

PHY 517 / AST 443: Observational Techniques in Astronomy

Lecture 5: Statistics Spectroscopy

Last Time:

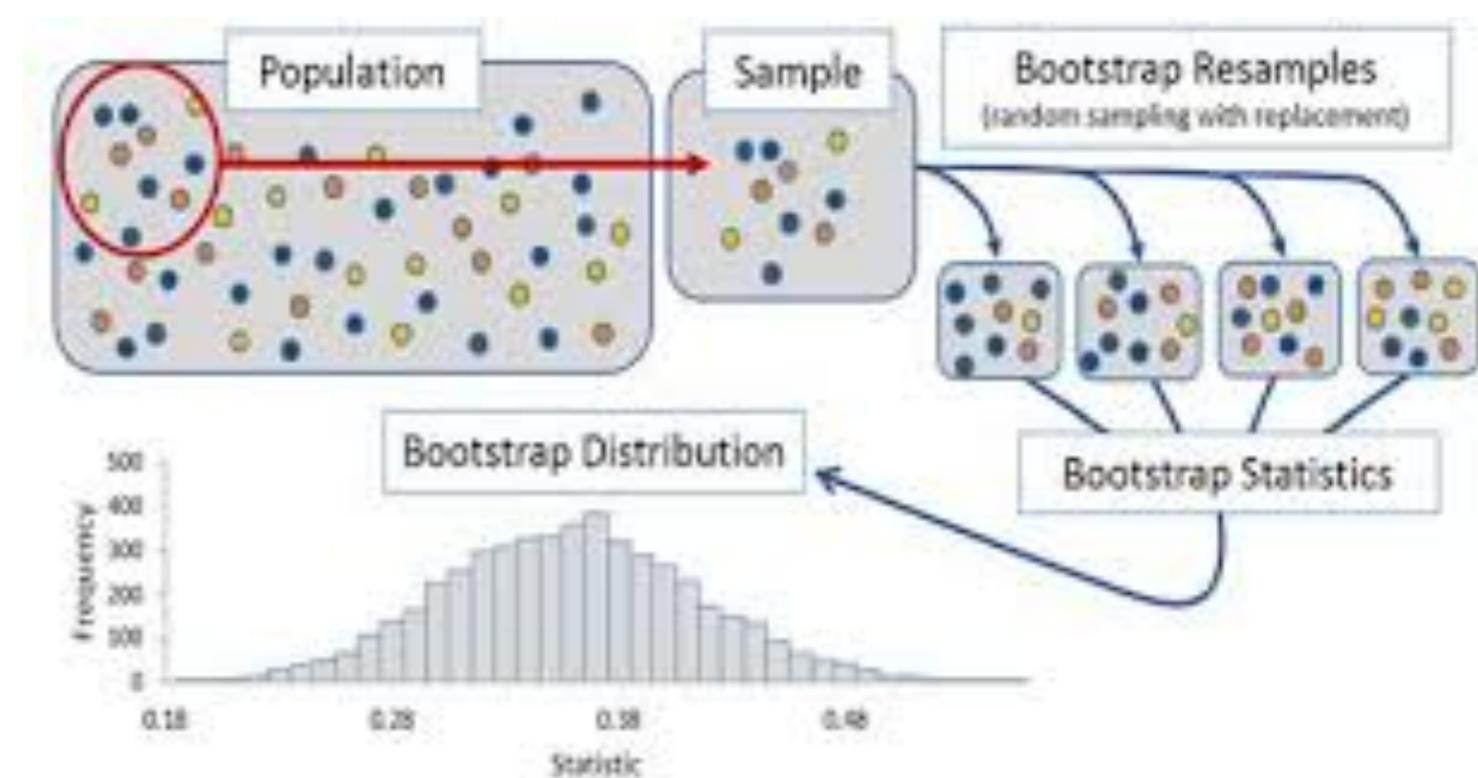
- sample distribution vs. parent population
- summary statistics
- uncertainty on the mean
- Binomial, Poisson, Gaussian distributions
- Central Limit Theorem
- statistical significance
- Gaussian error propagation

Resampling Methods

- suppose you have a sample distribution, and do not know the shape of the parent population
- ... and you want to measure *something* from your data, along with an uncertainty on *something*
- can almost always use a resampling method (bootstrap, jackknife, ...)
 - use subsets of the data
 - draw randomly from the data with replacement
 - swap labels on data points

Bootstrapping

- e.g. bootstrap: resampling with replacement
 - N measurements
 - draw from your measurements N times (can draw same measurement more than once)
 - determine derived quantity
 - repeat n times
 - quantify the bootstrap distribution



Model fitting

- to fit a model to a dataset, need to quantify how good the fit describes the data
- if errors are Gaussian, optimal statistic is χ^2 (“chi-squared”)

$$\chi^2 = \sum_i \frac{(D[x_i] - M[x_i])^2}{\sigma_i^2}$$

$D[x_i]$ are the data values; $M[x_i]$ are the values of the model evaluated at positions x_i

(note similarity to normal probability distribution!)

Model fitting

- the best-fitting model is the one that minimizes the χ^2 value

$$\chi_{\min}^2 = \sum_i \frac{(D[x_i] - M_{\text{best}}[x_i])^2}{\sigma_i^2}$$

how to find the best-fit model:

- brute force: make a grid of parameter values, calculate χ^2 for each
- use a minimization algorithm

Model fitting

- you have found the “best-fit” parameters of the model that minimize the χ^2 , but is that model actually a good fit?

$$\chi_{\nu}^2 = \frac{\chi_{\min}^2}{\nu}$$

- reduced chi-square: scale best-fit chi-square by ν , the number of free parameters = number of data points minus the number of free model parameters
- example: fitting a line: two model parameters (slope and intercept)

$$\nu = \text{number of data points} - 2$$

Model fitting

- given a random realization of an experiment with ν degrees of freedom, the probability to obtain χ_{\min}^2/ν or larger is described by the chi-squared distribution
- comparing the measured reduced chi-squared to the expectation (from the chi-squared distribution) is an indication whether the model is an acceptable fit to the data
- for an acceptable model, the remaining deviations should be well described by a random (Gaussian) process

$\chi_{\min}^2/\nu \approx 1$ model is a good fit

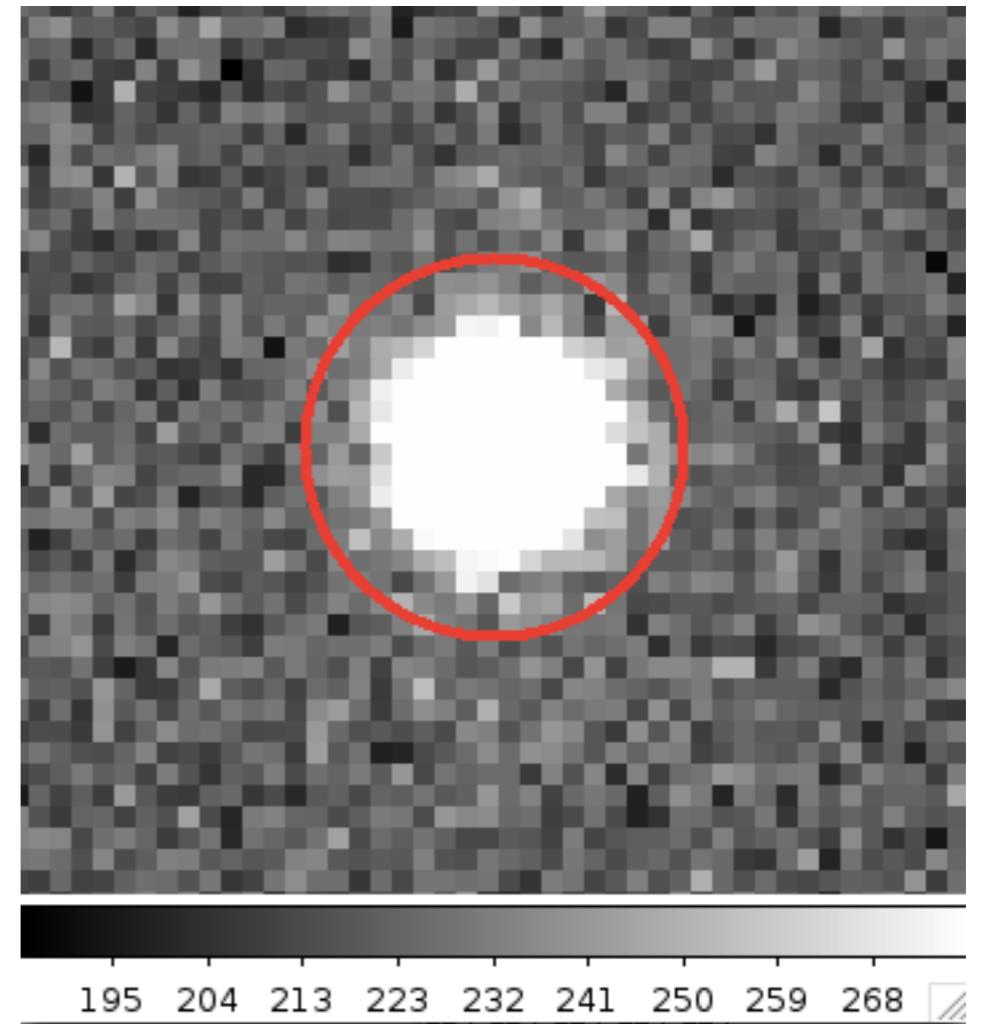
$\chi_{\min}^2/\nu \gg 1$ model is a bad fit

$\chi_{\min}^2/\nu \ll 1$ model is overfitting the data

Signal-to-Noise

flux measured in an aperture:

total electrons = electrons from
object + electrons from background
(sky, dark current, etc.)



signal = total electrons - background electrons

noise: counting statistics (i.e. Poisson distribution, \sqrt{N}),
have to consider noise from all sources

Signal-to-Noise

signal:

$$N_{\text{object}} = N_{\text{total}} - N_{\text{background}}$$

noise: if the noise contributions are independent of each other, can add quadratically:

$$\sigma = \sqrt{\sum_{i \in \text{noise terms}} \sigma_i^2}$$

Note: the “counting” processes apply to the **number of registered electrons**. The counts reported in the image have been rescaled by the gain, $N_{\text{counts}} = N_{\text{electrons}}/G$

Signal-to-Noise

noise contributions:

- shot noise from source

$$\begin{aligned}\sigma_{\text{object}} &= \sqrt{N_{\text{object}}} \\ &= \sqrt{S_{\text{object}} \times t}\end{aligned}$$

- sky noise

$$\begin{aligned}\sigma_{\text{sky}} &= \sqrt{N_{\text{sky}}} \\ &= \sqrt{s_{\text{sky}} \times n_{\text{pix}} \times t}\end{aligned}$$

- dark current noise

$$\begin{aligned}\sigma_{\text{dk}} &= \sqrt{N_{\text{dk}}} \\ &= \sqrt{s_{\text{dk}} \times n_{\text{pix}} \times t}\end{aligned}$$

- read-out noise

$$\sigma_{\text{ro}} = \text{RON} \times \sqrt{n_{\text{pix}}}$$

already a std. dev.

