OTTO VON GUERICKE
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INF

FAKULTÄT FÜR
INFORMATIK

Comparing Systems

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How to Compare Systems

Perform the same number of replications for both systems and record the desired result sequences Y_1 and Y_2, then compute confidence intervals.

Y_1	Y_2
1	3
4	5
2	1
…	…

Approach 1:

- Compute C.I. for each sequence separately
- If the two C.I. do not overlap, evaluate positions

Problem: difficult to interpret combined level of confidence

Y_1-Y_2
-2
-1
1
…

Approach 2:

- Compute single C.I. for the difference sequence
- If C.I. does not include the origin, evaluate position

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Correlated Sampling – Why does it help?

Approach 2 is easier to compute and potentially more efficient

- C.I. width depends on data variance
- So, for A2 it depends on the variance of the *differences* :

$$\mathrm{var}(\bar{Y}_1 - \bar{Y}_2) = \frac{\sigma_1^2}{R} + \frac{\sigma_2^2}{R} - \frac{2\rho_{12}\sigma_1\sigma_2}{R}$$

Variance of Y_1 *Variance of Y_2*

Correlation coefficient between Y_1 and Y_2

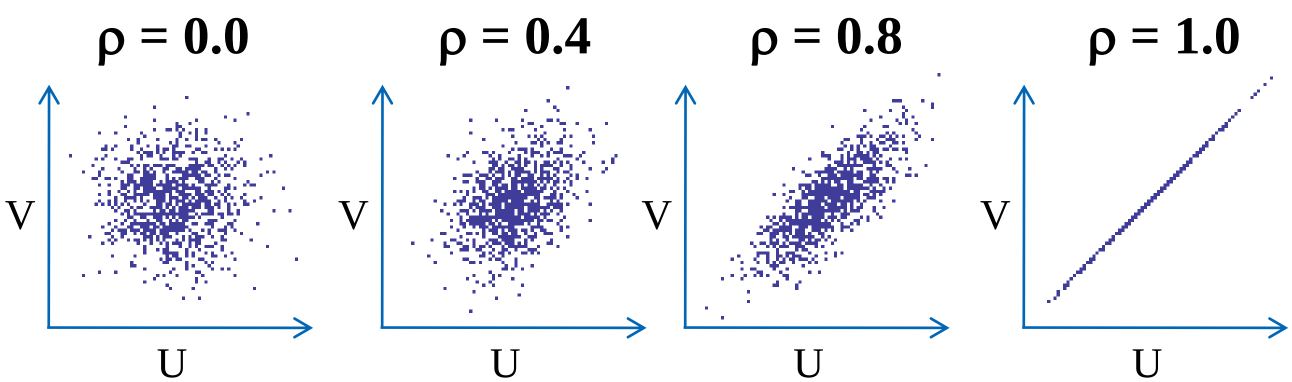
→ If results between both models correlate, variance and width of difference C.I. get smaller

Consequence: Fewer replications for desired level of accuracy

Review: Correlation

The correlation coefficient ρ describes the statistical relationship between two data sequences U and V

- ρ > 0 for a positive linear relationship between U and V



How do we create a positive correlation?

- Ensure that the models to be compared behave similarly in the same replication
- Do **NOT** manipulate the models or re-order the results!

Y_1	Y_2
100	105
345	314
57	64
…	…

Correlated Sampling –Achieving a Positive Correlation

Idea:

- Use the same random numbers in both models

More precisely:

- For each model, each replication uses different random numbers
- In each replication, both models use the same RNs

Then,

- Different replications will be independent (Required for C.I.)
- Every pair of results Y_{r1}, Y_{r2} will be positively correlated

