PHY 517 / AST 443:
Observational Techniques in Astronomy

Lecture 5:
Statistics /
Spectroscopy

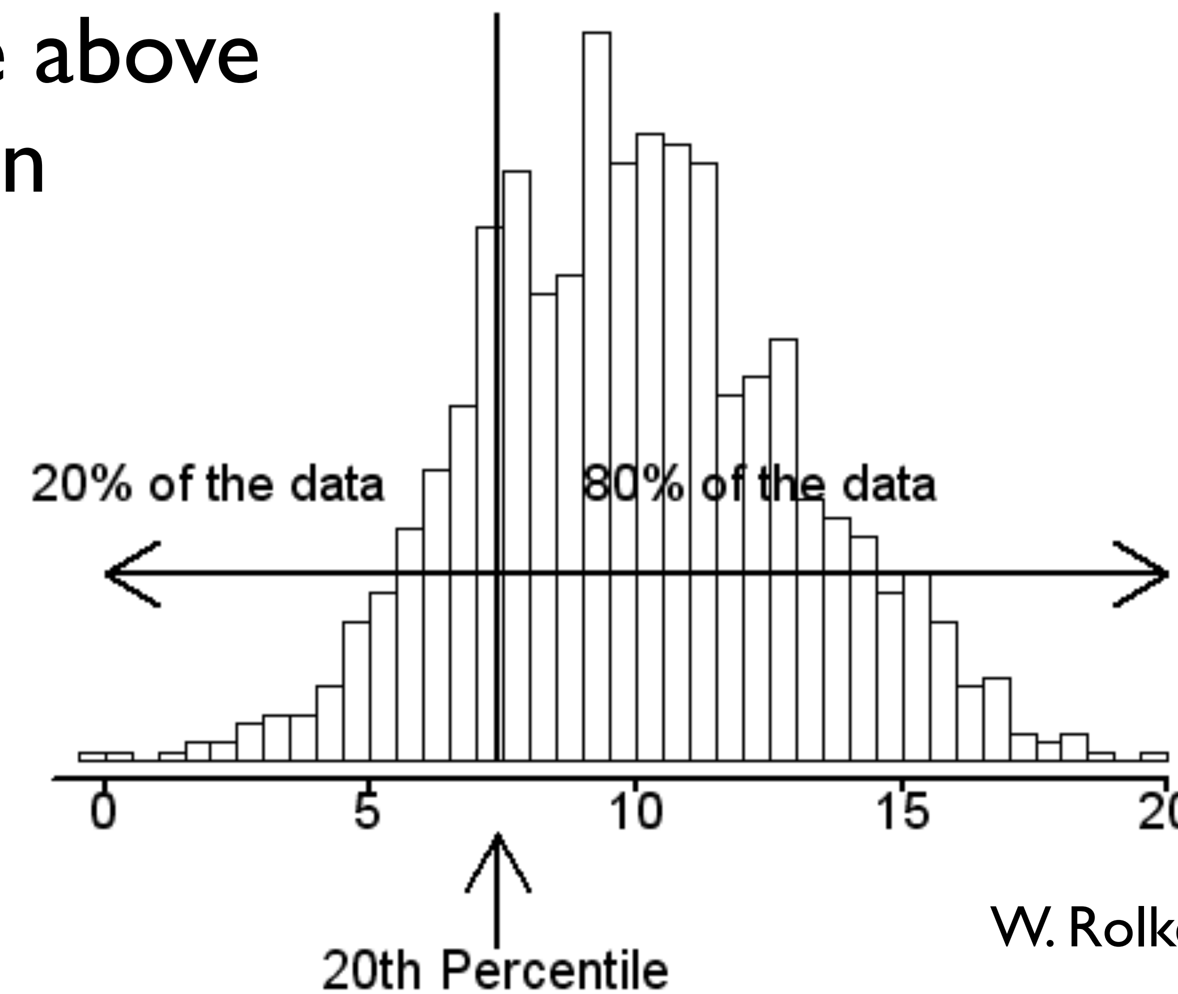
Last Time:

- sample distribution vs. parent population
- summary statistics
- uncertainty on the mean
- Binomial, Poisson, Gaussian distributions
- Central Limit Theorem
- statistical significance

Non-Gaussian distributions

- what if your distribution is non-Gaussian?
- have to decide on case-by-case basis
- percentiles (quartiles): can always sort your data, quote values that are above certain percentage of population
- median: 50th percentile; half the data above, half below
- can quote measurement + uncertainty with percentiles, e.g.:

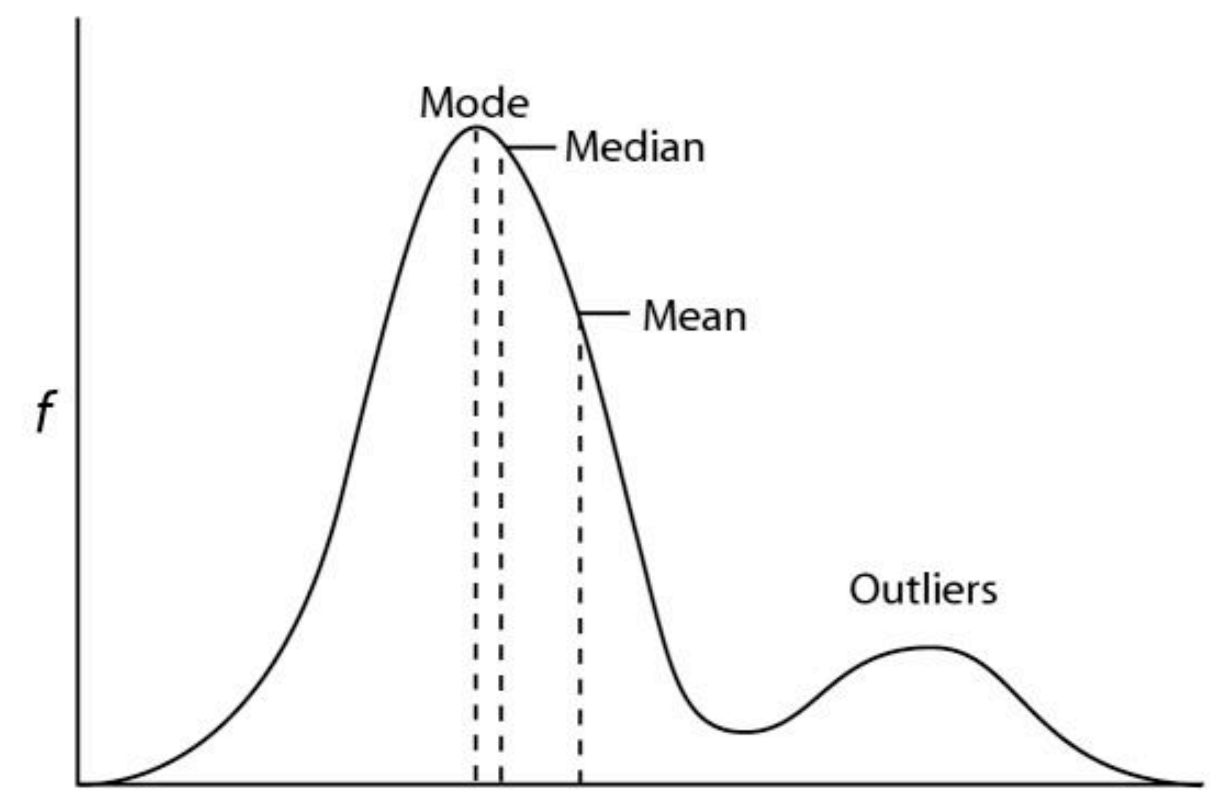
$$99.123^{+0.005}_{-0.004}$$



W. Rolke

Outliers

- for normal distribution, median = mean
- what if distribution is "almost" normal, but has a few outliers? *e.g. cosmic rays in dark frame*
- mean: significantly affected by outliers
- median: robust against (small number of) outliers
- sometimes, it's ok to remove gross outliers ("sigma-clipping"), but need to make sure not to bias your results!



Hedges & Shah 2003

Gaussian error propagation

- often, want to determine dependent variable *x* that is a function of one or more measurements

e.g. $x = f(u, v)$ *u* and *v* have (measured variances):

$$\sigma_u^2 = \frac{1}{N-1}\sum_i (u_i - \bar{u})^2 \qquad \sigma_v^2 = \frac{1}{N-1}\sum_i (v_i - \bar{v})^2$$

covariance between *u* and *v*:

$$\sigma_{uv}^2 = \frac{1}{N-1}\sum_i (u_i - \bar{u})(v_i - \bar{v})$$

note: if *u* and *v* are independent, covariance vanishes for large N

Gaussian error propagation

- Gaussian case: variance in *x* can be expressed in terms of variance in *u* and *v*, and the covariance between them:

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u}\right)\left(\frac{\partial x}{\partial v}\right)$$

- if u and v independent:

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2$$

e.g. $x = a\,u\,v$

with a = constant: $\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2}$

Note: if *f* is non-linear in *x* or *y*, and uncertainty is large, this approximation breaks down

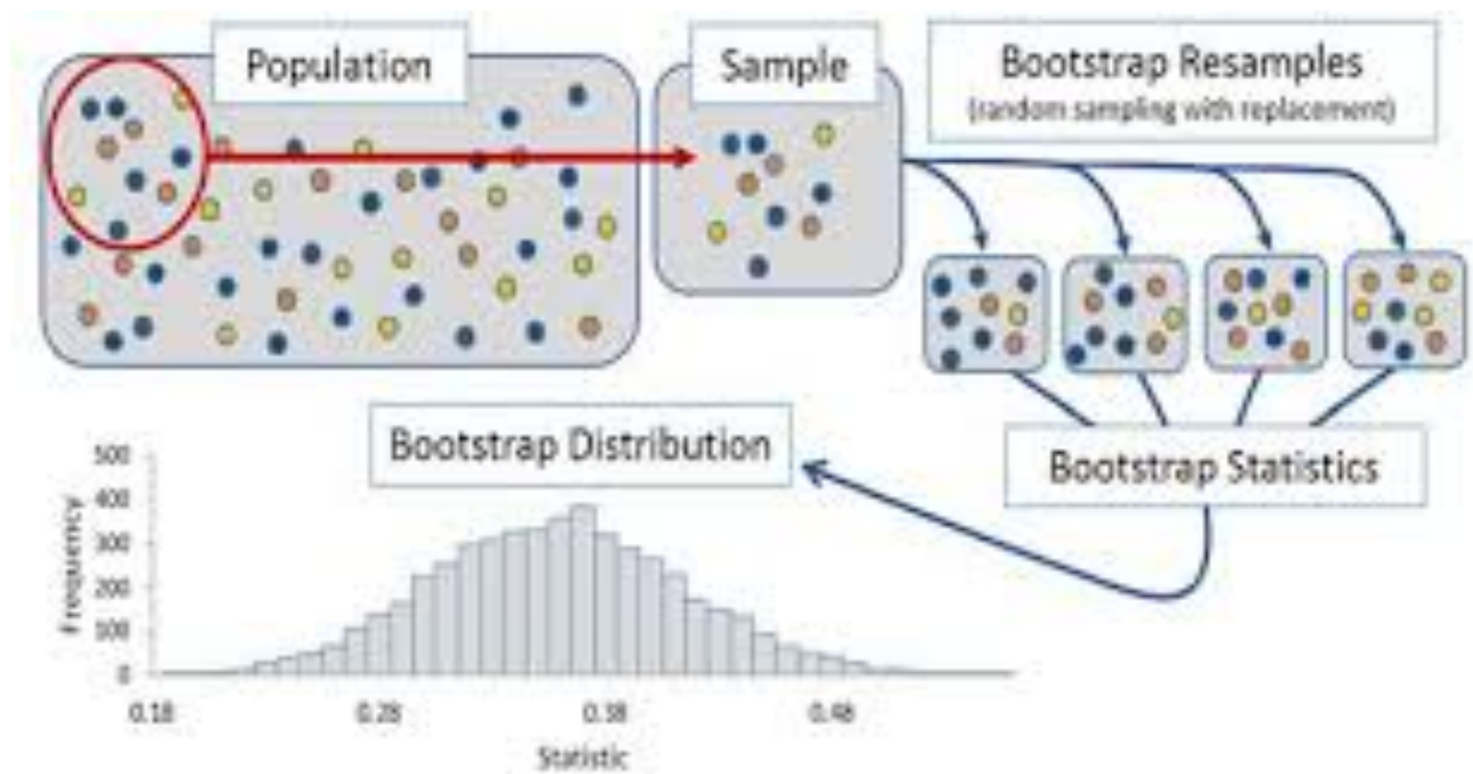
Resampling Methods

- suppose you have a sample distribution, and do not know the shape of the parent population
- … and you want to measure *something* from your data, along with an uncertainty on *something*
- *(and the CLT might not hold, e.g. n too small and/or distribution is skewed)*

