ASSIGNMENT 3

1

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Abstract—This document solves determinant of a matrix using its properties

Download all python codes from

https://github.com/anjanavasudevan/ grad_schoolwork/tree/master/EE5609/ Assignment3/code

and latex-tikz codes from

https://github.com/anjanavasudevan/ grad_schoolwork/tree/master/EE5609/ Assignment3/latex

1 Question No. 16 (ii)

By using properties of determinants, show that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c) \quad (1.0.1)$$

2 Answer

Using column operations to simplify the equation, we get:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^{3} & b^{3} & c^{3} \end{vmatrix} \xrightarrow{C_{1} \to C_{1} - C_{2}} \begin{vmatrix} 0 & 1 & 1 \\ a - b & b & c \\ a^{3} - b^{3} & b^{3} & c^{3} \end{vmatrix}$$
(2.0.1)

$$\stackrel{C_2 \to C_2 - C_3}{\longleftrightarrow} \begin{vmatrix} 0 & 0 & 1 \\ a - b & b - c & c \\ a^3 - b^3 & b^3 - c^3 & c^3 \end{vmatrix}$$
(2.0.2)

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1\\ 1 & 1 & c\\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$
 (2.0.3)

$$\stackrel{C_1 \to C_1 - C_2}{\longleftrightarrow} (a - b)(b - c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ |(a^2 - c^2) + b(a - c) & b^2 + bc + c^2 & c^3 \end{vmatrix}$$
(2.0.4)

$$\implies (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & c \\ k & b^2 + bc + c^2 & c^3 \end{vmatrix}$$
 (2.0.5)

Where

$$k = (a - c)(a + c) + b(a - c)$$
 (2.0.6)

$$(a-b)(b-c)(a-c)\begin{vmatrix} 0 & 0 & 1\\ 0 & 1 & c\\ a+b+c & b^2+bc+c^2 & c^3 \end{vmatrix}$$
 (2.0.7)

$$= (a-b)(b-c)(a-c)(-1)(a+b+c)$$
 (2.0.8)

$$= (a-b)(b-c)(c-a)(a+b+c)$$
 (2.0.9)