ASSIGNMENT 2

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Abstract—**This document examines the consistency of equations.**

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1 QUESTION No. 55

Examine the consistency of the system of given equations:

$$x + y + z = 1 \tag{1.0.1}$$
$$2x + 3y + 2z = 2 \tag{1.0.2}$$
$$ax + ay + 2az = 4 \tag{1.0.3}$$

2 ANSWER

Assume that a is any real number. The above system of equations can be expressed in the form of matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \tag{2.0.1}$$

This is in the form of:

$$\mathbf{Ax} = \mathbf{B} \tag{2.0.2}$$

The system defined above has a solution only when

$$rank(\mathbf{A|B}) = rank(\mathbf{A}) = dim(\mathbf{A})$$

Reducing the augmented matrix to row echelon form, we get:

$$\begin{pmatrix} 1 & 1 & 1 & \big| & 1 \\ 2 & 3 & 2 & \big| & 2 \\ a & a & 2a & \big| & 4 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 1 & \big| & 1 \\ 0 & 1 & 0 & \big| & 0 \\ a & a & 2a & \big| & 4 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 1 & 1 & \big| & 1 \\ 0 & 1 & 0 & \big| & 0 \\ a & a & 2a & \big| & 4 \end{pmatrix} \xrightarrow{R_3 \to R_3 - aR_1} \begin{pmatrix} 1 & 1 & 1 & \big| & 1 \\ 0 & 1 & 0 & \big| & 0 \\ 0 & 0 & a & \big| & 4 - a \end{pmatrix} \tag{2.0.4}$$

$$\implies rank(\mathbf{A}) = rank(\mathbf{A|B}) = dim(\mathbf{A}) \tag{2.0.5}$$

The system of equations is consistent and has a unique solution except at $a = 0$.