- can almost always use a resampling method (bootstrap, jackknife, …)
  - use subsets of the data
  - draw randomly from the data with replacement
  - swap labels on data points

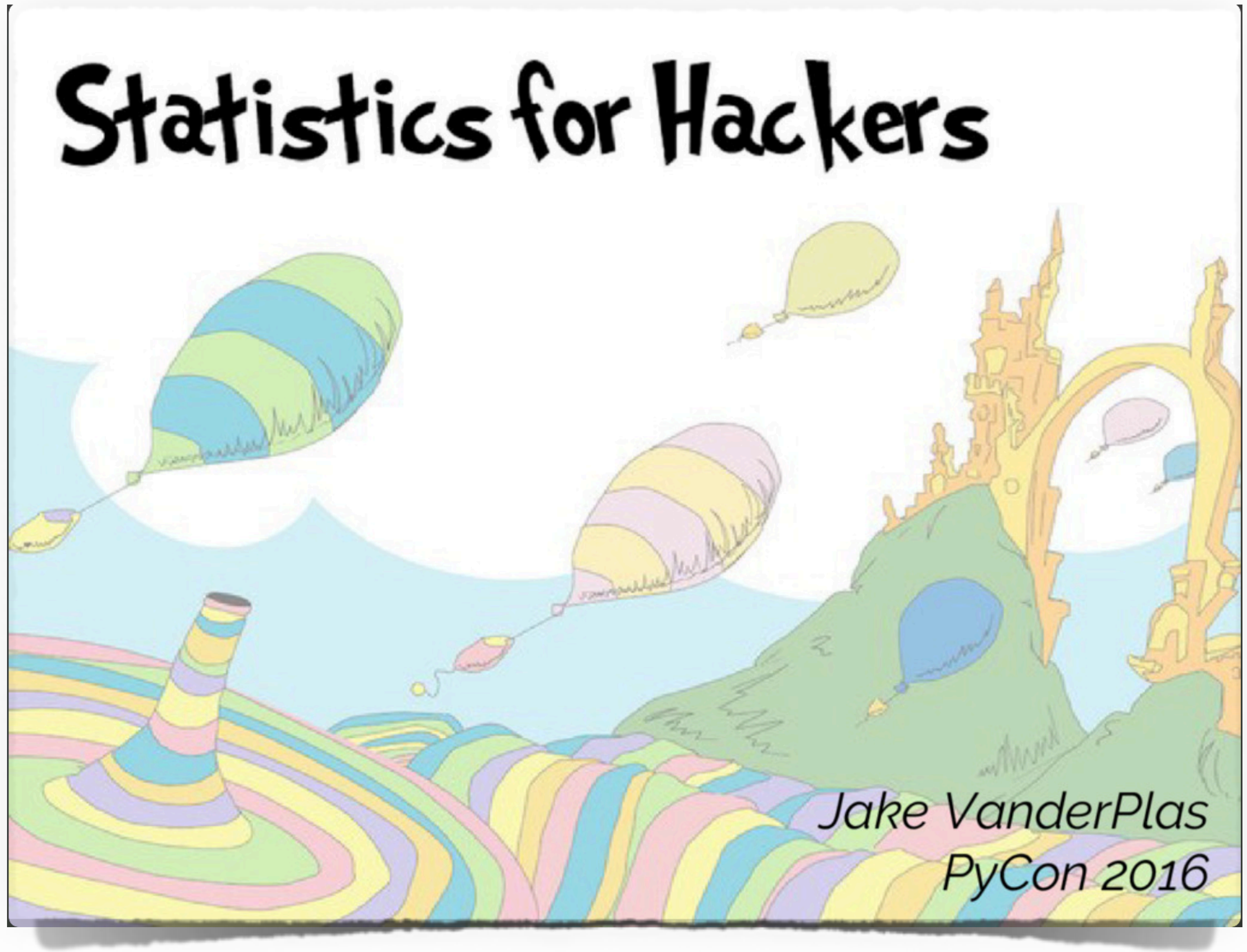
Bootstrapping

- e.g. bootstrap: resampling with replacement
- *N* measurements
- draw from your measurements *N* times (can draw same measurement more than once)
- determine derived quantity
- repeat *n* times
- quantify the bootstrap distribution



Banjanovic & Osborne

More on resampling techniques



https://speakerdeck.com/jakevdp/statistics-for-hackers

Model fitting

- to fit a model to a dataset, need to quantify how good the fit describes the data

- if errors are Gaussian, optimal statistic is χ^2 ("chi-squared")

$$\chi^2 = \sum_i \frac{(D[x_i] - M[x_i])^2}{\sigma_i^2}$$

$D[x_i]$ are the data values; $M[x_i]$ are the values of the model evaluated at positions x_i

(note similarity to normal probability distribution!)

Model fitting

- the best-fitting model is the one that minimizes the χ^2 value

$$\chi^2_{\min} = \sum_i \frac{(D[x_i] - M_{\text{best}}[x_i])^2}{\sigma_i^2}$$

how to find the best-fit model:

- brute force: make a grid of parameter values, calculate χ^2 for each
- use a minimization algorithm

Model fitting

- you have found the "best-fit" parameters of the model that minimize the χ^2, but is that model actually a good fit?

$$\chi^2_\nu = \frac{\chi^2_{\rm min}}{\nu}$$

- reduced chi-square: scale best-fit chi-square by ν, the number of free parameters = number of data points minus the number of free model parameters

- example: fitting a line: two model parameters (slope and intercept)

$$\nu = \text{number of data points} - 2$$

Model fitting

- given a random realization of an experiment with ν degrees of freedom, the probability to obtain $\chi^2_{\min}/\nu$ or larger is described by the chi-squared distribution
- comparing the measured reduced chi-squared to the expectation (from the chi-squared distribution) is an indication whether the model is an acceptable fit to the data
- for an acceptable model, the remaining deviations should be well described by a random (Gaussian) process

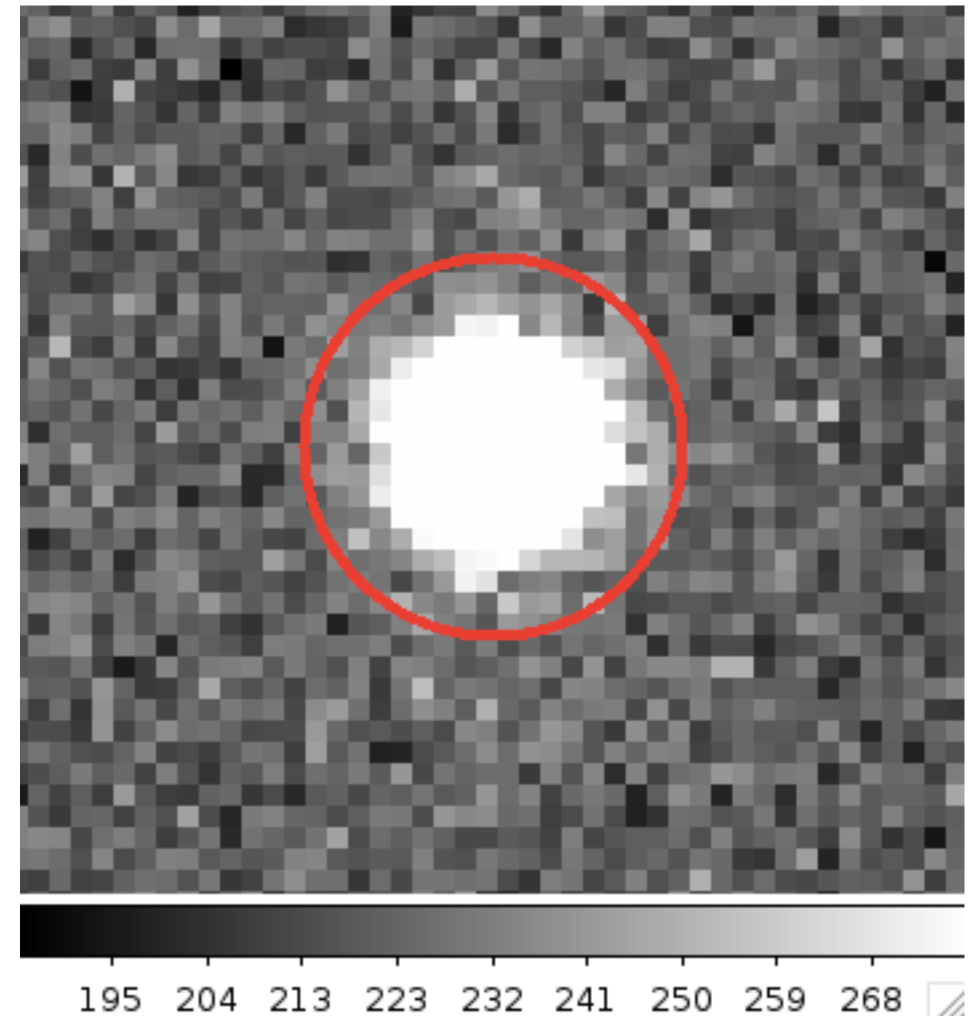
$\chi^2_{\min}/\nu \approx 1$	model is a good fit
$\chi^2_{\min}/\nu \gg 1$	model is a bad fit
$\chi^2_{\min}/\nu \ll 1$	model is overfitting the data

Signal-to-Noise in CCD images

Signal-to-Noise

flux measured in an aperture:

total electrons = electrons from object + electrons from background (sky, dark current, etc.)



signal = total electrons - background electrons

noise: counting statistics (i.e. Poisson distribution, $\sqrt{N}$), have to consider noise from all sources

Signal-to-Noise

signal: $$N_{\text{object}} = N_{\text{total}} - N_{\text{background}}$$

noise: if the noise contributions are independent of each other, can add quadratically:

$$\sigma = \sqrt{\sum_{i \,\in\, \text{noise terms}} \sigma_i^2}$$

Note: the "counting" processes apply to the number of registered electrons. The counts reported in the image have been rescaled by the gain, $N_{\text{counts}} = N_{\text{electrons}}/G$

Signal-to-Noise

noise contributions:

- shot noise from source

$$\sigma_{\rm object} = \sqrt{N_{\rm object}}$$
$$= \sqrt{S_{\rm object} \times t}$$

- sky noise

$$\sigma_{\rm sky} = \sqrt{N_{\rm sky}}$$
$$= \sqrt{s_{\rm sky} \times n_{\rm pix} \times t}$$

- dark current noise

$$\sigma_{\rm dk} = \sqrt{N_{\rm dk}}$$
$$= \sqrt{s_{\rm dk} \times n_{\rm pix} \times t}$$

- read-out noise

$$\sigma_{\rm ro} = \mathrm{RON} \times \sqrt{n_{pix}}$$

already a std. dev.

Signal-to-Noise

total signal-to-noise: can add noise components quadratically

$$SNR = \frac{N_{\text{object}}}{\sqrt{\sum_{\text{noise}} \sigma_i^2}}$$

$$= \frac{N_{\text{object}}}{\sqrt{N_{\text{object}} + N_{\text{sky}} + N_{\text{dk}} + N_{\text{ro}}}}$$

$$= \frac{s_{\text{object}} \times t}{\sqrt{s_{\text{object}} \times t + n_{\text{pix}} \times s_{\text{sky}} \times t + n_{\text{pix}} \times s_{\text{dk}} \times t + n_{\text{pix}} \times \text{RON}^2}}$$

“CCD signal-to-noise equation”

Signal-to-Noise

in general, you do not want to be limited by dark current and read-out noise!

limiting case 1: very bright object $N_{\rm object} >> N_{\rm other}$

$$SNR = \frac{N_{\rm object}}{\sqrt{N_{\rm object}}} = \frac{s_{\rm object} \times t}{\sqrt{s_{\rm object} \times t}}$$

$$\propto \sqrt{t}$$

Signal-to-Noise

in general, you do not want to be limited by dark current and read-out noise!

limiting case 2: faint objects $N_{\rm sky} >> N_{\rm other}$

$$SNR = \frac{N_{\rm object}}{\sqrt{N_{\rm sky}}} = \frac{s_{\rm object} \times t}{\sqrt{s_{\rm sky} \times n_{\rm pix} \times t}}$$

$$\propto \sqrt{t}$$

Sky Background

twilight:

Sun at -6° : “civil twilight”, still bright

Sun at -12° : “nautical twilight”, can see bright stars

Sun at -18° : “astronomical twilight”

twilight is scattered light (blue)

observations in different filters are affected differently

sky is “dark” in red filters before -18°

Sky Background

Patat 2004

moonlight:

detrimental in the very blue; not a big problem in the infrared

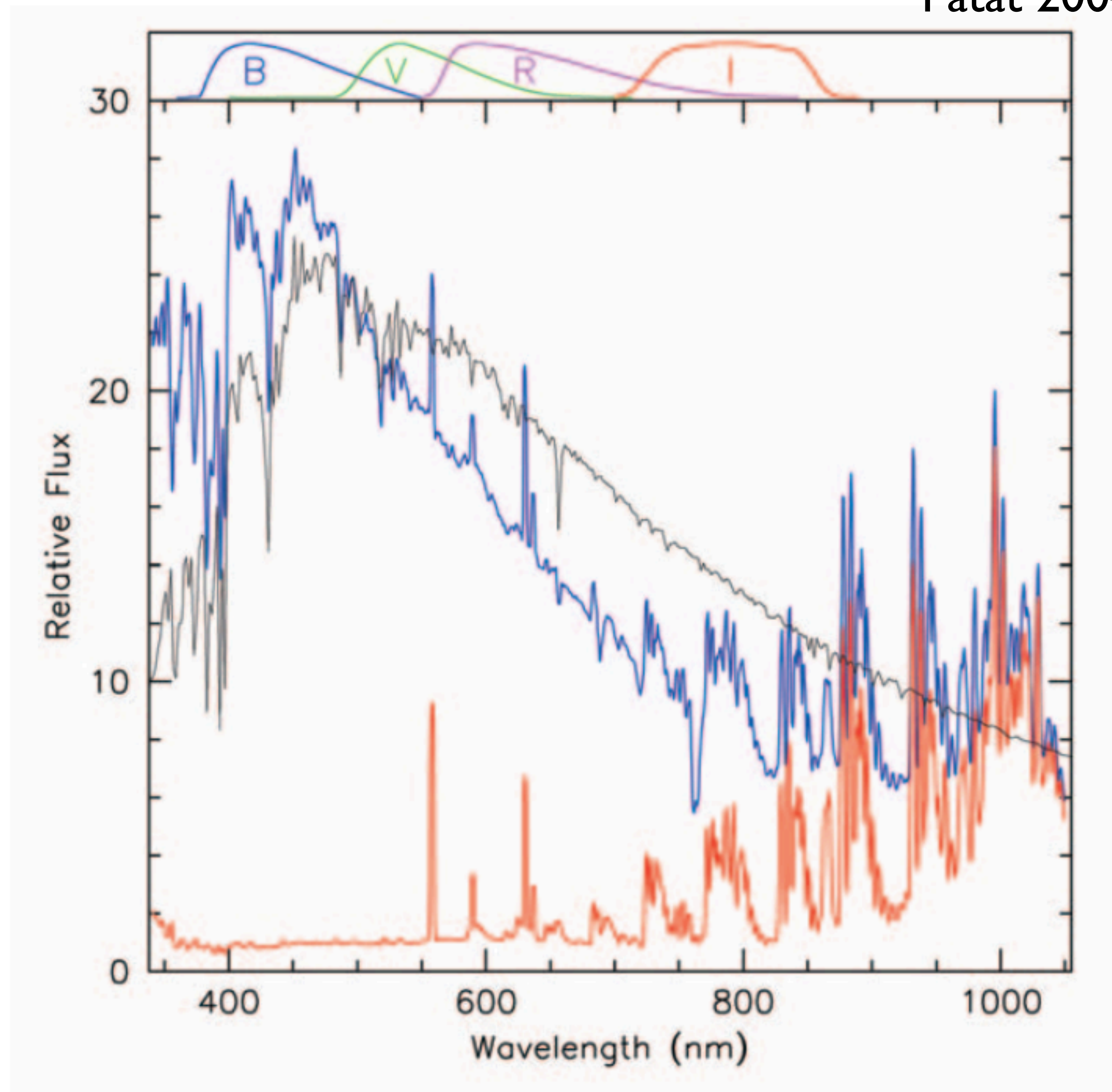
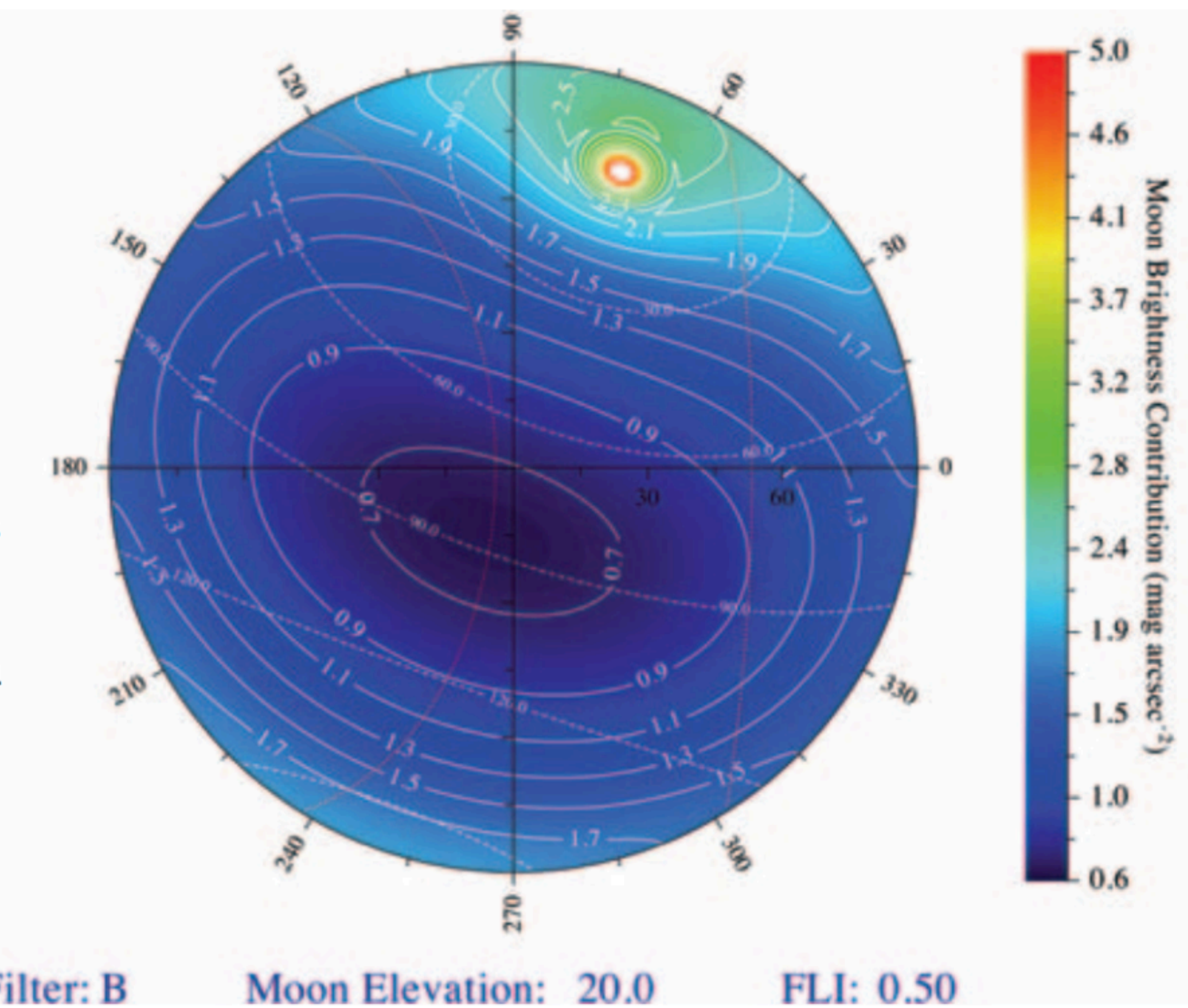


Figure 2: Comparison between the night sky spectrum during dark time (red line, Patat 2003) and bright time (blue line). The latter was obtained with FORS1 on September 1, 2004 using the low dispersion grism 150I and no order sorter filter. Due to the very blue continuum, the spectral region at wavelengths redder than 650 nm is probably contaminated by the grism second order. Both spectra have been normalized to the continuum of the first one at 500 nm. For comparison, the model spectrum of a solar-type star is also plotted (black line). For presentation, this has been normalized to the moonlit night sky spectrum at 500 nm. The upper plot shows the standard *BVRI* Johnson-Cousins passbands.

Sky Background

sky brightness from moonlight depends on distance from it, and from horizon

Figure 5: Example isophotal *alt-az* map for the expected moonlight contribution. The dashed white lines trace the loci at constant angular distance from the moon, while the two dotted red lines indicate the extreme apparent lunar paths during a full Saros cycle.

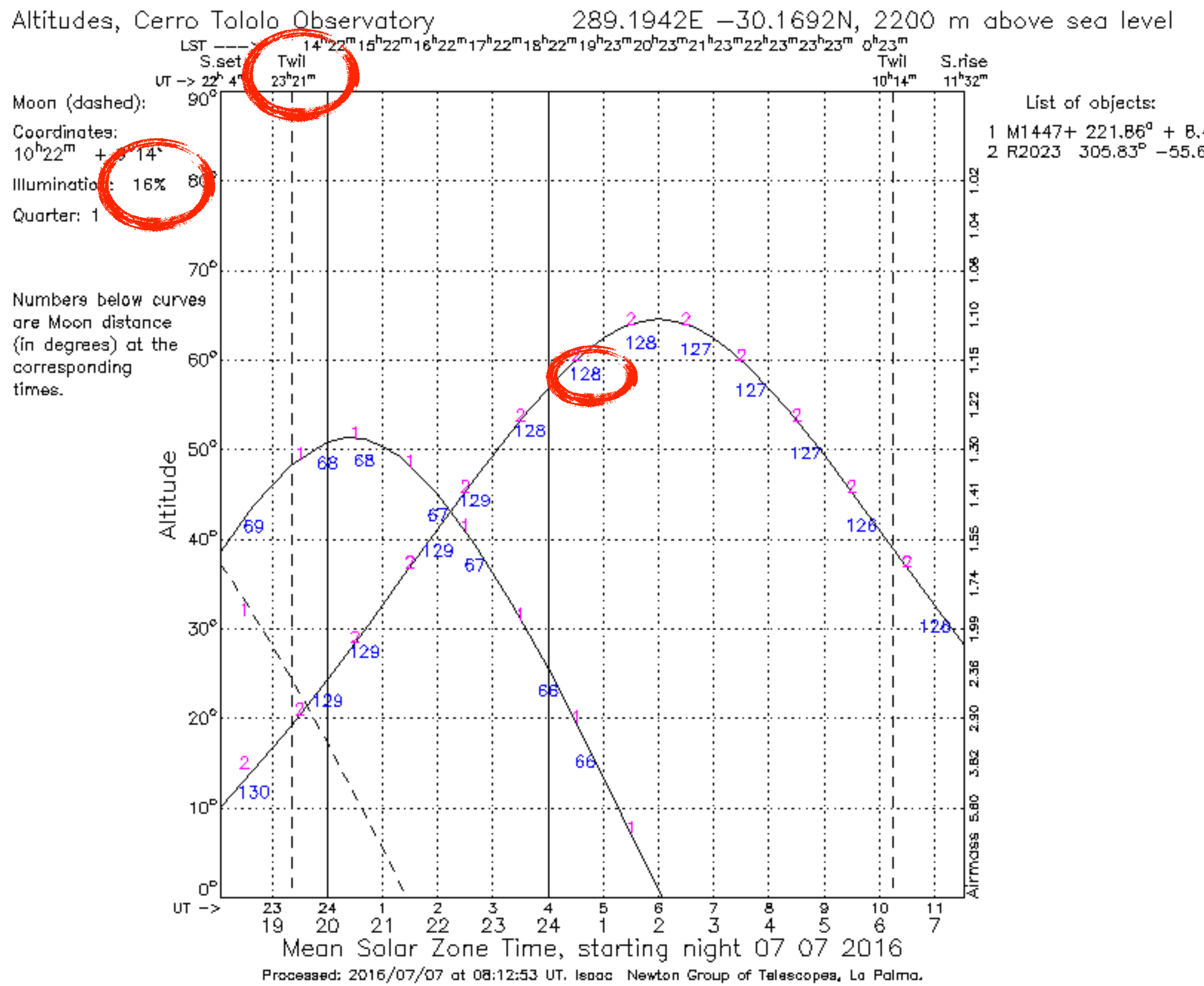


in addition: moonlight can cause reflections inside telescope, from dome, etc.

