INDIAN INSTITUTE OF TECHNOLOGY PATNA

EE520

DIGITAL SIGNAL PROCESSING LAB

Assignment

NAME: **ANJALI SHAW**

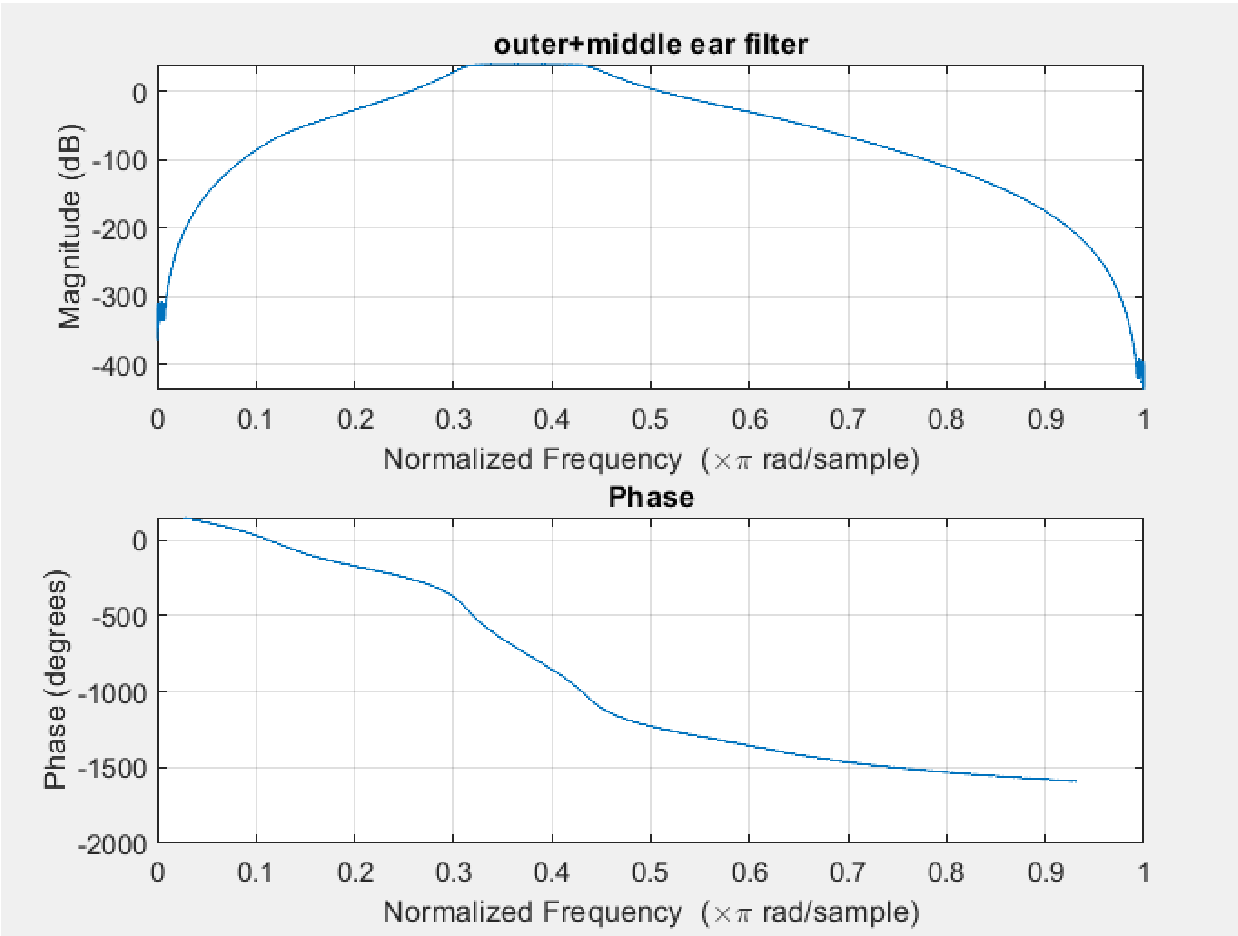
ROLL NUMBER: **2311EE12**

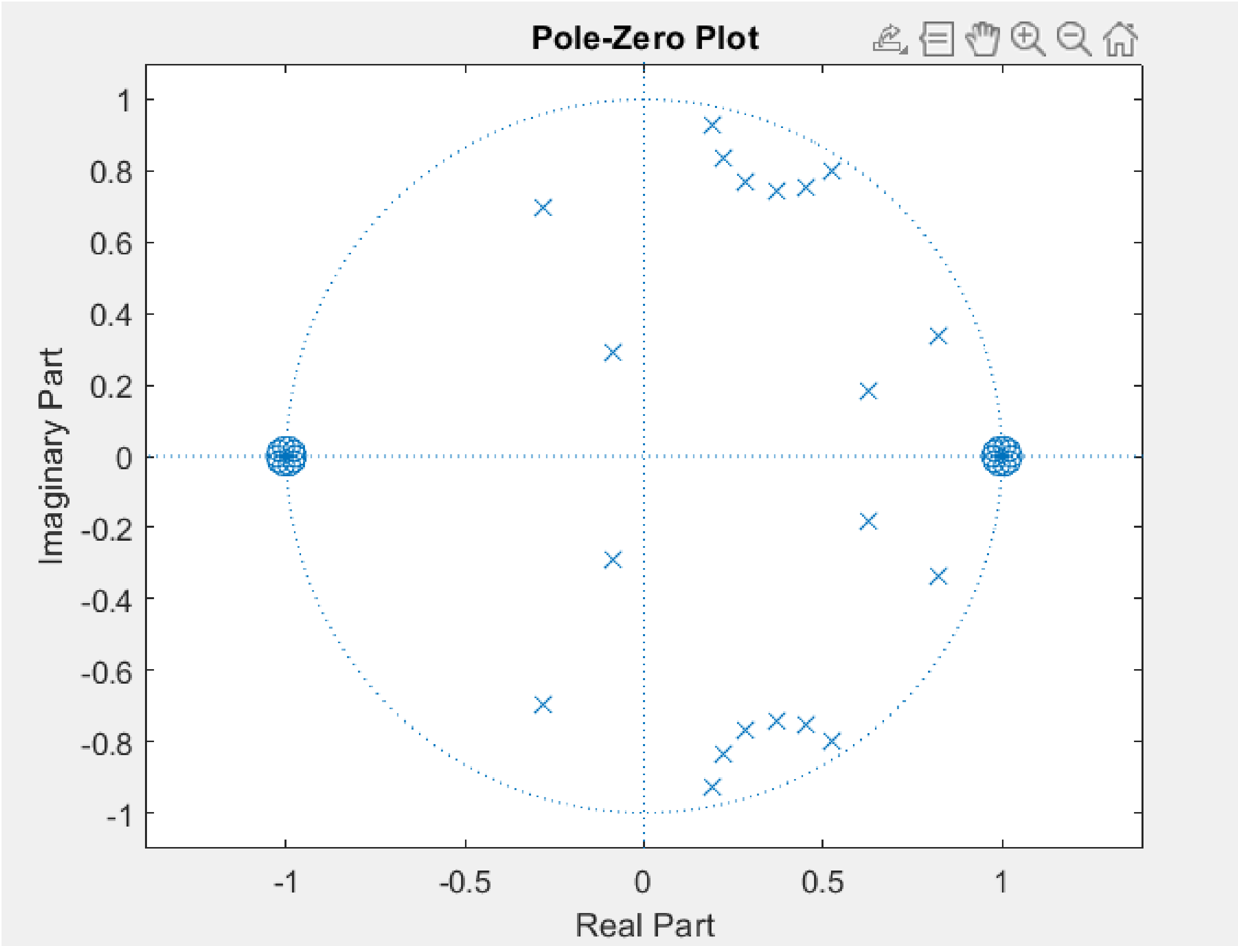
```
clear all;close all; clc;
fs=48000;
f=2000;n=0.01:0.01:100;
y=sin(2*pi*(f/fs)*n );
[y, fs]=audioread('Music.wav');
figure;spectrogram(y);
T=1/fs;
%% Bandpass Filter Design Parameters for mid and outer ear
% Bandpass Filter Design Parameters for outer ear
cf = 3000;
bw = 4000;
fo = 4;
dg = 20;
dgl = 10^(dg / 20);
ncf = cf / (fs / 2);
nbw = bw / (fs / 2);
% co-efficient of filter using butterworth filter
[bo, ao] = butter(fo, [ncf - nbw/2 ncf + nbw/2], 'bandpass');
bo = bo * dgl;
figure; freqz(bo, ao,  fs); xlabel('Frequency
(Hz)');ylabel('Magnitude');title('Bandpass Filter Frequency Response(OE)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Bandpass Filter Design Parameters for middle ear
cf = 3000;
bw = 1000;
fo = 6;
dg = 20;
dgl = 10^(dg / 20);
ncf = cf / (fs / 2);
nbw = bw / (fs / 2);
% co-efficient of filter using butterworth filter
[bm, am] = butter(fo, [ncf - nbw/2 ncf + nbw/2], 'bandpass');
bm = bm * dgl;
figure;freqz(bm, am,  fs); xlabel('Frequency
(Hz)');ylabel('Magnitude');title('Bandpass Filter Frequency Response(ME)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Cascaded filter of outer and middle ear
b=conv(bo, bm);
a=conv(ao, am);
y=filter(b,a,y);
