Recorrence Relation..

A sequence can be define by giving a general formula for its n8th term by writing a few of its term and alternative purpose approach is to represent the sequence by finding a relationship among its terms such relation are refer as reccurence.

A recurrence relation for the sequence $\{a_n\}$ is an equation that express $\{a_n\}$ in terms of one or more of the previous terms of the sequence mainly $a_0, a_1, a_2, \ldots a_{n-1}$ v integeis n, $n \geq n_0$ when n_0 is a non negative integeis.

A sequence is called a solution of a recurrence relation if its terms satisfied the recurrence Relation for

eg:-

The reaucerence relation $a_n = a_{n-1} + 3$, with $a_1 = 4$ recursively defines the seqn $4, 7, 10, 13, \ldots$

$n = 2, \ldots \infty$ $\quad a_1 = 4$

$$a_2 = a_{2-1} + 3$$
$$= a_{2-1} + 3$$
$$= a_1 + 3$$
$$= 4 + 3$$
$$= 7$$

$$a_3 = a_2 + 3 = 7 + 3 = 10$$

$f_n = f_{n-1} + f_{n-2}$, $f_1 = f_2 = 1$
defines the fibonacci seauense.

1, 1, 2, 3, 5, 8 . . .

Linear Recurrence Relation:-
A recurrence relation of the ~~form~~ form $C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + \dots + C_k a_{r-k} = f(r)$ where C_is are constants is called a liner recussence relation.
Here if C_0 & C_k are non zero than it is known k^{th} order recurrence relation.

Ex:-

$2a_r + 3a_{r-1} = 2^r$ is the first order linear recurrence relation with constant term or cofficient

If f of r is identic zero that is no terms occur that are not multiplus of a_i the relation is known as homonization otherwise it is non-homonization.

Ex:- $a_n = 2a_{n-1}$ (~~Homonization~~ Homogeneous recurrence relation with order one)

Solving homogeneous recurrence relation
an exclusive formula which satisfy the recurrence relation with initial condition is called a solution to the recurrence relation.

Finding the solution of recurrence Relation.
Back tracking
~~[illegible]~~

Back tracking
1) ~~B~~ Solve recourrence relation $a_n = a_{n-1} + 3$
with $a_1 = 2$ defines the sequence. 2, 5, 8, ...

Soln :- $a_n = a_{n-1} + 3$ $\qquad a_{n-1} = a_{n-1-1} + 3$

$= a_{n-2} + 3$

$a_n = a_{n-1} + 3$

$= (a_{n-2} + 3) + 3$

$= a_{n-2} + 3 + 3 = a_{n-2} + 2 \cdot 3$

$= (a_{n-3} + 3) + 3 + 3 = a_{n-3} + 3 \cdot 3$

$= (a_{n-4} + 3) + 3 + 3 + 3 = a_{n-4} + 4 \cdot 3$

$n = n-1$

$= a_{n-(n-1)} + (n-1)3$

$= a_1 + (n-1)3$

$a_n = 2 + (n-1)3$

$\boxed{a_{n-(n-1)} = a_{n-n+1} = a_1}$

$\therefore$ The exclusive formula for given recurrence relation is.

Find an exclusive formula for the sequence $b_n = 2b_{n-1} + 1$ with $b_1 = 7$

$b_n = 2b_{n-1} + 1$

$b_n = 2\{2b_{n-2} + 1\} + 1$

$= 2^2 b_{n-2} + 2 + 1$

$= 2^2 \{2 b_{n-3} + 1\} + 2 + 1$

$= 2^3 b_{n-3} + 2^2 + 2 + 1$

$= 2^3 \{2b_{n-4} + 1\} + 2^2 + 2 + 1$

$= 2^4 \{2b_{n-4} + 2^3 + 2^2 + 2 + 1$

$= 2^{n-1} b_{n-(n-1)} + 2^{n-2} + 2^{n-3} + 2 + 1$

$= 2^{n-1} b_1 + 2^{n-2} + 2^{n-3} + \dots + 2 + 1$

$= 2^{n-1}(7) + \dfrac{2^{n-1} - 1}{2-1}$

$\because 1 + a + a^2 + a^3 + \dots + a^{n}$

$= \dfrac{a^n - 1}{a - 1}$

$= 2^{n-1}(2^3) - 1$

$= 2^{n-1+3} - 1$

$= 2^{n+2} - 1$

$x^2 - x \ldots$

Find an exclusive formula for the sequence defined by $c_n = 3c_{n-1} - 2c_{n-2}$ with initial condition $c_1 = 5$ and $c_2 = 3$

$$c_n = 3c_{n-1} - 2c_{n-2}$$

into quadratic fuurm

$$x^2 - 3x + 2 = 0$$

sol^n.

