BAUHAUS UNIVERSITÄT WEIMAR

INTRODUCTION TO MACHINE LEARNING

ASSIGNMENT 6

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Exercise 1 : Probability Basics

A function that fulfills the Kolmogorov axioms is a probability measure.
Each subset A of a sample space Ω is an event.

Exercise 3 : Bayes' Rule

(a) Prior Probabilities

$P(C_i)$: i =1,2,3,4,5

$P(C_1) = 1/8$

$P(C_2) = 2/8$

$P(C_3) = 3/8$

$P(C_4) = 1/8$

$P(C_5) = 1/8$

(b) Posterior Probabilities

(i) $P(C_i \mid S_4) = (P(Ci) \ * \ P(S4 \mid Ci)) \div P(S4)$

$P(C_1) = 1/8$; $P(S_4) = 2/8$

$P(S4 \mid C1) = 0$ (from table)

$P(C_1 \mid S_4) = (1/8) \ * \ 0 \ \div \ (2/8) = 0$

(ii) $P(C_2 \mid S_4) = (P(C2) \ * \ P(S4 \mid C2)) \div P(S4)$

$P(C_2) = 2/8$; $P(S_4) = 2/8$

$P(S4 \mid C2) = 1/2$ (from table)

$P(C_2 \mid S_4) = ((2/8) \ * \ (1/2)) \ \div \ (2/8) = 1/2$

(iii) $P(C_3 \mid S_4) = (P(C3) \ * \ P(S4 \mid C3)) \div P(S4)$

$P(C_3) = 3/8$; $P(S_4) = 2/8$

$P(S4 \mid C3) = 0$ (from table)

$P(C_3 \mid S_4) = ((3/8) \ * \ 0) \ \div \ (2/8) = 0$

(iv) $P(C_4 \mid S_4) = (P(C4) \ * \ P(S4 \mid C4)) \div P(S4)$

$P(C_4) = 1/8$; $P(S_4) = 2/8$

$P(S4 \mid C4) = 1$ (from table)

$P(C_4 \mid S_4) = ((1/8) \ * \ 1) \ \div \ (2/8) = 1/2$

(v) $P(C_5 \mid S_4) = (P(C5) \ * \ P(S4 \mid C5)) \div P(S4)$

$P(C_5) = 1/8$; $P(S_4) = 2/8$

$P(S4 \mid C5) = 0$ (from table)

$P(C_4 \mid S_4) = ((1/8) \ * \ 0) \ \div \ (2/8) = 0$

Exercise 4 : Probability Basics

A → Box containing a green ball
B → Box containing a Blue ball
C → Box containing a yellow ball
D → Box containing a red ball

a) $P(A) = 4/8$
$P(B) = 4/8$
$P(C) = 3/8$
$P(D) = 4/8$

b) $P(A \cap B) = 2/8$
$P(A \cap C) = 1/8$
$P(B \cap C) = 2/8$
$P(B \cap D) = 2/8$

c) i) $P(A \cap B) = 2/8$
$P(A) * P(B) = \frac{4}{8} + \frac{4}{8} = \frac{2}{8}$
$P(A \cap B) = P(A) * P(B)$

ii) $P(A \cap C) = 1/8$
$P(A) * P(C) = \frac{4}{8} + \frac{3}{8} = \frac{6}{8} \rightarrow P(A \cap C) \neq P(A) + P(C)$

iii) $P(B \cap C) = 2/8$
$P(B) + P(C) = \frac{4}{8} + \frac{3}{8} = \frac{6}{8}$
$P(B \cap C) \neq P(B) + P(C)$

iv) $P(B \cap D) = 2/8$
$P(B) + P(D) = \frac{4}{8} + \frac{4}{8} = 2/8$
$P(B \cap D) = P(B) + P(D)$

