2. $5^{1-x}+5^{x-1}=5\frac{1}{5}$

or, $5^1.5^{-x}+5^x.5^{-1}=26/5$

or, $\frac{5}{5^x}+\frac{5^x}{5}=\frac{26}{5}$

let $5^x=a$

then, $\frac{5}{a}+\frac{a}{5}=\frac{26}{5}$

or, $\frac{25+a^2}{5a}=\frac{26}{5}$

or, $25+a^2=26a$

or, $a^2-26a+25=0$

or $a^2-25a-a+25=0$

or, $a(a-25)-1(a-25)=0$

or, $(a-25)(a-1)=0$

This gives, $a=25, 1$

for $a=1$,

$5^x=1$

or $5^x=5^0$

or $x=0$

and for $a=25$

or $5^x=5^2$

or $x=2$

$\therefore \boxed{x=0,2}$

(4) If $x - \frac{1}{x} = 5$, find $x^2 + \frac{1}{x^2}$

Here, $x - \frac{1}{x} = 5$

or, $\cancel{x^2 - \frac{1}{x^2}}$ $\left(x - \frac{1}{x}\right)^2 = x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2}$

or, $5^2 = x^2 - 1 + \frac{1}{x^2}$

or, $25 + 1 = x^2 + \frac{1}{x^2}$

$\therefore \boxed{x^2 + \frac{1}{x^2} = 26}$

Long:

1. Solve $3^{x+3} + \frac{1}{3^x} = 28$

or, $3^x \cdot 3^3 + \frac{1}{3^x} = 28$

or, $27 \cdot 3^x + \frac{1}{3^x} = 28$

let $3^x = a$

so, $27a + \frac{1}{a} = 28$

or, $\frac{27a^2 + 1}{a} = 28$

or, $27a^2 + 1 = 28a$

or, $27a^2 - 28a + 1 = 0$

or, $27a^2 - 27a - a + 1 = 0$

or, $27a(a-1) - 1(a-1) = 0$

or, $(a-1)(27a-1) = 0$

or, $a = 1, \frac{1}{27}$

for $a = 1$

$3^x = 1$

or, $3^x = 3^0$

$\therefore x = 0$

for $a = \frac{1}{27}$

$3^x = \frac{1}{27}$

or, $3^x = 3^{-3}$

$\therefore x = -3$

$\therefore \boxed{x = 0, -3}$

(2) $\sqrt{x} + \sqrt{x-15} = \dfrac{105}{\sqrt{x-15}}$

or, $\sqrt{x(x-15)} + \left(\sqrt{x-15}\right)^2 = 105$

or, $\sqrt{x^2-15x} + x - 15 = 105$

or, $\sqrt{x^2-15x} = 120 - x$

Squaring both sides

$\left(\sqrt{x^2-15x}\right)^2 = (120-x)^2$

or, $x^2 - 15x = 14400 - 240x + x^2$

or, $240x - 15x = 14400$

or, $225x = 14400$

or, $x = \dfrac{14400}{225}$

or, $x = 64$

$\therefore \boxed{x = 64}$

Indices & Exponential Equations:

* $a^m \times a^n = a^{m+n}$
* $\frac{a^m}{a^n} = a^{m-n}$
* $(a^m)^n = a^{mn}$
* $(ab)^m = a^m \cdot b^m$
* $a^{-x} = \frac{1}{a^x}$
* $\sqrt[n]{a} = a^{1/n}$
* $x^0 = 1$
* $x^a = x^b \Rightarrow a = b.$
* $\sqrt[m]{\sqrt[n]{a}} = a^{\frac{1}{mn}}$

Short Questions:

Simplify:

(1) $\frac{5^{n+2} - 2\cdot 5^n}{23 \cdot 5^n}$

$= \frac{5^n \cdot 5^2 - 2\cdot 5^n}{23 \cdot 5^n}$

$= \frac{5^n (25-2)}{23 \cdot 5^n}$

$= \frac{23}{23}$

$= 1.$

(2) $\frac{5^{x+2} - 20 \times 5^{x-1}}{7 \times 5^x}$

$= \frac{5^x \cdot 5^2 - 20 \times 5^x \cdot 5^{-1}}{7 \times 5^x}$

$= \frac{5^x \left(25 - 20 \times \frac{1}{5}\right)}{7 \times 5^x}$

$= \frac{25-4}{7}$

$= 21/7$

$= 3$

Surds:

