AI20MTECH14010

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***Abstract*—This document aims to plot a conic given its equation using matrix approach**

Download all python codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment6/Code
```

and latex-tikz codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment6/Latex
```

1 Question

Trace the following central conic:

$$2x^2 + 3xy - 2y^2 - 7x + y - 2 = 0 \tag{1.0.1}$$

2 Answer

Any second degree equation of the form:

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

Can be represented in matrix / vector form as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d & e \end{pmatrix} \tag{2.0.4}$$

Rewriting (1.0.1) in matrix form, we get:

$$\mathbf{x}^T \begin{pmatrix} 2 & \frac{3}{2} \\ \frac{3}{2} & -2 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -\frac{7}{2} & \frac{1}{2} \end{pmatrix} - 2 = 0 \tag{2.0.5}$$

where,

$$\mathbf{V} = \begin{pmatrix} 2 & \frac{3}{2} \\ \frac{3}{2} & -2 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{u} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{2.0.7}$$

$$f = -2 \tag{2.0.8}$$

$$det(\mathbf{V}) = \begin{vmatrix} 2 & \frac{3}{2} \\ \frac{3}{2} & -2 \end{vmatrix} = -\frac{25}{4} \tag{2.0.9}$$

As $det(\mathbf{V}) < 0$, the given conic represents a hyperbola.
The characteristic equation of $\mathbf{V}$ is given by the determinant:

$$\left|\mathbf{V} - \lambda\mathbf{I}\right| = 0 \tag{2.0.10}$$

$$\begin{vmatrix} 2-\lambda & \frac{3}{2} \\ \frac{3}{2} & -2-\lambda \end{vmatrix} = 0 \tag{2.0.11}$$

$$\implies \lambda^2 - \frac{25}{4} = 0 \tag{2.0.12}$$

The roots of (2.0.12) (the eigenvalues) are:

$$\lambda_1 = \frac{5}{2}, \lambda_2 = -\frac{5}{2} \tag{2.0.13}$$

The eigenvector $\mathbf{p}$ is defined as:

$$\mathbf{V}\mathbf{p} = \lambda\mathbf{p} \tag{2.0.14}$$

$$\implies (\mathbf{V} - \lambda\mathbf{I})\mathbf{p} = 0 \tag{2.0.15}$$

Evaluating (2.0.15) for $\lambda_1 = \frac{5}{2}$, we get: