AI20MTECH14010

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***Abstract*—This document aims to plot equation of tangent to a circle givenits parallel line using matrices**

Download all python codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment6/Code
```

and latex-tikz codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment5/Latex
```

1 Question

Find equation of the tangent to the circle

$$x^2 + y^2 = 4 \tag{1.0.1}$$

which is parallel to the line

$$x + 2y - 6 = 0 \tag{1.0.2}$$

2 Answer

The equations for the circle and line in (1.0.1) and (1.0.2) can be rewritten in vector form as:

$$\|\mathbf{x}\|^2 = 4 \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 6 \tag{2.0.2}$$

The center of the circle happens to be $(0, 0)$
Since the tangent is parallel to the line in (2.0.2), it will also have the same normal.

The point of contact for a conic is given by:

$$\mathbf{v} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.3}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{V^{-1}}\mathbf{u} - f}{\mathbf{n}^T\mathbf{V}^{-1}\mathbf{n}}} \tag{2.0.4}$$

For a circle,

$$\mathbf{V} = \mathbf{I} \tag{2.0.5}$$

Using properties of identity matrix, we get:

$$\mathbf{I}^{-1} = \mathbf{I} \tag{2.0.6}$$

$$\mathbf{IX} = \mathbf{X} \tag{2.0.7}$$

Therefore (2.0.3) and (2.0.4) simplify to:

$$\kappa = \pm\sqrt{\frac{\mathbf{u^T}\mathbf{u} - f}{\mathbf{n^T}\mathbf{n}}} \tag{2.0.8}$$

$$\implies \mathbf{v} = \kappa\mathbf{n} - \mathbf{u} \tag{2.0.9}$$

Substituting the values, we get:

$$\kappa = \pm\sqrt{\frac{4}{\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}}} \tag{2.0.10}$$

$$\implies \kappa = \pm\sqrt{\frac{4}{5}} \tag{2.0.11}$$

$$\mathbf{q} = \pm\sqrt{\frac{4}{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.12}$$

$$\implies \mathbf{q_1} = \begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix}, \mathbf{q_2} = -\begin{pmatrix} \sqrt{\frac{4}{5}} \\ \sqrt{\frac{16}{5}} \end{pmatrix} \tag{2.0.13}$$

Since there are two points of contact, there are two tangents parallel to (2.0.2) that have the same normal vector.

$$\implies \mathbf{n^T}\mathbf{q_1} = c_1 \tag{2.0.14}$$

$$\mathbf{n^T}\mathbf{q_2} = c_2 \tag{2.0.15}$$

Substituting the values, we get:

$$c_1 = 2\sqrt{5}, c_2 = -2\sqrt{5} \tag{2.0.16}$$

Therefore, the equation of the tangents are:

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = 2\sqrt{5} \tag{2.0.17}$$

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \mathbf{x} = -2\sqrt{5} \tag{2.0.18}$$

The plot of the circle with the tangents is given below:

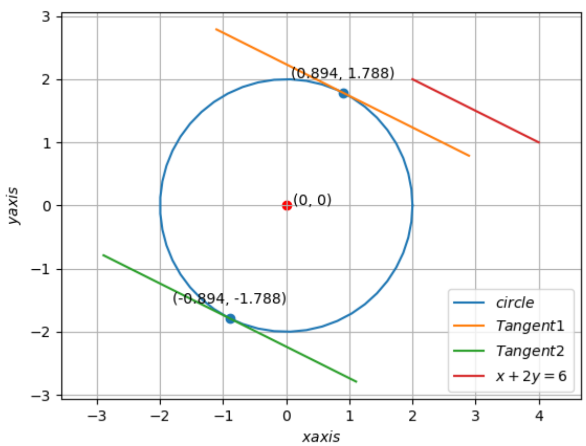

Fig. 0: Circle centered at $(0, 0)$ with tangents parallel to line $x + 2y = 6$.