John Subba

BIT – II

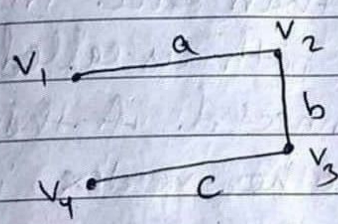
Unit 4. Graph and Tree

Q.1) what is graph, degree of vertex, adjacent vertices, path, circuit, simple path, discrete graph, complete graph, subgraph? Explain methods to represent graph in a computer

Ans:- A graph consists of a finite set V of objects called vertices, a finite set E of objects called edges & a function γ that assigns to each edge a subset $\{v, w\}$ where v & w are vertices. The graph is denoted by $G = (V, E, \gamma)$

for e.g:- let $V = \{1, 2, 3, 4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$



$\gamma(a) = \{v_1, v_2\}$

$\gamma(b) = \{v_2, v_3\}$

$\gamma(c) = \{v_3, v_4\}$

The degree of a vertex is the number of edges having that vertex as an end point.

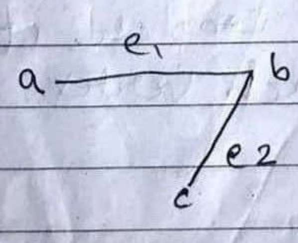
A graph may contain an edge from a vertex to itself, such an edge is called a loop.

A loop contributes ~~2~~ to the degree of a vertex.

A vertex with degree 0 is called an isolated vertex.

A pair of vertices that determine an edge are adjacent vertices.

for e.g.



here, a & b are adjacent vertices but a & c are not.

Date
Page

A path π in a graph G consists of a pair (V_π, E_π) of sequences: a vertex sequence $V_\pi: v_1, v_2 \ldots v_k$ & an edge sequence $E_\pi = e_1, e_2, e_{k-1}$ for which

① Each successive pair v_i, v_{i+1} of vertices is adja adjacent in G, & edge e_i has v_i & v_{i+1} as end points for $i = 1, \ldots k-1$.

② No edge occurs more than once in the edge sequence

A circuit is a path that begins and ends at the same vertex.

A simple path or circuit is any path where no vertex appear more than once in the vertex sequence. If $v_1 = v_k$ then the path is called a simple circuit.

A graph is called connected if there is a path from any vertex to any other vertex in the graph.

The various connected pieces are called ~~co~~ the components of the graph.

A graph in which all vertices are of equal degree is called regular graph.

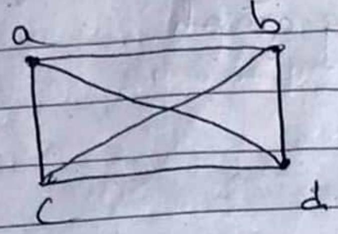
A graph with vertices & no edges is called discrete graph.

for e.g.: v_1 v_2 v_3 v_4

A graph is called complete if each vertex is connected with each vertex.

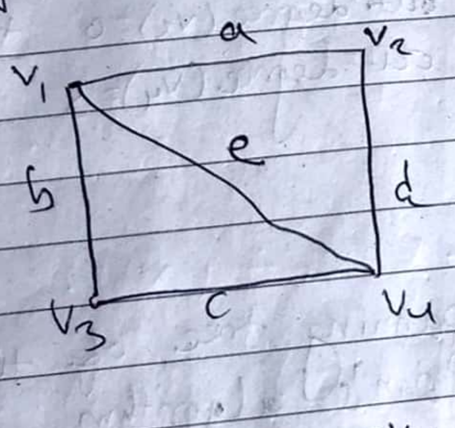
Date ______ Page ______

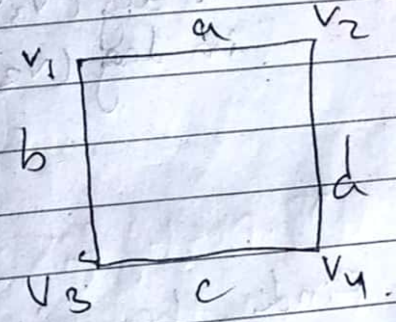
for. e.g.:-

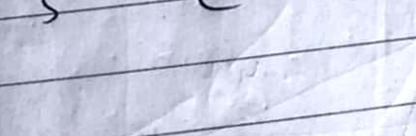


subgraph:- Suppose that $G = (V, \mathcal{E}, \varphi)$ is a graph & a subset $\mathcal{E}_1$ of the edges in $\mathcal{E}$ & a subset V_1 of the vertices in V, so that V_1 contains (at least) all the end points of edges in $\mathcal{E}_1$. Then $H = \{V_1, \mathcal{E}_1, \varphi_1\}$ is also a graph where φ_1 is φ restricted to edges in $\mathcal{E}_1$. Such a graph H is called a subgraph of G.

for e.g.:-

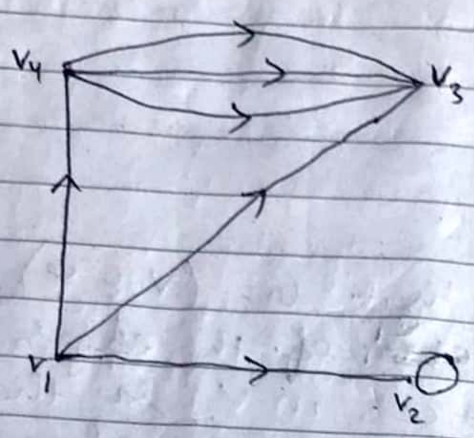






Date ______ Page ______

Q(x) Find the in degree & out degree and of total degree of each vertex of the following graph.

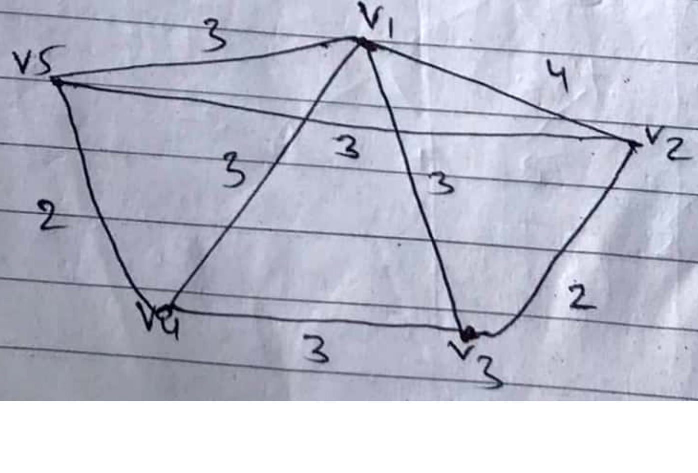


Solⁿ:-

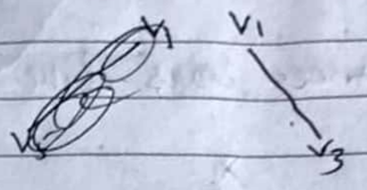
It is a directed graph

in degree $(v_1) = 0$ out degree $(v_1) = 3$ total degree$(v_1) = 4$
in degree $(v_2) = 2$ out degree $(v_2) = 1$ total degree$(v_2) = 3$
in deg $(v_3) = 4$ out degree $(v_3) = 0$ total degree$(v_3) = 4$
in deg $(v_3) = 1$ out degree $(v_4) = 3$ total degree$(v_4) =$

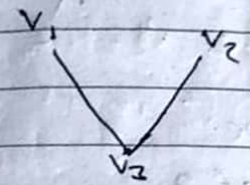
Q(x) Find the minimal spanning tree of the weighted graph of fig using Prim's algorithm.



