

AI20MTECH14010

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Abstract—This document aims to identify subspaces of a given vector space.

Download all latex-tikz codes from

[https://github.com/anjanavasudevan/
grad_schoolwork/tree/master/EE5609/
Assignment8](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment8)

1 QUESTION

Which of the following are subspaces of the vector space \mathbb{R}^3 ?

- 1) $(x, y, z) : x + y = 0$
- 2) $(x, y, z) : x - y = 0$
- 3) $(x, y, z) : x + y = 1$
- 4) $(x, y, z) : x - y = 1$

2 ANSWER

A subspace \mathbf{S} of a vector space is defined as a non-empty subset that is closed under addition and scalar multiplication, i.e

- 1) All possible linear combinations of the vectors in \mathbf{S} lie in the subspace.
- 2) Any vector in \mathbf{S} scaled by a scalar c lies in the subspace.

Using the above definition, :

2.1 Option 1:

Let $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \in \mathbf{S}$, and k_1 and k_2 be

some scalars.

The linear combination of \mathbf{A} and \mathbf{B} is:

$$k_1\mathbf{A} + k_2\mathbf{B} = k_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + k_2 \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \quad (2.1.1)$$

Verifying the property of the subspace:

$$\begin{aligned} (k_1 \ k_2) \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\} &= \\ k_1x_1 + k_2x_2 + k_1y_1 + k_2y_2 &= \\ k_1(x_1 + y_1) + k_2(x_2 + y_2) &= 0 \end{aligned} \quad (2.1.2)$$

It is also evident from above that

$$c\mathbf{A} = \begin{pmatrix} cx_1 \\ cy_1 \\ cz_1 \end{pmatrix} \quad (2.1.3)$$

$$\Rightarrow cx_1 + cy_1 = 0 \quad (2.1.4)$$

for some scalar c . Therefore, option 1 is a subspace of \mathbb{R}^3 .

It can also be proven that option 2 is also a valid subspace of \mathbb{R}^3 as:

$$c\mathbf{A} = \begin{pmatrix} cx_1 \\ cy_1 \\ cz_1 \end{pmatrix} \quad (2.1.5)$$

$$\Rightarrow cx_1 - cy_1 = c(x_1 - y_1) = 0 \quad (2.1.6)$$

$$\Rightarrow k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \in \mathbf{S} \quad (2.1.7)$$

for some scalars c, k_1 and k_2 and vectors \mathbf{A} and $\mathbf{B} \in \mathbf{S}$.

2.2 Option 3:

Option 3 is not a valid subspace of \mathbb{R}^3 as:

$$k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \quad (2.2.1)$$

$$\begin{aligned} \Rightarrow (k_1 \ k_2) \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\} &= \\ k_1x_1 + k_2x_2 + k_1y_1 + k_2y_2 &= \\ k_1(x_1 + y_1) + k_2(x_2 + y_2) &= k_1 + k_2 \neq 1 \end{aligned} \quad (2.2.2)$$

For some scalars k_1 and k_2 , \mathbf{A} and $\mathbf{B} \in \mathbf{S}$ in the option.

Similarly option 4 is also not a valid subspace of \mathbb{R}^3 as it can be shown in similar manner that

$$k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \quad (2.2.3)$$

$$\begin{aligned}
\Rightarrow (k_1 \quad k_2) \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \right\} &= \\
k_1 x_1 + k_2 x_2 - (k_1 y_1 + k_2 y_2) &= \\
k_1(x_1 - y_1) + k_2(x_2 - y_2) &= k_1 + k_2 \neq 1 \quad (2.2.4)
\end{aligned}$$

Therefore, Options 1 and 2 are valid subspaces of the vector space \mathbb{R}^3