figure; freqz(b,a,fs);title('outer+middle ear filter');
figure; zplane(b,a);
sys=filt(b,a);
%% inner ear (Basilar Membrance output)
div =128;d=3.5/div;
qp = linspace(10, 5.5, div);
qz = linspace(22, 12, div);
fp=zeros(1,div);
fz=zeros(1,div);
```

```
bwp=zeros(1, 128);
bwz=zeros(1, 128);
for i=1:div
   fp(i)=20000*10^(-(0.667*(i-1)*d));
   fz(i)=fp(i)*1.0429;
   bwp(i)=fp(i)/qp(i);
   bwz(i)=fz(i)/qz(i);
end
fil_coff=zeros(128, 9);
% k=fil_coff(i,1)
% g0=fil_coff(i,2) = 1-a0
% a0=fil_coff(i,3)
% gp=fil_coff(i,4) = 1-b1+b2
% b1=fil_coff(i,5)
% b2=fil_coff(i,6)
% gz=fil_coff(i,7) = 1/(1-a1+a2)
% a1=fil_coff(i,8)
% a2=fil_coff(i,9)
for i=1:div
   p1=bwp(i)/(2*qz(i));q1=p1*sqrt(4*qz(i)^2-1);
   fil_coff(i,8)=2*exp(-p1*T ).*cos(q1*T );fil_coff(i,9)=exp(-2*p1*T );
   p2=bwp(i)/(2*qp(i));q2=p2*sqrt(4*qp(i)^2-1);
   fil_coff(i,5)=2*exp(-p2*T )*cos(q2*T );fil_coff(i,6)=exp(-2*p2*T );

   flp=fz(i)*1.4;
   thetalp=2*pi*(flp/fs);
   fil_coff(i,3)=2-cos(thetalp)-sqrt((2-cos(thetalp))^2-1);

   fil_coff(i,1)=fp(i)/fz(i);
a0=fil_coff(i,3) ;
   fil_coff(i,2)=1-a0;
b1=fil_coff(i,5) ;
b2=fil_coff(i,6) ;
   fil_coff(i,4)=1-b1+b2;
a1=fil_coff(i,8) ;
a2=fil_coff(i,9) ;
   fil_coff(i,7)=(1/1-a1+a2);
num=[1-a0];
den=[1 -a0];
   y=filter(num,den,y);
num=[1-b1-b2];
den=[1 -b1 -b2];
   y=filter(num,den,y);
   dis_o(:,i)=y;
num=[1 -a1 -a2];
den=[1-a1-a2];
   y=filter(num,den,y);
   pre_o(:,i)=y;
end
```

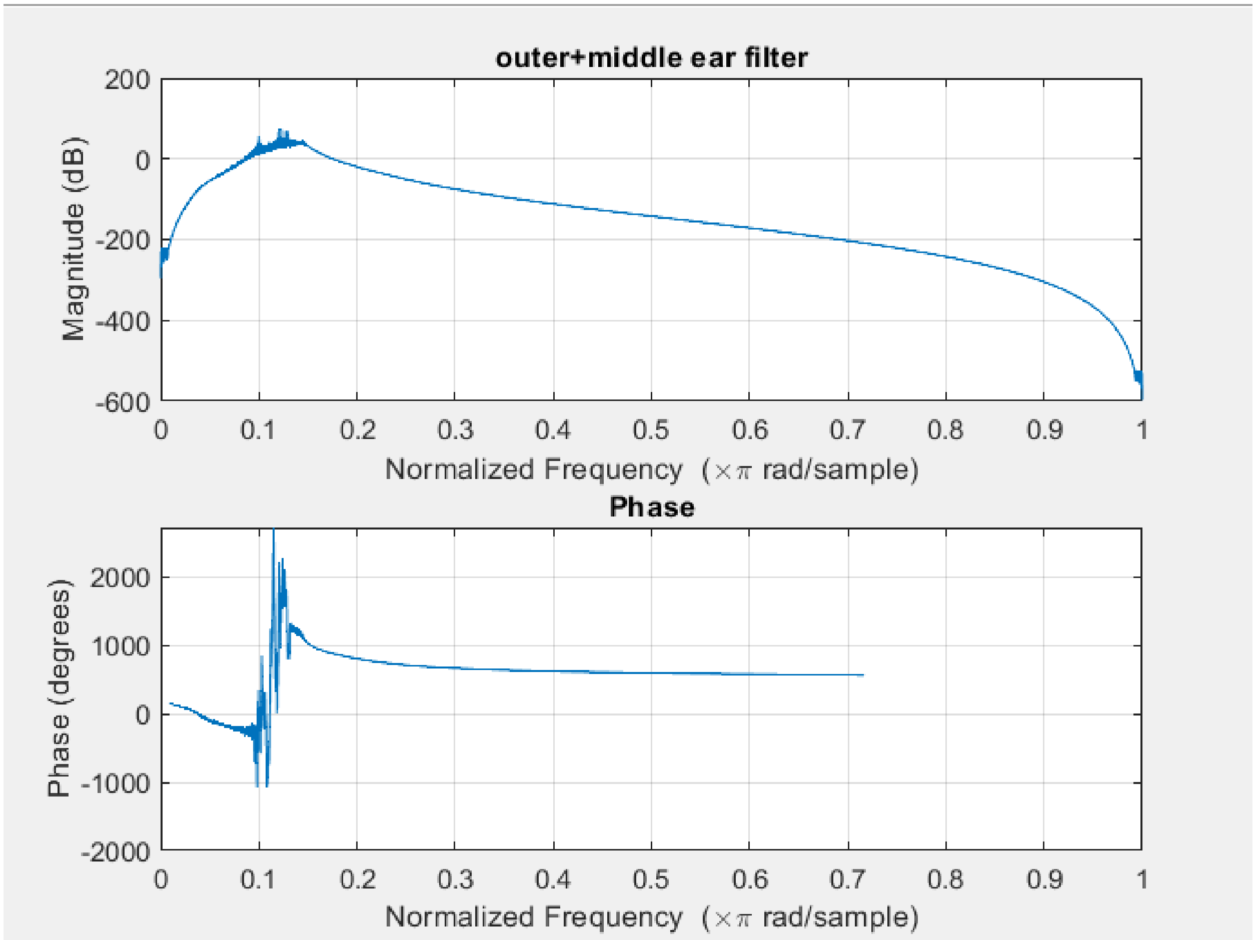
```
%% differentiation
d_dis_o=zeros(size(dis_o)); d_pre_o=zeros(size(pre_o));
d_dis_o(:, 1)=dis_o(:,1);
d_pre_o(:, 1)=pre_o(:,1);
for i=2:div
   d_dis_o(:, i)=dis_o(:,i)-dis_o(:, i-1);
   d_pre_o(:, i)=pre_o(:,i)-pre_o(:, i-1);
end
for i=2:length(dis_o)
   d_dis_o(i,:)=dis_o(i, :)-dis_o(i-1,:);
   d_pre_o(i,:)=pre_o(i, :)-pre_o(i-1,:);
end
clear a0;clear num;clear den; clear flp; clear thetalp;
clear b1;clear b2;clear a1;clear a2;
clear p1;clear p2;clear q1;clear q2;
h_o=zeros(size(dis_o));
%% hair cell
for i=1:128
   c0=exp(-30*(2*pi/48000));
   num=[1-c0];
   den=[1 -c0];
   h_o(:,i)=filter(num, den, dis_o(:,i));
end
```