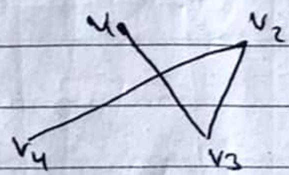
Soln:- We choose the vertex v_1. Now edge with smallest weight incident on v_1 is (v_1, v_3), so we choose the edge $[v_1, v_3]$ or $[v_1, v_5]$



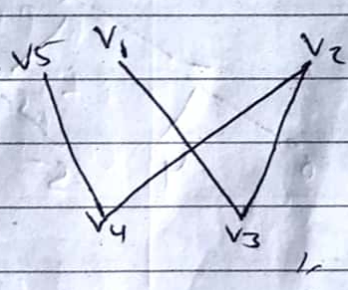
2. Now $w(v_1, v_2) = 4$, $w(v_1, v_4) = 3$, $w(v_1, v_5) = 3$, $w(v_3, v_2) = 2$ & $w(v_3, v_4) = 3$, we chose the edge (v_3, v_2) since it's minim



3. Again $w(v_1, v_5) = 3$, $w(v_2, v_4) = 1$ & $w(v_3, v_4) = 3$, we cho the edge (v_2, v_4)



4. Now we chose the edge (v_4, v_5). Now all the vertic are covered. The minimal spanning tree is produced.

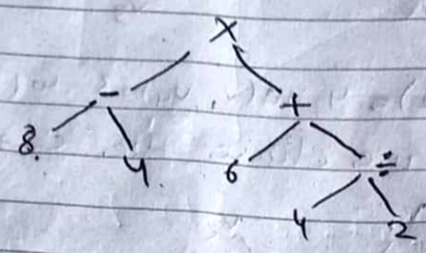


Q. what is the value of :-

(a) prefix expression $\times - 84 + 6 / 42$

Soln :-

the tree looks like :-



now, solving the tree :-

$\times 4 + 6 2$

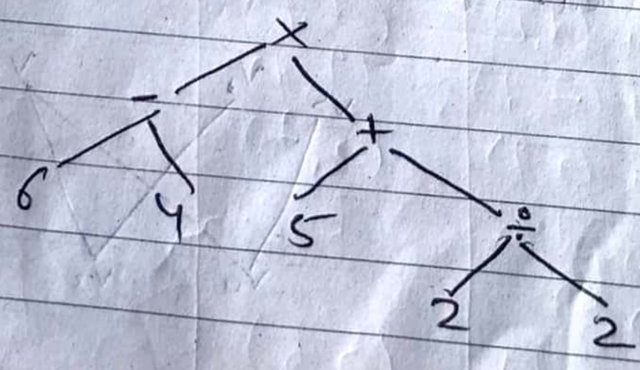
$= \times 4 \, 8$

$= 32$ // is the answer in prefix.

(b) post fix :- $64 - 522 \div + \times$

Soln :-

the tree looks like :-



Date ______ Page ______

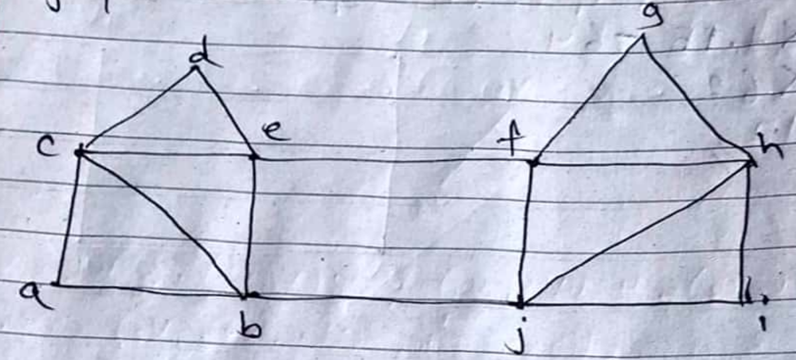
∴ when solving it we get :-

$= \times + 15 - 64$

$= \times 62$

$= 12$ //

Q(x) Using Fleury's algorithm, find Euler circuit in the graph.

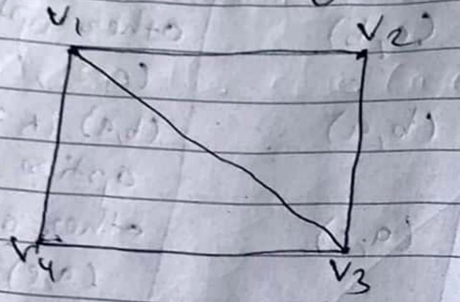


Sol^n:- The degrees of all the vertices are even. There exists an Euler circuit in it.

Current path	Next Edge	Remarks
π : a	(a, j)	no edge from a is a bridge.
π : aj	(j, f)	no edge from j is a bridge
π : ajf	(f, g)	(f,e) is a bridge & (f,g) is not bridge. other option (f, h)
π : ajfg	(g, h)	(g,h) is the only edge.
π : ajfgh	(h, i)	(h,i) is the other option
π : ajfghi	(i, j)	(i, j) is the only edge.
π : ajfghij	(j, h)	(j,h) is the only edge
π : ajfghijh	(h, f)	(h,f) is the only edge.
π : ajfghijhf	(f, e)	(f,e) is the only edge.
π : ajfghijhfe	(e, d)	other options are: (e,c), (e,a)
π : ajfghijhfed	(d, c)	(d,c) is the only

current path	Next edge	Remarks
π: ajfghijhgedc	(c,b)	other options:- (c,e), (c,a)
π: ajfghijkfedcb	(b,a)	(b,a) is the only option
π: ajfghijkfedcba	(a,c)	other options are (a,e)
π: ajfghijkfedcbac	(c,e)	(c,e) is the only option.
π: ajfghijkfedcbace	(e,a)	(e,a) is the only option.
π: ajfghijkfedcbacea	no. edge remaining in $\mathcal{E}$	This is Euler Circuit.

Q. 7 i) Use adjacency matrix to represent the graph:-



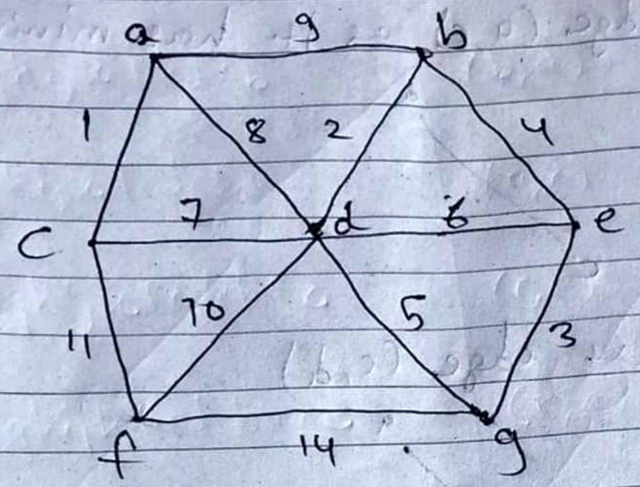
Sol^n:- We order the vertices as v_1, v_2, v_3 & v_4.