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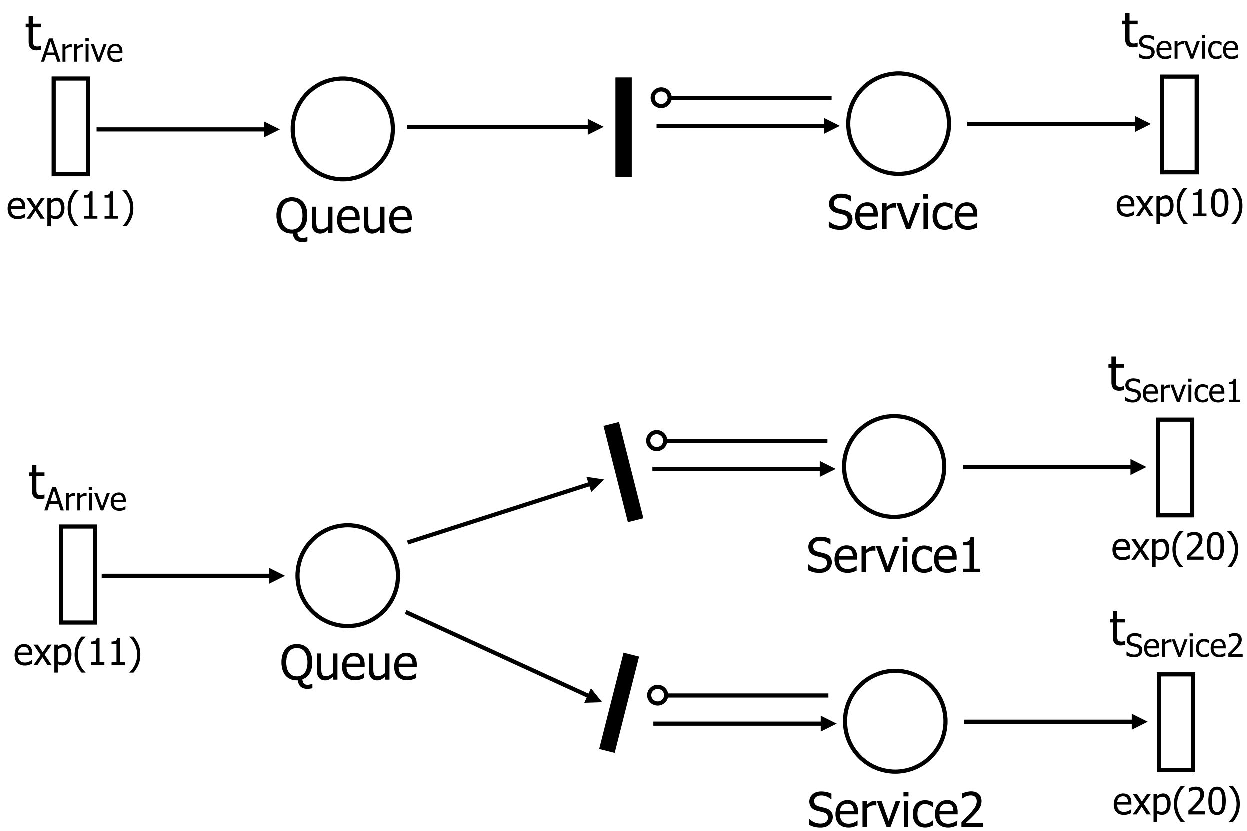
Correlated Sampling

Algorithm for comparing two systems:

- Compute $D_r = Y_{r1} - Y_{r2}$
- Compute $\overline{D} = \frac{1}{R} \sum D_r$
- Compute $S^2 = \frac{1}{R-1} \sum (D_r - \overline{D})^2$
- Compute $\sigma = \frac{S}{\sqrt{R}}$
- Choose α and compute the confidence interval

Example

Example: Compare one– and two–server models



Example

Do a simulation study to make the comparison

We will do the following experiment:

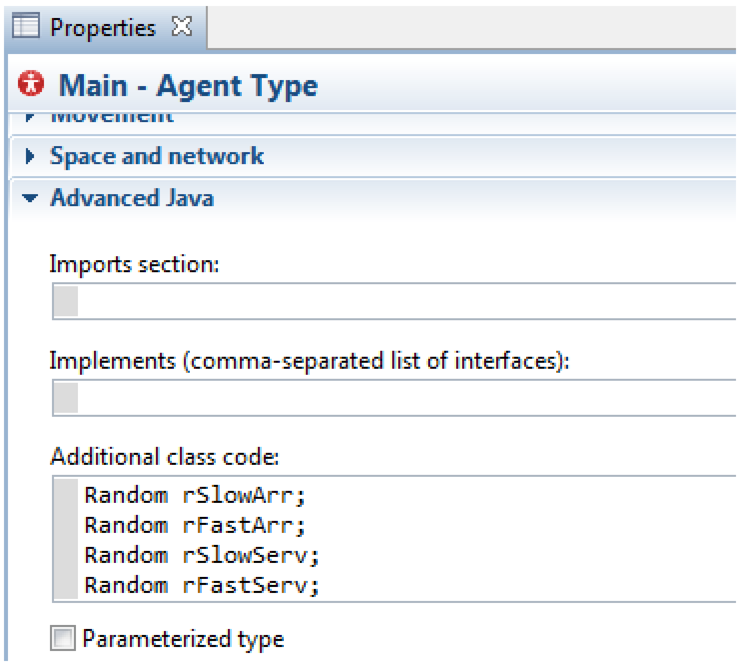
- Compute average queue length at $T = 10{,}000$
- Use $R = 15$ replications

