

# ASSIGNMENT 3

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**Abstract—This document solves determinant of a matrix using its properties**

Download all python codes from

[https://github.com/anjanavasudevan/  
grad\\_schoolwork/tree/master/EE5609/  
Assignment2](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment2)

and latex-tikz codes from

[https://github.com/anjanavasudevan/  
grad\\_schoolwork/tree/master/EE5609/  
Assignment3/latex](https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment3/latex)

## 1 QUESTION NO. 16 (II)

By using properties of determinants, show that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c) \quad (1.0.1)$$

## 2 ANSWER

The following properties of determinants will be used to solve the problem:

- 1) If a row or column of matrix is multiplied by a constant  $k$ , the determinant is multiplied by the same constant  $k$
- 2) Row or column transformations do not affect the determinant of a matrix.

Using column operations to simplify the equation, we get:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \xrightarrow{C_1 \rightarrow C_1 - C_2} \begin{vmatrix} 0 & 1 & 1 \\ a-b & b & c \\ a^3-b^3 & b^3 & c^3 \end{vmatrix} \quad (2.0.1)$$

$$\begin{vmatrix} 0 & 1 & 1 \\ a-b & b & c \\ a^3-b^3 & b^3 & c^3 \end{vmatrix} \xrightarrow{C_2 \rightarrow C_2 - C_3} \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix} \quad (2.0.2)$$

Taking out common factors  $a-b$  and  $b-c$  from  $C_1$  and  $C_2$  respectively, we get:

$$(a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix} \quad (2.0.3)$$

Further column operations give:

$$\xrightarrow{C_1 \rightarrow C_1 - C_2} (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a^2+ab-bc-c^2 & b^2+bc+c^2 & c^3 \end{vmatrix} \quad (2.0.4)$$

$$\Rightarrow (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ (a^2-c^2)+b(a-c) & b^2+bc+c^2 & c^3 \end{vmatrix} \quad (2.0.5)$$

$$\Rightarrow (a-b)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ k & b^2+bc+c^2 & c^3 \end{vmatrix} \quad (2.0.6)$$

Where

$$k = (a-c)(a+c) + b(a-c) \quad (2.0.7)$$

Taking  $a-c$  from  $C_1$ , we get

$$\Rightarrow (a-b)(b-c)(a-c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ a+b+c & b^2+bc+c^2 & c^3 \end{vmatrix} \quad (2.0.8)$$

$$= (a-b)(b-c)(a-c)(-1)(a+b+c) \quad (2.0.9)$$

$$= (a-b)(b-c)(c-a)(a+b+c) \quad (2.0.10)$$