Signal-to-Noise

total signal-to-noise: can add noise components quadratically

$$\begin{aligned} SNR &= \frac{N_{\text{object}}}{\sqrt{\sum_{\text{noise}} \sigma_i^2}} \\ &= \frac{N_{\text{object}}}{\sqrt{N_{\text{object}} + N_{\text{sky}} + N_{\text{dk}} + N_{\text{ro}}}} \\ &= \frac{s_{\text{object}} \times t}{\sqrt{s_{\text{object}} \times t + n_{\text{pix}} \times s_{\text{sky}} \times t + n_{\text{pix}} \times s_{\text{dk}} \times t + n_{\text{pix}} \times \text{RON}^2}} \end{aligned}$$

“CCD signal-to-noise equation”

Signal-to-Noise

in general, you do not want to be limited by dark current and read-out noise!

limiting case 1: very bright object $N_{\text{object}} \gg N_{\text{other}}$

$$SNR = \frac{N_{\text{object}}}{\sqrt{N_{\text{object}}}} = \frac{s_{\text{object}} \times t}{\sqrt{s_{\text{object}} \times t}}$$
$$\propto \sqrt{t}$$

Signal-to-Noise

in general, you do not want to be limited by dark current and read-out noise!

limiting case 2: faint objects

$$N_{\text{sky}} \gg N_{\text{other}}$$

$$\begin{aligned} SNR &= \frac{N_{\text{object}}}{\sqrt{N_{\text{sky}}}} = \frac{s_{\text{object}} \times t}{\sqrt{s_{\text{sky}} \times n_{\text{pix}} \times t}} \\ &\propto \sqrt{t} \end{aligned}$$

Sky Background

twilight:

Sun at -6° : “civil twilight”, still bright

Sun at -12° : “nautical twilight”, can see bright stars

Sun at -18° : “astronomical twilight”

twilight is scattered light (blue)

observations in different filters are affected differently

sky is “dark” in red filters before -18°

Sky Background

Patat 2004

moonlight:

detrimental in the very blue; not a big problem in the infrared

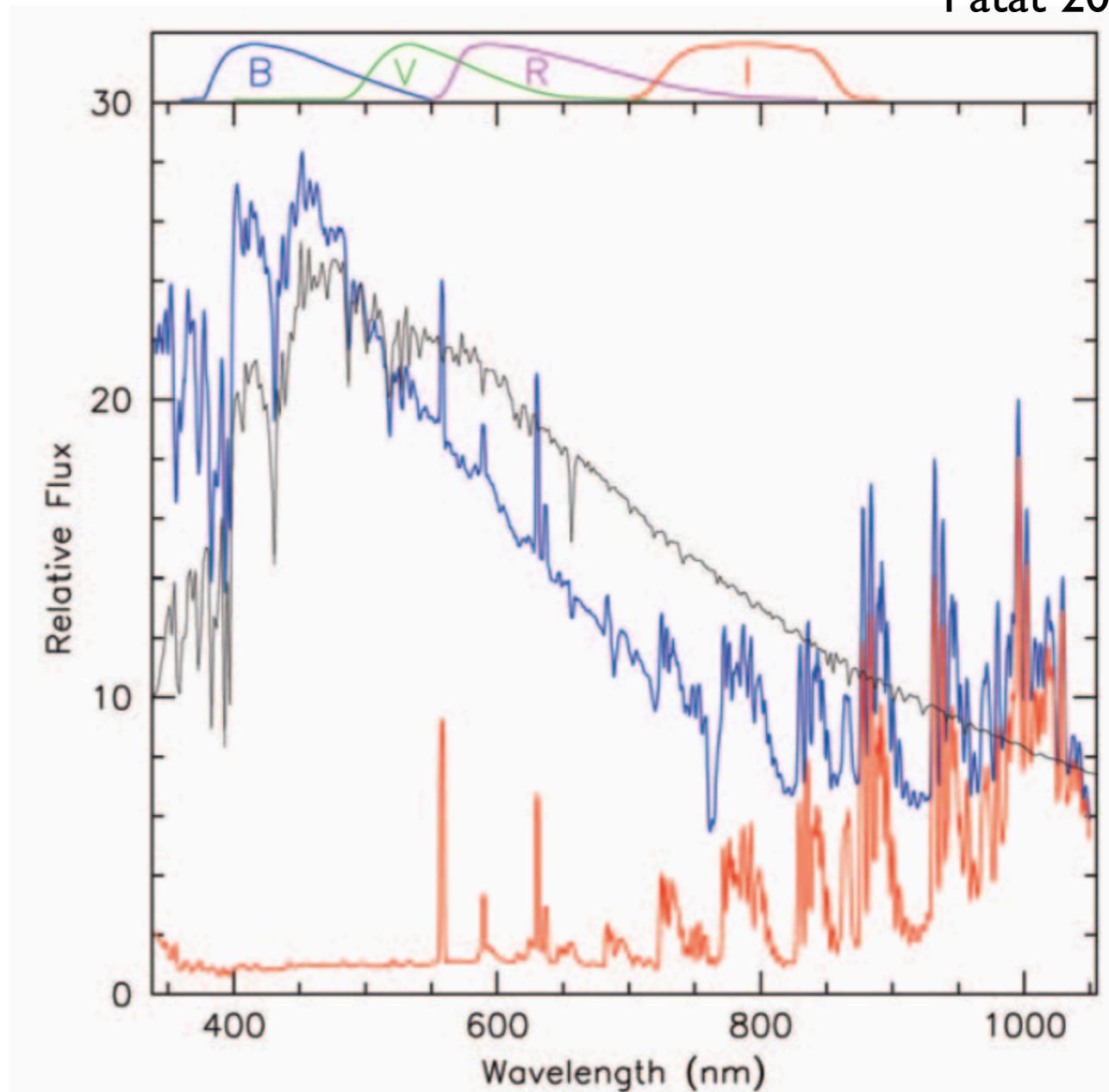
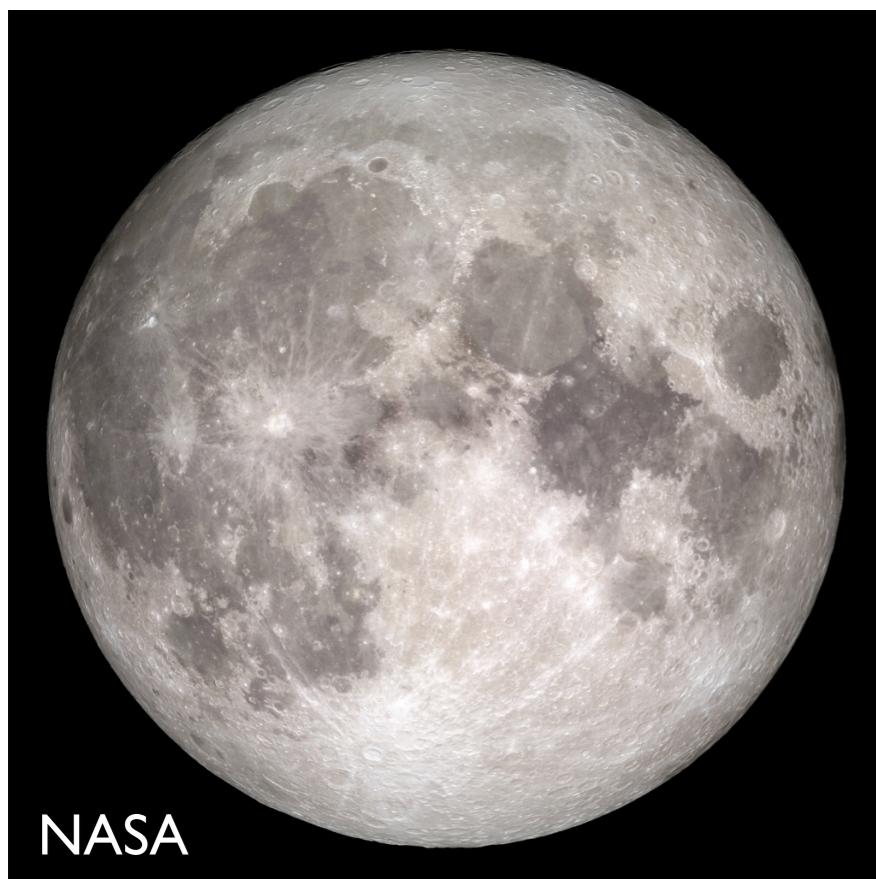
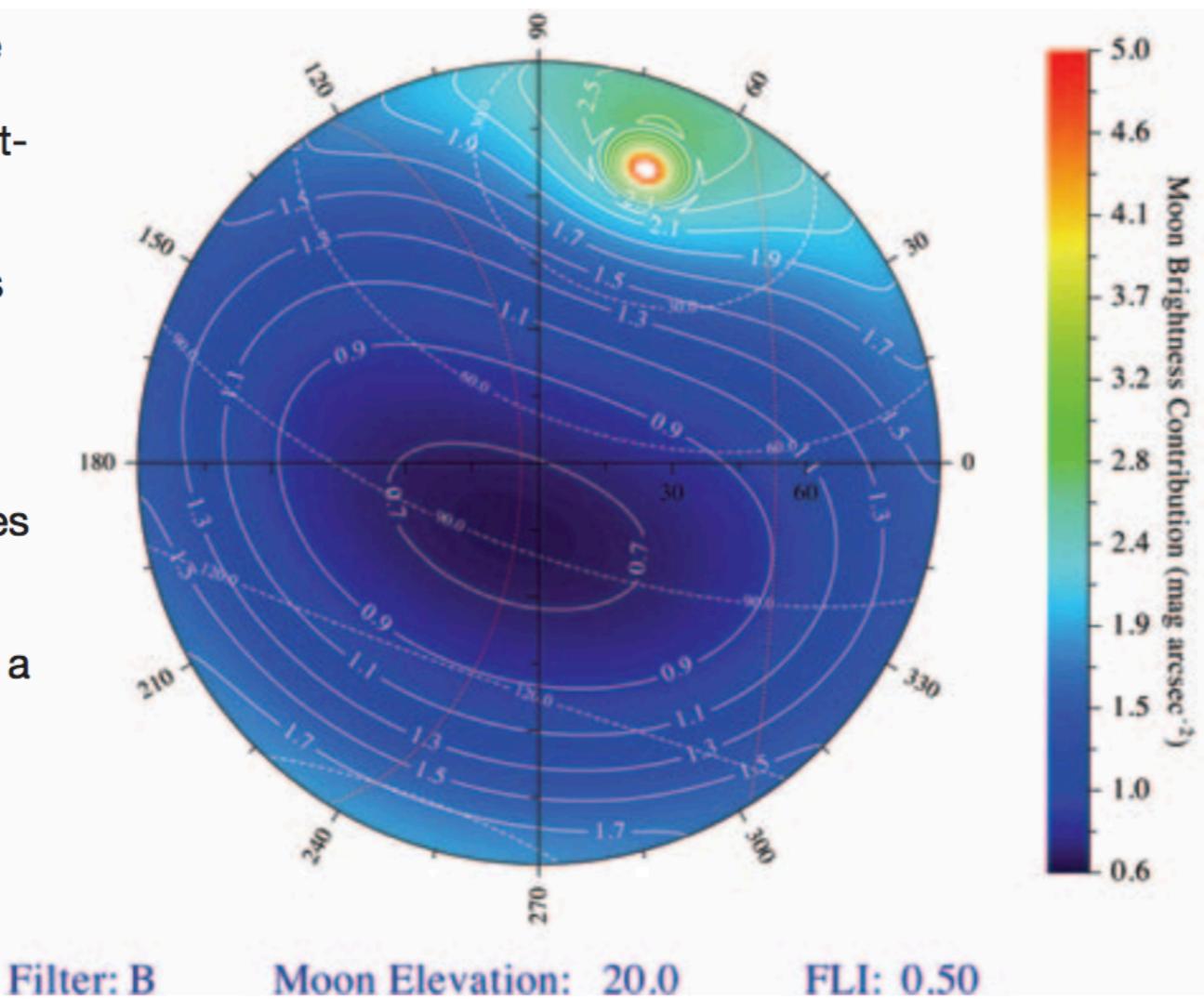


Figure 2: Comparison between the night sky spectrum during dark time (red line, Patat 2003) and bright time (blue line). The latter was obtained with FORS1 on September 1, 2004 using the low dispersion grism 150I and no order sorter filter. Due to the very blue continuum, the spectral region at wavelengths redder than 650 nm is probably contaminated by the grism second order. Both spectra have been normalized to the continuum of the first one at 500 nm. For comparison, the model spectrum of a solar-type star is also plotted (black line). For presentation, this has been normalized to the moonlit night sky spectrum at 500 nm. The upper plot shows the standard *BVRI* Johnson-Cousins passbands.

Sky Background

sky brightness from moonlight depends on distance from it, and from horizon

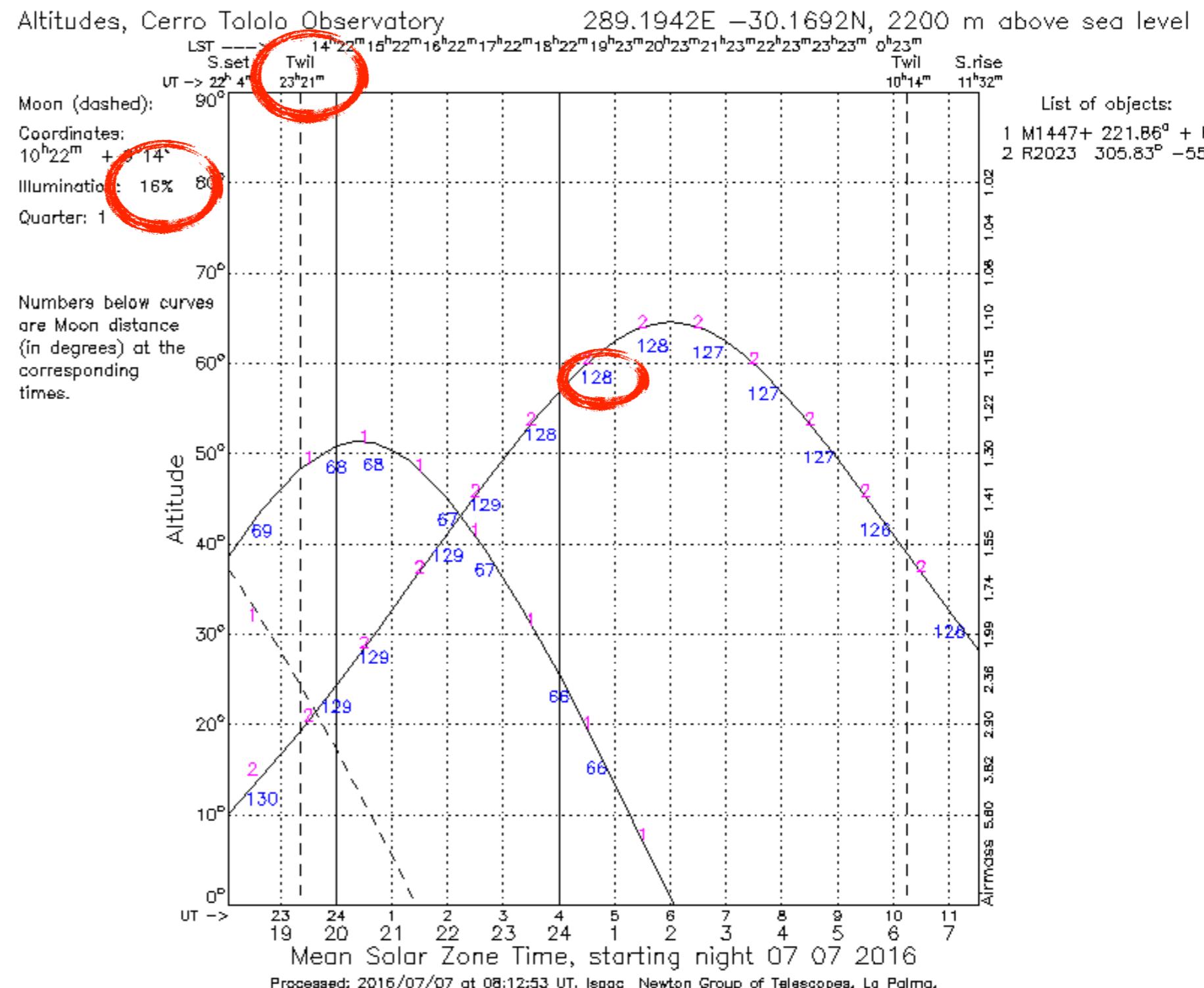
Figure 5: Example isophotal *alt-az* map for the expected moonlight contribution. The dashed white lines trace the loci at constant angular distance from the moon, while the two dotted red lines indicate the extreme apparent lunar paths during a full Saros cycle.



in addition:
moonlight can
cause reflections
inside telescope,
from dome, etc.

StarAlt

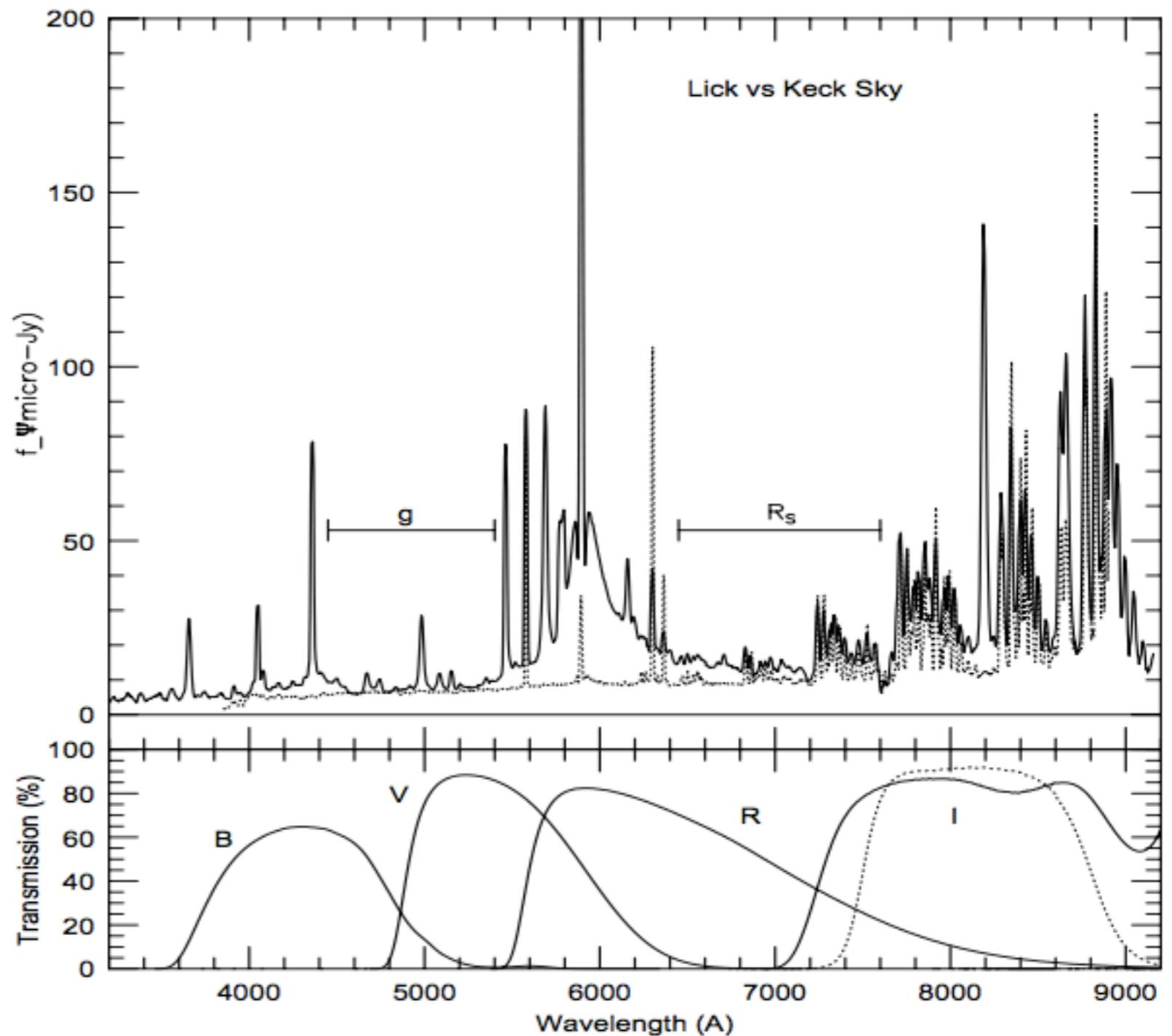
- indicates times of astronomical twilight
- indicates lunar illumination
- indicates distance to the Moon



Sky Background

limits most astronomical observations!

always present:
emission from
atmosphere
(+city lights)