1. Simplify: $\sqrt{50} + \sqrt{18} - 8\sqrt{2}$

$= \sqrt{2\times5\times5} + \sqrt{2\times3\times3} - 8\sqrt{2}$

$= 5\sqrt{2} + 3\sqrt{2} - 8\sqrt{2}$

$= (5+3-8)\sqrt{2}$

$= 0$

2. $\dfrac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} - \dfrac{\sqrt{x}-\sqrt{a}}{\sqrt{x}+\sqrt{a}}$

$= \dfrac{(\sqrt{x}+\sqrt{a})^2 - (\sqrt{x}-\sqrt{a})^2}{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})}$

$= \dfrac{x + 2\sqrt{xa} + a - (x - 2\sqrt{xa} + a)}{(\sqrt{x})^2 - (\sqrt{a})^2}$

$= \dfrac{x + 2\sqrt{xa} + a - x + 2\sqrt{xa} - a}{x-a}$

$= \dfrac{4\sqrt{xa}}{x-a}$

3. $\dfrac{x-1}{\sqrt{x}+1} = 1$

or, $\dfrac{(\sqrt{x})^2 - (1)^2}{\sqrt{x}+1} = 1$

or, $\dfrac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}+1)} = 1$

or, $\sqrt{x} - 1 = 1$

or, $\sqrt{x} = 2$

Squaring both sides,

$(\sqrt{x})^2 = (2)^2$

$\therefore \boxed{x = 4}$

Long:

1. $\frac{5x-4}{\sqrt{5x}+2} = 2 + \frac{\sqrt{5x}-2}{2}$

or, $\frac{(\sqrt{5x})^2-(2)^2}{(\sqrt{5x}+2)} = 2 + \frac{\sqrt{5x}-2}{2}$

or, $\frac{(\sqrt{5x}-2)(\sqrt{5x}+2)}{(\sqrt{5x}+2)} = 2 + \frac{\sqrt{5x}-2}{2}$

or, $\sqrt{5x}-2 = 2 + \frac{\sqrt{5x}-2}{2}$

or, $\sqrt{5x}-4 = \frac{\sqrt{5x}-2}{2}$

or, $2\sqrt{5x}-8 = \sqrt{5x}-2$

or, $\sqrt{5x} = 8-2$

or, $\sqrt{5x} = 6$

Squaring both sides

$5x = 36$

or, $x = \frac{36}{5}$

$\therefore \boxed{x = 36/5}$

5. $\frac{p^2}{(p-y)^y} - \frac{2p}{(p-y)^{y-1}} + \frac{1}{(p-y)^{y-2}}$

$= \frac{p^2}{(p-y)^y} - \frac{2p}{\frac{(p-y)^y}{(p-y)}} + \frac{1}{\frac{(p-y)^y}{(p-y)^2}}$

$= \frac{p^2}{(p-y)^y} - \frac{2p(p-y)}{(p-y)^y} + \frac{(p-y)^2}{(p-y)^y}$

$= \frac{p^2 - 2\cdot p\cdot (p-y) + (p-y)^2}{(p-y)^y}$ $\left[\because a^2 - 2ab + b^2 = (a-b)^2\right]$

$= \frac{\{p-(p-y)\}^2}{(p-y)^y}$

$= \frac{(\cancel{p}-\cancel{p}+y)^2}{(p-y)^y}$

$= \frac{y^2}{(p-y)^y}$ Ans.

Alternative method

$= \frac{p^2 - 2p^2 + \cancel{2py} + p^2 - \cancel{2py} + y^2}{(p-y)^y}$

$= \frac{\cancel{2p^2} - \cancel{2p^2} + y^2}{(p-y)^y}$

$= \frac{y^2}{(p-y)^y}$ Ans.