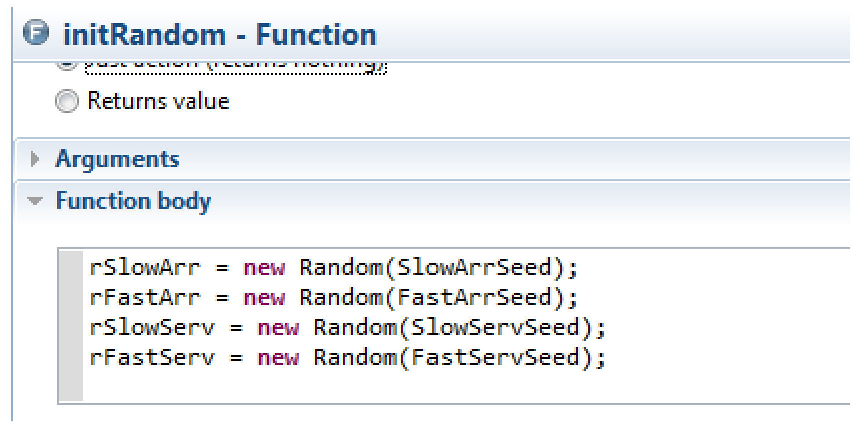
AnyLogic Model Using Correlated Sampling