Since there are 4 vertices, the adjacency matrix representing the graph will be a square matrix of order four. The required adjacency matrix A is:-

$$A \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} //$$

Date ____ Page ____

Q.(7) Show how Kruskal's algorithm finds a minimal spanning tree of the graph of fig.



Sol :-

We collect edges with their weights :-

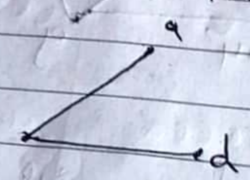
edge	weight
(a,c)	1
(b,d)	2
(e,g)	3
(b,e)	4
(d,g)	5
(d,e)	6
(d,c)	7
(a,d)	8
(a,b)	9
(d,f)	10
(c,f)	11
(f,g)	14

Date ____ Page ____

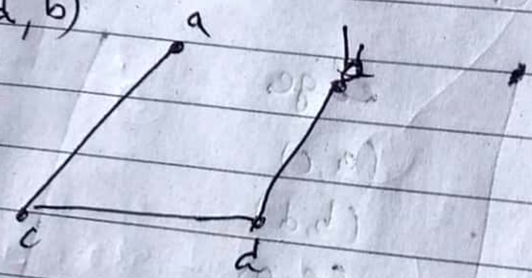
• The steps of finding a minimal spanning tree are shown below.

1) Choose the edge (a, c) as it has minimal weight.

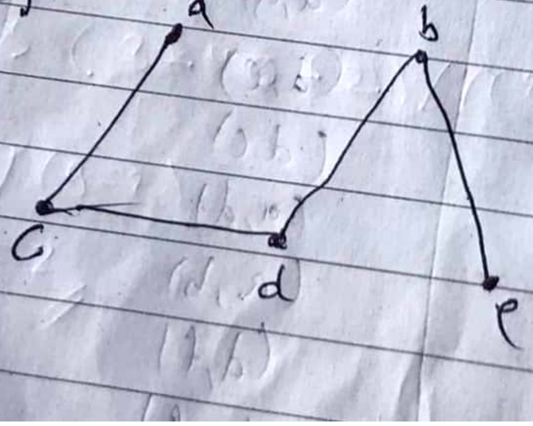
② Add the next edge (c, d)



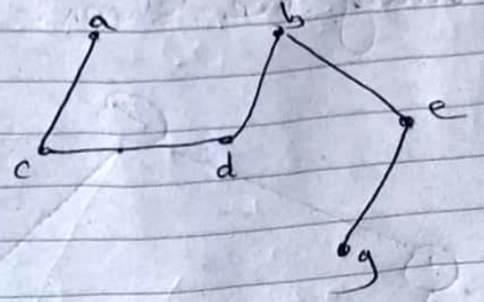
③ Add the edge (d, b)



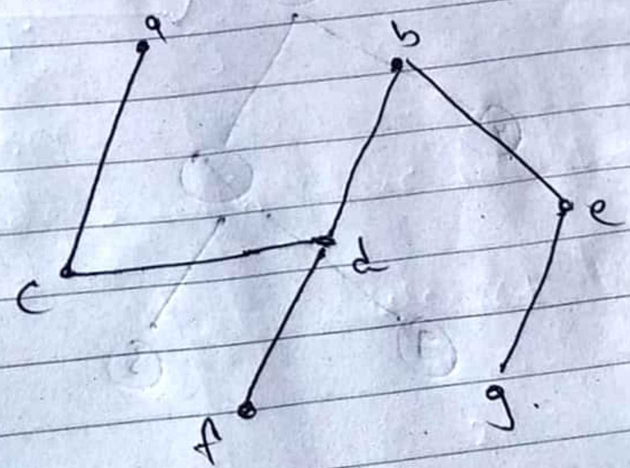
Add the edge (b, e)



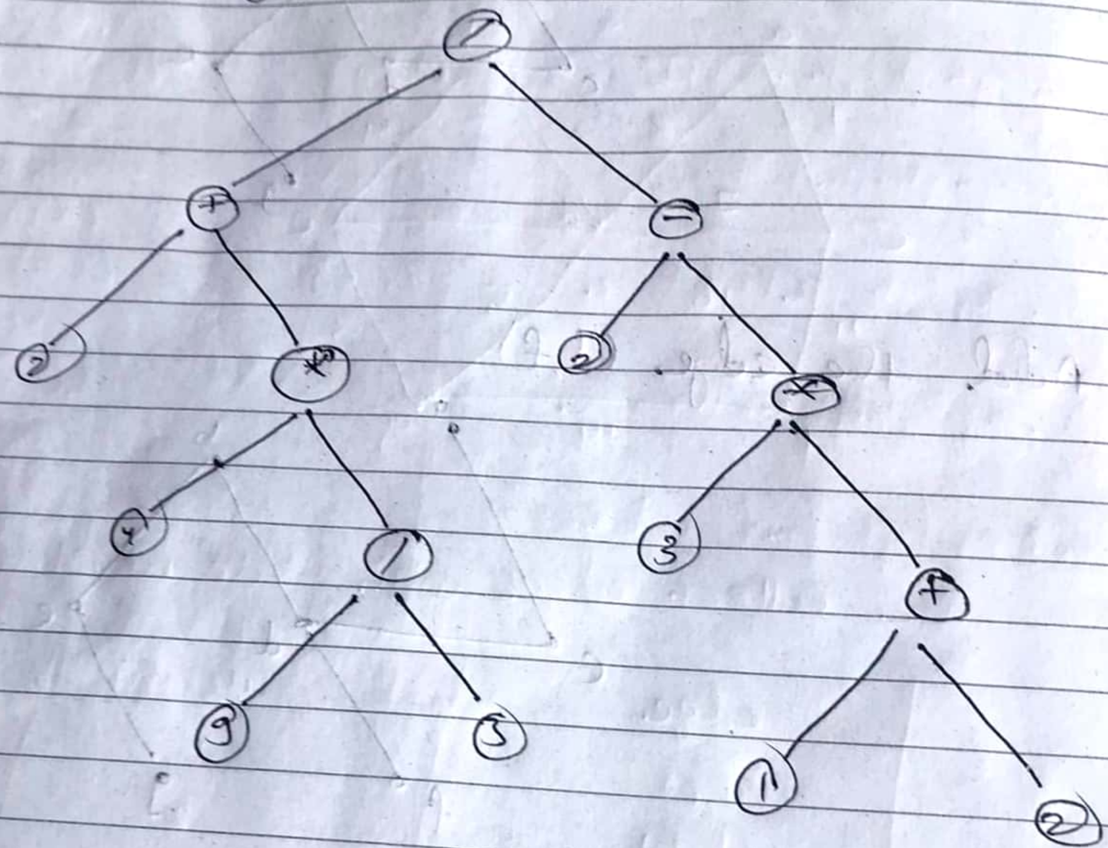
⑤ Add the edge (e,g)



⑥ Add the edge (d,f)