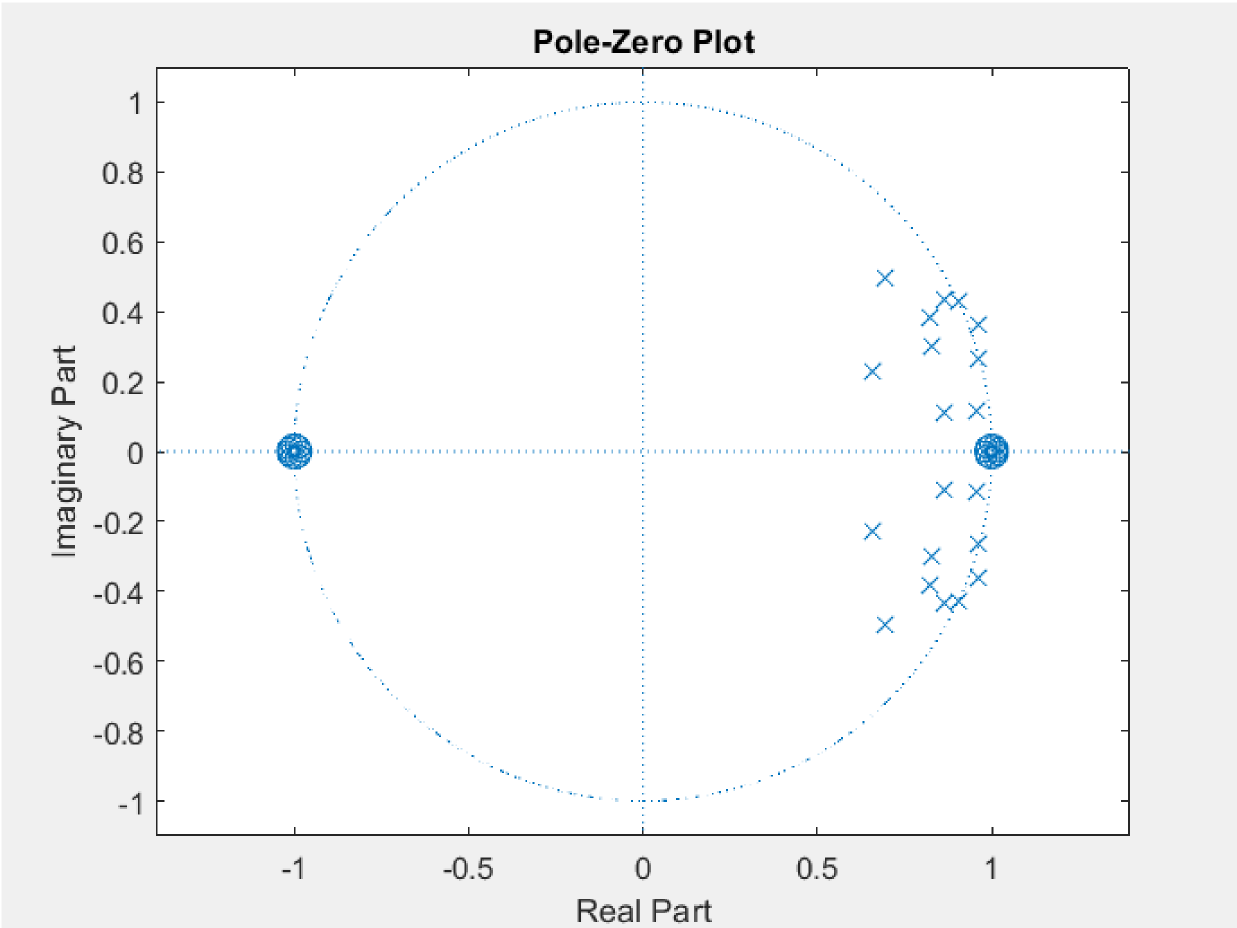
For audio file:





For sinusoid input:

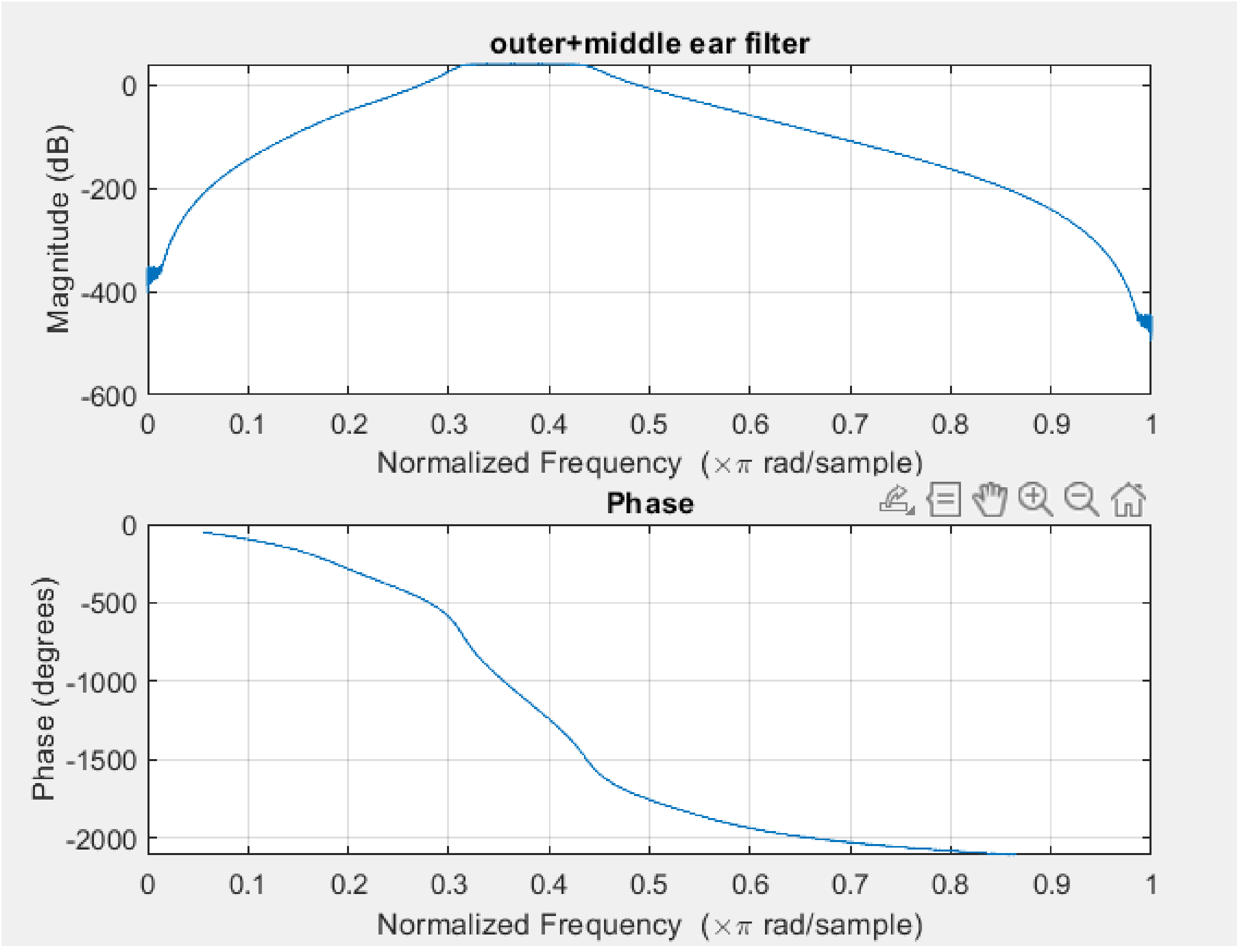

outer+middle ear filter
Magnitude (dB)
200
0
-200
-400
-600
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
2000
1000
0
-1000
-2000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)