Controlling Random Numbers in AnyLogic

Declare four separate random number streams

Initialize the random number streams in a function





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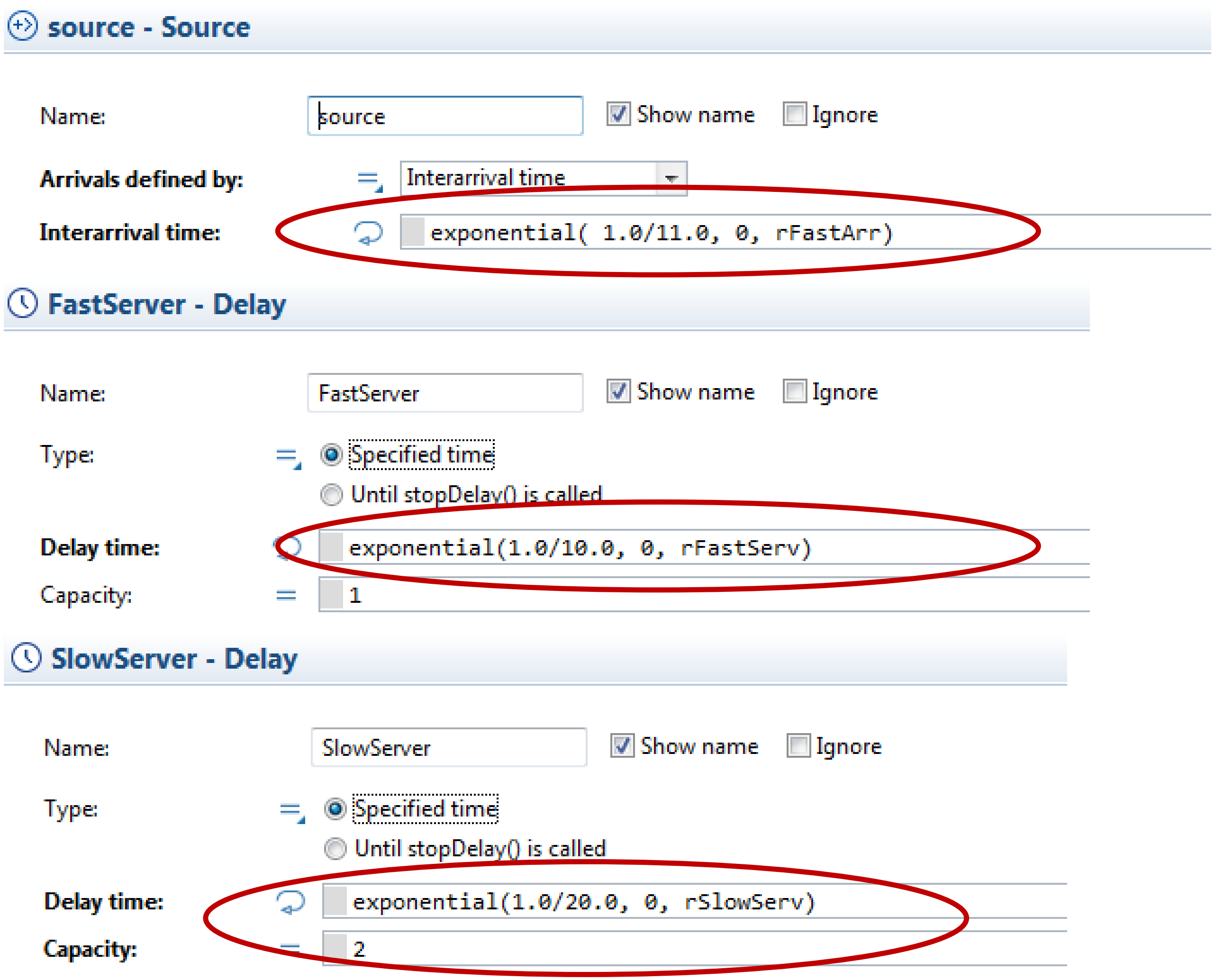
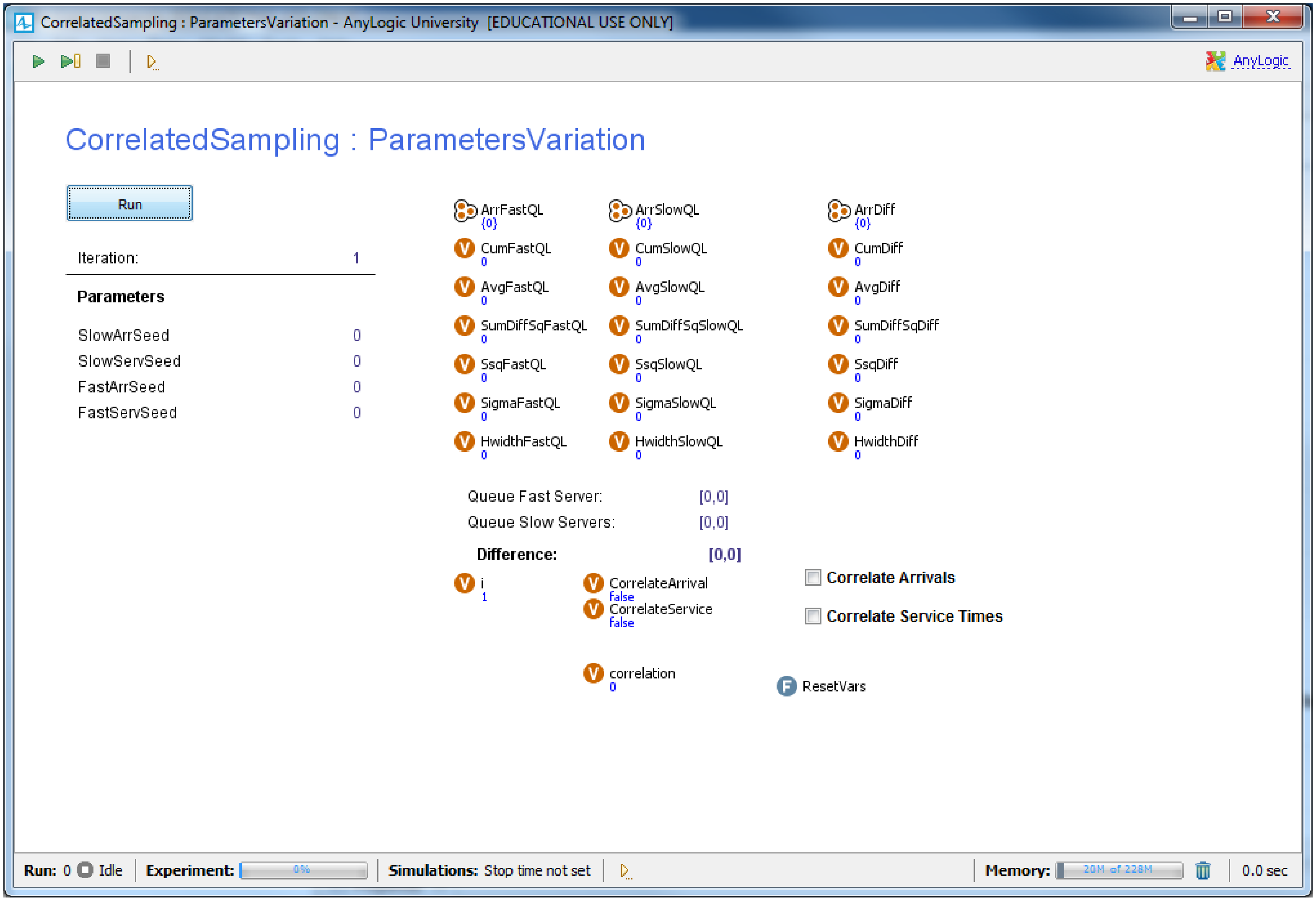
Controlling Random Numbers in AnyLogic

