

AI20MTECH14010

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Abstract—This document aims to plot equation of tangent to a circle given its parallel line using matrices

Download all python codes from

[https://github.com/anjanavasudevan/
grad_schoolwork/tree/master/EE5609/
Assignment6/Code](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment6/Code)

and latex-tikz codes from

[https://github.com/anjanavasudevan/
grad_schoolwork/tree/master/EE5609/
Assignment6/Latex](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment6/Latex)

1 QUESTION

Find equation of the tangent to the circle

$$x^2 + y^2 = 4 \quad (1.0.1)$$

which is parallel to the line

$$x + 2y - 6 = 0 \quad (1.0.2)$$

2 ANSWER

The equations for the circle and line in (1.0.1) and (1.0.2) can be rewritten in vector form as:

$$\|\mathbf{x}\|^2 = 4 \quad (2.0.1)$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 6 \quad (2.0.2)$$

The center of the circle happens to be (0,0) Since the tangent is parallel to the line in (2.0.2), it will also have the same normal.

The point of contact for a conic is given by:

$$\mathbf{v} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (2.0.3)$$

where,

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (2.0.4)$$

For a circle,

$$\mathbf{V} = \mathbf{I} \quad (2.0.5)$$

Using properties of identity matrix, we get:

$$\mathbf{I}^{-1} = \mathbf{I} \quad (2.0.6)$$

$$\mathbf{I}\mathbf{x} = \mathbf{x} \quad (2.0.7)$$

Therefore (2.0.3) and (2.0.4) simplify to:

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{u} - f}{\mathbf{n}^T \mathbf{n}}} \quad (2.0.8)$$

$$\Rightarrow \mathbf{v} = \kappa \mathbf{n} - \mathbf{u} \quad (2.0.9)$$

Substituting the values, we get:

$$\kappa = \pm \sqrt{\frac{4}{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}} \quad (2.0.10)$$

$$\Rightarrow \kappa = \pm \sqrt{\frac{4}{5}} \quad (2.0.11)$$

$$\mathbf{q} = \pm \sqrt{\frac{4}{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.12)$$

$$\Rightarrow \mathbf{q}_1 = \begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix}, \mathbf{q}_2 = -\begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix} \quad (2.0.13)$$

Since there are two points of contact, there are two tangents parallel to (2.0.2) that have the same normal vector.

$$\Rightarrow \mathbf{n}^T \mathbf{q}_1 = c_1 \quad (2.0.14)$$

$$\mathbf{n}^T \mathbf{q}_2 = c_2 \quad (2.0.15)$$

Substituting the values, we get:

$$c_1 = 2\sqrt{5}, c_2 = -2\sqrt{5} \quad (2.0.16)$$

Therefore, the equation of the tangents are:

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 2\sqrt{5} \quad (2.0.17)$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = -2\sqrt{5} \quad (2.0.18)$$

The plot of the circle with the tangents is given below:

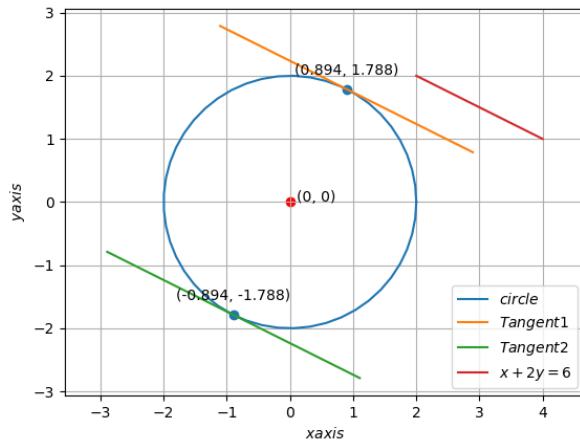


Fig. 0: Circle centered at (0, 0) with tangents parallel to line $x + 2y = 6$.