Q. (+) Determine the value of the expression represen
in a binary tree shown.



Sol^n: The expression represented by ~~by~~ the bin
tree is $(9/3 \times 4 + 2) / (((1+2) * 3) - 2)$

and the value is $(3 * 4 + 2) / ((3 \times 3) - 2$

$= (12+2) / (9-2)$

$= 14/7 = 2$

Find postfix expression of:-

(a) 823* − 2↑63/+

1. 823x − 2↑63/+
2. 86−2↑63/+ replacing 23x by 2×3=6
3. 22↑63/+ 〃 86− by 8−6=2
4. 463/ 〃 2↑2 by $2^2=4$
5. 42+ 〃 63/ by 6/3=2
6. 6 〃 42+ by 4+2=6

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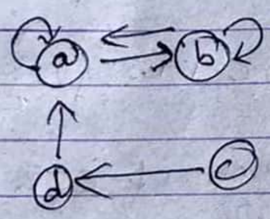
unit 5:- Relation & Diagraphs

DATE

Q.1. what do you mean by diagraph of a relation? let $A = \{a, b, c, d\}$ & $R = \{(a,b), (a,c), (b,a), (b,c), (c,d), (d,a)\}$. Find the transitive closure of R using warshall's algorithm.

Ans:- A diagraph or directed graph consists of a set V of vertices (or nodes) together with a set E for its edge (a, b), a is called the initial vertex and b is called terminal vertex.
An edge of the form (a,a) is called a loop.

$R = \{(a,a), (a,b), (b,a), (b,b), (c,d), (d,a)\}$



finding transitive closure of $R = \{(a,b), (a,c), (b,a), (b,c), (c,d), (d,a)\}$ of $A = \{a, b, c, d\}$ using warshall algorith.

1st, we will represent the relation R in matrix form.

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}_{4\times4} \end{array}$$

PAGE

since there are 4 elements in set A, therefore four steps are required to find the transitive closure of the relation R.

→ step 1, we will consider 1st column and 1st row of the above matrix i.e. C_1 & R_1.

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}_{4\times 4} \end{array}$$

Writing all positions where 1 is present in column 1.

$C_1 = \{b, d\}$

also, writing all position where 1 is present in row 1

$R_1 = \{b, c\}$

Now, taking the cross product of C_1 & R_1.

$C_1 \times R_1 = \{b, d\} \times \{b, c\}$

$= \{(b,b), (b,c), (d,b), (d,c)\}$

Therefore new additions = $\{(b,b), (d,b), (d,c)\}$.

now, placing 1 in the above new additions.

∴ new ~~Relation~~ matrix.

$$W_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{4\times 4} \end{array}$$

∴

DATE

Step 2, we will consider 2nd Column & 2nd row of the above matrix i.e. C_2 & R_2.

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

writing all positions where 1 is present in column 2.

$C_2 = \{a, b, d\}$

also, writing all positions where 1 is present in Row 2.

$R_2 = \{a, b, c\}$

Now, taking the cross product of C_2 & R_2.

$C_2 \times R_2 = \{a, b, d\} \times \{a, b, c\}$

$= \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (d,a), (d,b), (d,c)\}$

$\therefore$, new additions $= \{(a,a)\}$

now, placing 1 in the above new additions.

new matrix W_2

$$= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{4\times4} \end{array}$$

Step 3, we will consider 3rd column & 3rd row of the above matrix i.e. C_3 & R_3

writting all positions where 1 is present in column 3.

$\therefore$ $C_3 = \{a, b, d\}$

PAGE

also writting all positions where 1 is present in Row 3.

$R_3 = \{d\}$.

Now, taking the cross product of C_3 & R_3.

$$C_3 \times R_3 = \{a, b, d\} \times \{d\}$$
$$= \{(a,d), (b,d), (d,d)\}$$

$\therefore$ new additions $= \{(a,d), (b,d), (d,d)\}$

now, placing 1 in the above new additions.

new matrix :-

$$W_3 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}_{4\times4} \end{array}$$

Step 4, Finally we consider Column 4 & Row 4 a

$C_4 = \{a, b, c, d\}$ & $R_4 = \{a, b, c, d\}$

Now, cross product of C_4 & R_4

$$C_4 \times R_4 = \{a, b, c, d\} \times \{a, b, c, d\}$$
$$= \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d)\}$$

$\therefore$ new additions $= \{(c,a), (c,b), (c,c),\}$

new matrix :-

$$W_4 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

∴ transitive closure :-

our final matrix. W_4 has ones in all places

$\therefore R_t^+ = \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,a), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d)\}$

is the required transitive closure using warshall algorithm.

Q.2. what are equivalence relation.

let $A = \{1,2,3,4,5\}$,

$R = \{(1,1), (2,2), (1,2), (2,1), (3,3), (3,4), (4,3), (4,4), (5,5), (5,4), (4,5)\}$

Determine whether R is a equivalence relation or not.

Ans:- A relation R on a set A is an equivalence relation iff R is reflexive, symmetric & transitive.

E.g.:-

$A = \{0,1,2,3\}$

$R_1 = \{(0,0), (1,1), (2,2), (3,3)\}$.

Is R_1 an equivalence relation? yes,

as it satisfies, reflexive, symmetric & transitive

In our given relation:-

$R = \{(1,1), (2,2), (1,2), (2,1), (3,3), (3,4), (4,3), (4,4), (5,5), (5,4), (4,5)\}$

DATE

R is reflexive as it has:- $\{(1,1),(2,2),(3,3),(4,4),(5,5)\}$
R is symetric as it has:- $\{(1,2),(2,1),(3,4),(4,3),(5,4),(4,5)\}$

R is transitive as it has:- $\{(1,2),(1,1),(2,1),(3,4),(3,3),(4,3),(5,4),(5,5),(4,5)$

Hence, it satisfies all the above condition of reflexive, symmetric & transitive.
Therefore the given relation R is equivalence relation

Q.3. Define transitive closure of a relation.

Ans:- let R be a relation on a set A then R^∞ is the transitive closure of R. The transitive closure of a relation R is the smallest transitive relation containing R.

E.g:- $A = \{1,2,3,4\}$
$R = \{(1,2),(2,3),(3,4),(2,1)\}$
$R^\infty = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),($
$(3,4)\}$

*) what do you mean by closure of Relations?
→ let R be a relation on a set A. R may or may not have some property P, such as reflexivity, symmetry or transitivity. If there is a relation S with property P

such that S is a subset of every relation with P containing R, then S is called the closure of R with respect to P.

Q(✱) Define Reflexive closure:-

The reflexive closure $R^{(r)}$ of a relation R is the smallest reflexive relation that contains R as a subset. To find the reflexive closure, one there fore, has to know what pairs have to be added to the relation to make it reflexive. Given a relation R on a set A, the reflexive closure of R can be obtained by adding to R all pairs of the form (a, a) with $A \in R$, not already in R (since for reflexive closure xRx be true for all x). Thus,

$$R^{(r)} = R \cup I_A$$

where $I_A = \{(a,a) : a \in A\}$ is the diagonal relation on A.