Assign specific random number stream to each random variable



Experiment Setup



Controlling Random Numbers in AnyLogic

Set seeds and initialize the random number streams from the experiment

- Use the same seeds for correlated runs
- Use different seeds for independent runs

Before simulation run:

```
i+=1;
root.SlowArrSeed = i;
if(CorrelateArrival)    root.FastArrSeed = i;
else                    root.FastArrSeed = i+10;
root.SlowServSeed = i+20;
if(CorrelateService) root.FastServSeed = i+20;
else                 root.FastServSeed = i+30;
root.initRandom();
```

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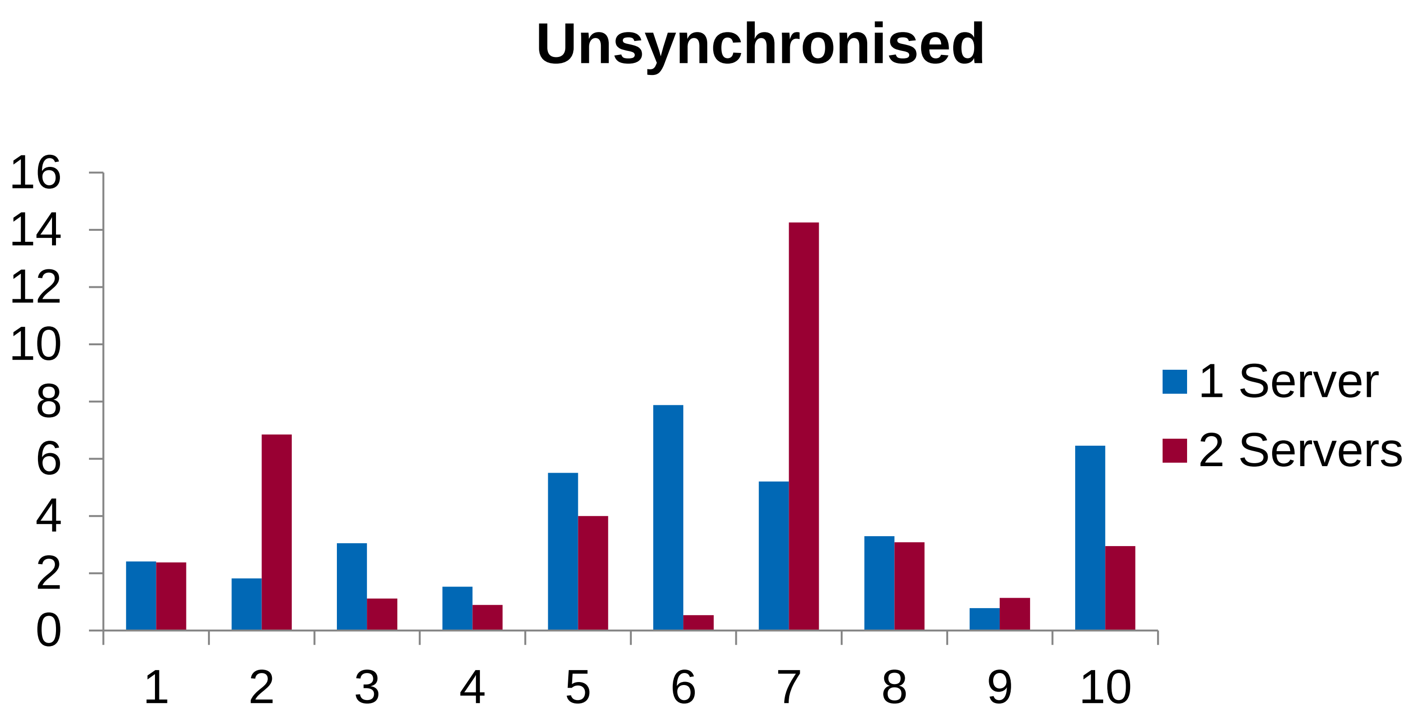
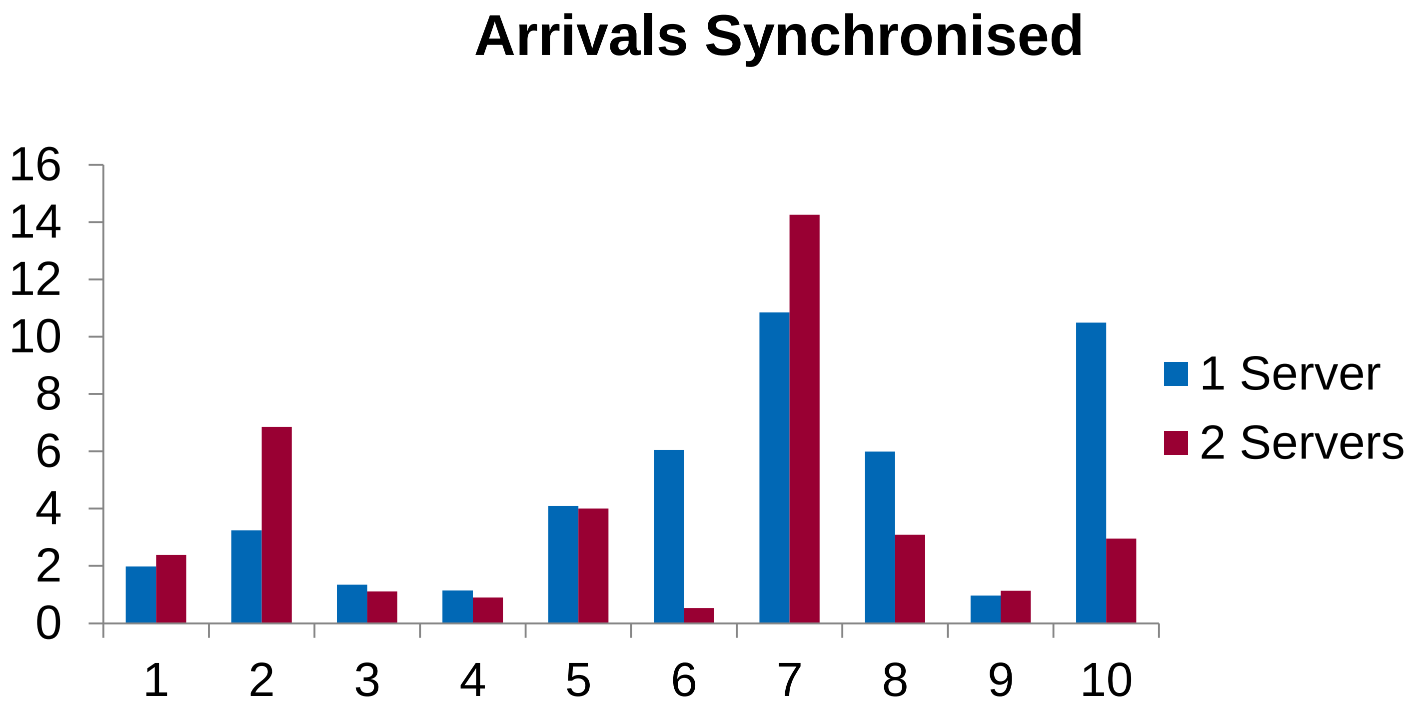
Experimental Results