Patat 2004

StarAlt

- indicates times of astronomical twilight
- indicates lunar illumination
- indicates distance to the Moon

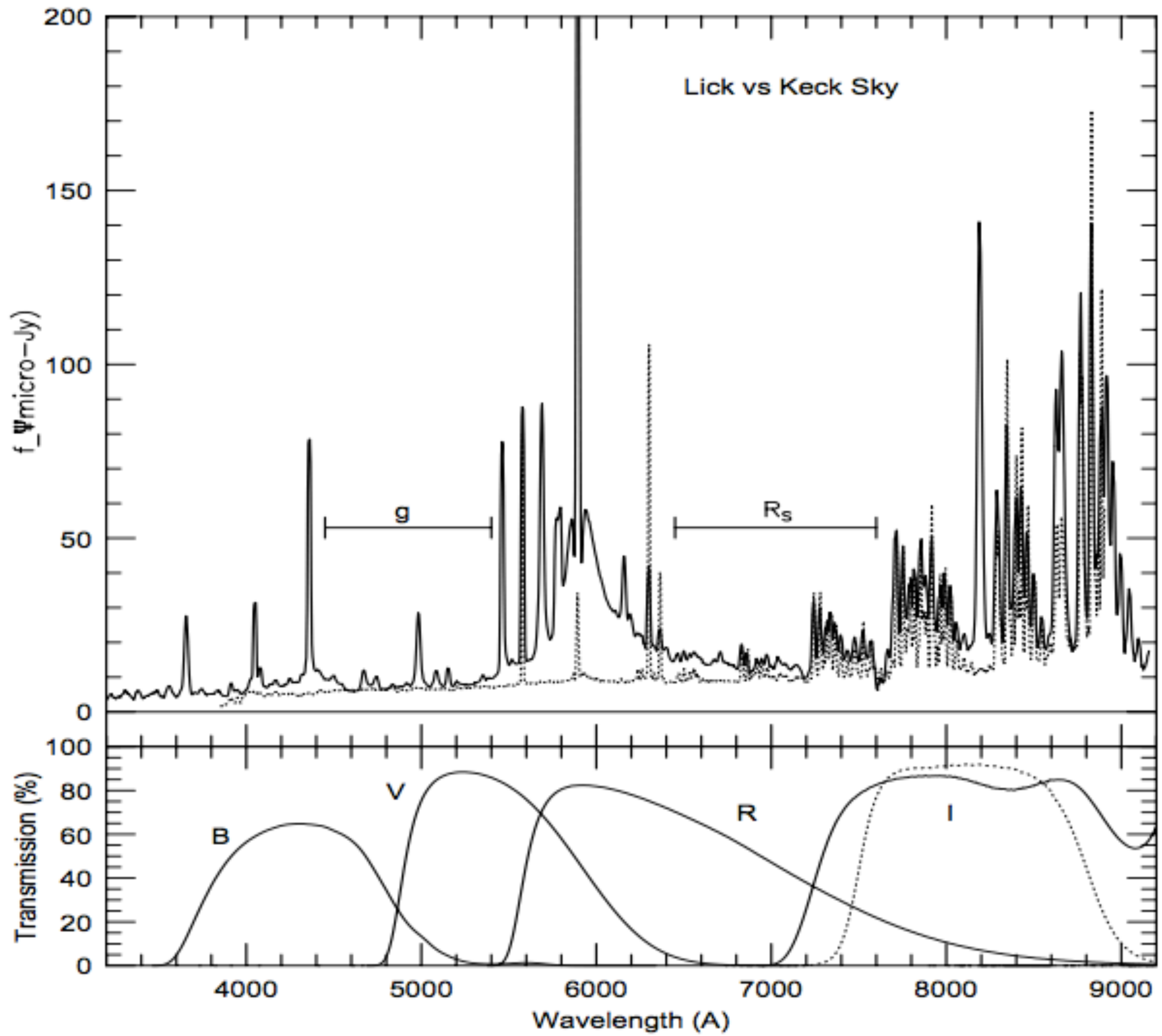


Sky Background

limits most astronomical observations!

always present:
emission from
atmosphere

(+city lights)



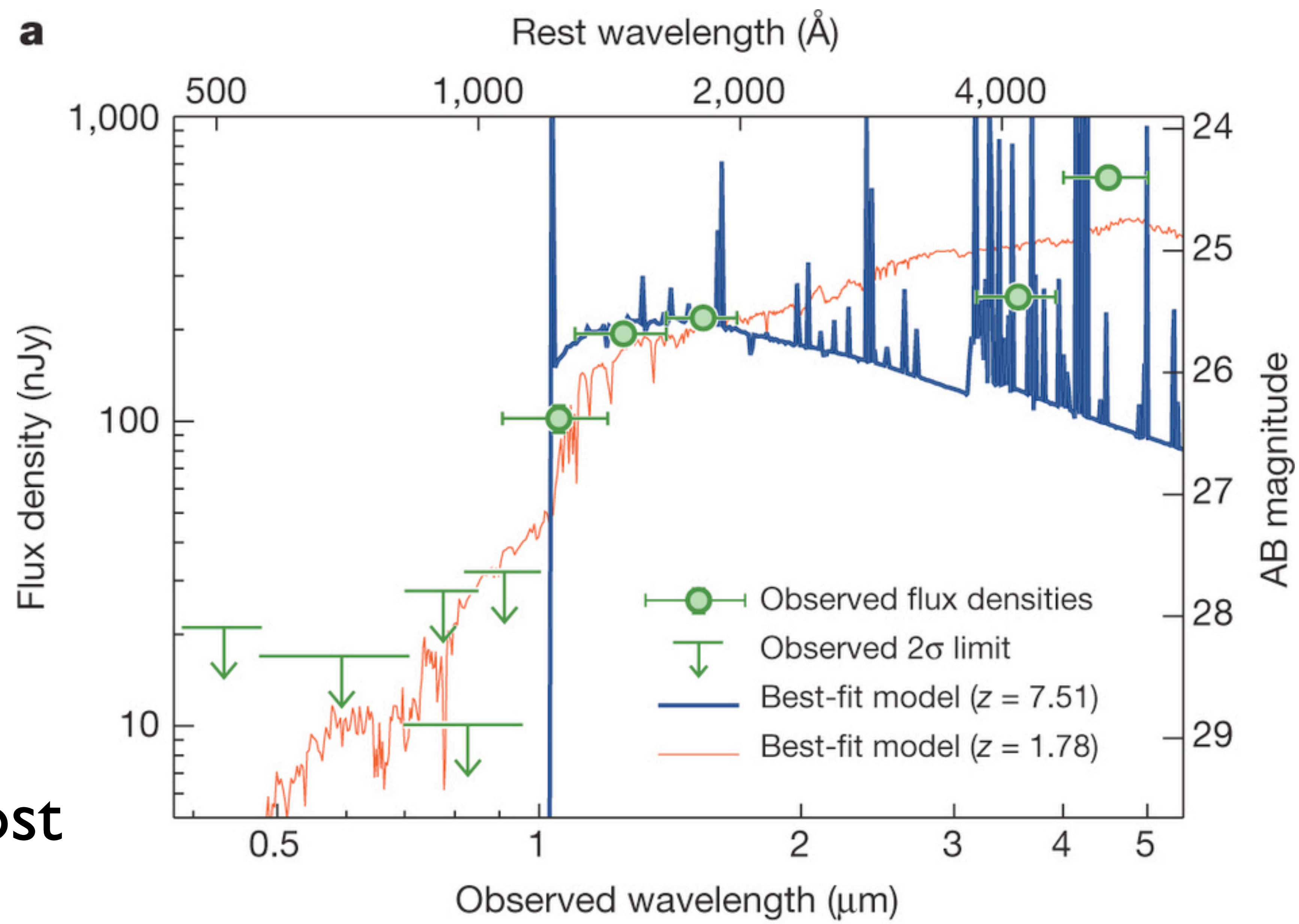
M. Bolte / UCO Lick

Spectroscopy

Motivation

photometry (measuring flux from images) only measures integrated flux

gives some information about the object properties, but often not enough



e.g.: finding the most distant galaxies

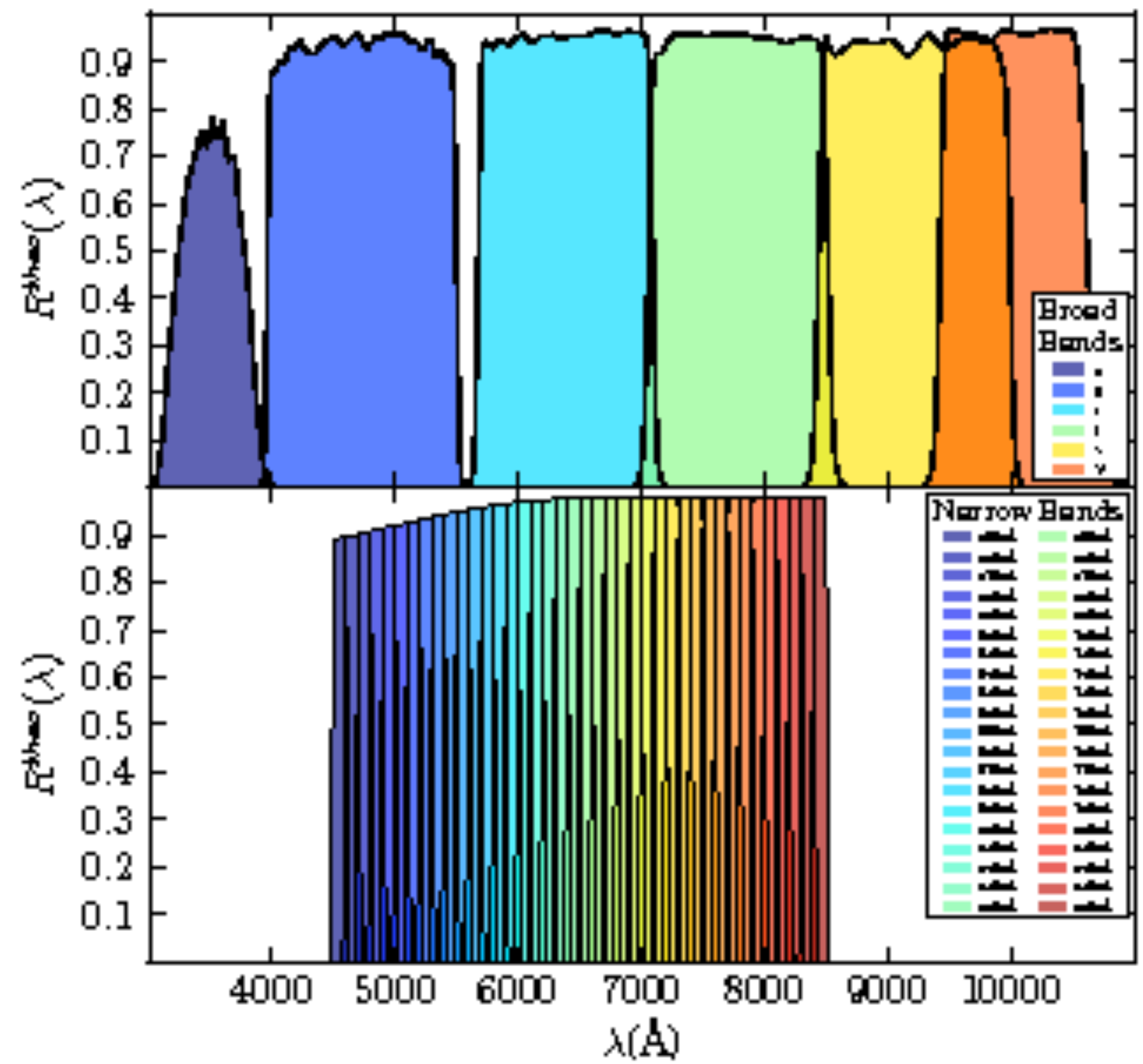
Finkelstein et al. 2013

Narrow-band imaging

can determine spectrum of object with images in many narrow-band filters

advantage: can determine spectra of all objects in the same FOV

disadvantage: have to take a lot of images!