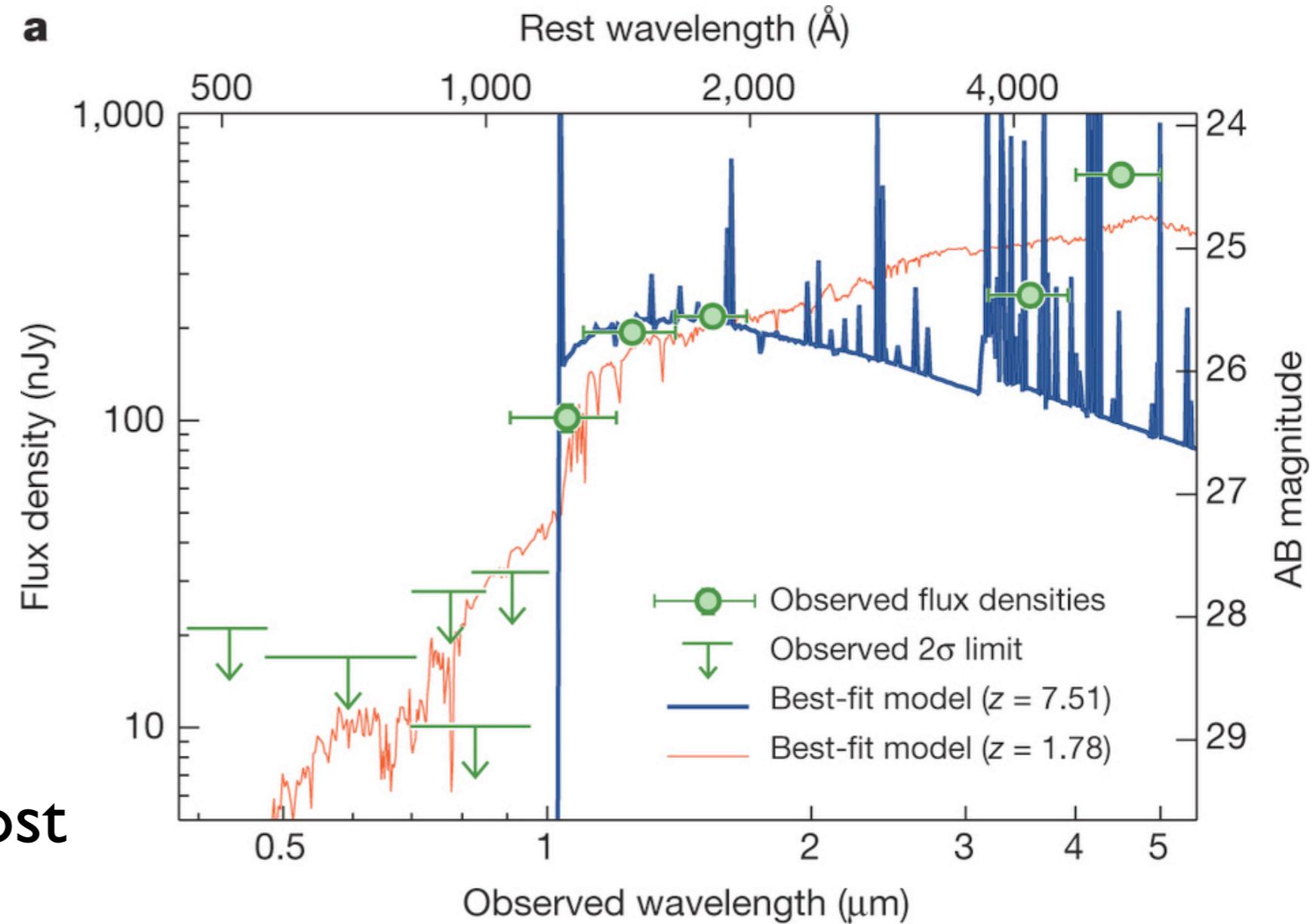
Spectroscopy

Motivation

photometry (measuring flux from images) only measures integrated flux

gives some information about the object properties, but often not enough

e.g.: finding the most distant galaxies

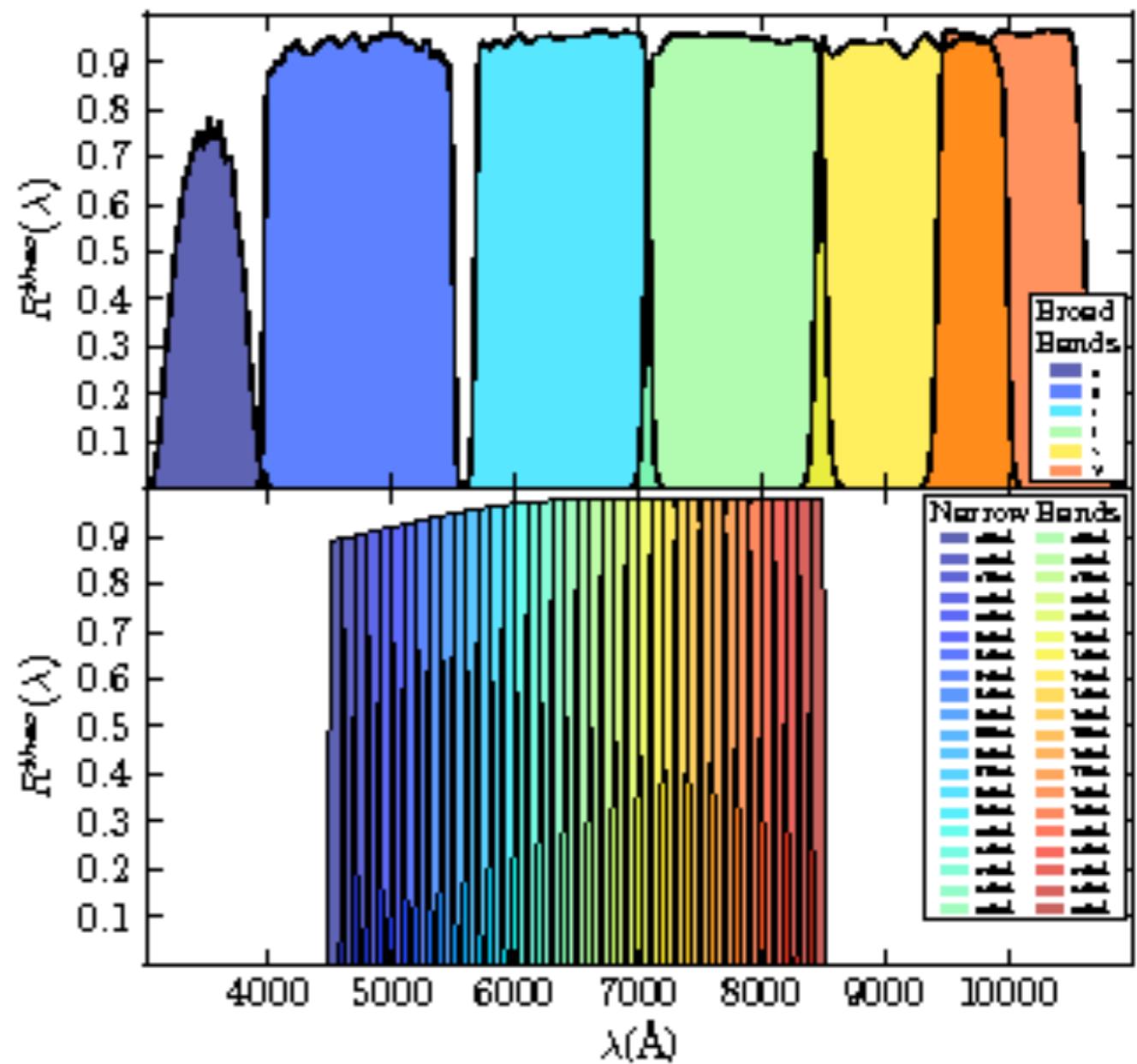


Narrow-band imaging

can determine spectrum of object with images in many narrow-band filters

advantage: can determine spectra of all objects in the same FOV

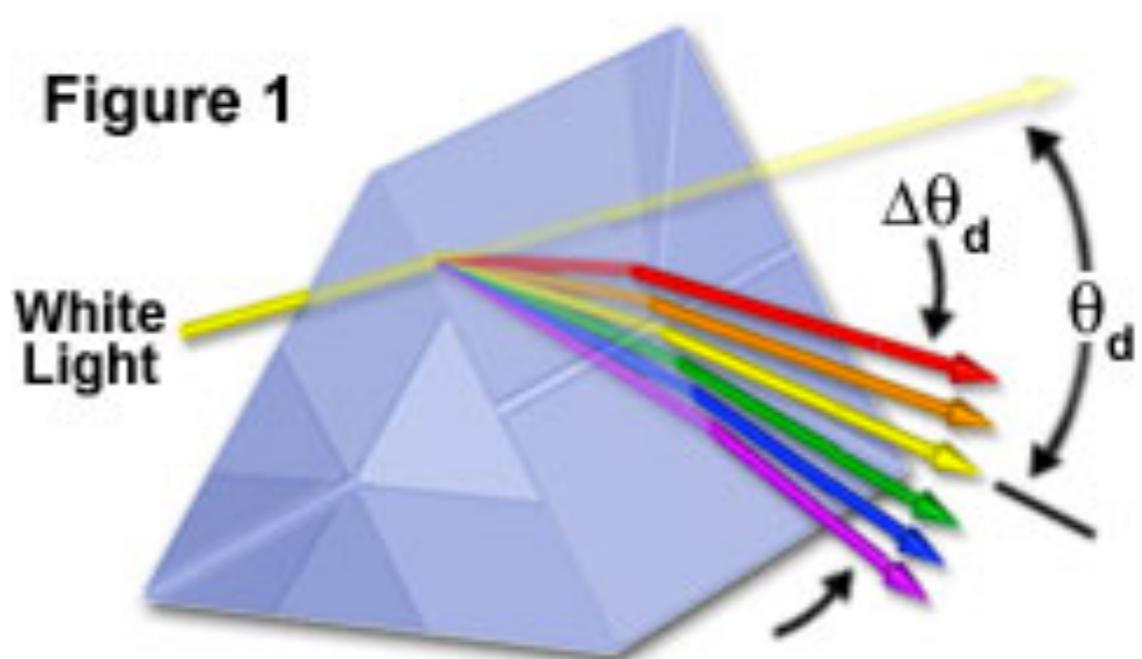
disadvantage: have to take a lot of images!



Spectroscopy

add a dispersing element
to split up the light from
an object: measure the
spectrum directly

e.g. a prism:

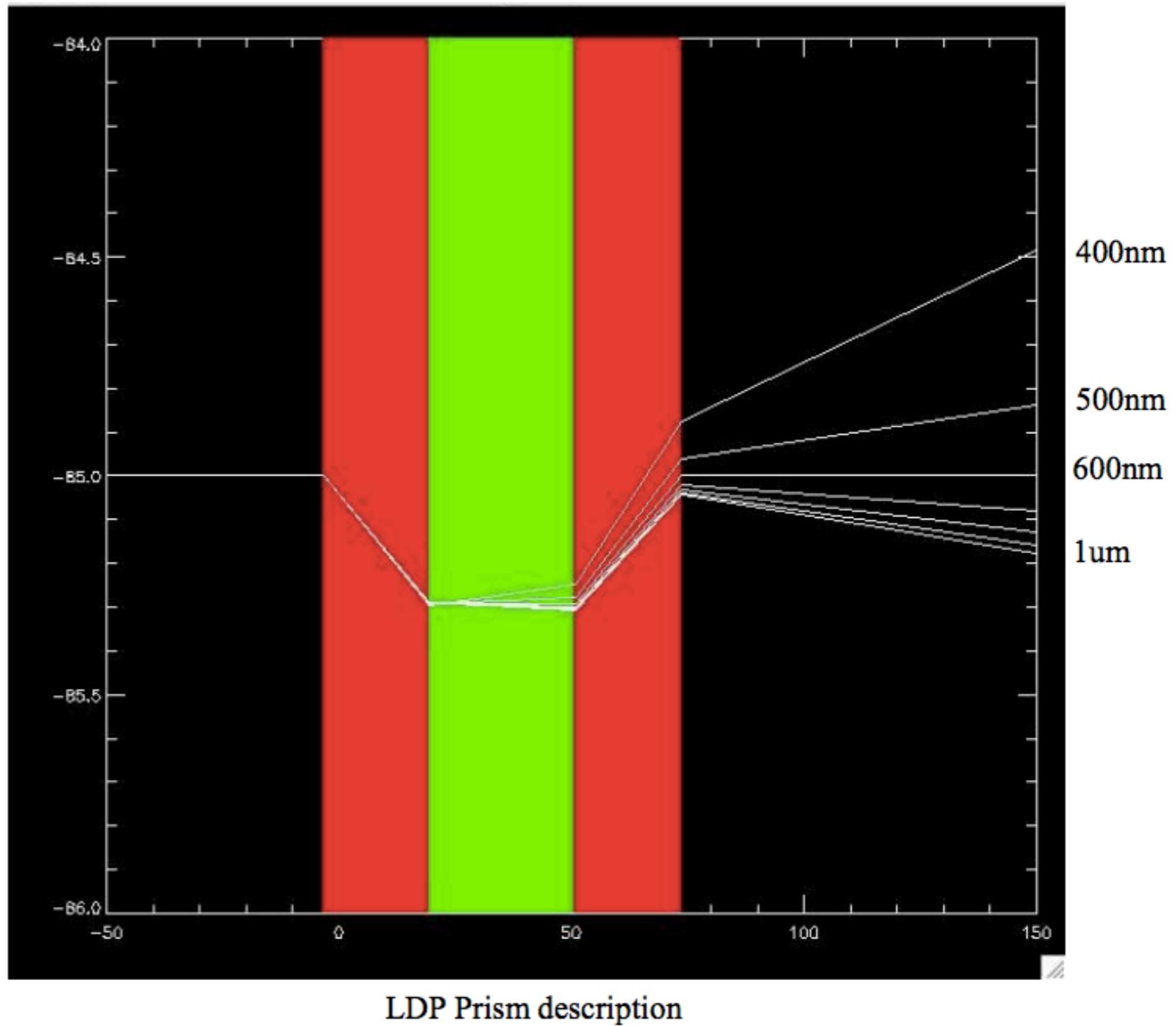


Olympus

Prism Spectroscopy

only few astronomical spectrographs use prisms

- low dispersion (resolution)
- dispersion varies with wavelength



“low dispersion prism” for IMACS spectrograph on Magellan 6-m telescope; uses 3 prisms