Individual replication results, unsynchronised



Experimental Results

Individual replication results, only arrivals synchronised



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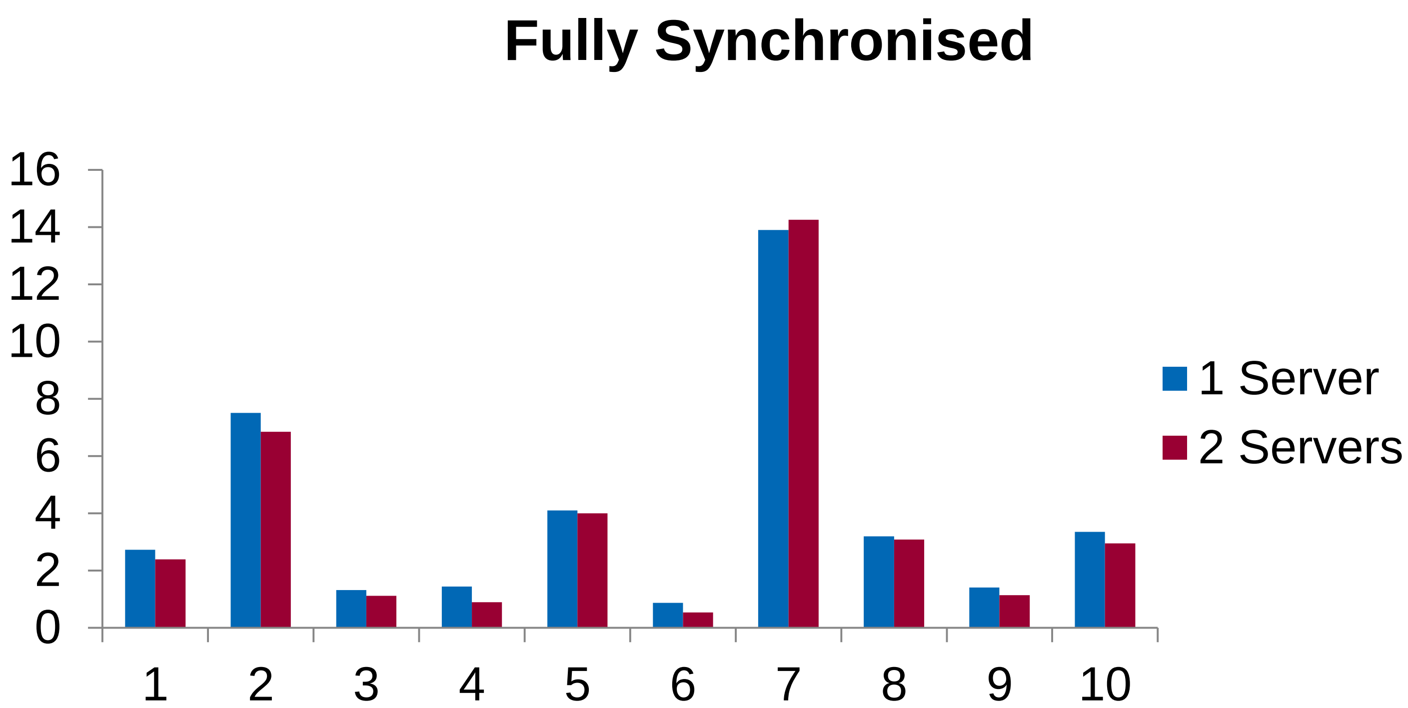
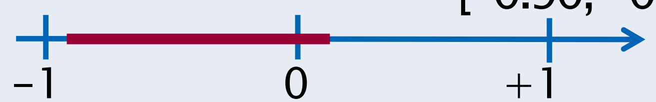
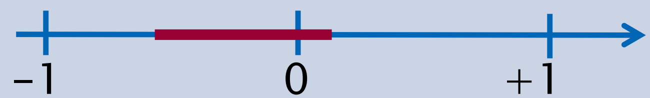
Experimental Results

Individual replication results, fully synchronised



Results

For 100 Replications:

	Sample		C.I. ($\alpha = 0.05$)	
RN	Corr.	Variance	Width	Interval
Independent	0.02	6.46	1.01	[−0.90, 0.11] 
Arrivals Synchronized	0.54	2.75	0.66	[−0.54, 0.12] 
Fully Synchronized	0.99	0.05	0.09	[−0.35, −0.26]