

# AI20MTECH14010

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**Abstract**—This document aims to plot equation of tangent to a circle given its parallel line using matrices where,

Download all python codes from

[https://github.com/anjanavasudevan/  
grad\\_schoolwork/tree/master/EE5609/  
Assignment6/Code](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment6/Code)

and latex-tikz codes from

[https://github.com/anjanavasudevan/  
grad\\_schoolwork/tree/master/EE5609/  
Assignment6/Latex](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment6/Latex)

## 1 QUESTION

Find equation of the tangent to the circle

$$x^2 + y^2 = 4 \quad (1.0.1)$$

which is parallel to the line

$$x + 2y - 6 = 0 \quad (1.0.2)$$

## 2 ANSWER

The equations for the circle and line in (1.0.1) and (1.0.2) can be rewritten in vector form as:

$$\|\mathbf{x}\|^2 = 4 \quad (2.0.1)$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 6 \quad (2.0.2)$$

The center of the circle happens to be (0,0)

The equation of a line is of the form:

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.3)$$

Where  $\mathbf{n}$  is the normal to the line.

Comparing (2.0.3) to (2.0.2),

$$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.4)$$

Since the tangent is parallel to the line in (2.0.2), it will also have the same normal.

The point of contact for a conic is given by:

$$\mathbf{v} = \mathbf{V}^{-1}(\kappa \mathbf{n} - \mathbf{u}) \quad (2.0.5)$$

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\mathbf{n}^T \mathbf{V}^{-1} \mathbf{n}}} \quad (2.0.6)$$

For a circle,

$$\mathbf{V} = \mathbf{I} \quad (2.0.7)$$

Using properties of identity matrix, we get:

$$\mathbf{I}^{-1} = \mathbf{I} \quad (2.0.8)$$

$$\mathbf{I} \mathbf{x} = \mathbf{x} \quad (2.0.9)$$

Therefore (2.0.5) and (2.0.6) simplify to:

$$\kappa = \pm \sqrt{\frac{\mathbf{u}^T \mathbf{u} - f}{\mathbf{n}^T \mathbf{n}}} \quad (2.0.10)$$

$$\Rightarrow \mathbf{v} = \kappa \mathbf{n} - \mathbf{u} \quad (2.0.11)$$

Substituting the values, we get:

$$\kappa = \pm \sqrt{\frac{4}{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}} \quad (2.0.12)$$

$$\Rightarrow \kappa = \pm \sqrt{\frac{4}{5}} \quad (2.0.13)$$

$$\mathbf{q} = \pm \sqrt{\frac{4}{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.14)$$

$$\Rightarrow \mathbf{q}_1 = \begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix}, \mathbf{q}_2 = -\begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix} \quad (2.0.15)$$

Since there are two points of contact, there are two tangents parallel to (2.0.2) that have the same normal vector.

$$\Rightarrow \mathbf{n}^T \mathbf{q}_1 = c_1 \quad (2.0.16)$$

$$\mathbf{n}^T \mathbf{q}_2 = c_2 \quad (2.0.17)$$

Substituting the values, we get:

$$c_1 = 2\sqrt{5}, c_2 = -2\sqrt{5} \quad (2.0.18)$$

Therefore, the equation of the tangents are:

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 2\sqrt{5} \quad (2.0.19)$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = -2\sqrt{5} \quad (2.0.20)$$

The plot of the circle with the tangents is given below:

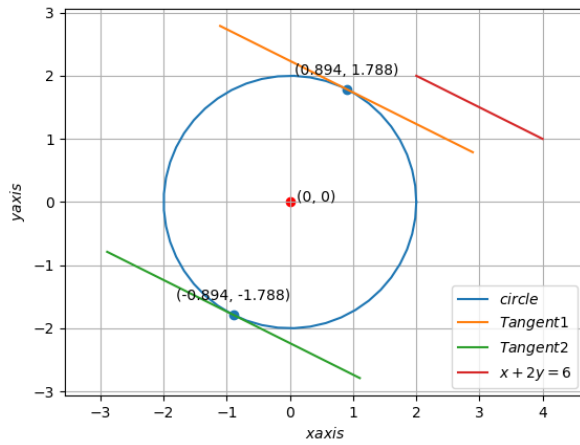


Fig. 0: Circle centered at (0,0) with tangents parallel to line  $x + 2y = 6$ .