ASSIGNMENT 2

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***Abstract*—This document examines the consistency of equations.**

Download all python codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment2
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and latex-tikz codes from

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https://github.com/anjanavasudevan/
    grad_schoolwork/tree/master/EE5609/
    Assignment2/latex
```

1 Question No. 55

Examine the consistency of the system of given equations:

$$x + y + z = 1 \tag{1.0.1}$$
$$2x + 3y + 2z = 2 \tag{1.0.2}$$
$$ax + ay + 2az = 4 \tag{1.0.3}$$

2 Answer

Assume that a is any real number. The above system of equations can be expressed in the form of matrix:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \tag{2.0.1}$$

This is in the form of:

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{2.0.2}$$

The system defined above is consistent and has a solution only when

$$rank(\mathbf{A}|\mathbf{B}) = rank(\mathbf{A}) = dim(\mathbf{A})$$

Reducing the augmented matrix to row echelon form, we get:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ a & a & 2a & 4 \end{array}\right) \xleftrightarrow{R_2 \rightarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ a & a & 2a & 4 \end{array}\right) \tag{2.0.3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ a & a & 2a & 4 \end{array}\right) \xleftrightarrow{R_3 \rightarrow R_3 - aR_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & 4 - a \end{array}\right) \tag{2.0.4}$$

$$\implies rank(\mathbf{A}) = rank(\mathbf{A}|\mathbf{B}) = dim(\mathbf{A}) \tag{2.0.5}$$

The system of equations is consistent and has a unique solution except at $\mathbf{a} = \mathbf{0}$.

Back-solving the above set of equations, we get:

$$\mathbf{x} = \begin{pmatrix} \frac{2(a-2)}{a} \\ 0 \\ \frac{4-a}{a} \end{pmatrix} \tag{2.0.6}$$