Eg:- On the set $S = \{1,2,3,4\}$, the relation $R = \{(1,2),(2,1),(1,1),(2,2)\}$ is not reflexive since for e.g. $3R3$. we can fortify this relation with reflexivity by adding (3,3) and (4,4) to R, since these are the only pairs of the form (a, a) that are not in R.

$$R^{(r)} = R \cup \{(3,3),(4,4)\}$$

$R^{(r)}$ is obtained by supplementing with exactly what is essential (no more, no less) in order to get a reflexive relation containing R. So, $R^{(r)}$ is

DATE

Q(x) What is matrix of relation?

Ans:- If $A = \{a_1, a_2, a_3, \ldots, a_m\}$ and $B = \{b_1, b_2, b_3 \ldots b_n\}$ are finite sets containing m and n elements and R is a relation from A to B.

we represent R by ~~___~~ $m \times n$ matrix:-

$M_r = [m_{ij}]$ which is defined by:-

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

e.g:- $A = \{1, 2, 3\}$, $B = \{r, s\}$

$R = \{(1,r), (2,s), (3,r)\}$ is a relatio[n] from A to B.

$$\begin{array}{c|cc} & r & s \\ \hline 1 & (1,r) & (1,s) \\ 2 & (2,r) & (2,s) \\ 3 & (3,r) & (3,s) \end{array}$$

$$M_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$ //

DATE

Q(x) If $R = \{(1,2),(4,3),(2,2),(2,1),(3,1)\}$ be a relation on $S = \{1,2,3,4\}$. Find the symmetric closure.

Sol^n :-

The symmetric closure can be found by taking the union of R and R^{-1}.

Now, $R^{-1} = \{(2,1),(3,4),(2,2),(1,2),(1,3)\}$

so, $R^{(S)} = (R \cup R^{-1} = \{(1,2),(2,1),(4,3),(3,4),(3,1),(1,3)\}$

John Subba

BIT = II

DATE

Unit 2:- Function & Counting

Q.1a) State pigeonhole principle. There are 15 staffs in the office. Find the minimum number of staffs that can have their joining in the same months.

Soln:-

Theorem:- If n pigeons are assigned to m pigeonholes and ~~m>n~~ $m < n$, then atleast one pigeonhole contains 2 or more pigeons.

proof:- let each pigeonhole contains at most one pigeon. Then at most m pigeon have been assigned. Since $m<n$, not all pigeons have been assigned pigeonholes which is a contradiction that each pigeonhole contains at must 1 pigeon. so there are at least one pigeonhole contains two or more pigeons.

let the joining month play the role of the pigeons and the calender months the pigeonholes.

Then there are 15 different pigeons and 12 pigeonholes. By the pigeonhole principle, at least 2 or more staffs should join in the same months.

Hence the minimum number of staffs that can have their joining in the same months is:-

if all other pigeonholes contains at most $\left\lfloor \frac{15}{12} \right\rfloor$ i.e 1 pigeon

then one pigeonholes will have 15 − 11 (i.e 4 pigeons) which is the minimum number of staffs that can have their joining in the same months.

PAGE

b) In a group of 7 boys and 5 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Sol^n. Here we need to select atleast one boy.

our 1st case :- we need to select 1 boys & 3 girls.

so the possible ways are $7C1 \times 5C3$

$= \frac{7!}{(7-1)!} \times \frac{5\times4\times3\times2!}{(5-3)!\,3!}$

$= 7 \times \frac{60}{6}$

= 70 ways

Second case :- we need to select 2 boys & 2 girls

so the possible ways are $7C2 \times 5C2$

$= \frac{7\times6\times5!}{5!\,2\times1} \times \frac{5\times4\times3!}{3!\,2\times1}$

$= 21 \times 10$

= 210 ways.

Third case :- we need to select 3 boys & 1 girl

so the possible ways are $7C_3 \times 5C_1$

$= \frac{7\times6\times5\times4!}{4!\times3\times2} \times \frac{5\times4!}{4!\times1!}$

$= 35 \times 5$

= 175 ways.

4th case :- we need to select 4 boys & 0 girl.

So the possible ways are $7C_4$

$= \frac{7\times6\times5\times4!}{3!\times4!}$ = 35 ways.

Q.x) What are functions? Explain one-one and into function with example.

Sol:-

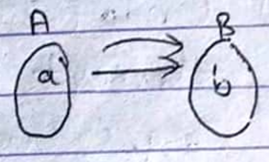
A function from a set A to a set B is a relation or rule that associates each element of A with a unique element of B.

Let A & B be two non-empty sets.

A function from a set A to a set B denoted by $f: A \to B$ is a relation such that for each $a \in A$, $\exists$ (there exists) a unique $b \in B$ such that $(a, b) \in f$.

If $b \in B$ is associated with an element a of A, we write as $b = f(a)$

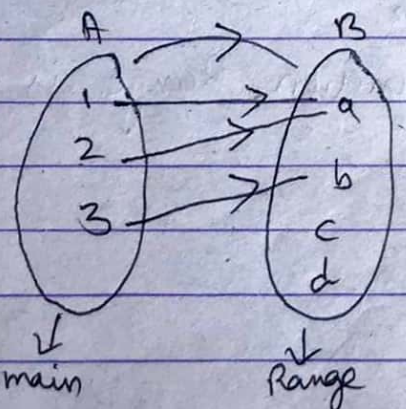
Here $f(a)$ is known as the image of f at a or value of f at a.



If b is the unique element of B corresponding to an element a of A under the function f, a is called the pre-image of b under f.

It is to be noted that if f is a function from A to B then no element of A is related to more than one element of B although more than one element of A can be related to the same element of B.

Functions are also called mappings or transformations.



$f(1) = a$

$f(2) = a$

$f(3) = b$

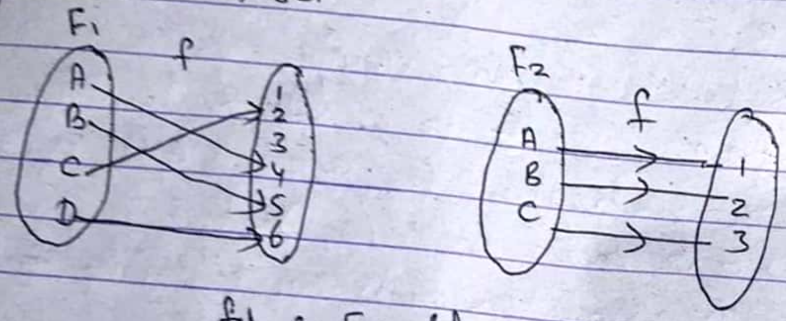
Domain

Range

DATE

Types of functions:-

(1) One to One function:- A function from A to B is one to one or injective if ~~for all elements x_1, x_2 in A is connected to~~ ~~or~~ one element of Domain set is connected to one element of Co-Domain set.

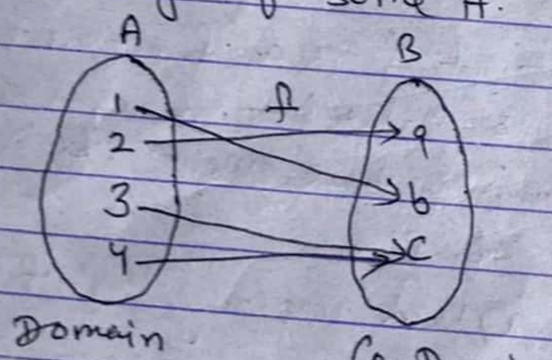


f1 & F_2 show one to one function

(2) Onto (Surjective) function:- A function in which every element of Co-Domain set has one pre-image.

e.g.:- consider, $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ & $f = \{(1, b), (2, a), (3, c), (4, c)\}$.

