AI20MTECH14010

Anjana Vasudevan

***Abstract*—This document aims to identify subspaces of a given vector space.**

Download all latex-tikz codes from

https://github.com/anjanavasudevan/grad_schoolwork/tree/master/EE5609/Assignment8

1 Question

Which of the following are subspaces of the vector space $\mathbb{R}^3$?

1) $(x, y, z) : x + y = 0$
2) $(x, y, z) : x - y = 0$
3) $(x, y, z) : x + y = 1$
4) $(x, y, z) : x - y = 1$

2 Answer

A subspace **S** of a vector space is defined as a non-empty subset that is closed under addition and scalar multiplication, i.e

1) All possible linear combinations of the vectors in **S** lie in the subspace.
2) Any vector in **S** scaled by a scalar c lies in the subspace.

Using the above definition, :

2.1 Option 1:

Let $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \in \mathbf{S}$, and k_1 and k_2 be some scalars.

The linear combination of **A** and **B** is:

$$k_1\mathbf{A} + k_2\mathbf{B} = k_1 \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + k_2 \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \tag{2.1.1}$$

Verifying the property of the subspace:

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \{k_1\mathbf{x_1} + k_2\mathbf{x_2}\} = \begin{pmatrix} 1 & 1 \end{pmatrix} \left\{ k_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + k_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\} \tag{2.1.2}$$

$$\implies = \begin{pmatrix} 1 & 1 \end{pmatrix} k_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \end{pmatrix} k_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \tag{2.1.3}$$

$$\implies k_1 \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + k_2 \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = 0 \tag{2.1.4}$$

As

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{2.1.5}$$

It is also evident from above that

$$c\mathbf{A} = \begin{pmatrix} cx_1 \\ cy_1 \\ cz_1 \end{pmatrix} \tag{2.1.6}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} cx \\ cy \end{pmatrix} = 0 \tag{2.1.7}$$

for some scalar c. Therefore, option 1 is a subspace of $\mathbb{R}^3$.

It can also be proven that option 2 is also a valid subspace of $\mathbb{R}^3$ as:

$$c\mathbf{A} = \begin{pmatrix} cx_1 \\ cy_1 \\ cz_1 \end{pmatrix} \tag{2.1.8}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} cx_1 \\ cy_1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \end{pmatrix} c \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = c \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 0 \tag{2.1.9}$$

$$\implies k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \tag{2.1.10}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_1x_1 \\ k_1y_1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_2x_2 \\ k_2y_2 \end{pmatrix} = 0 \in \mathbf{S} \tag{2.1.11}$$

for some scalars c, k_1 and k_2 and vectors **A** and **B** $\in$ **S**.

2.2 Option 3:

Option 3 is not a valid subspace of $\mathbb{R}^3$ as:

$$k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \tag{2.2.1}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} k_1x_1 \\ k_1y_1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} k_2x_2 \\ k_2y_2 \end{pmatrix} =$$

$$k_1 \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x_1} + k_2 \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x_2} = k_1 + k_2 \neq 1 \tag{2.2.2}$$

For some scalars k_1 and k_2, $\mathbf{A}$ and $\mathbf{B} \in \mathbf{S}$ in the option.
Similarly option 4 is also not a valid subspace of $\mathbb{R}^3$ as it can be be shown in similar manner that

$$k_1\mathbf{A} + k_2\mathbf{B} = \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \\ k_1z_1 + k_2z_2 \end{pmatrix} \tag{2.2.3}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_1x_1 + k_2x_2 \\ k_1y_1 + k_2y_2 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_1x_1 \\ k_1y_1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} k_2x_2 \\ k_2y_2 \end{pmatrix} =$$

$$k_1 \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x_1} + k_2 \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x_2} = k_1 + k_2 \neq 1 \tag{2.2.4}$$

Therefore, Options 1 and 2 are valid subspces of the vector space $\mathbb{R}^3$