3econd :3lack i:3 40 becau:3e the factm:y con:3ume:3 60 unit:3 of r:aw matei:::ial A (15 unit:3 for: the :fir:3t p:i::oduct plu:3 45 for the third) out of apotential 100 unit::3.

The thii::d:3lack i:3 0, whim mea.n:3 -cha-c the :factoi::y con:3ume:3 all 90 unit:3 o,f the i::aw matei::ial B. Thi:3 entii::e amount i:3 con:3umed :fm: the thii::dpr:oduct. That':3 why the :factoi::y can't pr:oduce the :3econd or: fourthpr:oduct at all a.nd ca.n't p:::oduce mo:::e than 45 unit:3 of the thi:::dproduct. It lack:3 the i::aw material B.

opt .:3tatu:3 i:3 0 and opt

i:3 True, indicating that the op-cimization problem wa:3 :3ucce:3:3:fully :301·..red with the

INFEASIBILITY IN SIMPLEX METHOD

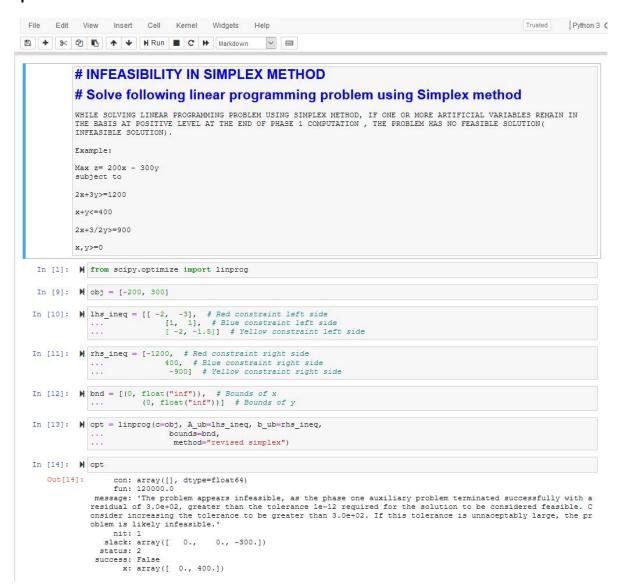
Solve following linear programming problem using Simplex method

WHILE SOLVING LINEAR PROGRAMMING PROBLEM USING SIMPLEX METHOD, IF ONE OR MORE ARTIFICIAL VARIABLES REMAIN IN THE BASIS AT POSITIVE LEVEL AT THE END OF PHASE 1 COMPUTATION, THE PROBLEM HAS NO FEASIBLE SOLUTION(INFEASIBLE SOLUTION).

```
Example:
Max z = 200x - 300y
subject to
2x+3y>=1200
x+y<=400
2x+3/2y>=900
x,y>=0
from scipy.optimize import linprog
obj = [-200, 300]
lhs ineq = [[ -2, -3], # Red constraint left side
... [1, 1], # Blue constraint left side
        [-2, -1.5]] # Yellow constraint left side
rhs_ineq = [-1200, # Red constraint right side
        400, # Blue constraint right side
         -900] # Yellow constraint right side
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
         bounds=bnd,
```

... method="revised simplex")

opt



DUAL SIMPLEX METHOD

##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R PROGRAMMING

Max z=40x1+50x2

#subject to

#2x1 + 3x2 <= 3

#8x1 + 4x2 <= 5

x1, x2>=0

Import IpSolve package

library(lpSolve)

Set coefficients of the objective function

f.obj <- c(40, 50)

Set matrix corresponding to coefficients of constraints by rows

Do not consider the non-negative constraint; it is automatically assumed

f.con <- matrix(c(2, 3,

Set unequality signs

```
"<=")
```

Set right hand side coefficients

f.rhs <- c(3,

5)

Final value (z)

lp("max", f.obj, f.con, f.dir, f.rhs)

Variables final values

lp("max", f.obj, f.con, f.dir, f.rhs)\$solution

Sensitivities

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.from lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.to

Dual Values (first dual of the constraints and then dual of the variables)

Duals of the constraints and variables are mixed

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals

Duals lower and upper limits

lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.from lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.to