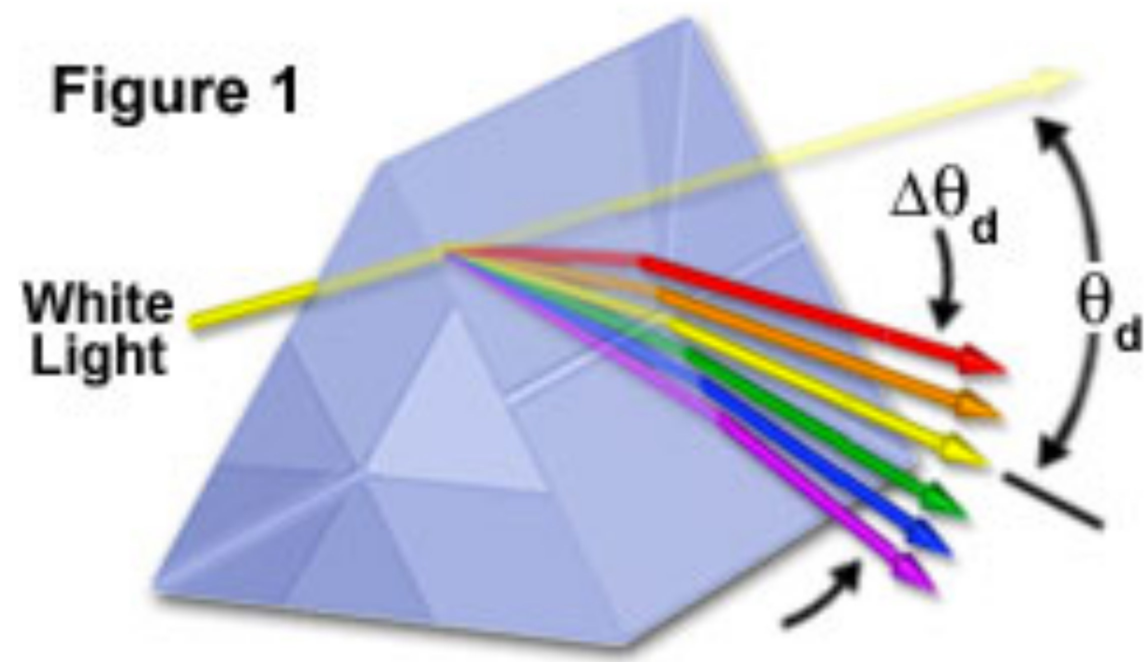


The PAU Survey

Spectroscopy

add a dispersing element to split up the light from an object: measure the spectrum directly

e.g. a prism:

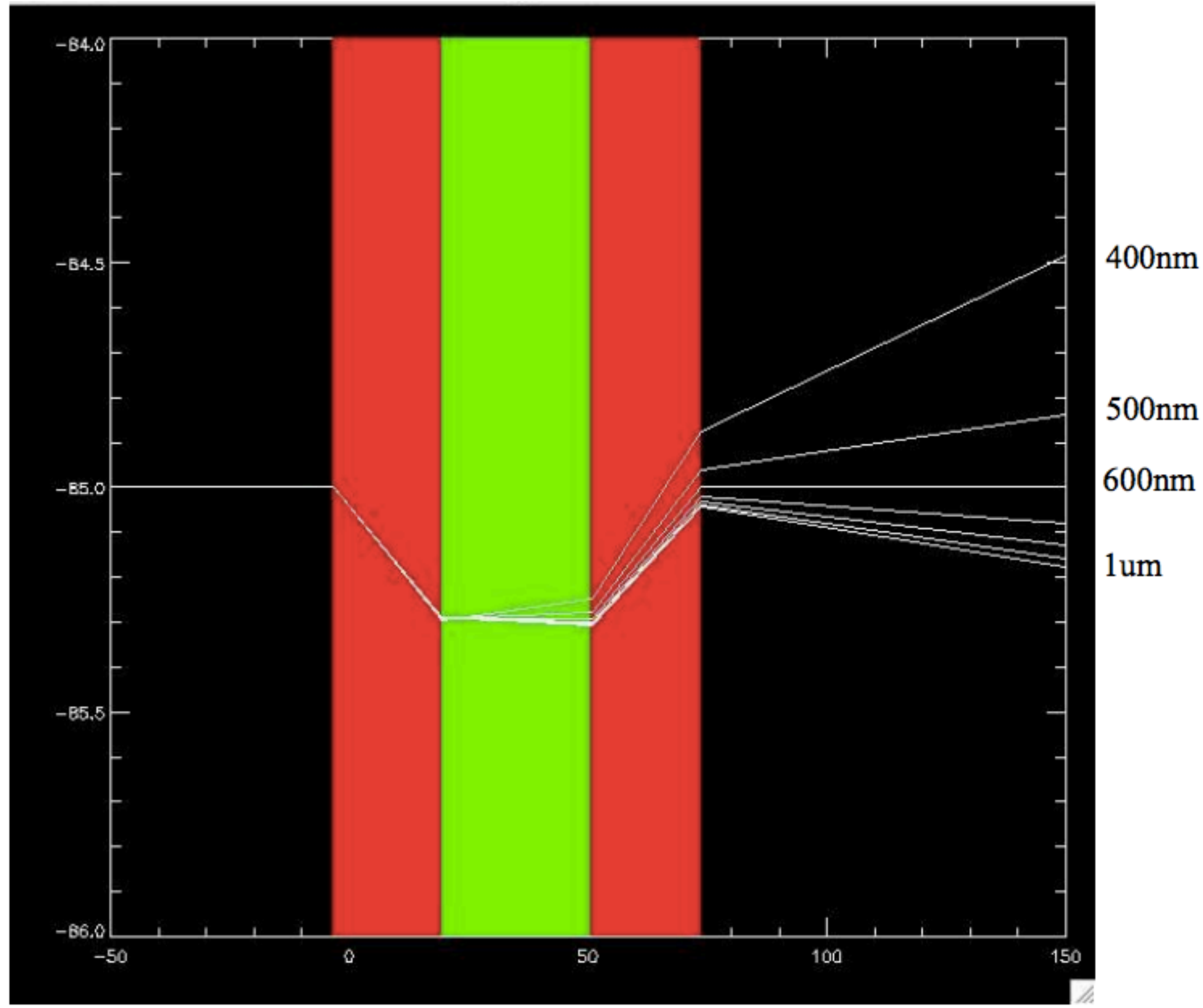


Olympus

Prism Spectroscopy

only few astronomical spectrographs use prisms

- low dispersion (resolution)
- dispersion varies with wavelength



LDP Prism description

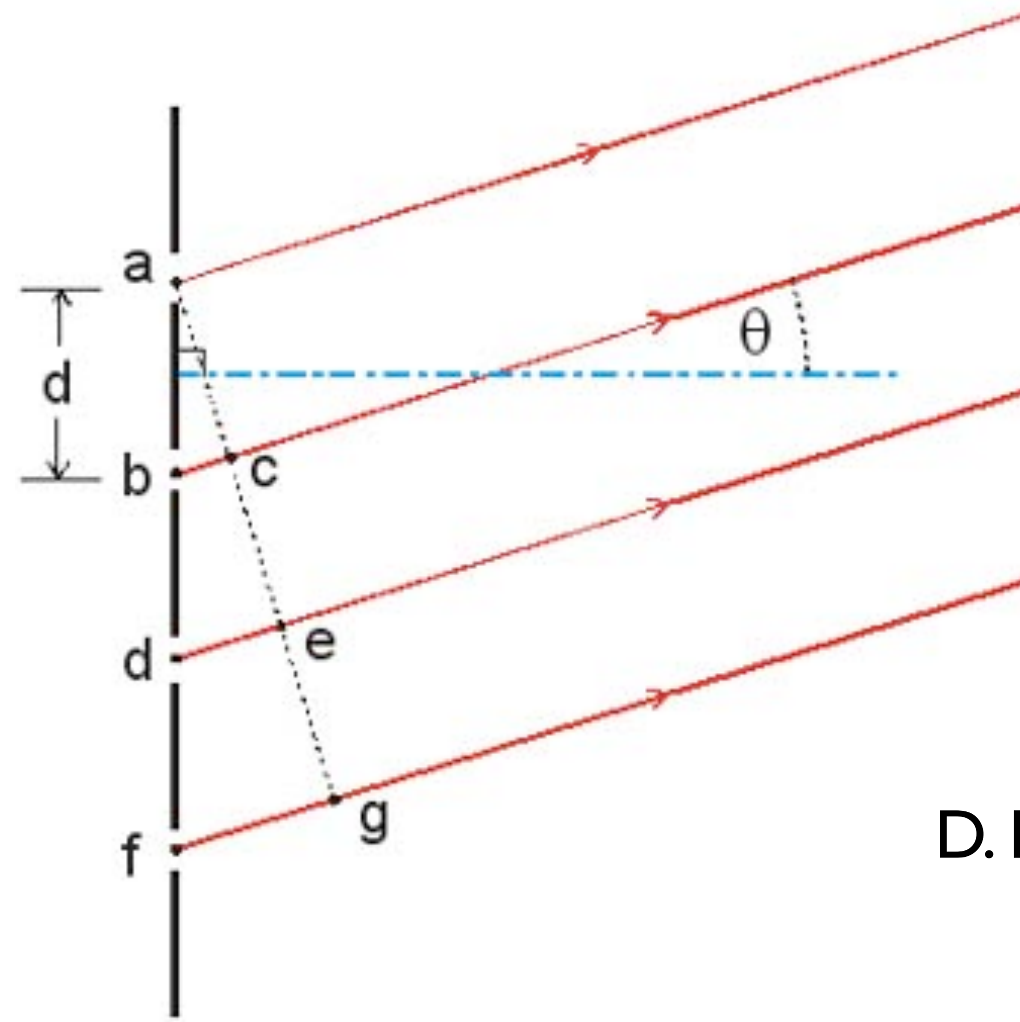
"low dispersion prism" for IMACS spectrograph on Magellan 6-m telescope; uses 3 prisms

Diffraction gratings

make use of wave properties of light: interference

grating: many parallel lines (~500/mm)

similar to single-slit and double-slit experiments

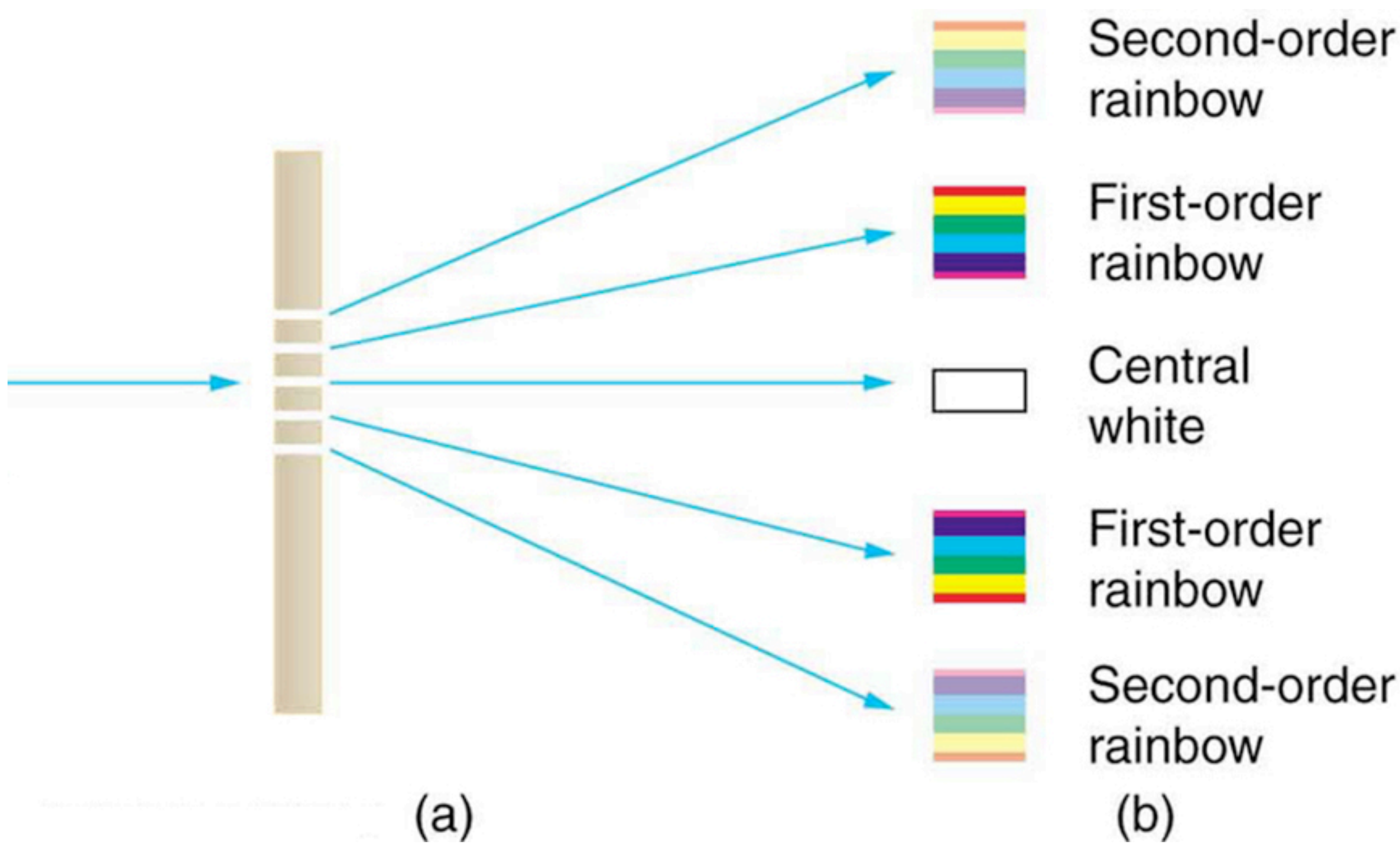


D. Pogosian

if b-c = λ: maximum at θ
and d-e = 2λ, etc.