If $a+b+c=0$

$\therefore a=-b-c$

$\therefore -a=b+c$

Now, $\dfrac{1}{1+x^{a}+x^{-b}}+\dfrac{1}{1+x^{b}+x^{-c}}+\dfrac{1}{1+x^{c}+x^{-a}}=1$

$$\text{LHS}=\frac{1}{1+x^{a}+x^{-b}}+\frac{1}{1+x^{b}+x^{-c}}+\frac{1}{1+x^{c}+x^{-a}}$$

$$=\frac{1}{1+x^{-b-c}+x^{-b}}+\frac{1}{1+x^{b}+x^{-c}}+\frac{1}{1+x^{c}+x^{b+c}}$$

$$=\frac{1}{1+\frac{1}{x^{b}\cdot x^{c}}+\frac{1}{x^{b}}}+\frac{1}{1+x^{b}+\frac{1}{x^{c}}}+\frac{1}{1+x^{b}\cdot x^{c}+x^{c}}$$

$$=\frac{1}{\frac{x^{b}\cdot x^{c}+1+x^{c}}{x^{b}\cdot x^{c}}}+\frac{1}{\frac{x^{c}+x^{b}\cdot x^{c}+1}{x^{c}}}+\frac{1}{1+x^{b}\cdot x^{c}+x^{c}}$$

$$=\frac{x^{b}\cdot x^{c}}{1+x^{b}\cdot x^{c}+x^{c}}+\frac{x^{c}}{1+x^{b}\cdot x^{c}+x^{c}}+\frac{1}{1+x^{b}\cdot x^{c}+x^{c}}$$

$$=\cancel{\frac{(1+x^{b}\cdot x^{c}+x^{c})}{(1+x^{b}\cdot x^{c}+x^{c})}}$$

$$=1$$

Hence RHS Proved

Long:

(1) $$\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}+\frac{1}{1+x^{c-a}+x^{b-a}}$$

$$=\frac{1}{1+\frac{x^{a}}{x^{b}}+\frac{x^{c}}{x^{b}}}+\frac{1}{1+\frac{x^{b}}{x^{c}}+\frac{x^{a}}{x^{c}}}+\frac{1}{1+\frac{x^{c}}{x^{a}}+\frac{x^{b}}{x^{a}}}$$

$$=\frac{1}{\frac{x^{b}+x^{a}+x^{c}}{x^{b}}}+\frac{1}{\frac{x^{c}+x^{b}+x^{a}}{x^{c}}}+\frac{1}{\frac{x^{a}+x^{c}+x^{b}}{x^{a}}}$$

$$=\frac{x^{b}}{x^{a}+x^{b}+x^{c}}+\frac{x^{c}}{x^{c}+x^{b}+x^{a}}+\frac{x^{a}}{x^{a}+x^{c}+x^{b}}$$

$$=\frac{(x^{a}+x^{b}+x^{c})}{(x^{a}+x^{b}+x^{c})}$$

$$=1.$$

(2) $$\frac{p^2}{(p-y)^y}-\frac{2p}{(p-y)^{y-1}}+\frac{1}{(p-y)^{y-2}}$$

$$=\frac{p^2}{(p-y)^y}-\frac{2p}{(p-y)^y.(p-y)^{-1}}+\frac{1}{(p-y)^y.(p-y)^{-2}}$$

$$=\frac{p^2}{(p-y)^y}-\frac{2p.(p-y)}{(p-y)^y}+\frac{(p-y)^2}{(p-y)^y}$$

$$=\frac{p^2-2p(p-y)+(p-y)^2}{(p-y)^y}$$

$$=\frac{\{p-(p-y)\}^2}{(p-y)^y}$$

$$=\frac{y^2}{(p-y)^y}$$

(4) $$\frac{\left(a^2-\frac{1}{b^2}\right)^a \left(a-\frac{1}{b}\right)^{b-a}}{\left(b^2-\frac{1}{a^2}\right)^b \left(b+\frac{1}{a}\right)^{a-b}}$$

