

AI20MTECH14010

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Abstract—This document aims to identify subspaces of a given vector space.

Download all latex-tikz codes from

https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment8

1 QUESTION

Which of the following are subspaces of the vector space \mathbb{R}^3 ?

- 1) $(x, y, z) : x + y = 0$
- 2) $(x, y, z) : x - y = 0$
- 3) $(x, y, z) : x + y = 1$
- 4) $(x, y, z) : x - y = 1$

2 ANSWER

A subspace \mathbf{S} of a vector space is defined as a non-empty subset that is closed under addition and scalar multiplication, i.e

- 1) All possible linear combinations of the vectors in \mathbf{S} lie in the subspace.
- 2) Any vector in \mathbf{S} scaled by a scalar c lies in the subspace.

We define any vector $\mathbf{V} \in \mathbf{S}$ for each of the subspaces defined in the options as:

$$\mathbf{V} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (2.0.1)$$

2.1 Option 1:

Let $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x_1 \\ y_1 \\ z_2 \end{pmatrix} \in \mathbf{S}$, and k_1 and k_2 be some scalars. As per definition:

$$(1 \ 1 \ 0)\mathbf{A} = (1 \ 1 \ 0)\mathbf{B} = 0 \quad (2.1.1)$$

Verifying the property of the subspace by using the linear combination of \mathbf{A} and \mathbf{B} :

$$\begin{aligned} (1 \ 1 \ 0)\{k_1\mathbf{A} + k_2\mathbf{B}\} = \\ (1 \ 1 \ 0)k_1\mathbf{A} + (1 \ 1 \ 0)k_2\mathbf{B} \end{aligned} \quad (2.1.2)$$

$$\Rightarrow k_1(1 \ 1 \ 0)\mathbf{A} + k_2(1 \ 1 \ 0)\mathbf{B} = 0 \quad (2.1.3)$$

It is also evident from above that

$$(1 \ 1 \ 0)c\mathbf{A} = c(1 \ 1 \ 0)\mathbf{A} = 0 \quad (2.1.4)$$

for some scalar c . Therefore, option 1 is a subspace of \mathbb{R}^3 .

It can also be proven that option 2 is also a valid subspace of \mathbb{R}^3 as:

$$(1 \ -1 \ 0)(c\mathbf{A}) = c(1 \ -1 \ 0)\mathbf{A} = 0 \quad (2.1.5)$$

From the definition that $x - y = 0$

$$\begin{aligned} \Rightarrow (1 \ -1 \ 0)\{k_1\mathbf{A} + k_2\mathbf{B}\} = \\ (1 \ -1 \ 0)(k_1\mathbf{A}) + (1 \ -1 \ 0)(k_2\mathbf{B}) = 0 \in \mathbf{S} \end{aligned} \quad (2.1.6)$$

for some scalars c, k_1 and k_2 and vectors \mathbf{A} and $\mathbf{B} \in \mathbf{S}$.

2.2 Option 3:

Option 3 is not a valid subspace of \mathbb{R}^3 as it can be shown that for some scalars k_1 and k_2 , \mathbf{A} and $\mathbf{B} \in \mathbf{S}$ in the option:

$$\begin{aligned} \Rightarrow (1 \ 1 \ 0)\{k_1\mathbf{A} + k_2\mathbf{B}\} = \\ k_1(1 \ 1 \ 0)\mathbf{A} + k_2(1 \ 1 \ 0)\mathbf{B} = k_1 + k_2 \neq 1 \end{aligned} \quad (2.2.1)$$

Because

$$(1 \ 1 \ 0)\mathbf{A} = (1 \ 1 \ 0)\mathbf{B} = 1 \quad (2.2.2)$$

from definition.

Similarly option 4 is also not a valid subspace of \mathbb{R}^3 as it can be shown in similar manner that

$$\begin{aligned} (1 \ -1 \ 0) \{k_1 \mathbf{A} + k_2 \mathbf{B}\} &= \\ (1 \ -1 \ 0)(k_1 \mathbf{A}) + (1 \ -1 \ 0)(k_2 \mathbf{B}) &= \\ k_1 + k_2 \neq 1 &\quad (2.2.3) \end{aligned}$$

$$(1 \ -1 \ 0) \mathbf{A} = (1 \ -1 \ 0) \mathbf{B} = 1 \quad (2.2.4)$$

Therefore, Options 1 and 2 are valid subspaces of the vector space \mathbb{R}^3