So, option 1 and option 4 are correct

d) $P(A|C) = \frac{P(A\cap C)}{P(C)} = \frac{1/8}{3/8} = 1/3$

$P(B|C) = \frac{P(B\cap C)}{P(C)} = \frac{2/8}{3/8} = 2/3$

$P(A\cap B|C) = \frac{P(A\cap B\cap C)}{P(C)} = \frac{1/8}{3/8} = 1/3$

e) $P(B|D) = \frac{P(B\cap D)}{P(D)} = \frac{2/8}{4/8} = \frac{2}{4}$

$P(C|D) = \frac{P(C\cap D)}{P(D)} = \frac{2/8}{4/8} = \frac{2}{4}$

$P(B\cap C|D) = \frac{P(B\cap C\cap D)}{P(D)} = \frac{1/8}{4/8} = \frac{1}{4}$

f)

i) $P(A\cap B|C) = 1/3$

$P(A|C) = 1/3$, $P(B|C) = 2/3$

$P(A|C) + P(B|C) = \frac{1}{3} + \frac{2}{3} = \frac{2}{9}$

So, $\underline{\underline{P(A\cap B|C) \neq P(A|C) + P(B|C)}}$

ii) $P(B\cap C|D) = 1/4$

$P(B|D) + P(C|D) = \frac{2}{4} + \frac{2}{4}$

$= \frac{1}{4}$

So, $\underline{\underline{P(B\cap C|D) = P(B|D) * P(C|D)}}$

$\therefore$ option 2 is correct.

Exercise 5 : Naïve Bayes

Color	Fur	Size	Character (C)
brown	ragged	small	well-behaved
black	ragged	big	dangerous
black	smooth	big	dangerous
black	curly	small	well-behaved
white	curly	small	well-behaved
white	smooth	small	dangerous
red	ragged	big	well-behaved

(a)

i)Color-

Color	P(colorval|WB)	P(Colorval|D)
Brown	1/4	0/3
Black	1/4	2/3
White	1/4	1/3
Red	1/4	0/3

ii) Fur

Fur	P(Furval|WB)	P(Furval|D)
Ragged	2/4	1/3
Smooth	0/4	2/3
Curly	2/4	0/3

iii) Size

Size	P(Sizeval|WB)	P(Sizeval|D)
Small	3/4	1/3
Big	1/4	2/3

(b)

New example - color = black, Fur = ragged, size = Small

$$A_{NB} = \underset{A_i \in \{A_1 \ldots A_k\}}{\operatorname{argmax}} P(A_i) \cdot \prod_{\gamma=1}^{P} P(B_\gamma | A_i)$$

$$= \underset{A_i \in \{WB, D\}}{\operatorname{argmax}} P(A_i) * P(\text{Color} = \text{black} | A_i) * P(\text{Fur} = \text{ragged} | A_i) * P(\text{Size} = \text{Small} | A_i)$$

$P(WB) = 4/7$

$P(D) = 3/7$

$$A_{NB}(WB) = P(WB) * P(\text{black} | WB) * P(\text{ragged} | WB) * P(\text{Small} | WB)$$

$$= \frac{4}{7} * \frac{1}{4} * \frac{2}{4} * \frac{3}{4}$$

$(WB) = \underline{\underline{0.054}}$

$$(D) = P(D) * P(\text{black} | D) * P(\text{ragged} | D) * P(\text{Small} | D)$$

$$= \frac{3}{7} * \frac{2}{3} * \frac{1}{3} * \frac{1}{3}$$

$A_{NB}(D) = \underline{\underline{0.032}}$

From this, it is observed that $A_{NB}(WB) > A_{NB}(D)$.

$\therefore$ {color = black, Fur = ragged, size = Small} is classified as well-behaved.

To verify A_{NB} values, normalized probabilities can be calculated -

$$\left.\begin{array}{l} A_{NB}(WB) = \dfrac{0.054}{0.054 + 0.032} = 0.63 \\[2em] A_{NB}(D) = \dfrac{0.032}{0.054 + 0.032} = 0.37 \end{array}\right\} \text{Sum} = \underline{\underline{1}}$$