$$= \frac{\left(a-\frac{1}{b}\right)^a \left(a+\frac{1}{b}\right)^a \left(a-\frac{1}{b}\right)^{b-a}}{\left(b-\frac{1}{a}\right)^b \left(b+\frac{1}{a}\right)^b \left(b+\frac{1}{a}\right)^{a-b}}$$

$$= \frac{\left(a-\frac{1}{b}\right)^{a+b-a} \left(a+\frac{1}{b}\right)^a}{\left(b+\frac{1}{a}\right)^{b+a-b} \left(b-\frac{1}{a}\right)^b}$$

$$= \frac{\left(a-\frac{1}{b}\right)^b \left(a+\frac{1}{b}\right)^a}{\left(b+\frac{1}{a}\right)^a \left(b-\frac{1}{a}\right)^b}$$

$$= \frac{\left(\frac{ab-1}{b}\right)^b \left(\frac{ab+1}{b}\right)^a}{\left(\frac{ab-1}{a}\right)^b \left(\frac{ab+1}{a}\right)^a}$$

$$= \left(\frac{\frac{ab-1}{b}}{\frac{ab-1}{a}}\right)^b \left(\frac{\frac{ab+1}{b}}{\frac{ab+1}{a}}\right)^a$$

$$= \left(\frac{a}{b}\right)^b \left(\frac{a}{b}\right)^a$$

$$= \left(\frac{a}{b}\right)^{a+b}$$

(8) $\dfrac{\left(a^2-\frac{1}{b^2}\right)^a \cdot \left(a-\frac{1}{b}\right)^{b-a}}{\left(b^2-\frac{1}{a^2}\right)^b \cdot \left(b+\frac{1}{a}\right)^{a-b}}$ (question correction gurus!)

$= \dfrac{\left(\frac{a^2b^2-1}{b^2}\right)^a \cdot \left(\frac{ab-1}{b}\right)^{b-a}}{\left(\frac{a^2b^2-1}{a^2}\right)^b \left(\frac{ab+1}{a}\right)^{a-b}}$

$= \dfrac{\frac{(ab+1)^a(ab-1)^a}{b^{2a}} \times \frac{(ab-1)^{b-a}}{b^{b-a}}}{\frac{(ab+1)^b(ab-1)^b}{a^{2b}} \cdot \frac{(ab+1)^{a-b}}{a^{a-b}}}$

$= \dfrac{(ab+1)^a \cdot (ab-1)^a \cdot (ab-1)^{b-a} \cdot a^{2b} \cdot a^{a-b}}{b^{2a} \cdot b^{b-a} \cdot (ab+1)^b (ab-1)^b \cdot (ab+1)^{a-b}}$

$= (ab+1)^{\cancel{a}-\cancel{b}-\cancel{a}+\cancel{b}} \cdot (ab-1)^{\cancel{a}+\cancel{b}-\cancel{a}-\cancel{b}} \cdot \dfrac{a^{2b+a-b}}{b^{2a+b-a}}$

$= (ab+1)^0 \times (ab-1)^0 \cdot \dfrac{a^{a+b}}{b^{a+b}}$

$= \left(\dfrac{a}{b}\right)^{a+b}$ Ans

(3) $\sqrt[3]{3x^7y^{11}z^{-1}} \times \sqrt[3]{72x^{-1}yz^4}$

$= \sqrt[3]{72\times3\,x^{7-1}\,y^{11+1}\,z^{-1+4}}$

$= \sqrt[3]{216\,x^6y^{12}z^3}$

$= \sqrt[3]{6^3(x^2)^3(y^4)^3(z)^3}$

$= \sqrt[3]{(6x^2y^4z)^3}$

$= 6x^2y^4z$

(4) $\sqrt[4]{216a^9b^5} \div \sqrt[4]{6^{-1}a^{-3}b}$

$= \sqrt[4]{\dfrac{216a^9b^5}{6^{-1}a^{-3}b}}$

$= \sqrt[4]{216\times6\,a^{9+3}\,b^{5-1}}$

$= \sqrt[4]{6^4a^{12}b^4}$

$= \sqrt[4]{6^4(a^3)^4(b)^4}$

$= \sqrt[4]{(6a^3b)^4}$

$= 6a^3b$

2. $\frac{3a+1}{9a^2+3a+1}+\frac{3a-1}{9a^2-3a+1}-\frac{2}{81a^4+9a^2+1}$

$= \frac{(3a+1)(9a^2-3a+1)+(3a-1)(9a^2+3a+1)}{(9a^2+3a+1)(9a^2-3a+1)}-\frac{2}{81a^4+9a^2+1}$