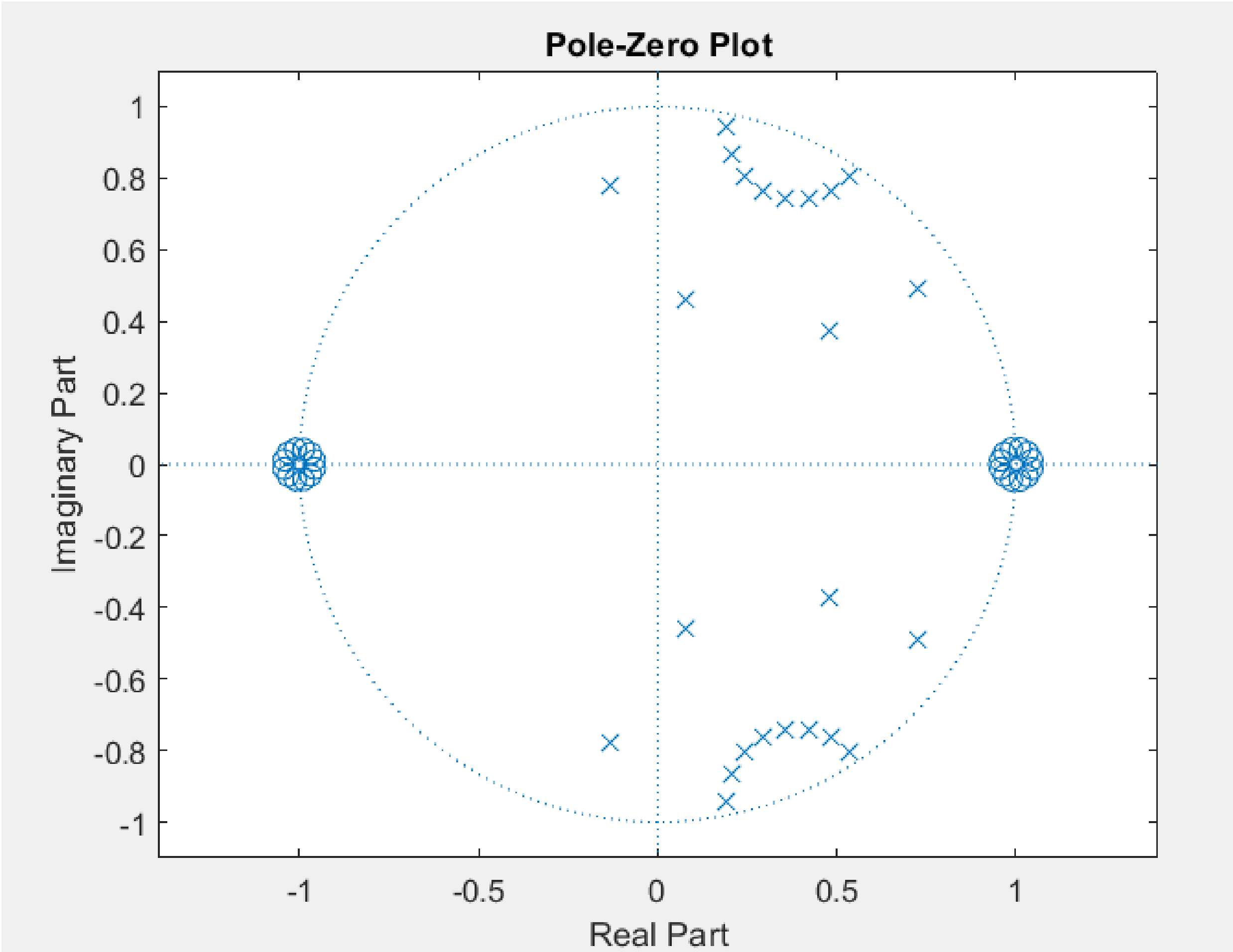
With changed filter co-efficient

```
%% Bandpass Filter Design Parameters for mid and outer ear
% Bandpass Filter Design Parameters for outer ear
cf = 3000;
bw = 2980;
fo = 4;
dg = 20;
dgl = 10^(dg / 20);
ncf = cf / (fs / 2);
nbw = bw / (fs / 2);
% co-efficient of filter using butterworth filter
[bo, ao] = butter(fo, [ncf - nbw/2 ncf + nbw/2], 'bandpass');
bo = bo * dgl;
figure; freqz(bo, ao,  fs); xlabel('Frequency
(Hz)');ylabel('Magnitude');title('Bandpass Filter Frequency Response(OE)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Bandpass Filter Design Parameters for middle ear
cf = 3000;
bw = 1000;
fo = 8;
dg = 20;
```

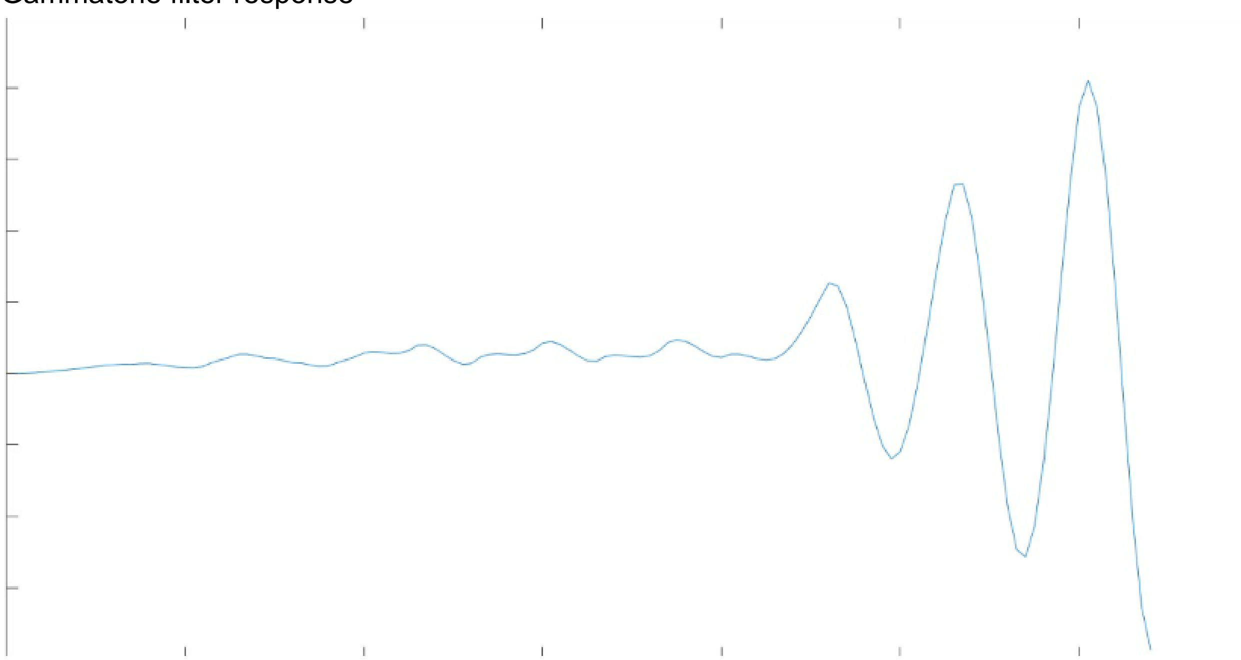
```
dgl = 10^(dg / 20);
ncf = cf / (fs / 2);
nbw = bw / (fs / 2);
% co-efficient of filter using butterworth filter
[bm, am] = butter(fo, [ncf - nbw/2 ncf + nbw/2], 'bandpass');
bm = bm * dgl;
figure;freqz(bm, am,  fs); xlabel('Frequency
(Hz)');ylabel('Magnitude');title('Bandpass Filter Frequency Response(ME)');
```



outer+middle ear filter
Magnitude (dB)
0
-200
-400
-600
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)
Phase
Phase (degrees)
0
-500
-1000
-1500
-2000
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)