position of *n*th order: $$n\lambda = d\sin\theta$$

Diffraction gratings

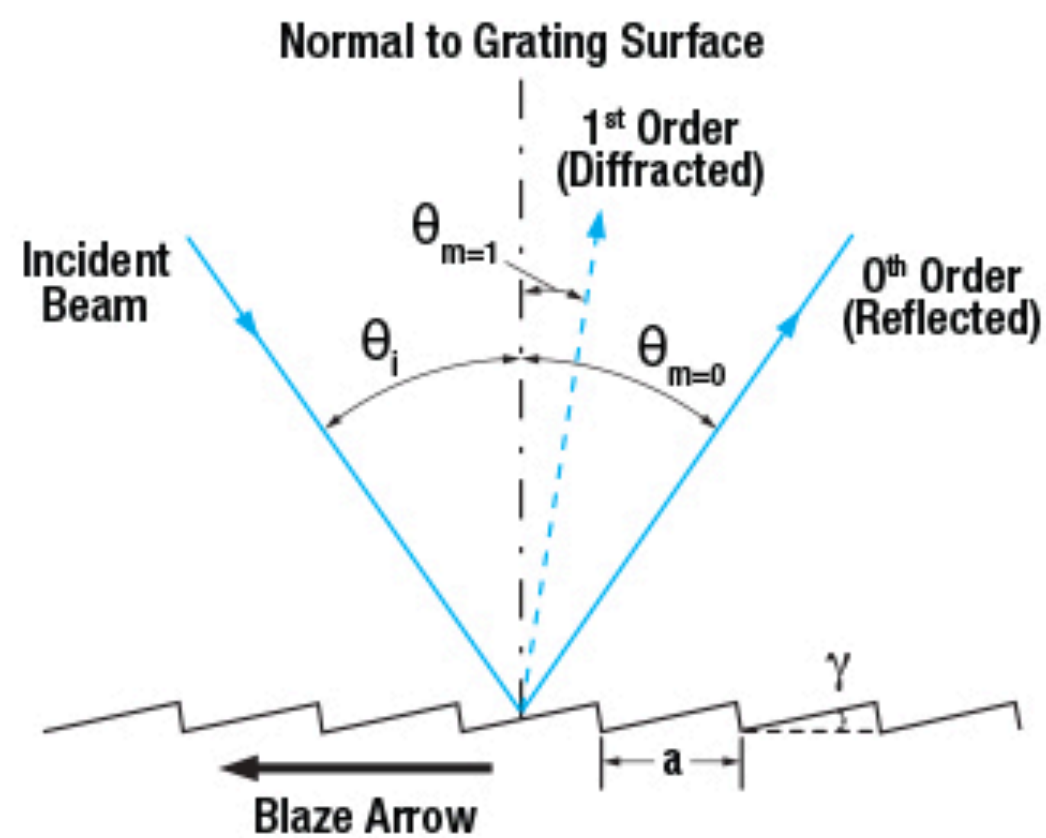


OpenStax University

Diffraction gratings

can be transmission gratings or reflection gratings

most astronomical spectrographs use reflection gratings



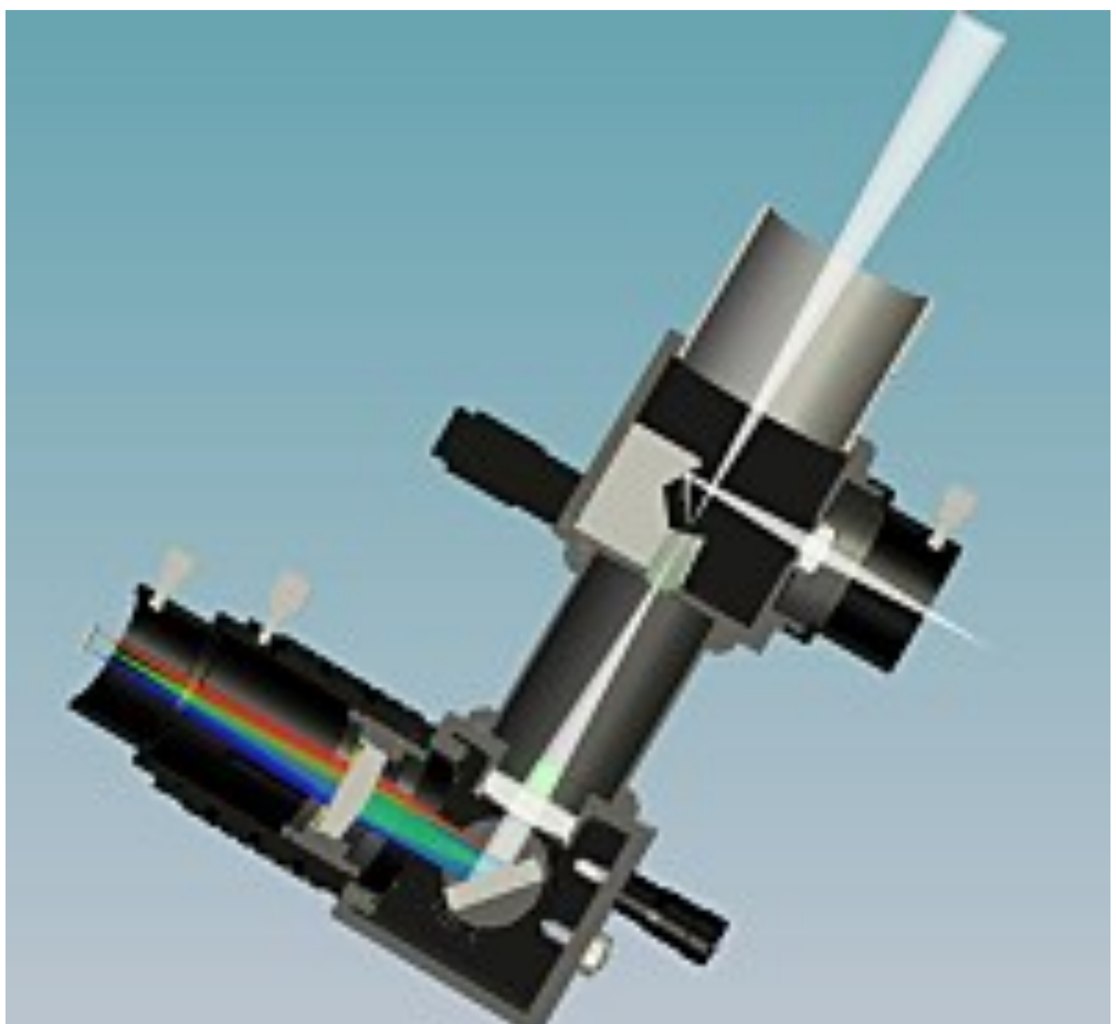
Thor Labs

blaze wavelength: wavelength for direction of reflection coincides with desired spectral order
→ maximal efficiency

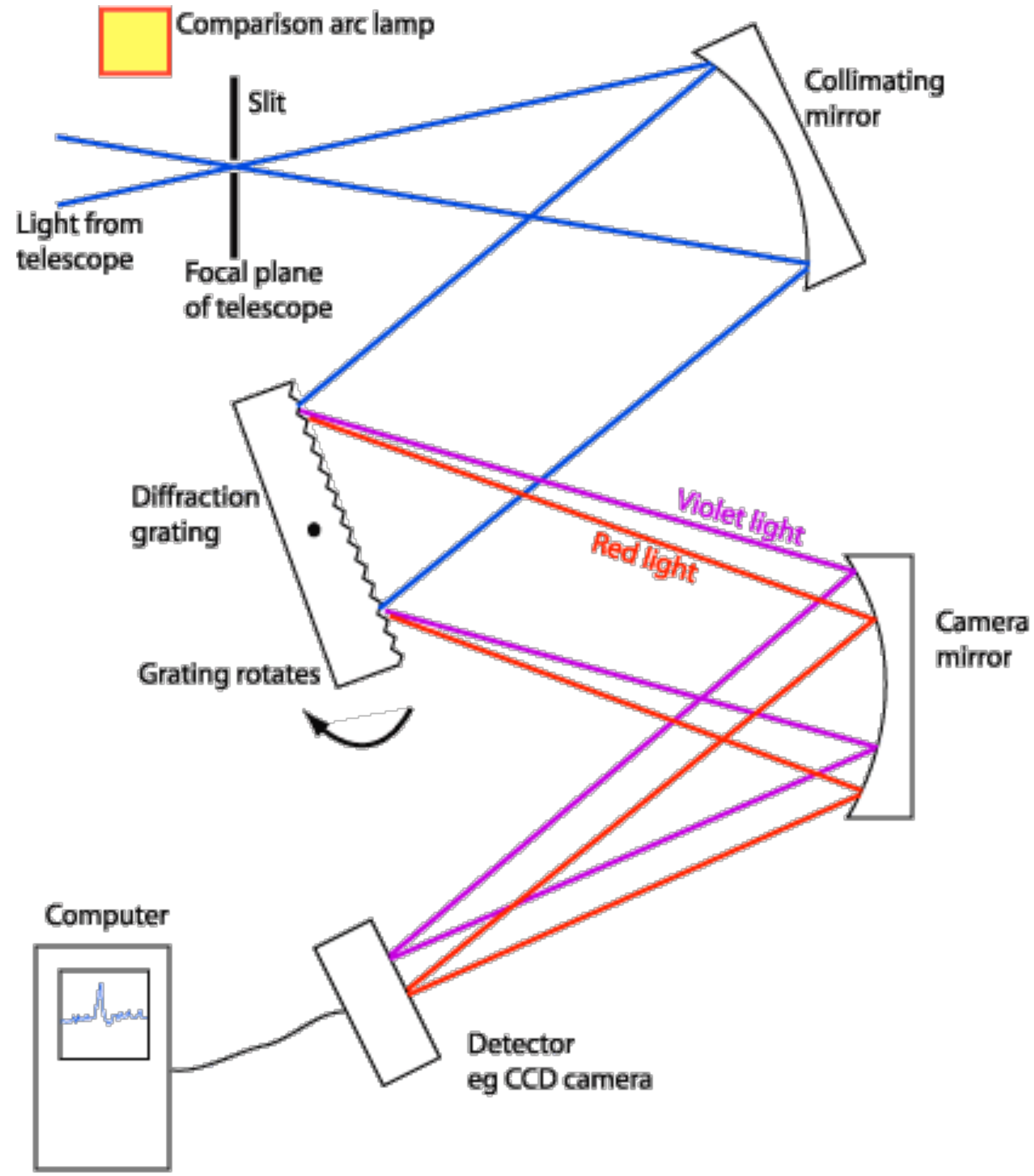
Typical spectrograph

entrance: usually a slit, similar to seeing size

collimator: converts a diverging beam to a parallel beam



Baader Planetarium



A Schematic Diagram of a Slit Spectrograph

Kaler, *Stars and their Spectra*

Spectral Resolution

defined by smallest wavelength difference $\Delta\lambda$ that can be distinguished at wavelength λ

$$R = \frac{\lambda}{\Delta\lambda}$$

determined by:

- grating (line density)
- width of entrance slit
- seeing

resolution: R or $\Delta\lambda$

dispersion: length $\Delta\lambda'$ of spectrum over single pixel, [Å/px]