It is a surjective function, as every element of B is the image of some A.



note:- In an Onto function, Range is equal to Co-Domain.

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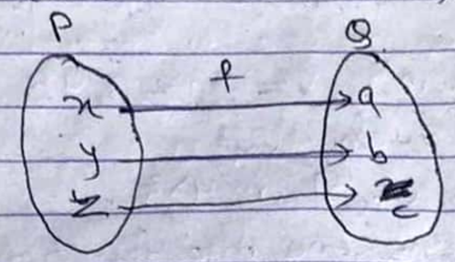
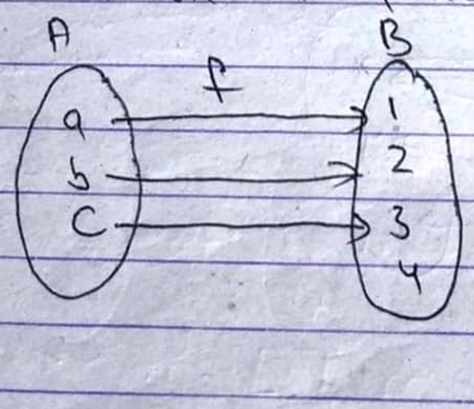
3. Bijective (One-to-One, Onto) functions:- A function which is both injective (one to-one) and surjective (onto) is called bijective (one-to-one onto) Function.



E.g:-
Consider $P = \{x, y, z\}$ & $Q = \{a, b, c\}$
& $f : P \to Q$ such that $f = \{(x,a), (y,b), (z,c)\}$.
The f is a one-to-one function also it is onto. So a bijective function.

4) Into Functions:- A function in which there must be element of co-domain Y does not have a pre-i in domain x.

Eg:- Consider, $A = \{a, b, c\}$ & $B = \{1, 2, 3, 4\}$
& $f : A \to B$ such that
$f = \{(a,1), (b,2), (c,3)\}$
In the function f, the range i.e., $\{1, 2, 3\} \neq$ co-d of y i.e $\{1, 2, 3, 4\}$
$\therefore$, it is an into function.



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5) One-One Into Functions:- Let $f: X \to Y$. The function f is called one-one into function if different elements of X have different unique images of Y.

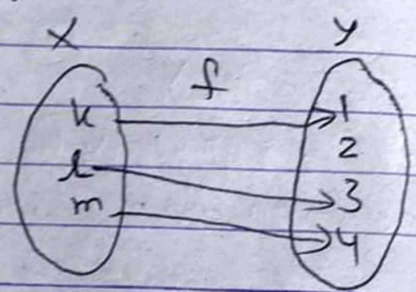
E.g.:-

Consider, $X = \{k, l, m\}$, $Y = \{1, 2, 3, 4\}$ &

$f: X \to Y$ such that

$f = \{(k,1), (l,3), (m,4)\}$

The function f is a one-one into function



⑥ Many-One functions: Let $f: X \to Y$. The function f is said to be many-one functions if there exists two or more than two different elements in X having same image in Y.

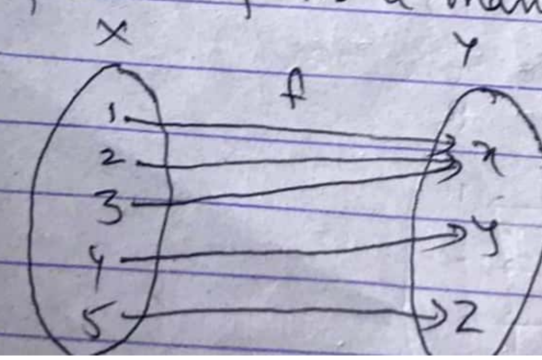
E.g.:-

Consider $X = \{1, 2, 3, 4, 5\}$

$Y = \{x, y, z\}$ & $f: X \to Y$ such that

$f = \{(1,x), (2,x), (3,x), (4,y), (5,z)\}$

The function f is a many-one function



7. Many - One Into Functions :- let $f : X \to Y$. The function f is called the many-one function if and only if is both many-one & into function.

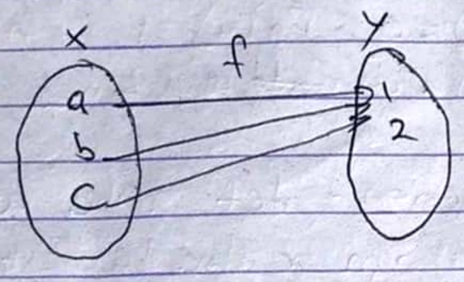
E.g :-

Consider $X = \{a, b, c\}$

$Y = \{1, 2\}$ & $f : X \to Y$ such that

$f = \{(a,1), (b,1), (c,1)\}$

As the function f is a many-one & into, so it is a many - one into function.



(8) Many - One Onto functions :- let $f : X \to Y$. The function f is called many-one onto function if and only if both many one & onto.

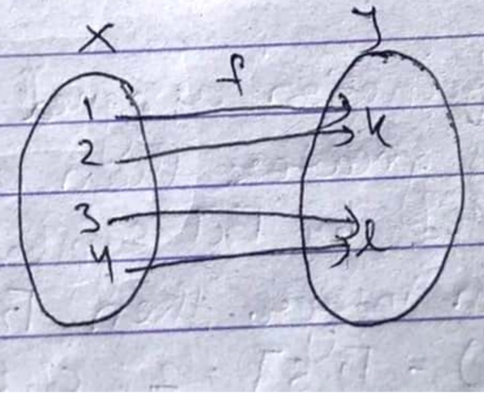
E.g.

consider $X = \{1, 2, 3, 4\}$

$Y = \{k, l\}$ & $f : X \to Y$ such that

$f = \{(1,k), (2,k), (3,l), (4,l)\}$

The function f is a many- one (as the two elements have the same image in Y) and it is onto (as every element of Y is the image of some element X). So, it is many-one onto function.



(a) Short notes on:

(i) Functions of Computer Science:-

The functions of computer science includes:-

(i) Characteristic functions:- Let A be a subset of the universal set $U = \{u_1, u_2, u_3, \dots\}$. The characteristic functio[n] of A is defined as a function from U to $\{0,1\}$ by the follow[ing]

$$F_A(u_i) = \begin{cases} 1 \text{ if } u_i \in A \\ 0 \text{ if } u_i \notin A \end{cases}$$

E.g. If $A = \{4,7,9\}$ & $U = \{1,2,3,\dots 10\}$

$F_A(2) = 0$, $F_A(4) = 1$, $F_A(7) = 1$

$F_A(12) =$ is ~~undd~~ undefined.

Floor and ceiling function:- Let x be a real number the floor function assign x the largest integer that is less than or equal to x.

The floor of x is denoted by $\lfloor x \rfloor$ symbolically, if x is a real number & n is an integer then

$\lfloor x \rfloor = n \Leftrightarrow n \le x < n+1$ the floor function is often called the greatest integer function.