Gammatone filter response



Given v/s calculated values

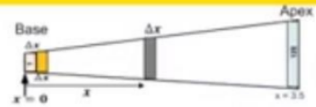
Filter Number	K	G_0	a_0	G_p	b_1	b_2	G_z	a_1	a_2
1	0.9589	0.8137	0.1863	3.2607	-1.5053	0.7553	0.2778	-1.7195	0.8797
2	0.9589	0.8199	0.1801	3.1751	-1.4112	0.7639	0.2847	-1.6286	0.8841
3	0.9589	0.8242	0.1758	3.0767	-1.3044	0.7723	0.2923	-1.5218	0.8884
.									
.									
128	0.9589	0.0183	0.9817	0.0002	1.9975	0.9977	5749	1.9987	0.9989

	1	2	3	4	5	6	7	8	9
1	0.9589	0.6329	0.3671	0.0155	1.9721	0.9876	0.0156	1.9788	0.9943
2	0.9589	0.7320	0.2680	0.0144	1.9736	0.9880	0.0144	1.9801	0.9945
3	0.9589	0.7854	0.2146	0.0133	1.9751	0.9884	0.0133	1.9814	0.9947
4	0.9589	0.8135	0.1865	0.0123	1.9765	0.9888	0.0124	1.9825	0.9949
5	0.9589	0.8262	0.1738	0.0114	1.9778	0.9892	0.0115	1.9836	0.9951
6	0.9589	0.8279	0.1721	0.0106	1.9790	0.9896	0.0106	1.9846	0.9952
7	0.9589	0.8203	0.1797	0.0098	1.9801	0.9899	0.0098	1.9856	0.9954
8	0.9589	0.8032	0.1968	0.0091	1.9812	0.9903	0.0091	1.9865	0.9956
9	0.9589	0.7750	0.2250	0.0084	1.9822	0.9906	0.0084	1.9873	0.9957
10	0.9589	0.7327	0.2673	0.0078	1.9831	0.9909	0.0078	1.9880	0.9959
11	0.9589	0.6715	0.3285	0.0072	1.9840	0.9912	0.0072	1.9888	0.9960
12	0.9589	0.5847	0.4153	0.0067	1.9848	0.9915	0.0067	1.9894	0.9961
13	0.9589	0.4639	0.5361	0.0062	1.9856	0.9918	0.0062	1.9901	0.9963
14	0.9589	0.2995	0.7005	0.0057	1.9863	0.9921	0.0057	1.9906	0.9964
15	0.9589	0.0829	0.9171	0.0053	1.9870	0.9923	0.0053	1.9912	0.9965
16	0.9589	0.1605	0.8395	0.0049	1.9877	0.9926	0.0049	1.9917	0.9966
17	0.9589	0.3431	0.6569	0.0046	1.9883	0.9929	0.0046	1.9922	0.9967
18	0.9589	0.4748	0.5252	0.0042	1.9889	0.9931	0.0042	1.9926	0.9969

128x9 double

	1	2	3	4	5	6	7	8	9
112	0.9589	0.1026	0.8974	3.7924e-06	1.9997	0.9997	3.7927e-06	1.9998	0.9999
113	0.9589	0.0986	0.9014	3.5280e-06	1.9997	0.9997	3.5283e-06	1.9999	0.9999
114	0.9589	0.0948	0.9052	3.2822e-06	1.9997	0.9997	3.2825e-06	1.9999	0.9999
115	0.9589	0.0911	0.9089	3.0538e-06	1.9997	0.9997	3.0541e-06	1.9999	0.9999
116	0.9589	0.0875	0.9125	2.8415e-06	1.9997	0.9997	2.8417e-06	1.9999	0.9999
117	0.9589	0.0841	0.9159	2.6441e-06	1.9997	0.9997	2.6443e-06	1.9999	0.9999
118	0.9589	0.0808	0.9192	2.4606e-06	1.9997	0.9997	2.4608e-06	1.9999	0.9999
119	0.9589	0.0776	0.9224	2.2901e-06	1.9997	0.9997	2.2902e-06	1.9999	0.9999
120	0.9589	0.0745	0.9255	2.1315e-06	1.9997	0.9997	2.1316e-06	1.9999	0.9999
121	0.9589	0.0716	0.9284	1.9840e-06	1.9998	0.9998	1.9841e-06	1.9999	0.9999
122	0.9589	0.0687	0.9313	1.8469e-06	1.9998	0.9998	1.8470e-06	1.9999	0.9999
123	0.9589	0.0660	0.9340	1.7194e-06	1.9998	0.9998	1.7195e-06	1.9999	0.9999
124	0.9589	0.0634	0.9366	1.6008e-06	1.9998	0.9998	1.6009e-06	1.9999	0.9999
125	0.9589	0.0608	0.9392	1.4905e-06	1.9998	0.9998	1.4906e-06	1.9999	0.9999
126	0.9589	0.0584	0.9416	1.3879e-06	1.9998	0.9998	1.3880e-06	1.9999	0.9999
127	0.9589	0.0561	0.9439	1.2925e-06	1.9998	0.9998	1.2926e-06	1.9999	0.9999
128	0.9589	0.0538	0.9462	1.2037e-06	1.9998	0.9998	1.2038e-06	1.9999	0.9999
129									