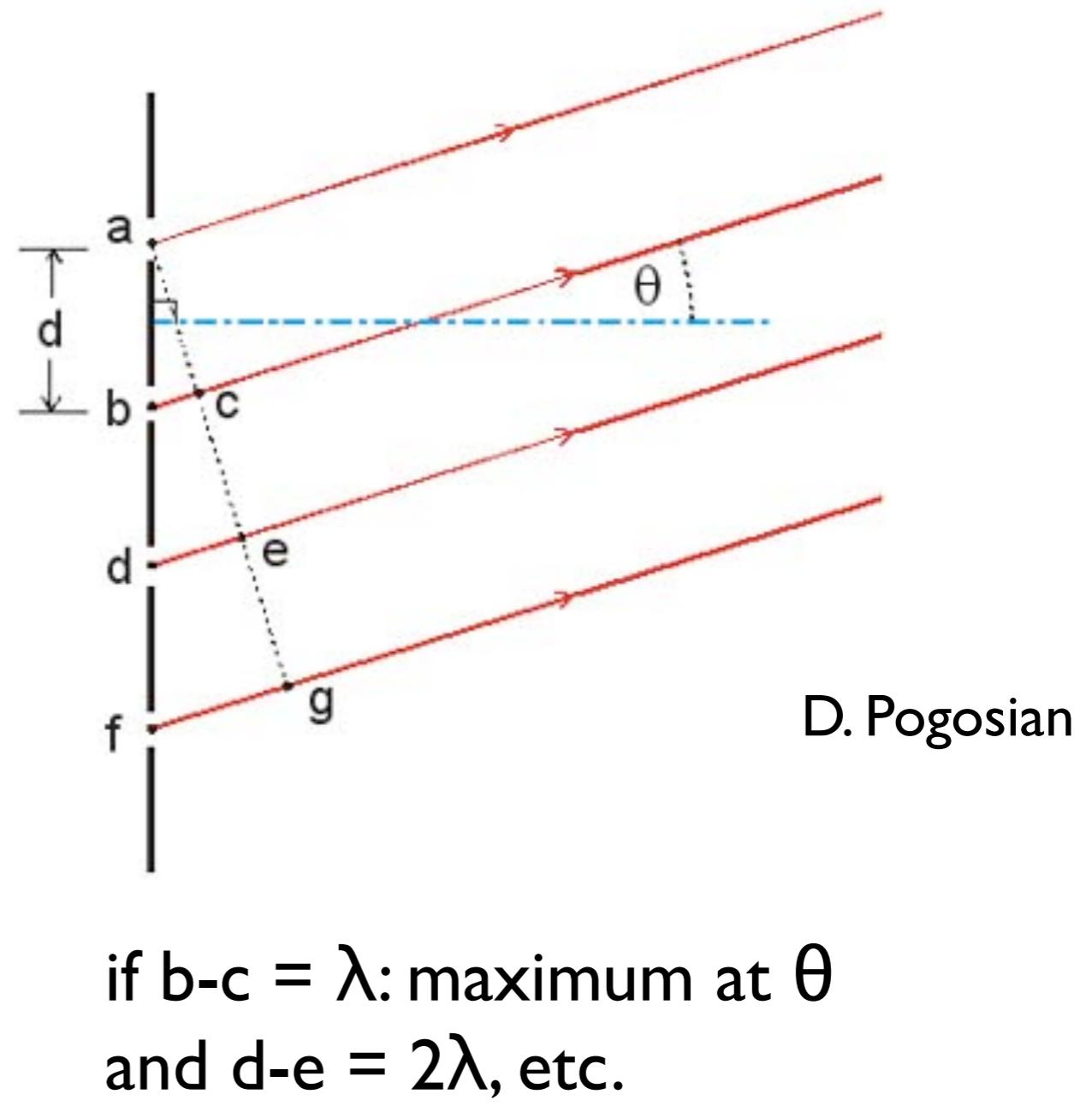
Diffraction gratings

make use of wave properties of light:
interference

grating: many parallel lines ($\sim 500/\text{mm}$)

similar to single-slit and double-slit experiments

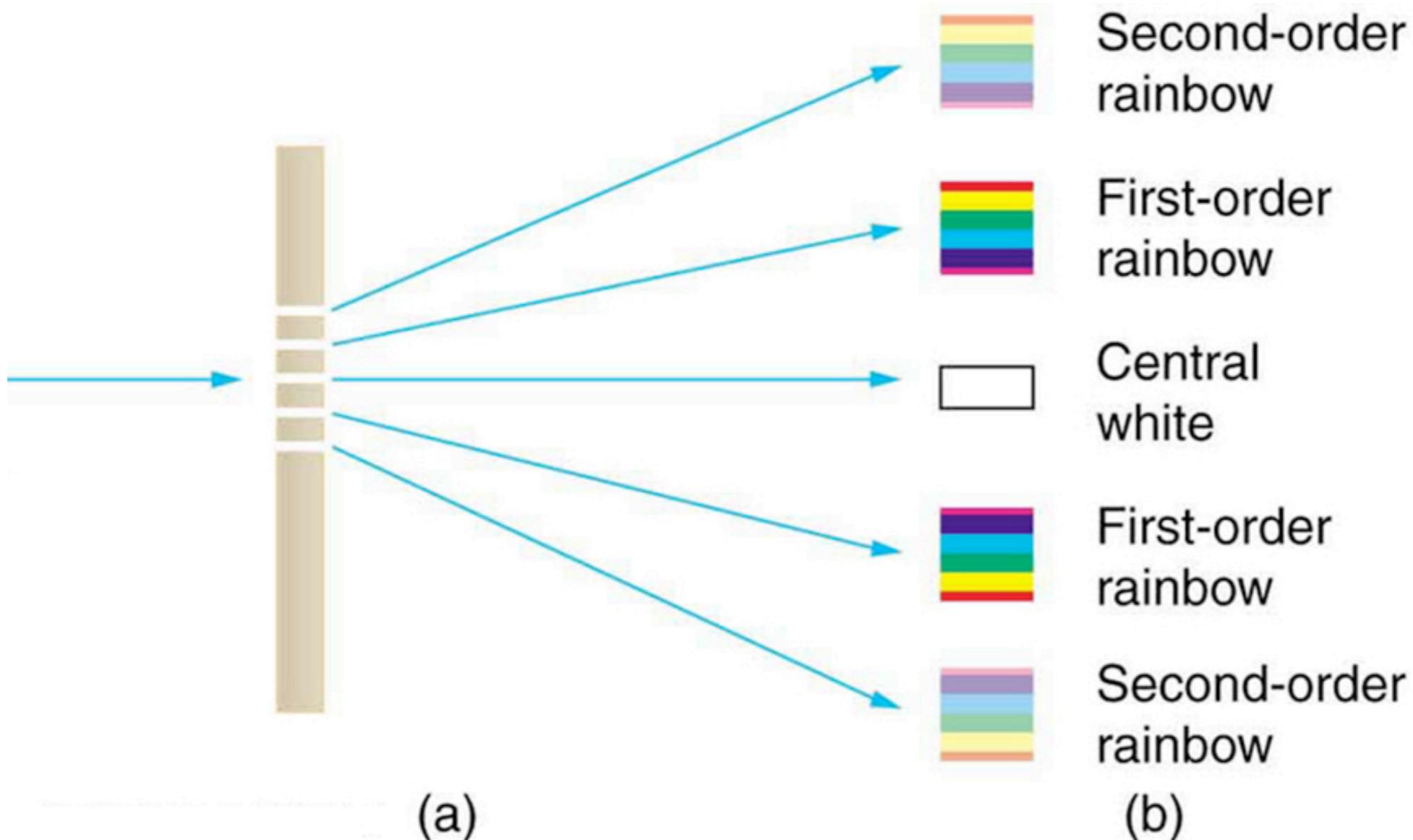
position of n th order:



if $b-c = \lambda$: maximum at θ
and $d-e = 2\lambda$, etc.