E.g.:- ① $f(1.5) = \lfloor 1.5 \rfloor$
$= 1$

② $f(-3) = \lfloor -3 \rfloor$
$= -3$

③ $f(-2.7) = \lfloor -2.7 \rfloor$
$= -3$

The ceiling function assigned to x the smallest integer that is greater than or equal to x. The ceilin[g] of x is denoted by $\lceil x \rceil$ symbolically, if x is a real number & n is an integer then $\lceil x \rceil = n \Leftrightarrow n+1$.

E.g.: $C(1.5) = \lceil 1.5 \rceil = 2$; $C(-3) = \lceil -3 \rceil = -3$;

$C(-2.7) = \lceil -2.7 \rceil = -2$

The floor of x "rounds x down" while the ceiling of x "rounds x up". The floor and ceiling functions are useful in data storage & data transmission.

Define permutation & combination. In how many wa[ys] the word Biratnagar can be be arranged?

Permutation:- Any arrangement of a set of n objects in [a] given order is called Permutation. Any arrangement of a[ny] $r \le n$ of these objects in a given order is called an r-p[erm]utation or a permutation of n object taken r at a time. ~~Ded~~ denoted by $P(n,r) = \frac{n!}{(n-r)!}$

Combination:- It is a selection of some or all, objects [of] a set of given objects, where the order of the objects [does] not matter. The number of combinations of n objects [taken] r at a time represented by nCr or $C(n,r)$.

$$nCr = \frac{n!}{r!(n-r)!}$$

The word Biratnager can be arranged in:

total letters$(n) = 10$

repeated letters$(r) = 2, 2$.

$\therefore$ The arrangements $= \frac{10!}{2!2!}$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4}^{2} \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$$

$= 907200$

DATE

Q.*) The shirts numbered from 1 to 20 are worn by 20 members of bowling league. when any 3 of these members are chosen to be a team, the sum of their shirts number is used as code number of team. show that if any 8 of 20 are selected, then from these 8 we may form at least two different teams having the same code number.

Sol^n: while team containing 3 persons is selected & number inscribed on the shirt is added the possible ~~mo~~ minimum number is $(1+2+3)=6$. and the maximum number is $(18+19+20)=57$. so a team of 3 can have a code number from the possible range of 52 codes from 6 to 57 both inclusive. Now after selecting 8 from 20 members any 3 out of 8 can be selected in ~~$^8C_3$~~ ~~56 ways~~ $^8C_3$ ways.

$$\therefore \frac{8!}{5! \times 3!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times \cancel{3 \times 2}}$$

$$= 56 \text{ ways.}$$

Now using the pigeonhole principle, let 56 pigeons are placed into 52 holes marked ~~to~~ with codes between 6 & 57, then there are at least two teams will be in the same hole, implying that these two teams will have ~~sa~~ the same code number.

Q.(x) In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y? How many of them do not begin with M but end with?

Sol^n.-

The word MONDAY can be arranged in P(6,6) ways:.

i.e 6!

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ ways.

If M and Y are not allowed to move then there will (O, N, D, A) i.e 4!

$= 4 \times 3 \times 2 \times 1$

$= 24$ ways.

If M is not allowed to start but Y will be fixed a

place then the starting letters would be any of O

$= 4$ ways.

For 2nd place again 4 letters are available including

4 ways

For 3rd place, again 3 letters are available 3

for 4th place again 2 letters are available 2

for 5th place only 1 letter is available 1.

So,

our total will be $= \cancel{24+24+6+2}+1$

$= 4 \times 4 \times 3 \times 2 \times 1$

$= 96$ ways

DATE

Q.(*) A bit is either 0 or 1 ∴ a byte is a sequence of 8 bi[ts]. Find (a) the number of bytes that can be formed (b) the number of bytes that begin with 11 and end wi[th] 11, (c) the number of bytes that begin will 11 & do n[ot] end with 11, & (d) the number of bytes that begin wi[th] 11 or end with 11.

Soln:-

(a) Since the bits 0 or 1 can repeat, the eight positions can be filled up either by 0 or 1 in 2^8 ways. Hence the number of bytes can be formed in 256.

(b) keeping 11 in start & 11 in last two space there is 4 open positions :- can be filled in 2^4 ways.
$= 16$ ways.

(c) Keeping 2 positions at the beginning by 11, the remaining 6 open position can be filled by $2^6 = 64$ ways. also we need to subtract $2^4 = 16$ (since 4 places the top short begin with 11 & last should be 00)
∴ $64 - 16 = 48$ ways.

(d) $2^6 = 64$ i.e 64 bytes begin with 11 & 64 bytes ~~begin~~ end with 11.
total is $64 + 64 = 128$. In both the case each byte that begin & end will 11 is counted twice.
Hence the reqd number is $128 - 16 = 112$ bytes.

DATE

A Computer password consists of a letter of the alphabets followed by 3 or 4 digits.
find (a) the total number of pw that can be formed, and (b) the number of p.w in which no digits repeats.

(2) 1st place alphabet = 26 alphabets.

no. of digits = 10

∴ for 4 digits password = $26 \times 10 \times 10 \times 10$ (since digits can be repeated)

$= 26000$.

∴ for 5 digits password = $26 \times 10 \times 10 \times 10 \times 10$

$= 260000$

∴ total number of passwords is:- $260000 + 26000$

$= 2,86,000$

If the digits do not repeat, the new ways:-

for 4 digits password = $26 \times 10 \times 9 \times 8$

$= 18720$

for 5 digits password = $26 \times 10 \times 9 \times 8 \times 7$

$= 131040$.

∴ total number of passwords = $131040 + 18720$

$= 149760$

PAGE

Q.(x) In how many ways can 12 balloons be distributed at a Birthday party among 10 children?

Q(x) solve the recurrence relation.

$$a_n = 4(a_{n-1} - a_{n-2})$$

The given relation is $a_n - 4a_{n-1} + 4a_{n-2} = 0$ .... (i)

let, $a_n = r^n$ is a solution of (i)

The characteristic eqn is $r^2 - 4r + 4 = 0$.

$\Rightarrow r^2 - (2+2)r + 4 = 0$

$(r-2)(r-2) = 0$.

$\therefore r = 2, 2$.

$\therefore$ The general soln is $a_n = (c_1 + x c_2)2^x$

so, $a_0 = c_1$ and $a_1 = (c_1 + c_2)2$

$\therefore a_0 = 1$ gives $c_1 = 1$

and $c_1 = 1$ gives $2(c_1 + c_2) = 1$

$\therefore c_2 = -\frac{1}{2}$

so, the general soln is:-

$$a_n = \left(1 - \frac{1}{2}n\right)2^n$$ //

Q(x) Solve the recurrence relation :-

$a_{n+2} - 5a_{n+1} \div 6a_n = 2$ with initial condition $a_0 = 1$ & $a_n = -1$.

Sol:- The associated homogeneous recurrence relation is:-

$a_{n+2} - 5a_{n+1} + 6a_n = 0$. . . . (1)

let $a_n = r^n$ be a soln of (1)

The characteristic eqn is $r^2 - 5r + 6 = 0$.

