Math Document Template

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***Abstract*—This document solves determinant of a matrix using properties of determinants.**

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1 Problem

By using properties of determinants, show that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c) \tag{1.0.1}$$

2 Construction

3 Solution

Using column operations to simplify the equation, we get:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \xleftrightarrow{C_1 \to C_1 - C_2} \begin{vmatrix} 0 & 1 & 1 \\ a-b & b & c \\ a^3-b^3 & b^3 & c^3 \end{vmatrix} \tag{3.0.1}$$

$$\begin{vmatrix} 0 & 1 & 1 \\ a-b & b & c \\ a^3-b^3 & b^3 & c^3 \end{vmatrix} \xleftrightarrow{C_2 \to C_2 - C_3} \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \tag{3.0.2}$$

Taking out common factors $a-b$ and $b-c$ from C_1 and C_2 respectively, we get:

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix} \tag{3.0.3}$$

Further column operations give:

$$\xleftrightarrow{C_1 \to C_1 - C_2} (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a^2+ab-bc-c^2 & b^2+bc+c^2 & c^3 \end{vmatrix} \tag{3.0.4}$$

$$\implies (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a^2-c^2)+b(a-c) & b^2+bc+c^2 & c^3 \end{vmatrix} \tag{3.0.5}$$

$$\implies (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a-c)(a+c)+b(a-c) & b^2+bc+c^2 & c^3 \end{vmatrix} \tag{3.0.6}$$

$$\tag{3.0.7}$$

Taking $a-c$ from C_1, we get

$$\implies (a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a+b+c & b^2+bc+c^2 & c^3 \end{vmatrix} \tag{3.0.8}$$

$$= (a-b)(b-c)(a-c)(-1)(a+b+c) \tag{3.0.9}$$

$$= (a-b)(b-c)(c-a)(a+b+c) \tag{3.0.10}$$