$$n\lambda = d \sin \theta$$

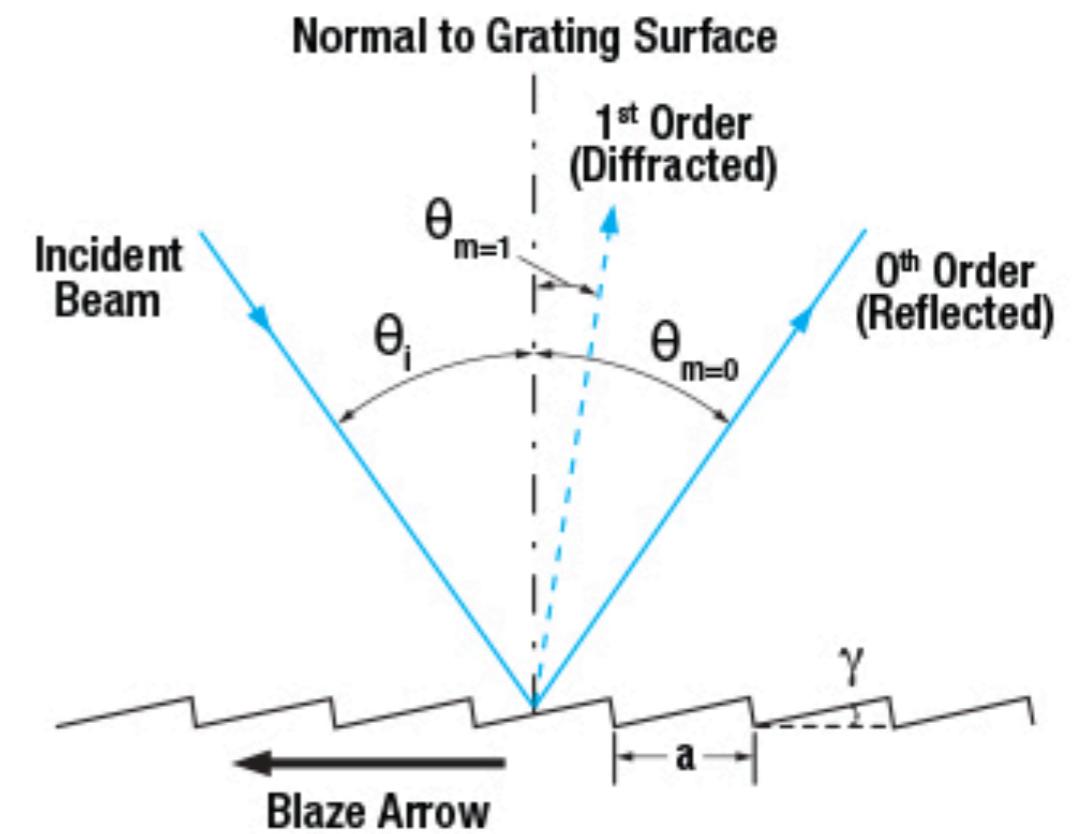
Diffraction gratings



Diffraction gratings

can be transmission gratings
or reflection gratings

most astronomical
spectrographs use reflection
gratings



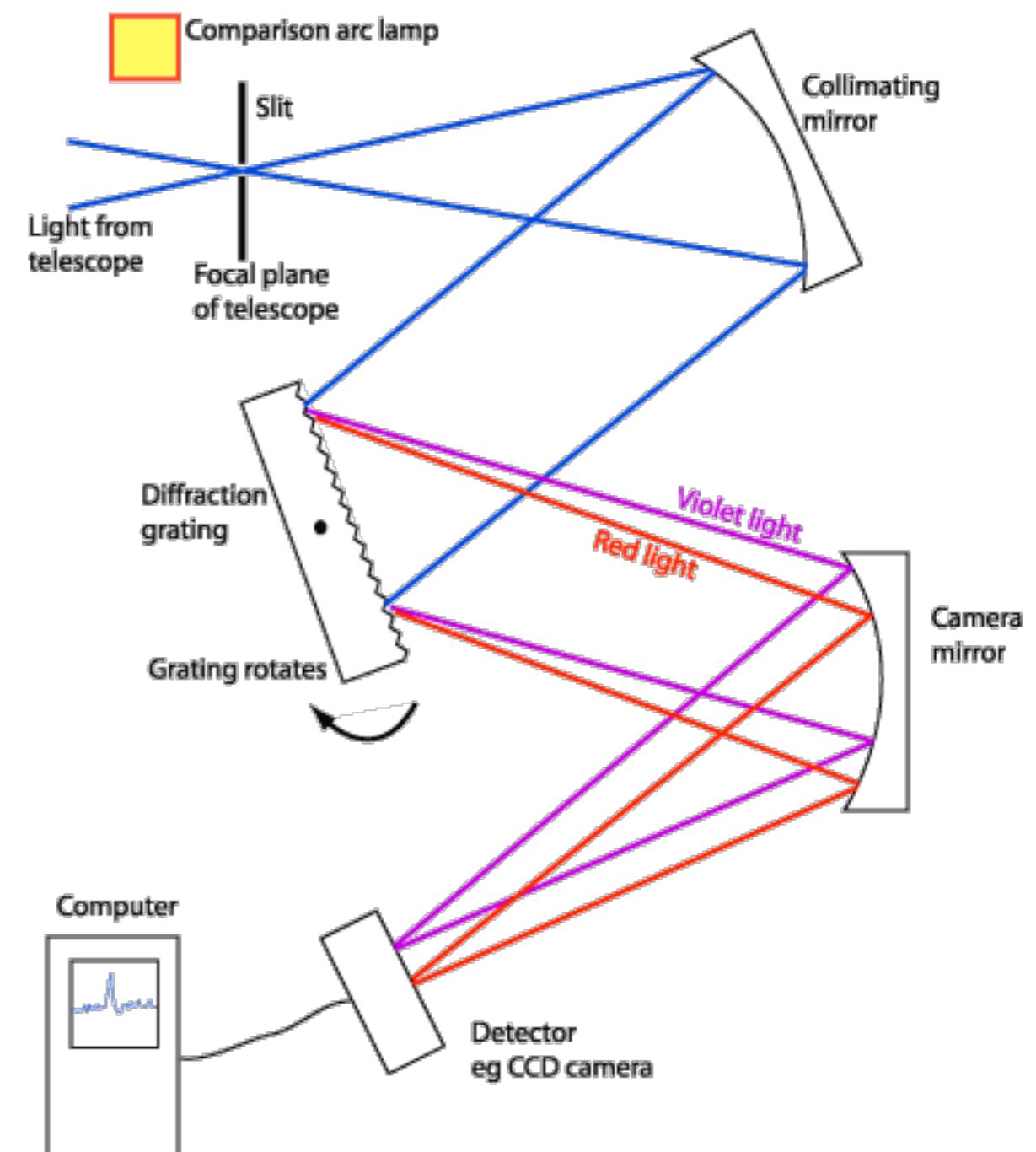
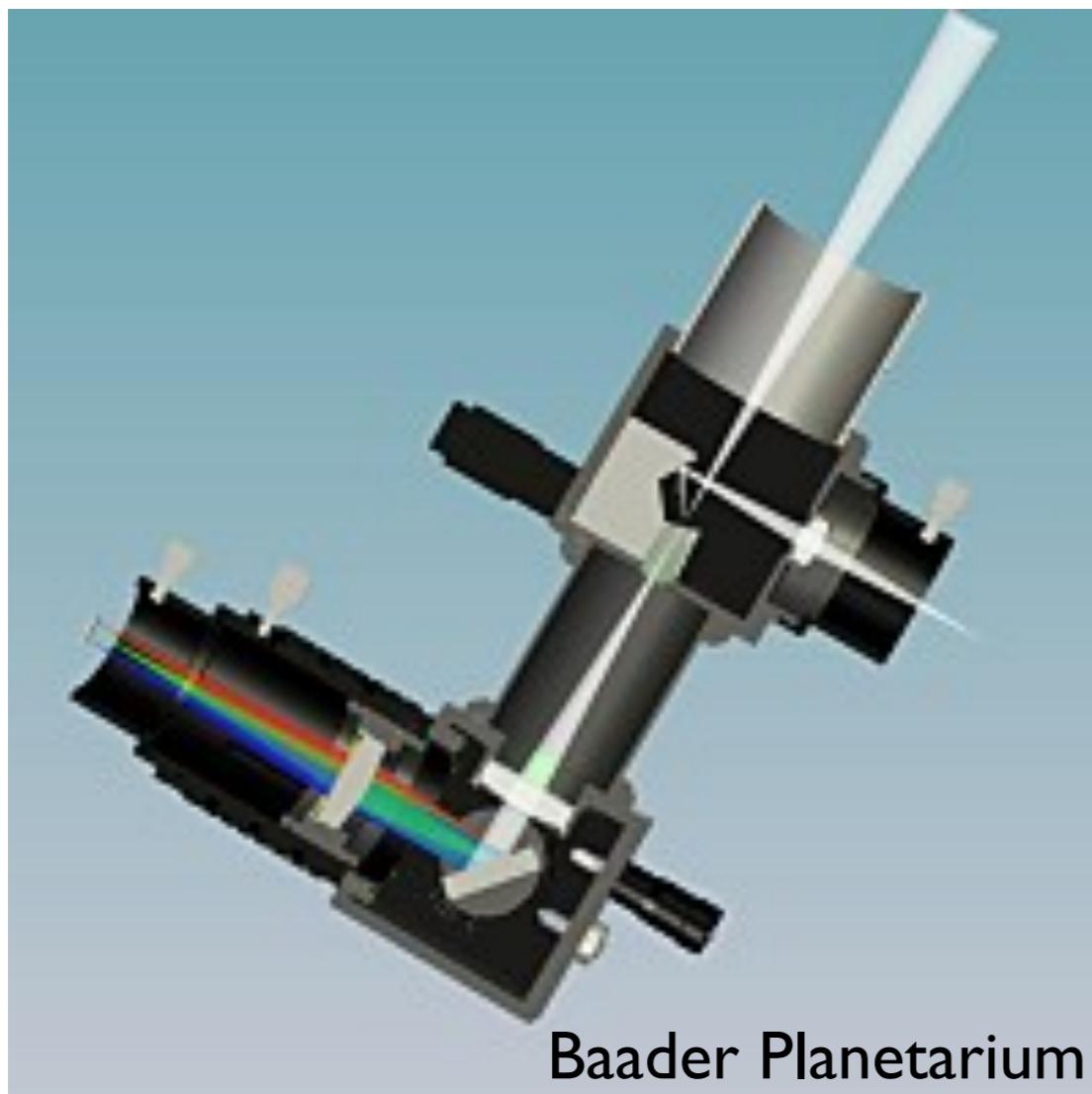
Thor Labs

blaze wavelength: wavelength for direction of reflection
coincides with desired spectral order
→ maximal efficiency

Typical spectrograph

entrance: usually a slit, similar to seeing size

collimator: converts a diverging beam to a parallel beam



A Schematic Diagram of a Slit Spectrograph

Spectral Resolution

defined by smallest wavelength difference $\Delta\lambda$ that can be distinguished at wavelength λ

$$R = \frac{\lambda}{\Delta\lambda}$$

determined by:

- grating (line density)
- width of entrance slit
- seeing

resolution: R or $\Delta\lambda$

dispersion: length $\Delta\lambda'$ of spectrum over single pixel, [$\text{\AA}/\text{px}$]

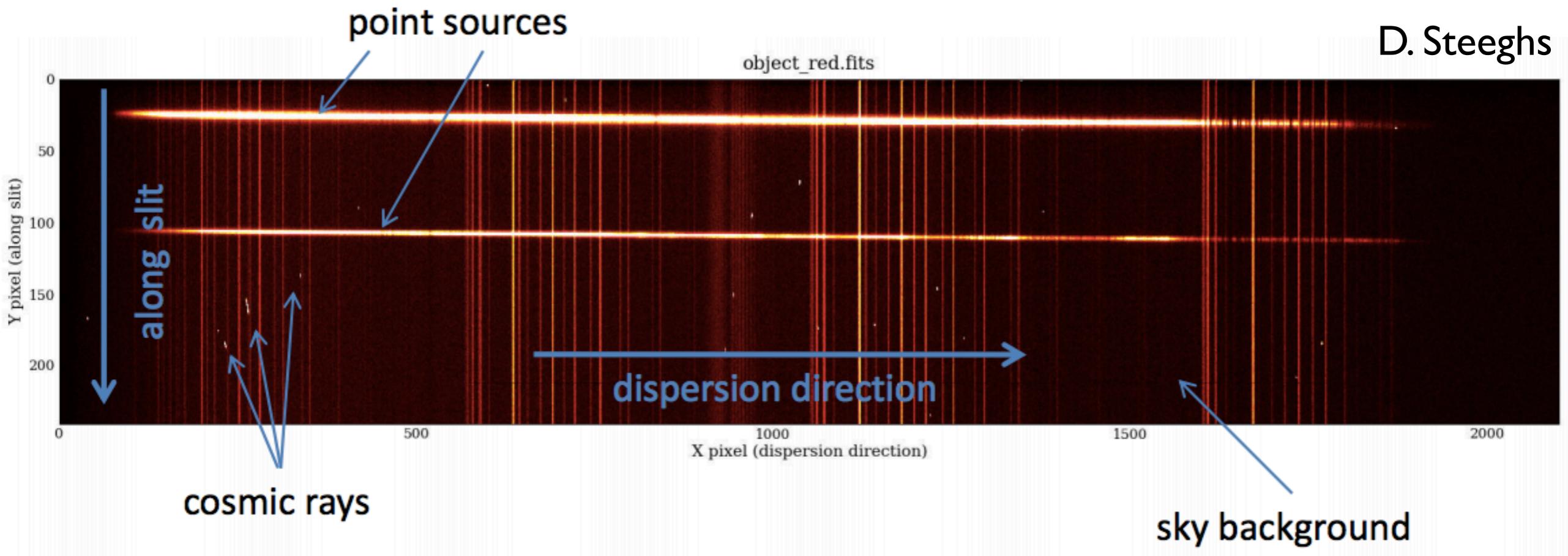
to properly sample the spectrum:

$$\Delta\lambda \sim 2 - 3 \Delta\lambda'$$

Spectral Resolution

$R < 1000$	low-resolution	e.g. our “low-resolution” spectrograph
$1000 < R < 10,000$	medium-resolution	e.g. our “high-resolution” spectrograph
$R > 10,000$	high-resolution	Echelle spectrographs

A long-slit observation



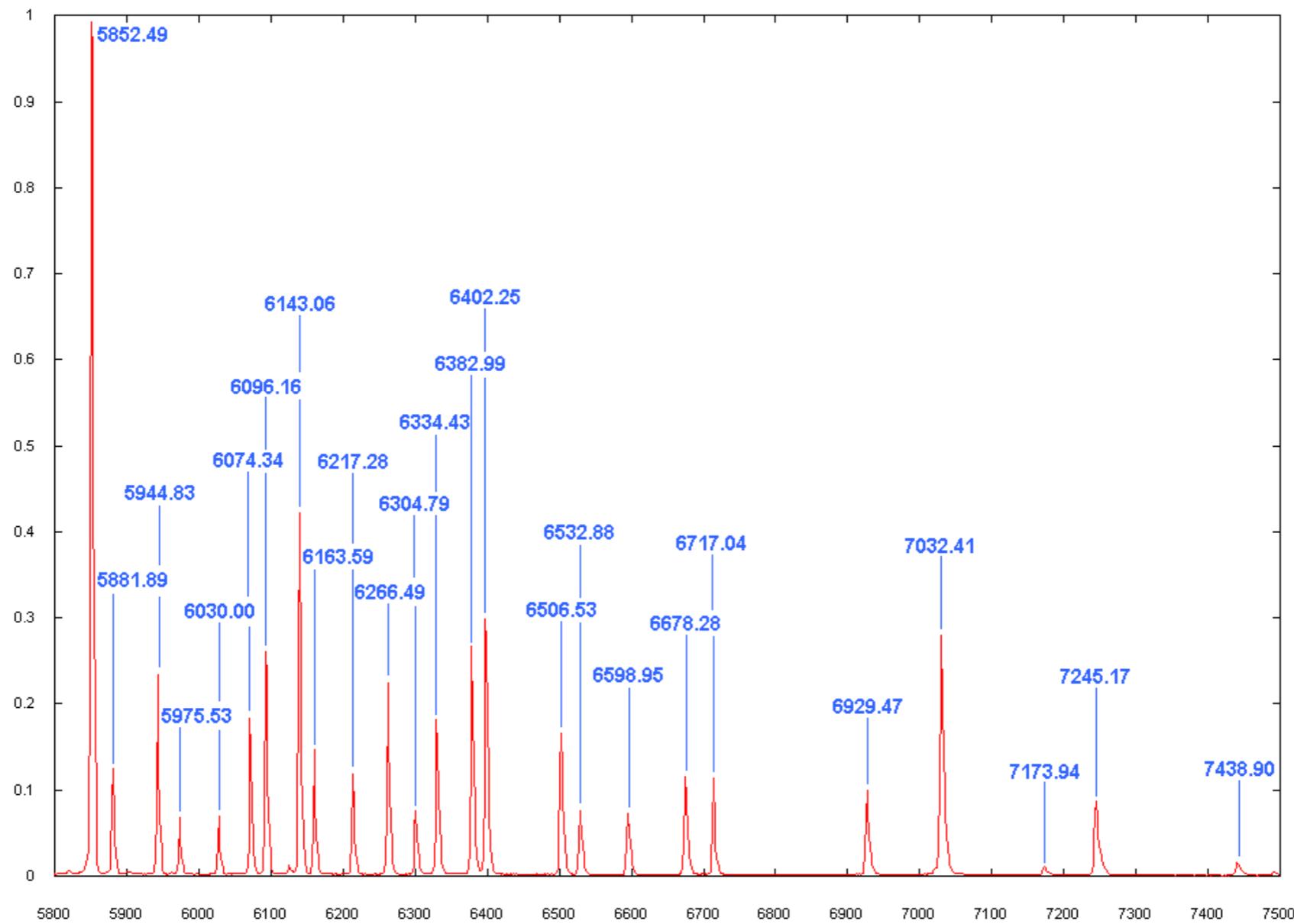
- long axis of CCD used to sample spectrum
- spatial information along slit still available: two objects, lots of sky
- sky background has a lot of emission lines!