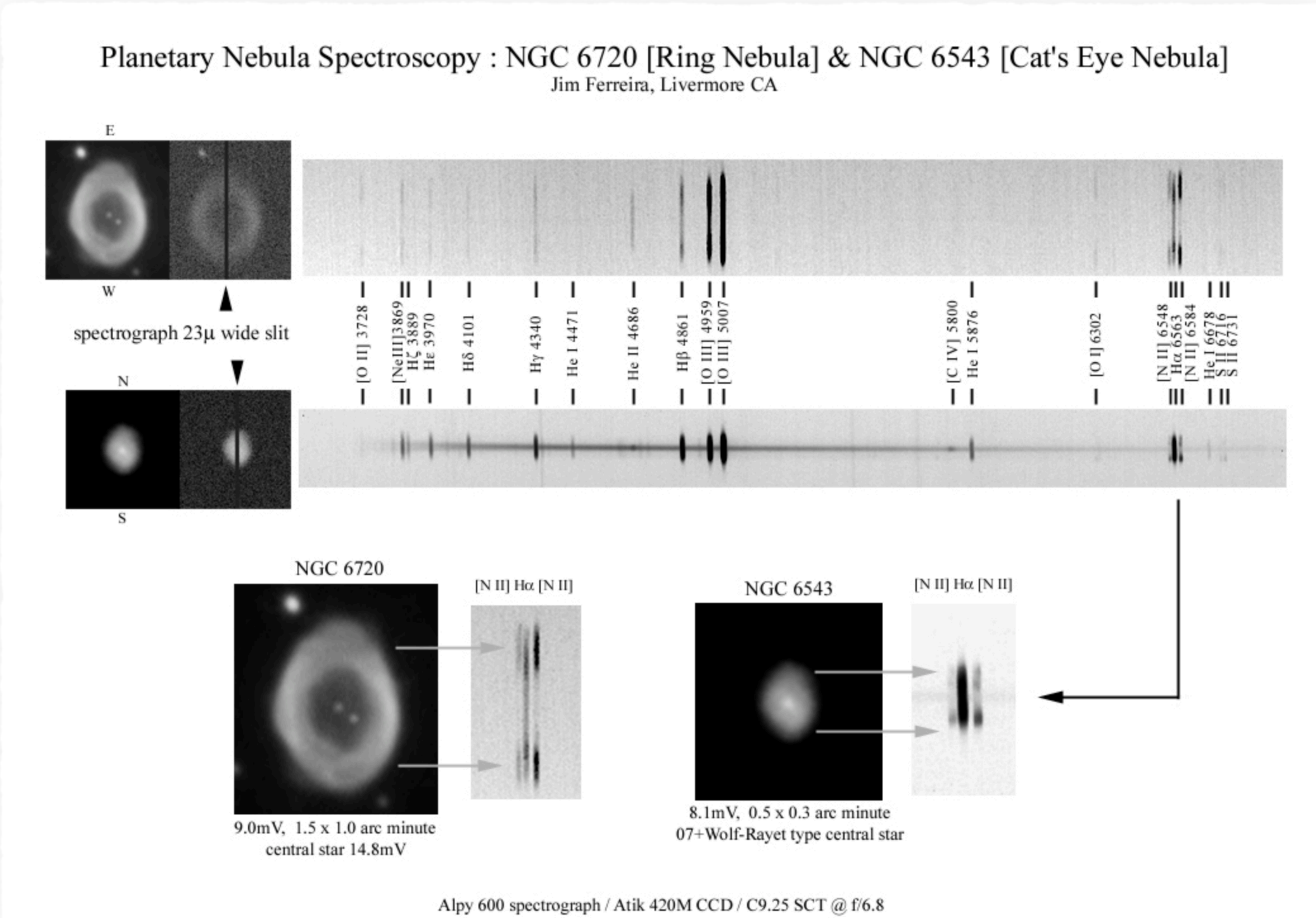
to properly sample the spectrum:

$$\Delta\lambda \sim 2 - 3\,\Delta\lambda'$$

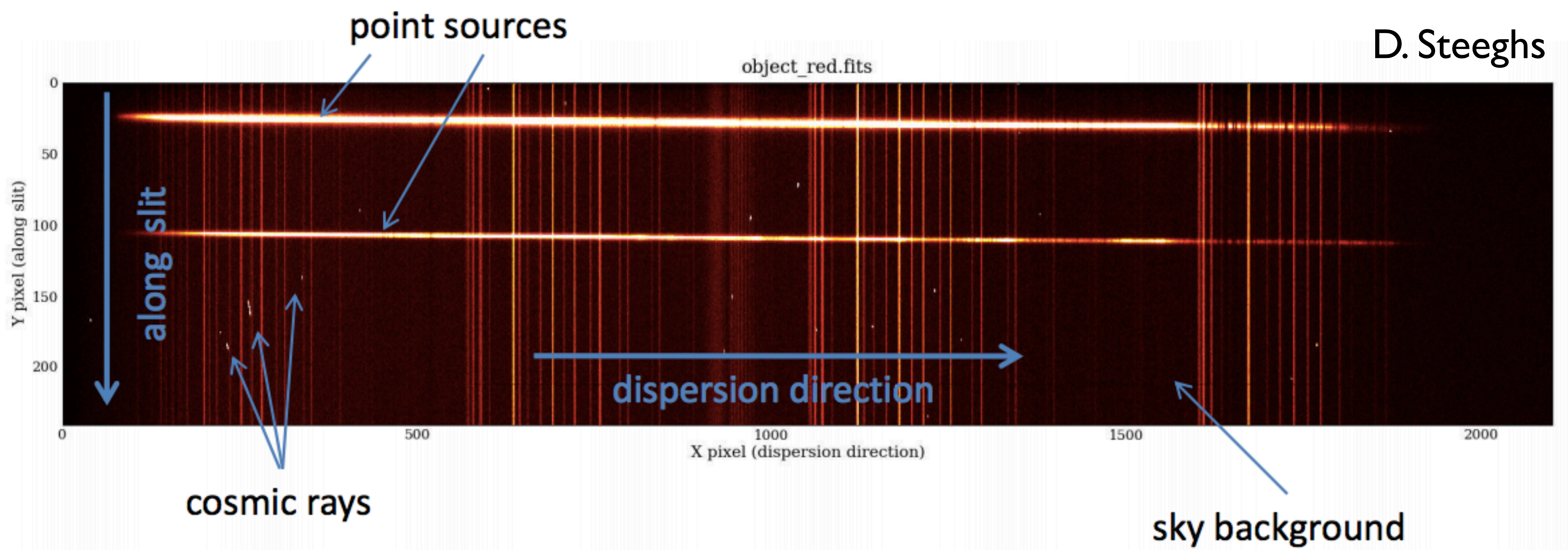
Spectral Resolution

$R < 1000$	low-resolution	e.g. our "low-resolution" spectrograph
$1000 < R < 10{,}000$	medium-resolution	e.g. our "high-resolution" spectrograph
$R > 10{,}000$	high-resolution	Echelle spectrographs

Long-slit observations



Long-slit observations



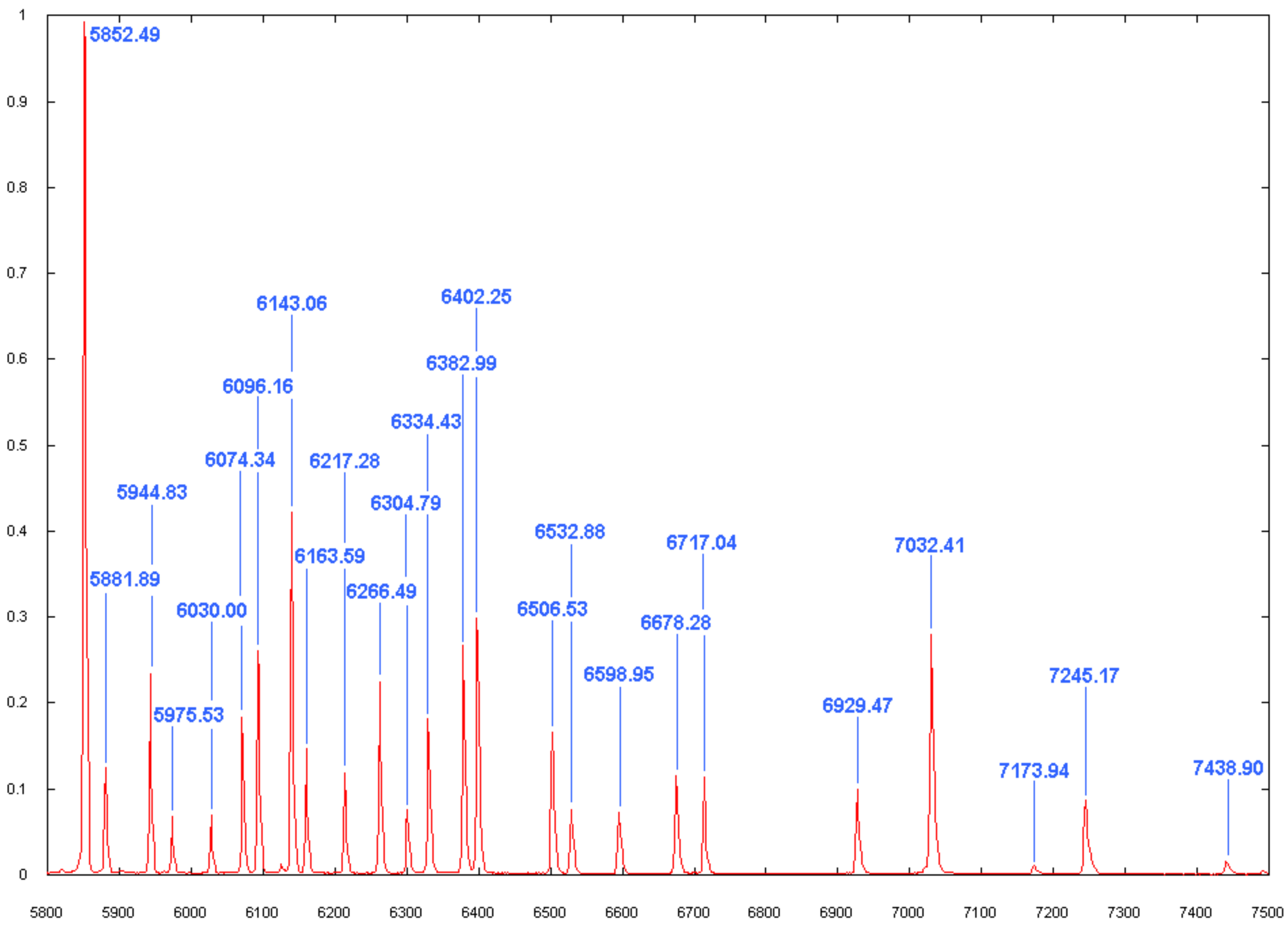
- long axis of CCD used to sample spectrum
- spatial information along slit still available: two objects, lots of sky
- sky background has a lot of emission lines!

Spectroscopic Calibration

- dark frames!
- flat field: use bright continuum source
  - small-scale pixel sensitivity variation
  - variations in slit width
- wavelength calibration: which position on the CCD corresponds to which wavelength?
  - use "arc" lamps with discrete emission lines
  - can also use sky emission lines
- flux calibration:
  - "spectrophotometric" standard stars: stars with known spectral shapes, smooth continua