Filter coefficient



- ✓ Number of filters N = 128; Length of the BM = 3.5 cm. $\Delta x = \frac{3.5}{128} = 0.0273\ cm$
- ✓ $x = 0, \Delta x, 2\Delta x, 3\Delta x, \dots\dots\dots, 127\Delta x;$ $Resonant\ frequency$: $f_p(n)$ = $(20000)\ 10^{-0.667n\Delta x}$

Filter No (n)	Distance (x) cm	$f_p(n)$: Resonant Frequency (Hz)	$\frac{f_p(n)}{f_p(n+1)}$	$f_z(n)$: Resonant zero (Hz) (Notch filer)
1	0	$f_p(1)$ = 20000	$\frac{f_p(1)}{f_p(2)}$ = 1.0429	$f_z(1)$ = 1.0429× $f_p(1)$ = 20858
2	Δx	$f_p(2)$ = 19177	$\frac{f_p(2)}{f_p(3)}$ = 1.0429	$f_z(2)$ = 1.0429× $f_p(2)$ = 20000
3	$2\Delta x$	$f_p(3)$ = 18389	$\frac{f_p(3)}{f_p(4)}$ = 1.0429	$f_z(3)$ = 1.0429× $f_p(3)$ = 19178
4	$3\Delta x$	$f_p(4)$ = 17633	.	.
.		.	.	.
128	$127\Delta x$	$f_p(128)$ = 96.55	-	$f_z(128)$ = 1.0429× $f_p(128)$=100.70

- ✓ Calculate the quality factor values, Q_p and Q_z; and bandwidths, BW_p and BW_z.
- ✓ Q_p varies linearly from 10 (first filter) to 5.5 (128th filter)
- ✓ Q_z varies linearly from 22 (first filter) to 12 (128th filter)
- ✓ You can change these around and observe what happens but ensure $Q_z > Q_p$.
- ✓ $BW_p(n) = \frac{f_p(n)}{Q_p(n)}$ and $BW_z(n) = \frac{f_z(n)}{Q_z(n)}$
- ✓ Where $a_0, a_1, a_2, b_1, and\ b_2$ are the digital filter coefficients;

$$\boldsymbol{a_1} = \boldsymbol{2e^{-p_1T}}\cos(\boldsymbol{q_1T}); \quad \boldsymbol{a_2} = \boldsymbol{e^{-2p_1T}}; \quad \boldsymbol{p_1} = \frac{\boldsymbol{\omega_z}}{\boldsymbol{2Q_z}}; \quad \boldsymbol{q_1} = \boldsymbol{p_1}\sqrt{4Q_z^{\ 2} - \boldsymbol{1}}\ ;$$

$$\boldsymbol{b_1} = \boldsymbol{2e^{-p_2T}}\cos(\boldsymbol{q_2T}); \quad \boldsymbol{b_2} = \boldsymbol{e^{-2p_2T}}; \quad \boldsymbol{p_2} = \frac{\boldsymbol{\omega_p}}{\boldsymbol{2Q_p}}; \quad \boldsymbol{q_2} = \boldsymbol{p_2}\sqrt{4Q_p^{\ 2} - \boldsymbol{1}}\ ;$$

- ✓ Determine the 3dB cut-off frequency, f_{LP} of the low pass filter and determine the filter coefficient, a_0.
- ✓ $f_{LP} = f_z \times 1.4$; $\theta_{LP} = 2\pi \frac{f_{LP}}{f_s}$

  (Note: $f_{LP} \geq f_z$ in order to have a small attenuation after the notch frequency. You can change the scaling factor of 1.4 to other values and observe the effect on the roll-off of the basilar membrane response at various sections).
- ✓ $H_{LP}(z) = \frac{1-a_0}{1-a_0 z^{-1}}$ (Transfer function of a low pass filter with DC gain of 1)
- ✓ $H_{LP}(\theta) = \frac{1-a_0}{1-a_0 e^{-j\theta}}$; $|H_{LP}(\theta)|^2 = \frac{(1-a_0)^2}{1+a_0^2-2a_0\cos\theta}$; When $\theta = \theta_{LP}$ (3dB cut-off), $|H_{LP}(\theta)|^2 = \frac{1}{2}$
- ✓ $\therefore \frac{1}{2} = \frac{(1-a_0)^2}{1+a_0^2-2a_0\cos\theta_{LP}}$; $a_0 = (2 - cos\theta_{LP}) - \sqrt{(2 - cos\theta_{LP})^2 - 1}$

Inner Hair Cell Model

Medium 6
(1000 Hz)

- ✓ The final stage of this project is the implementation of the inner hair cell model where spatially differentiated displacement is translated into electrical energy by the neural transduction mechanism.
- ✓ This model is given by the following input-output relationship:

$$v(n) = (1 - c_0)\tilde{s}(n) + c_0 v(n-1)$$

Where, $v(n)$ is the output electrical energy, $\tilde{s}(n)$ is the spatially differentiated displacement **after half-wave rectification** and

$$c_0 = e^{-30 \cdot \frac{2\pi}{48000}}$$