Spectroscopic Calibration

- dark frames!
- flat field: use bright continuum source
 - small-scale pixel sensitivity variation
 - variations in slit width
- wavelength calibration: which position on the CCD corresponds to which wavelength?
 - use “arc” lamps with discrete emission lines
 - can also use sky emission lines
- flux calibration:
 - “spectrophotometric” standard stars: stars with known spectral shapes, smooth continua

Spectroscopic Calibration

wavelength calibration: map pixel position to emission lines

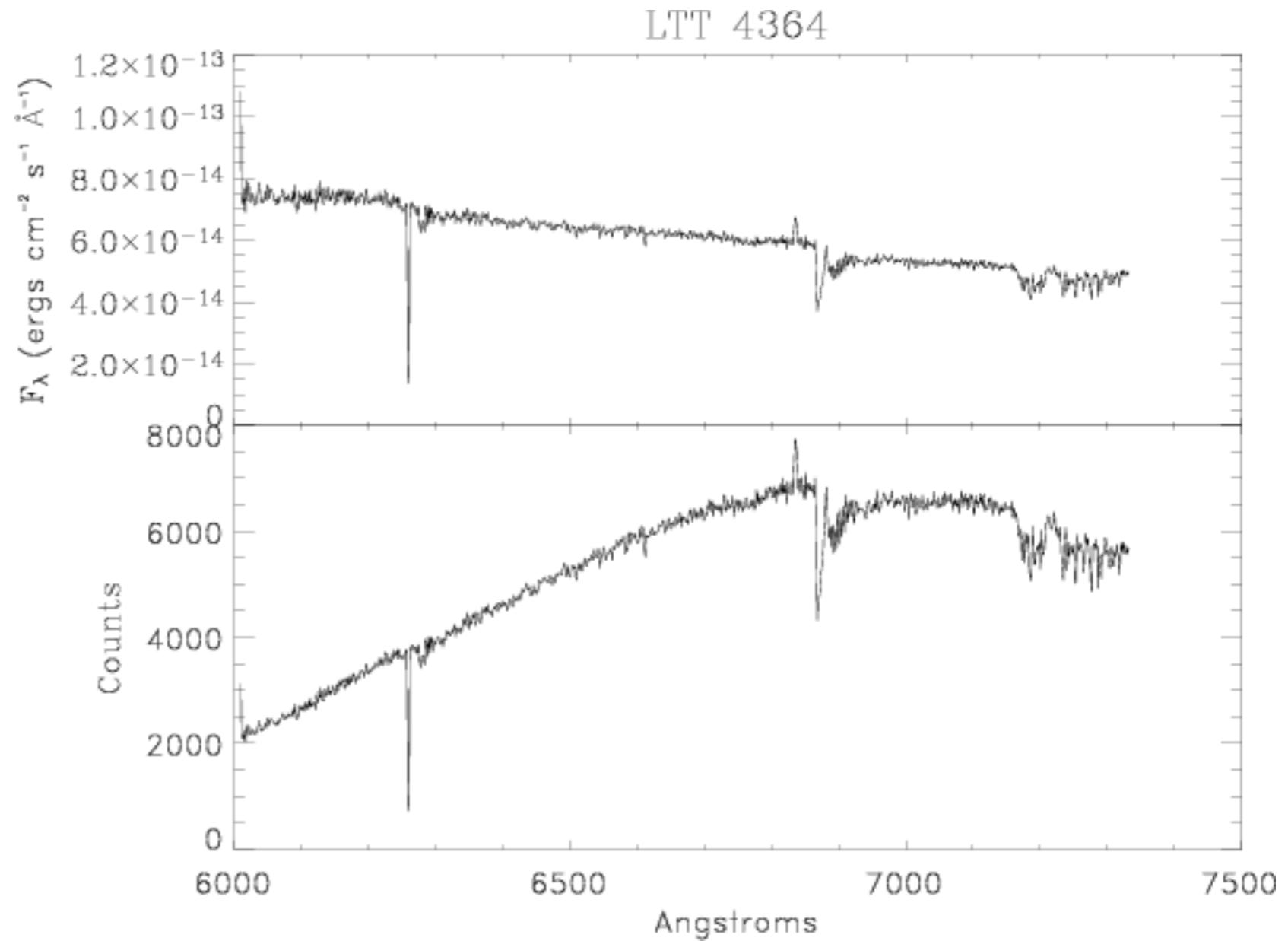


C. Buil /
astrosurf

Spectroscopic Calibration

flux calibration:
observe
spectrophotometric
standard star

compare observed
spectrum (counts)
to known spectrum



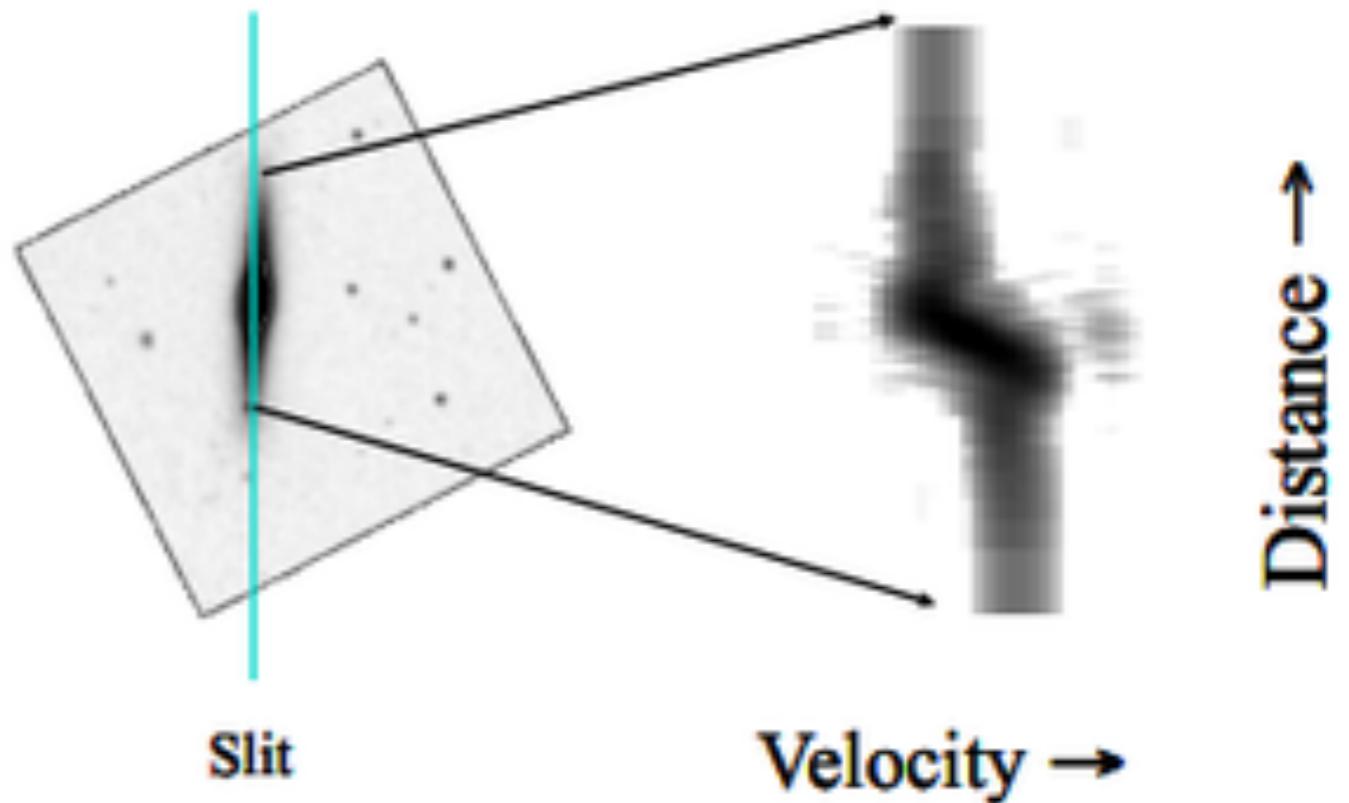
Long-slit spectrographs

most common spectrograph

can only target one (or a few) objects

gives spatial variation

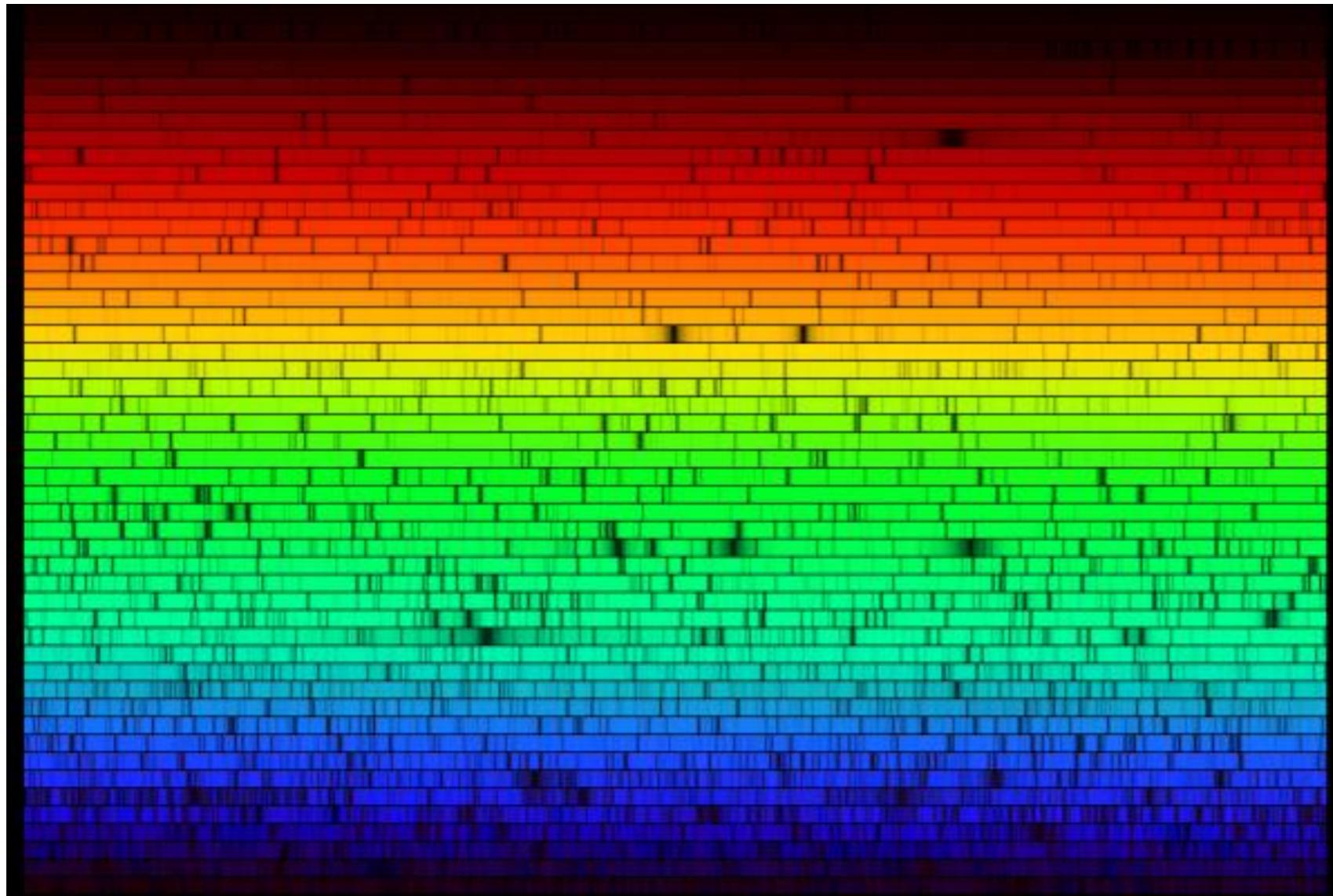
very good estimate of sky background



wikipedia

Echelle spectrographs

- very high resolution long-slit spectrographs
- have additional elements to fit entire spectrum onto CCD
- only for bright objects

