ASSIGNMENT 5

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***Abstract*—This document solves determinant of a matrix using its properties**

Download all python codes from

https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment3/code

and latex-tikz codes from

https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment3/latex

1 Question

Find the equation to the circle that passes through the points:

$$\mathbf{x_1} = \begin{pmatrix}1\\2\end{pmatrix}, \mathbf{x_2} = \begin{pmatrix}3\\-4\end{pmatrix}, \mathbf{x_3} = \begin{pmatrix}5\\-6\end{pmatrix} \tag{1.0.1}$$

2 Answer

The equation of circle in vector form is given by:

$$\mathbf{x^T x} + 2\mathbf{x^T u} + f = 0 \tag{2.0.1}$$

Using $\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}$ in (2.0.1),

$$\mathbf{x_1}^T\mathbf{x_1} + 2\mathbf{x_1}^T\mathbf{u} + f = 0 \tag{2.0.2}$$
$$\mathbf{x_2}^T\mathbf{x_2} + 2\mathbf{x_2}^T\mathbf{u} + f = 0 \tag{2.0.3}$$
$$\mathbf{x_3}^T\mathbf{x_3} + 2\mathbf{x_3}^T\mathbf{u} + f = 0 \tag{2.0.4}$$

The above system can be written in matrix form as:

$$\begin{pmatrix}2\mathbf{x_1}^T & 1\\2\mathbf{x_2}^T & 1\\2\mathbf{x_3}^T & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = \begin{pmatrix}-\mathbf{x_1}^T\mathbf{x_1}\\-\mathbf{x_1}^T\mathbf{x_1}\\-\mathbf{x_3}^T\mathbf{x_3}\end{pmatrix} \tag{2.0.5}$$

Substituting the values for $\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}$ in (2.0.5),

$$\begin{pmatrix}2 & 4 & 1\\6 & -8 & 1\\10 & -12 & 1\end{pmatrix}\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = \begin{pmatrix}-5\\-25\\-61\end{pmatrix} \tag{2.0.6}$$

Using row echelon form to reduce (2.0.6), we get:

$$\xleftrightarrow[R_3\to R_3-5R_1]{R_2\to R_2-3R_1}\left(\begin{array}{ccc|c}2 & 4 & 1 & -5\\0 & -20 & -2 & -10\\0 & -32 & -4 & -36\end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow[R_3\to -\frac{1}{4}R_3]{R_2\to -\frac{1}{2}R_2}\left(\begin{array}{ccc|c}2 & 4 & 1 & -5\\0 & 10 & 1 & 5\\0 & 8 & 1 & 9\end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_3\to 5R_3-4R_2}\left(\begin{array}{ccc|c}2 & 4 & 1 & -5\\0 & 10 & 1 & 5\\0 & 0 & 1 & 25\end{array}\right) \tag{2.0.9}$$

$$\xleftrightarrow[R_2\to \frac{1}{10}R_2]{R_1\to \frac{1}{2}R_1}\left(\begin{array}{ccc|c}1 & 2 & \frac{1}{2} & -\frac{5}{2}\\0 & 1 & \frac{1}{10} & \frac{2}{5}\\0 & 0 & 1 & 25\end{array}\right) \tag{2.0.10}$$

Back solving the system using (2.0.5) and (2.0.10), we get:

$$\begin{pmatrix}\mathbf{u}\\f\end{pmatrix} = \begin{pmatrix}-11\\-2\\25\end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{u} = \begin{pmatrix}-11\\-2\end{pmatrix}, f = 25 \tag{2.0.12}$$

The equation of the circle that passes through the points $\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}$ is given by:

$$\mathbf{x^T x} + 2\begin{pmatrix}-11\\-2\end{pmatrix}^T\mathbf{x} + f = 0 \tag{2.0.13}$$

$$\implies x^2 + y^2 - 22x - 4y + 25 = 0 \tag{2.0.14}$$

The plot of the circle is given below: