ASSIGNMENT 5

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Abstract—**This document aims to plot equation of circle with given points using matrices**

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1 QUESTION

Find the equation to the circle that passes through the points:

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}, \mathbf{x_3} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \qquad (1.0.1)$$

2 ANSWER

The equation of circle in vector form is given by:

$$\mathbf{x^T x} + 2\mathbf{x^T u} + f = 0 \qquad (2.0.1)$$

Using $\mathbf{x_1, x_2, x_3}$ in (2.0.1),

$$\mathbf{x_1}^T \mathbf{x_1} + 2\mathbf{x_1}^T \mathbf{u} + f = 0 \qquad (2.0.2)$$
$$\mathbf{x_2}^T \mathbf{x_2} + 2\mathbf{x_2}^T \mathbf{u} + f = 0 \qquad (2.0.3)$$
$$\mathbf{x_3}^T \mathbf{x_3} + 2\mathbf{x_3}^T \mathbf{u} + f = 0 \qquad (2.0.4)$$

The above system can be written in matrix form as:

$$\begin{pmatrix} 2\mathbf{x_1}^T & 1 \\ 2\mathbf{x_2}^T & 1 \\ 2\mathbf{x_3}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\mathbf{x_1}^T\mathbf{x_1} \\ -\mathbf{x_1}^T\mathbf{x_1} \\ -\mathbf{x_3}^T\mathbf{x_3} \end{pmatrix} \qquad (2.0.5)$$

Substituting the values for $\mathbf{x_1, x_2, x_3}$ in (2.0.5),

$$\begin{pmatrix} 2 & 4 & 1 \\ 6 & -8 & 1 \\ 10 & -12 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -5 \\ -25 \\ -61 \end{pmatrix} \qquad (2.0.6)$$

Using row echelon form to reduce (2.0.6), we get:

$$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 3R_1} \begin{pmatrix} 2 & 4 & 1 & -5 \\ 0 & -20 & -2 & -10 \\ 0 & -32 & -4 & -36 \end{pmatrix} \qquad (2.0.7)$$

$$\xrightarrow[R_3 \to -\frac{1}{4}R_3]{R_2 \to -\frac{1}{2}R_2} \begin{pmatrix} 2 & 4 & 1 & -5 \\ 0 & 10 & 1 & 5 \\ 0 & 8 & 1 & 9 \end{pmatrix} \qquad (2.0.8)$$

$$\xrightarrow{R_3 \to 5R_3 - 4R_2} \begin{pmatrix} 2 & 4 & 1 & -5 \\ 0 & 10 & 1 & 5 \\ 0 & 0 & 1 & 25 \end{pmatrix} \qquad (2.0.9)$$

$$\xrightarrow[R_2 \to \frac{1}{10}R_2]{R_1 \to \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 & \frac{1}{2} & -\frac{5}{2} \\ 0 & 1 & \frac{1}{10} & \frac{2}{5} \\ 0 & 0 & 1 & 25 \end{pmatrix} \qquad (2.0.10)$$

Back solving the system using (2.0.5) and (2.0.10), we get:

$$\begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -11 \\ -2 \\ 25 \end{pmatrix} \qquad (2.0.11)$$

$$\implies \mathbf{u} = \begin{pmatrix} -11 \\ -2 \end{pmatrix}, f = 25 \qquad (2.0.12)$$

The equation of the circle that passes through the points $\mathbf{x_1, x_2, x_3}$ is given by:

$$\mathbf{x^T x} + 2\begin{pmatrix} -11 \\ -2 \end{pmatrix}^T \mathbf{x} + f = 0 \qquad (2.0.13)$$

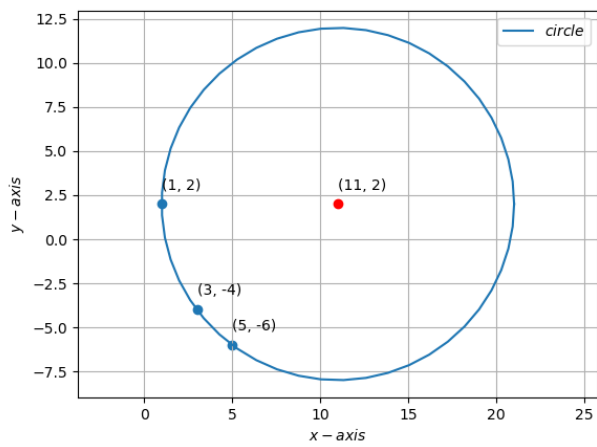$$\implies x^2 + y^2 - 22x - 4y + 25 = 0 \qquad (2.0.14)$$

The plot of the circle is given below:

Fig. 1. Circle centered at $(11, 2)$ with radius 10.