$r^2 - (3+2)r + 6 = 0$

$(r-3)(r-2) = 0$

$\therefore r = 3, 2$.

$\therefore$ The soln of (1) is $a_n(h) = c_1 3^n + c_2 2^n$.

To find the particular soln :-

$\cancel{A - 5D + 6 = 2}$

$\cancel{\frac{1}{D^2 - 5D + 6} \cdot 2}$

$\cancel{= \frac{1}{D^2 - 5D + 6} 2 \times e^{0x}}$

$\cancel{= \frac{2}{-5D + 6} \cdot e^{0x}}$

$\cancel{= \frac{2 \times (5D + 6)}{(5D)^2 - (6)^2} e^{0x}}$

$\cancel{= \frac{+2 \times (5D + 6)}{-36}}$

$= 10D + 60$.

DATE

Q(x) Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for each value of x.

(i) 8 (ii) 6.01 (iii) -6.2 (iv) $1/2$ (v) $-1/2$

Soln:-

(i) $\lfloor 8 \rfloor = 8$ & $\lceil 8 \rceil = 8$

(ii) $\lfloor 6.01 \rfloor = 6$ & $\lceil 6.01 \rceil = 7$

(iii) $\lfloor -6.2 \rfloor = -7$ & $\lceil -6.2 \rceil = -6$

(iv) $\lfloor 1/2 \rfloor = 0$ & $\lceil 1/2 \rceil = 1$

(v) $\lfloor -1/2 \rfloor = -1$ & $\lceil -1/2 \rceil = 0$

Q(x) How many bytes are required to encode n bits of data where n equals (each byte is made up of 8 bits) for 1001.

Sol: The number of required bytes is the smallest integer that is greater than or equal to 1001/8. Consequently $\lceil 1001/8 \rceil = \lceil 125.0125 \rceil = 126$

DATE

Q(*) Solve by mathematical induction:-

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

Sol^n:-

Basic step:-

$n=1$

$$1^3 = \frac{1^2(1+1)^2}{4}$$

$$1 = \frac{1(1+1)^2}{4}$$

$$= \frac{\cancel{2^2}}{\cancel{4}}$$

$$1 = 1$$ //

$$P(k) = 1^3 + 2^3 + 3^3 + \dots + k^3$$
$$= \frac{k^2(k+1)^2}{4}$$

$$P(k+1) = 1^3 + 2^3 + 3^3 + \dots + k^3 + (k+1)^3$$
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$
$$= (k+1)^2\left(\frac{k^2}{4} + k+1\right)$$
$$= (k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$
$$= (k+1)^2\left(\frac{k^2+(2+2)k+4}{4}\right)$$
$$= (k+1)^2\left(\frac{(k+2)(k+2)}{4}\right)$$
$$= \frac{(k+1)^2(k+2)^2}{4}$$

∴ P(k+1) resembles the ~~formula~~ explicit formula:-

$$\frac{n^2(n+1)^2}{4}$$

of ~~where~~ $\frac{(k+1)^2((k+1)+1)^2}{4}$ where $n = k+1$ //

Q8) $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol :-

$n = 1$

Basic step :-

$$1^2 = \frac{1(1+1)(2\times1+1)}{6}$$

$$1 = \frac{2\times3}{6}$$

$$1 = \frac{\cancel{6}}{\cancel{6}} 1$$

$\therefore 1 = 1$ // proved.

for, $P(k) = 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

$$P(k+1) = 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left(\frac{k(2k+1)}{6} + (k+1)\right)$$

$$= (k+1)\left(\frac{2k^2 + k + 6k + 6}{6}\right)$$

$$= (k+1)\left(\frac{2k^2 + 7k + 6}{6}\right)$$

$$= (k+1)\left(\frac{2k^2 + (4+3)k + 6}{6}\right) = (k+1)\left(\frac{2k^2 + 4k + 3k + 6}{6}\right)$$

$$= (k+1)\left(\frac{2k(k+2) + 3(k+2)}{6}\right) = (k+1)\left(\frac{(2k+3)(k+2)}{6}\right)$$

DATE

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

is the equivalent form of $\frac{n(n+1)(2n+1)}{6}$

where, $n = k+1$ then our explicit formula for $k+1$ is:-

$$\frac{(k+1)((k+1)+1)(2k+2+1)}{6}$$

$$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

//

Q(x) Find the solution to the recurrence relation

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

with initial conditions $a_0 = 2$, $a_1 = 5$, & $a_2 = 15$.

Soln.- The characteristic polynomial of this recurrence relation is $r^3 - 6r^2 + 11r - 6$.

The characteristic roots are :-

$$r^3 - 6r^2 + 11r - 6 = 0.$$

$$\text{or, } r^2(r-6) + 11(r-6)$$

Q. (*) A man 7 relatives, 4 of them ladies & 3 gentlemen, his wife has 7 relatives of which 3 are ladies & 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relatives & 3 of wife's relatives

Soln:- possible ways:-

(i) 3 ladies from husband's side & 3 from wife's side.

$$\text{ways} = C(4,3) \times C(4,3)$$

$$= \frac{4!}{(4-3)!\,3!} \times \frac{4!}{(4-3)!\,3!}$$

$$= \frac{4\times\cancel{3!}}{1!\,\cancel{3!}} \times \frac{4\times\cancel{3!}}{1!\,\cancel{3!}}$$

$$= 16$$

(ii) 3 gents from husband's side & 3 ladies from wife's side

$$\text{no. of ways} = C(3,3) \times C(3,3) = 1$$

$$= \frac{\cancel{3!}}{(3-3)!\,\cancel{3!}} \times \frac{\cancel{3!}}{(3-3)!\,\cancel{3!}}$$

$$= 1$$

(ii) (2 ladies & 1 gent) from husband's side & (one lady & 2 gents) f

wife's side.

$$\text{no. of ways:-} \{C(4,2) \times C(3,1)\} \times \{C(3,1) \times C(4,2)\}$$

$$= \frac{4!}{2!\times 2!} \times \frac{3!}{2!\,1!} \times \frac{3!}{2!\times 1!} \times \frac{4!}{2!\times 2!}$$

$$= \frac{4\times3\times2}{\cancel{2\times2}} \times \frac{3\times\cancel{2!}}{\cancel{2!}} \times \frac{3\times\cancel{2!}}{\cancel{2!}} \times \frac{4\times3\times2}{\cancel{2\times2}}$$

$$= 6\times3\times3\times6$$

$$= 351 \text{ ways}$$

DATE

(iv) (1 lady & 2 gents) husband's side & (2 ladies + 1 gent) from wife's side.

no. of ways :-

$= \{(C(4,1) \times C(3,2)) \times (C(3,2) \times C(4,1)\}$

$= \frac{4!}{3! \times 1!} \times \frac{3!}{1! \times 2!} \times \frac{3!}{1! \, 2!} \times \frac{4!}{3! \times 1!}$

$= \frac{4 \times 3!}{3!} \times \frac{3 \times 2!}{2!} \times \frac{3 \times 2!}{2!} \times \frac{4 \times 3!}{3!}$

$= 4 \times 3 \times 3 \times 4$

$= 144$

$\therefore$ The total no. of ways $= 16 + 1 + 351 + 144$

$= 512$ ways.