$= \frac{(3a)^3+\cancel{1^3}+(3a)^3-\cancel{1^3}}{81a^4+9a^2+1}-\frac{2}{81a^4+9a^2+1}$

$= \frac{54a^3-2}{81a^4+9a^2+1}$

$= \frac{2(27a^3-1)}{81a^4+9a^2+1}$

$= \frac{2\{(3a)^3-1^3\}}{81a^4+9a^2+1}$

$= \frac{2(3a-1)\cancel{(9a^2+3a+1)}}{\cancel{(9a^2+3a+1)}(9a^2-3a+1)}$

$= \frac{2(3a-1)}{9a^2-3a+1}$ Ans

$\because (3a+1)(9a^2-3a+1)=(3a)^3+1^3$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$\because 81a^4+9a^2+1=(9a^2+3a+1)(9a^2-3a+1)$

3. $\frac{2xy}{x^2-y^2}+\frac{2xy}{x^2+y^2}+\frac{4x^3y}{x^4+y^4}$

$= \frac{2xy(x^2+y^2)+2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)}+\frac{4x^3y}{x^4+y^4}$

$= \frac{2x^3y+\cancel{2xy^3}+2x^3y-\cancel{2xy^3}}{x^4-y^4}+\frac{4x^3y}{x^4+y^4}$

$= \frac{4x^3y(x^4+y^4)+4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$

$= \frac{4x^7y+\cancel{4x^3y^5}+4x^7y-\cancel{4x^3y^5}}{x^8-y^8}$

$= \frac{8x^7y}{x^8-y^8}$ Ans

Exponential Equations

Short questions:

* Solve

(1) $3^{x+1}+3^{x}=108$

or, $3^{x}.3^{1}+3^{x}=108$

or, $3^{x}(3+1)=108$

or, $3^{x}.4=108$

or, $3^{x}=108/4$

or, $3^{x}=27$

or $3^{x}=3^{3}$

$\therefore \boxed{x=3}$

(2) $2^{x}-2^{x-2}=6$

or, $2^{x}-2^{x}.2^{-2}=6$

or, $2^{x}(1-\frac{1}{4})=6$

or, $2^{x}.\frac{3}{4}=6$

or, $2^{x}=\frac{6\times4}{3}$

or, $2^{x}=8$

or $2^{x}=2^{3}$

$\therefore \boxed{x=3}$

(3) If $a=b^{x}$, $b=c^{y}$, $c=a^{z}$, prove, $xyz=1$.

Here, $a=b^{x}$

or, $a=(c^{y})^{x}$

or $a=c^{yx}$

or $a=(a^{z})^{yz}$

or $a^{1}=a^{xyz}$

$\therefore xyz=1$.

proved.

(9) $\frac{m+(mn^2)^{1/3}+(m^2n)^{1/3}}{m-n} \times \left(1-\frac{n^{1/3}}{m^{1/3}}\right)$

$= \frac{m+m^{1/3}n^{2/3}+m^{2/3}.n^{1/3}}{m-n} \times \left(\frac{m^{1/3}-n^{1/3}}{m^{1/3}}\right)$

let, $m^{1/3}=a \Rightarrow m=a^3$

and $n^{1/3}=b \Rightarrow n=b^3$

Now, $\frac{a^3+ab^2+a^2b}{a^3-b^3} \times \frac{a-b}{a}$

$= \frac{a(a^2+ab+b^2)}{a^3-b^3} \times \frac{a-b}{a}$

$= \frac{(a-b)(a^2+ab+b^2)}{a^3-b^3}$

$= \frac{a^3-b^3}{a^3-b^3}$

$= 1.$