OUTPUT:

```
##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R PROGRAMM
> # Max z=40x1+50x2
> #subject to
> #2x1 + 3x2 <= 3
> #8x1 + 4x2 <= 5
> \# x1, x2>=0
> # Import lpSolve package
> library(lpSolve)
> # Set coefficients of the objective function
> f.obj <- c(40, 50)
> # Set matrix corresponding to coefficients of constraints by rows
> # Do not consider the non-negative constraint; it is automatically assumed
> f.con <- matrix(c(2, 3,
                    8, 4), \text{ nrow} = 2, \text{ byrow} = \text{TRUE})
> # Set unequality signs
> f.dir <- c("<=",
             "<=")
> # Set right hand side coefficients
> f.rhs <- c(3,
             5)
> # Final value (z)
> lp("max", f.obj, f.con, f.dir, f.rhs)
Success: the objective function is 51.25
> # Variables final values
> lp("max", f.obj, f.con, f.dir, f.rhs)$solution
[1] 0.1875 0.8750
> # Sensitivities
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.from
[1] 33.33333 20.00000
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.to
[1] 100 60
> # Dual Values (first dual of the constraints and then dual of the variables)
> # Duals of the constraints and variables are mixed
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals
[1] 15.00 1.25 0.00 0.00
> # Duals lower and upper limits
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.from
[1] 1.25e+00 4.00e+00 -1.00e+30 -1.00e+30
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE) $duals.to
[1] 3.75e+00 1.20e+01 1.00e+30 1.00e+30
```

TRANSPORTATION PROBLEM

##sOLVE FOLLOWING TRANSPORTATION PROBLEM IN WHICH CELL ENTRIES REPRESENT UNIT COSTS USING R PROGRAMMING.

"Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY

#sUPPLIER 1	10	2	20	11	15
#sUPPLIER 1	12	7	9	20	25
#sUPPLIER 1	4	14	16	18	10
#DEMAND	5	15	15	15	

Import IpSolve package

library(lpSolve)

Set transportation costs matrix

Set customers and suppliers' names

Set unequality/equality signs for suppliers

row.signs <- rep("<=", 3)

```
# Set right hand side coefficients for suppliers

row.rhs <- c(15, 25, 10)

# Set unequality/equality signs for customers

col.signs <- rep(">=", 4)

# Set right hand side coefficients for customers

col.rhs <- c(5, 15, 15, 15)

# Final value (z)

TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)

# Variables final values

lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution

print(TotalCost)
```

OUTPUT:

```
> ##solve following transportation problem in which cell entries represent unit costs u
             "Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY
> #sUPPLIER 1 10
                                             20
                                                           11
                                                                       15
                12
> #sUPPLIER 1
                             7
                                             9
                                                           20
                                                                       25
> #sUPPLIER 1
                4
                            14
                                             16
                                                           18
                                                                       10
> #DEMAND
                 5
                             15
                                             15
                                                           15
> # Import lpSolve package
> library(lpSolve)
> # Set transportation costs matrix
> costs <- matrix(c(10, 2, 20, 11,
                   12, 7, 9, 20,
                   4, 14, 16, 18), nrow = 3, byrow = TRUE)
> # Set customers and suppliers' names
> colnames(costs) <- c("Customer 1", "Customer 2", "Customer 3", "Customer 4")
> rownames(costs) <- c("Supplier 1", "Supplier 2", "Supplier 3")</pre>
```

```
> # Set unequality/equality signs for suppliers
> row.signs <- rep("<=", 3)</pre>
> # Set right hand side coefficients for suppliers
> row.rhs <- c(15, 25, 10)
> # Set unequality/equality signs for customers
> col.signs <- rep(">=", 4)
> # Set right hand side coefficients for customers
> col.rhs <- c(5, 15, 15, 15)
> # Final value (z)
> TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)</pre>
> # Variables final values
> lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution
    [,1] [,2] [,3] [,4]
[1,]
      0 5 0 10
[2,]
      0 10 15 0
[3,]
      5 0
                0
                      5
> print(TotalCost)
Success: the objective function is 435
```

>

ASSIGNMENT PROBLEM

#SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMING

```
# Assignment Problem
```

```
# JOB1 JOB2 JOB3

#W1 15 10 9

#W2 9 15 10

#W3 10 12 8
```

Import IpSolve package

library(lpSolve)

Set assignment costs matrix

```
costs <- matrix(c(15, 10, 9,
9, 15, 10,
10, 12,8), nrow = 3, byrow = TRUE)
```

Print assignment costs matrix

costs

Final value (z)

lp.assign(costs)

Variables final values

lp.assign(costs)\$solution

OUTPUT:

- > #SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMI
- > # Assignment Problem
- > # JOB1 JOB2 JOB3

```
> #W1 15 10 9
> #W2 9 15 10
> #W3 10 12 8
                         10
> # Import lpSolve package
> library(lpSolve)
> # Set assignment costs matrix
> costs <- matrix(c(15, 10, 9,</pre>
                    9, 15, 10,
                    10, 12 ,8), nrow = 3, byrow = TRUE)
> # Print assignment costs matrix
> costs
    [,1] [,2] [,3]
[1,] 15 10 9
[2,]
      9 15 10
[3,] 10 12
                8
> # Final value (z)
> lp.assign(costs)
Success: the objective function is 27
> # Variables final values
> lp.assign(costs)$solution
    [,1] [,2] [,3]
[1,] 0 1 0
[2,] 1 0 0
[3,] 0 0 1
```

>