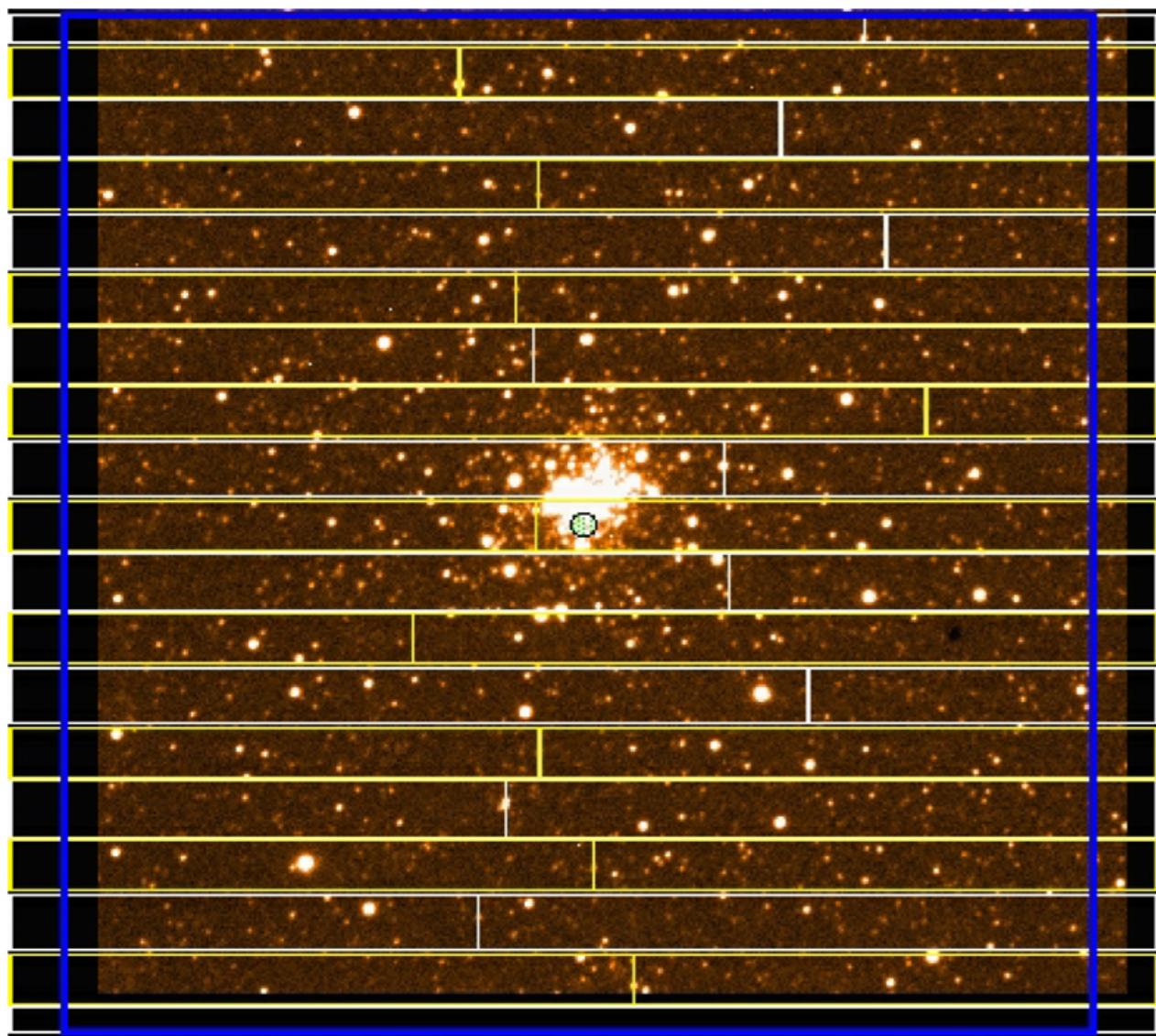


Echelle spectrum
of the Sun,
4000-7000Å

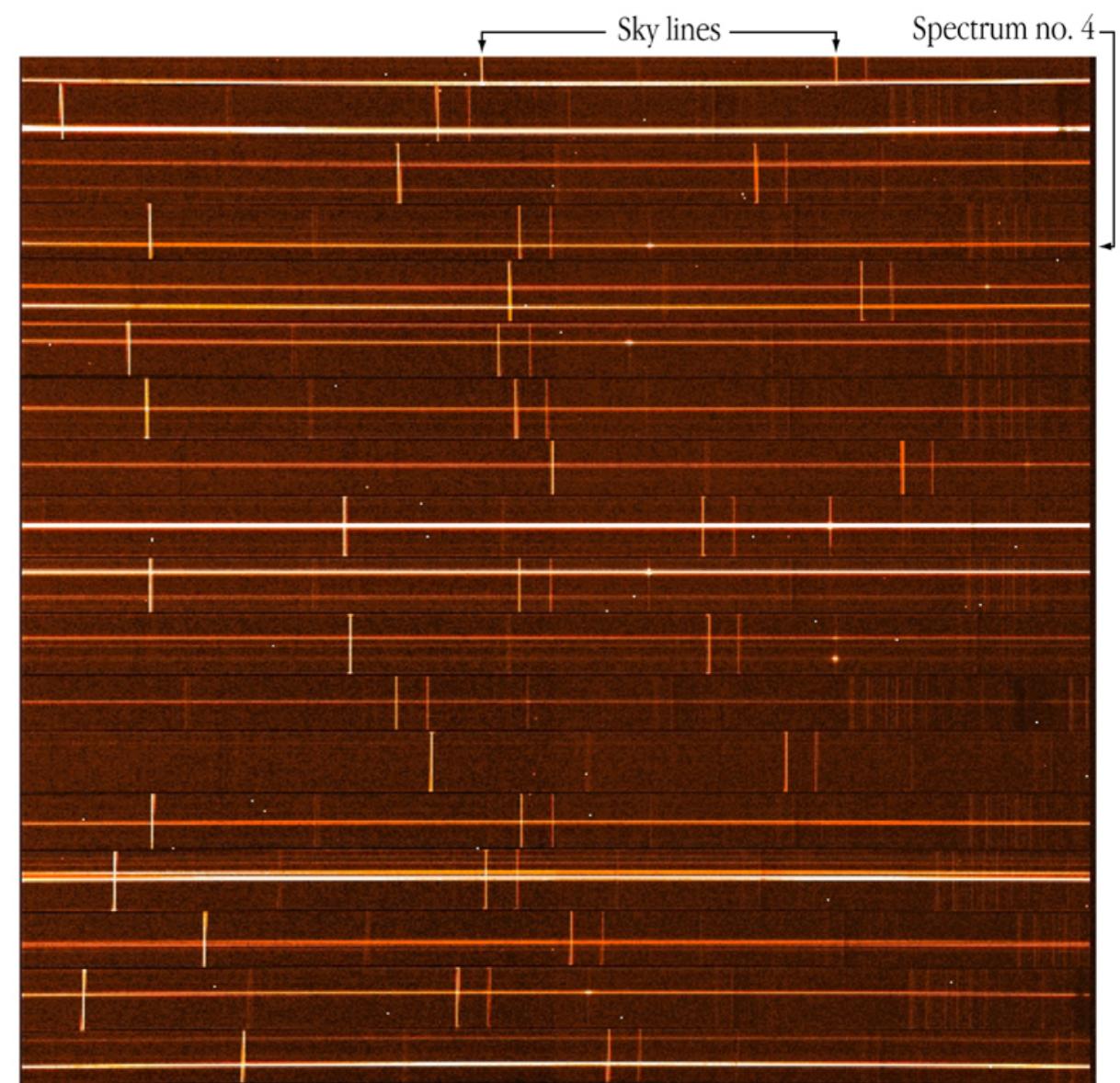
NOAO

Multi-object spectrographs

make a mask with multiple slits, one per target



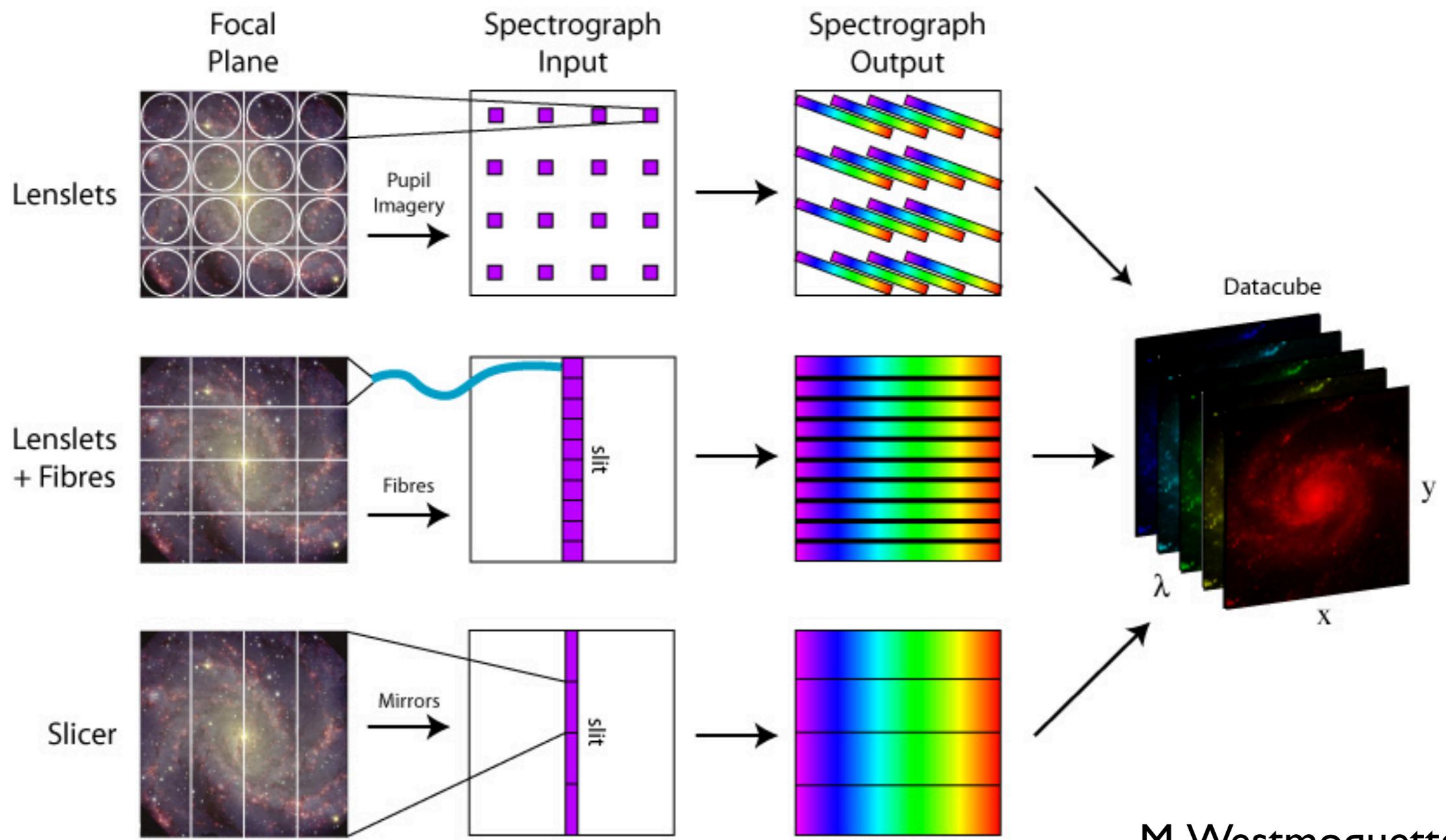
Open Cluster NGC 330 in SMC - VLT UT1 + FORS1 (MOS-mode)



Spectra of Stars in Open Cluster NGC 330 in SMC - VLT UT1 + FORS1 (MOS-mode)

Integral-Field Units

divide image into “spaxels” (spectroscopic pixels)



HW 3

- will be posted on github schedule (and blackboard) after the lecture
- due Feb. 24