Problem statement 3

Given Pass band attenuation $\alpha_p = 3dB$

Stop band attenuation $\alpha_s = 10dB$

Pass band frequency $\omega_p = 2\pi * 1000 = 2000\pi$ rad/sec

Stop band frequency $\omega_s = 2\pi * 350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2*10^{-4} \text{ sec.}$$

The characteristics are monotonic in both passband and stopband. Therefore, the filter is Butterworth

Prewarping the digit frequencies we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2*10^{-4}}\tan\frac{(2000\pi * 2*10^{-4})}{2} = 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2*10^{-4}}\tan\frac{(700\pi * 2*10^{-4})}{2} = 10^4 \tan(0.07\pi) = 2235 \text{ rad/sec}$$

The order of the filter

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log\sqrt{\frac{10^{0.1\times10}-1}{10^{0.1\times3}-1}}}{\log\frac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

$N = 1$

The first order Butterworth filter for $\Omega_c = 1$ rad/s is $H(s) = \frac{1}{s+1}$.

The high pass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$S \rightarrow \frac{\Omega_c}{S}$$

$$S \rightarrow \frac{7265}{S}$$

The transfer function of high pass filter

$$H(s) = \left.\frac{1}{S+1}\right|_{S=\frac{7265}{S}}$$

$$= \frac{S}{S+7265}$$

Using bilinear transformation.

$$H(z) = H(s)\Big|_{S=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \left.\frac{S}{S+7265}\right|_{S=\frac{2}{2*10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+7265}$$

$$H(z) = \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}.$$

Solving further to get the equation

$$\frac{Y(z)}{X(z)} = \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$

$$Y(z) = \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}} X(z)$$

$$Y(z) - 0.1584z^{-1}Y(z) = 0.5792X(z) - 0.5792z^{-1}X(z)$$

$$Y(z) = 0.5792X(z) - 0.5792z^{-1}X(z) + 0.1584z^{-1}Y(z)$$

$$Y(z) - 0.1584 z^{-1} Y(z) = 0.5792 X(z) - 0.592 z^{-1} X(z)$$

$$Y(z)\left[1 - 0.1584 z^{-1}\right] = 0.5792 X(z) - 0.59 z^{-1} X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{0.592 \left[1 - z^{-1}\right]}{1 - 0.01584 z^{-1}}$$