The given recurrence relation is

$$c_n = 3c_{n-1} - 2c_{n-2} \quad \ldots \text{ eqn (i)}$$

let

$$c_n = x^n$$

$$x^n = 3x^{n-1} - 2x^{n-2}$$

dividing both sides by x^{n-2}

$$x^2 = 3x - 2$$

$$x^2 - 3x + 2 = 0$$

which is a charact eqn of eqn (i)

or $x^2 - 2x - x + 2 = 0$

or $x(x-2) - (x-2) = 0$

$(x-2)(x-1) = 0$

$x = 1, 2$

$\therefore$ The roots are real and distinct

/ convert the eqn in the form of $a_n = us_1^n + vs_2^n$ /

$$c_n = u(1)^n + v(2)^n \ldots \text{ (ii)}$$

Satisfying initial condition

$n = 1$, $c_1 = u(1)^1 + v(2)^1$

$5 = u + 2v \quad \ldots \text{ eqn (iii)}$

when $n=2$

$n=2$

$$C_2 = u(1)^2 + v(2)^2$$

$$3 = u + 4v \quad \cdots \text{ eq}^n\ 4$$

Solving eq^n iii and iv

$$5 = \cancel{u} + 2v$$

$$\underline{- \; 3 = -\cancel{u} - 4v}$$

$$2 = -2v$$

or $v = -1$

placing the value of v in eq^n i

$$5 = u + 2v$$

or $5 = u - 2$

or $u = 7$

~~[illegible]~~

Find the soln of the recurrence relation

$a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 2$ $a_1 = 7$

Soln:

The given recurrence relation is.

$a_n = a_{n-1} + 2a_{n-2}$

let

$a_n = x^n$

$x^n = x^{n-1} + 2x^{n-2}$

diving both side by x^{n-2}

$\frac{x^n}{x^{n-2}} = \frac{x^{n-1}}{x^{n-2}} + \frac{2\cancel{x^{n-2}}}{\cancel{x^{n-2}}}$

or $x^{n-n+2} = x^{n-1-n+2} + 2$

or $x^2 = x + 2$

or $x^2 - x - 2 = 0$

or $\cancel{x(x-1)}$ $x^2 - 2x + x - 2 = 0$

or $x(x-2) + 1(x-2) = 0$

or $(x+1)(x-2) = 0$

$x = 2$ $x = -1$

∴ The roots are real and distinct

$a_n = u2^n + v(-1)^n$ ......(ii)

Satisfying initial condition.

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$n=0 \quad a_0 = u(2)^0 + v(-1)^0$

$2 = u+v \quad \ldots \text{iii}$

$n=1 \quad a_1 = u(2)^1 + v(-1)^1$

$7 = 2u-v \quad \ldots \text{iv}$

Substuting eqn iii & iv.
Adding eqn i

$2 = u + \cancel{v}$

$7 = 2u - \cancel{v}$

$9 = 3u$

or $u = 3$

placing vaue of u in iv.

$7 = 2u - v$

or $7 = 2 \times 3 - v$

or $7 = 6 - v$

or $v = -1$

placing vaue of u & v in ii

$a_n = u2^n + v(-1)^n$

$=$

Solve the recurrence ~~of the~~ relation of the fibonachi sequence

~~The~~ If the charactistic equation

$x^2 - rx - r_2 = 0$

as a single root S then the exclusive formul is

$a_n = us^n + vns^n$

$a_n = (u + vn) s^n$

then u and v depent on the initial condition.

Find the solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with the initial condition $a_0 = 1$, $a_1 = 6$

Soln.

$a_n = 6a_{n-1} - 9a_{n-2}$ .....eq^n i

$a_n = x^n$

or $x^2 = 6x - 9$

or $x^2 - 6x + 9 = 0$

or $x^2 - 3 \cdot 2x + 3^2 = 0$

or $(x - 3)^2 = 0$

or $x = 3, 3$

$a_n = u3^n + nv3^n$ ii

Substuting the initial condition $a_0 = 1$ and $a_1 = 6$

$n = 0$

$a_0 = u3^0 + 0$

$1 = u$.

$\therefore u = 1$

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$n = 1$

$a_1 = u3^1 + 1 \times v3^1$

or $6 = 3u + 3v$

or $\cancel{6 \neq 1}$ $\quad 2 = 1 + v$

or $\cancel{v =}$ $\quad$ or $v = 1$

placing the value of u and v in eqn. ii

$a_n = u3^n + nv3^n$

or $a_n = 13^n + n13^n$

or $a_n = 3^n + n3^n$

Q. $a_n = 4a_{n-1} - 4a_{n-2}$

Given $a_0 = 1$, $a_1 = 4$

soln.-

$a_n = 4a_{n-1} - 4a_{n-2}$ .... eqn i

Let $a_n = x^n$

$x^2 = 4x - 4$

or $x^2 - 4x + 4 = 0$

or $x^2 - 2 \cdot 2x + 2^2 = 0$

or $(x-2)^2 = 0$

Sim.

$x - 2 = 0$ or $x - 2 = 0$

$x = 2$ $\quad$ $x = 2$

$\therefore x = 2, 2$

Single root x

$a_n = u2^n + vn2^n$ .... eqn ii

Substituting the initial value

$a_0 = 1, \quad a_1 = 4$

when

$n = 0$

$a_0 = u \times 2^0 + v \times 0 \times 2^0$

$\Rightarrow 1 = u + 0$

$\Rightarrow u = 1$

when.

$n = 1$

$a_1 = u \times 2^1 + v \times 1 \times 2^1$

$\Rightarrow 4 = 2u + 2v$

$\Rightarrow 2 = u + v$

$\Rightarrow 2 = 1 + v$

$\Rightarrow v = 1$

Placing the value of u and v in eqn ii

$a_n = u 2^n + n v 2^n$

$\Rightarrow a_n = 2^n + 2^n n$