

ASSIGNMENT 2

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Abstract—This document examines the consistency of equations.

Download all python codes from

[https://github.com/anjanavasudevan/
grad_schoolwork/tree/master/EE5609/
Assignment2](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment2)

and latex-tikz codes from

[https://github.com/anjanavasudevan/
grad_schoolwork/tree/master/EE5609/
Assignment2/latex](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment2/latex)

Reducing the augmented matrix to row echelon form, we get:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ a & a & 2a & 4 \end{array}\right) \xrightarrow{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ a & a & 2a & 4 \end{array}\right) \quad (2.0.3)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ a & a & 2a & 4 \end{array}\right) \xrightarrow{R_3 \rightarrow R_3 - aR_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 4 - a \end{array}\right) \quad (2.0.4)$$

$$\Rightarrow \text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}|\mathbf{B}) = \dim(\mathbf{A}) \quad (2.0.5)$$

The system of equations is consistent and has a unique solution except at $\mathbf{a} = \mathbf{0}$.

Back-solving the above set of equations, we get:

$$\mathbf{x} = \begin{pmatrix} \frac{2(a-2)}{a} \\ 0 \\ \frac{4-a}{a} \end{pmatrix} \quad (2.0.6)$$

1 QUESTION NO. 55

Examine the consistency of the system of given equations:

$$x + y + z = 1 \quad (1.0.1)$$

$$2x + 3y + 2z = 2 \quad (1.0.2)$$

$$ax + ay + 2az = 4 \quad (1.0.3)$$

2 ANSWER

Assume that a is any real number. The above system of equations can be expressed in the form of matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \quad (2.0.1)$$

This is in the form of:

$$\mathbf{Ax} = \mathbf{B} \quad (2.0.2)$$

The system defined above has a solution only when

$$\text{rank}(\mathbf{A}|\mathbf{B}) = \text{rank}(\mathbf{A}) = \dim(\mathbf{A})$$