Spectroscopic Calibration

wavelength calibration: map pixel position to emission lines

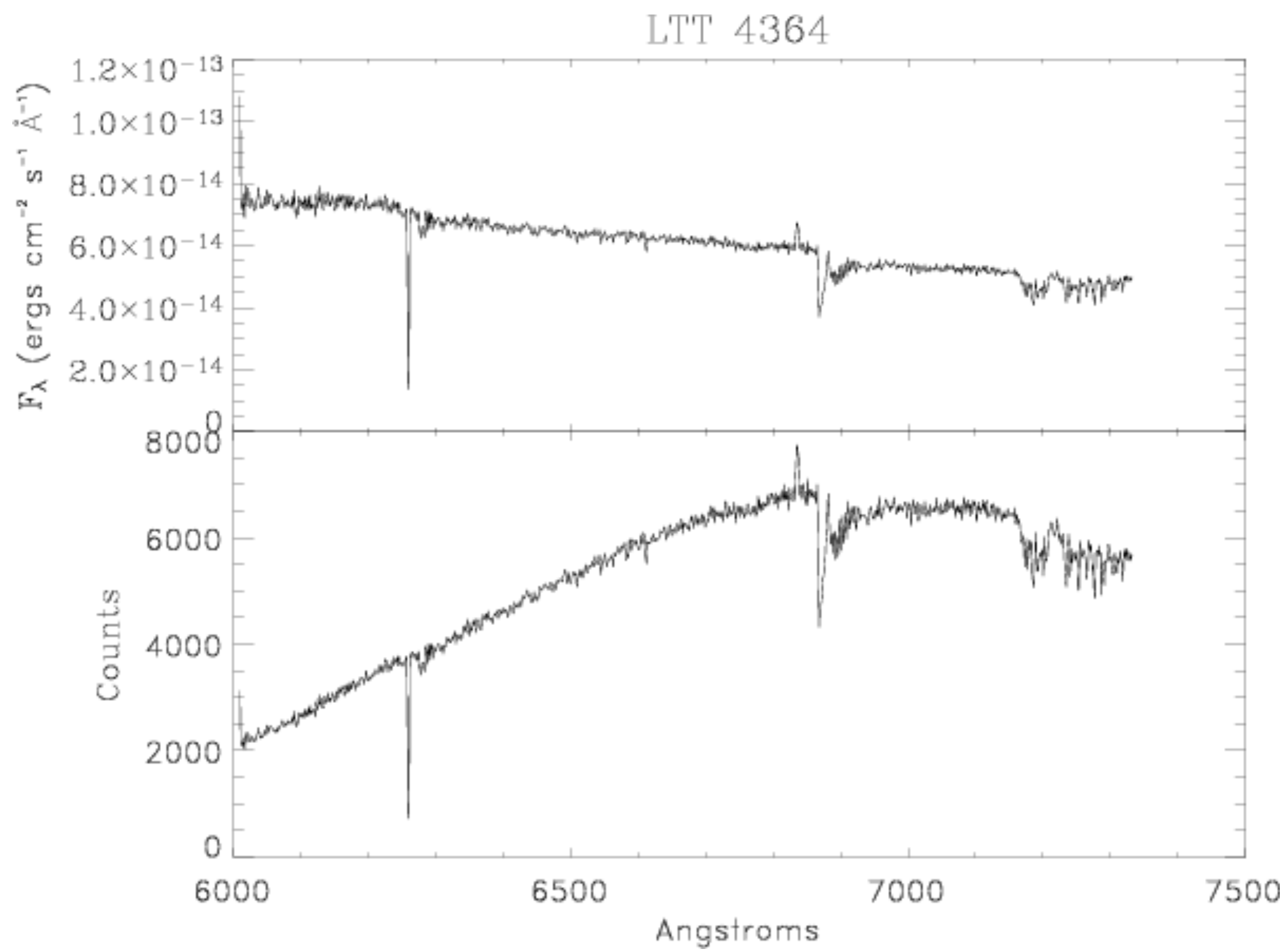


C. Buil / astrosurf

Spectroscopic Calibration

flux calibration:
observe spectrophotometric standard star

compare observed spectrum (counts) to known spectrum



F. Walter

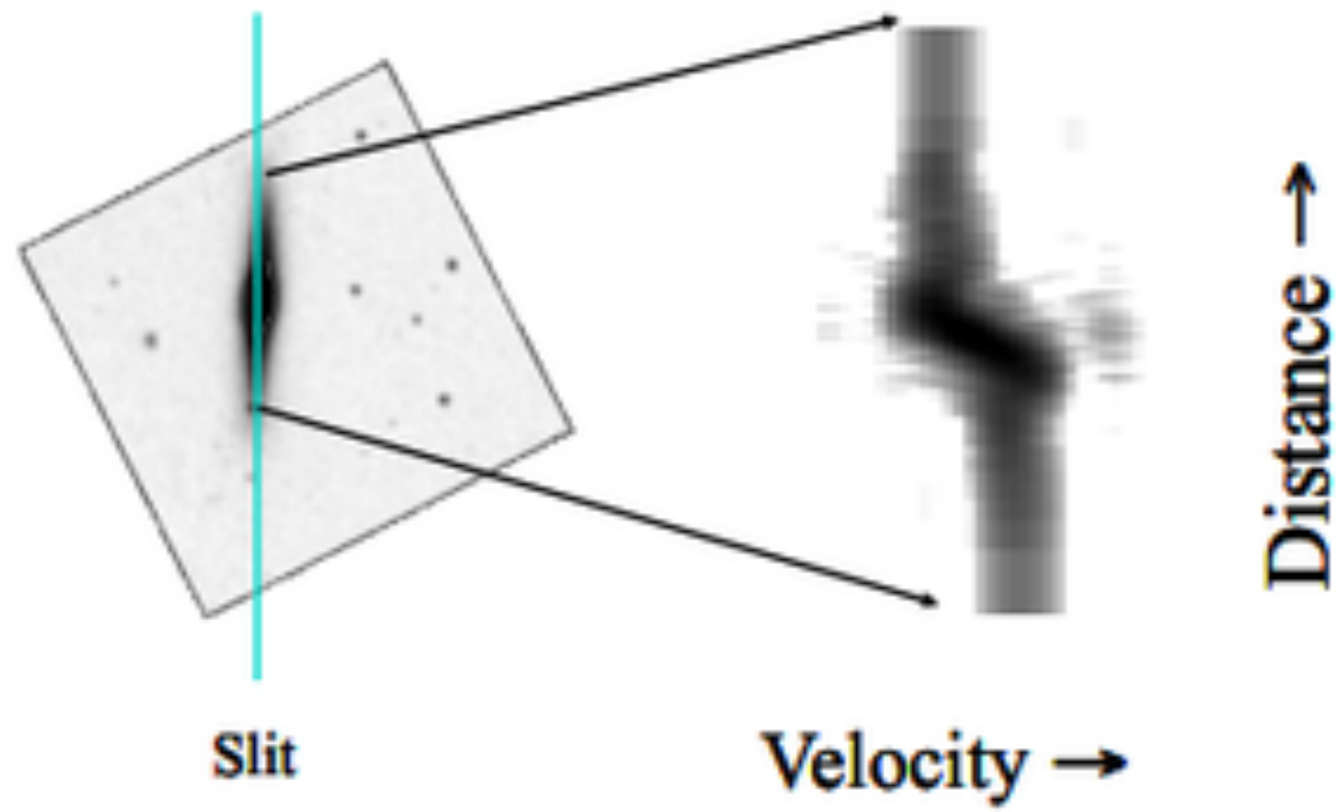
Long-slit spectrographs

most common spectrograph

can only target one (or a few) objects

gives spatial variation

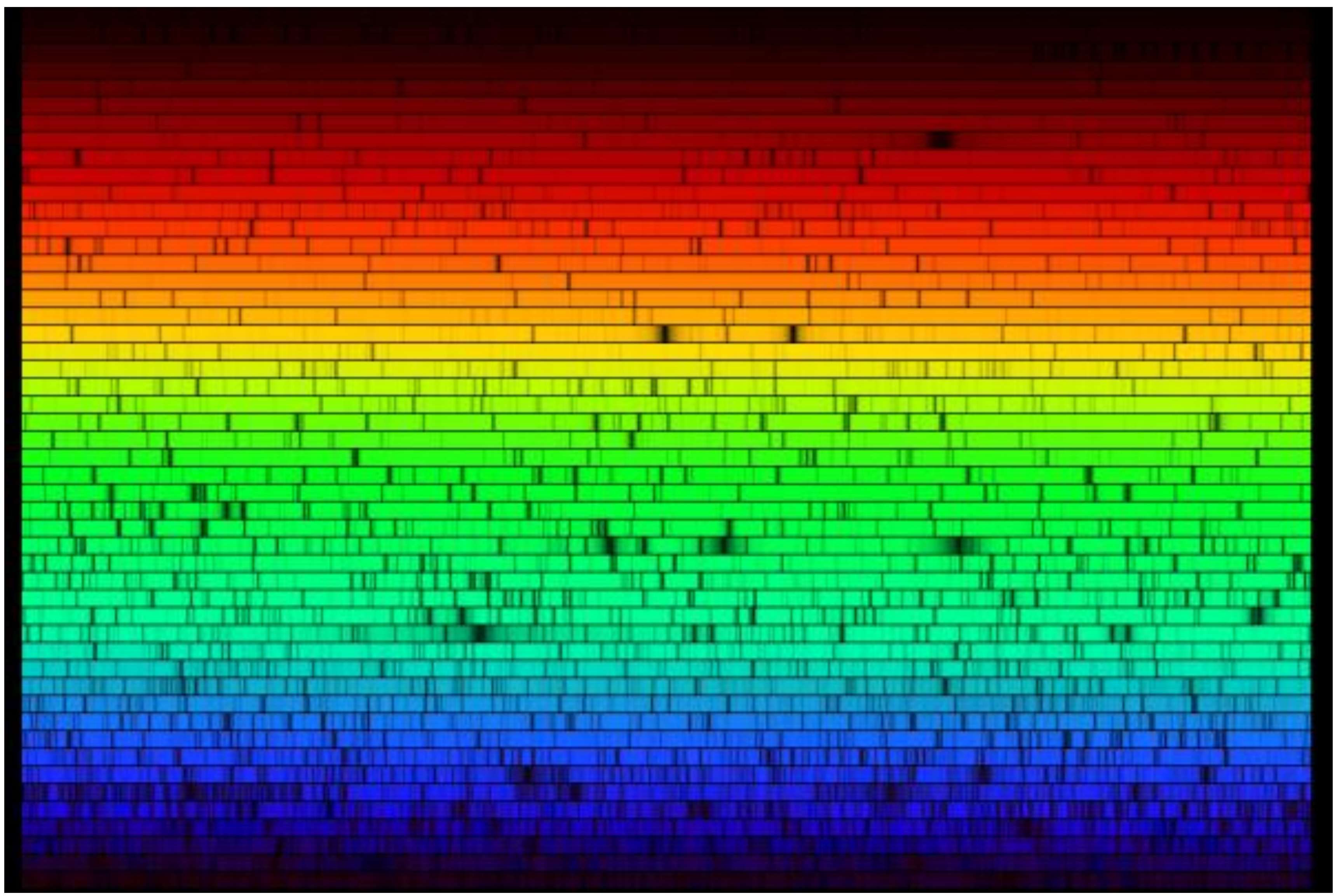
very good estimate of sky background



wikipedia

Echelle spectrographs

- very high resolution long-slit spectrographs
- have additional elements to fit entire spectrum onto CCD
- only for bright objects

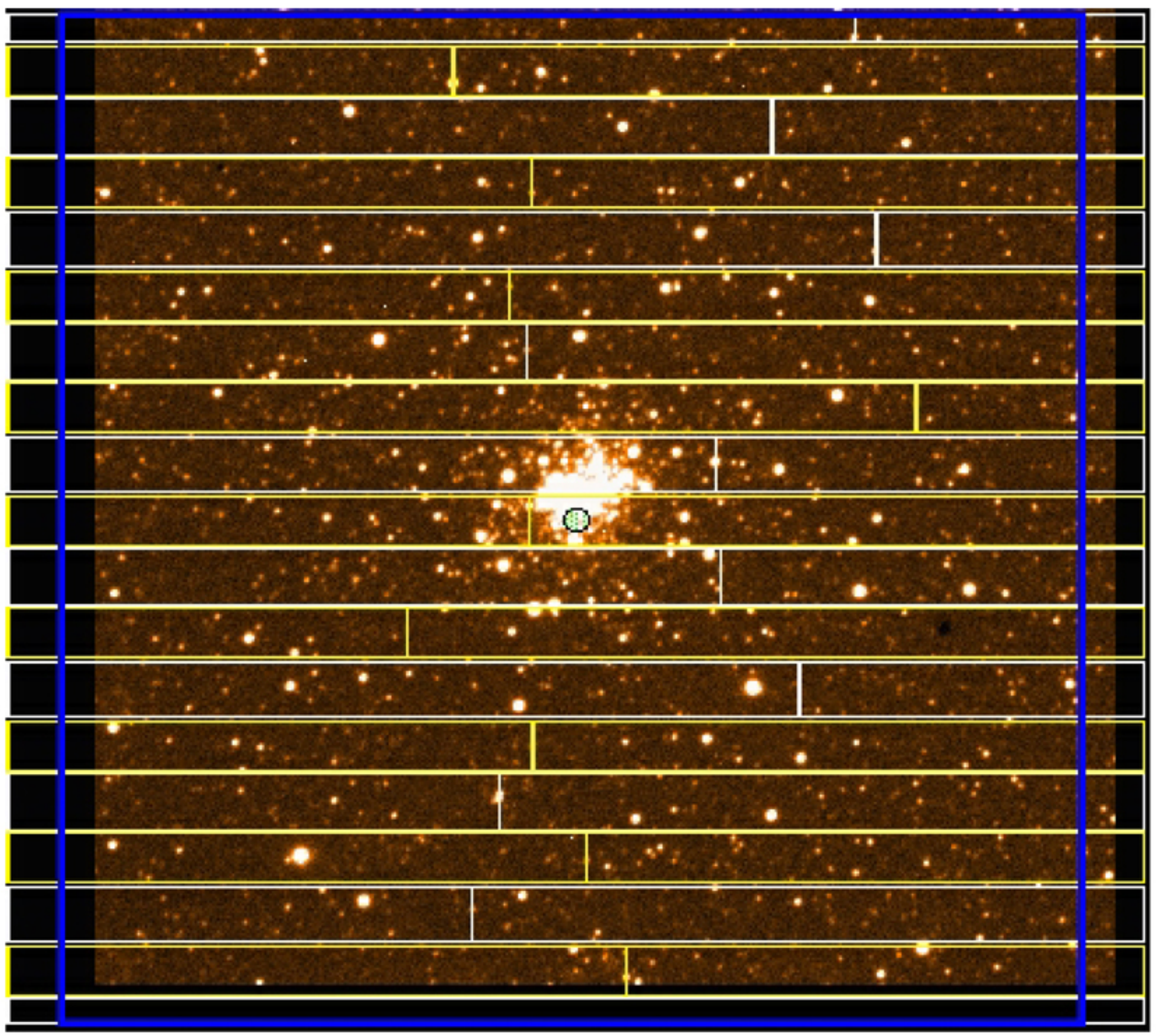


Echelle spectrum of the Sun, 4000-7000Å

NOAO

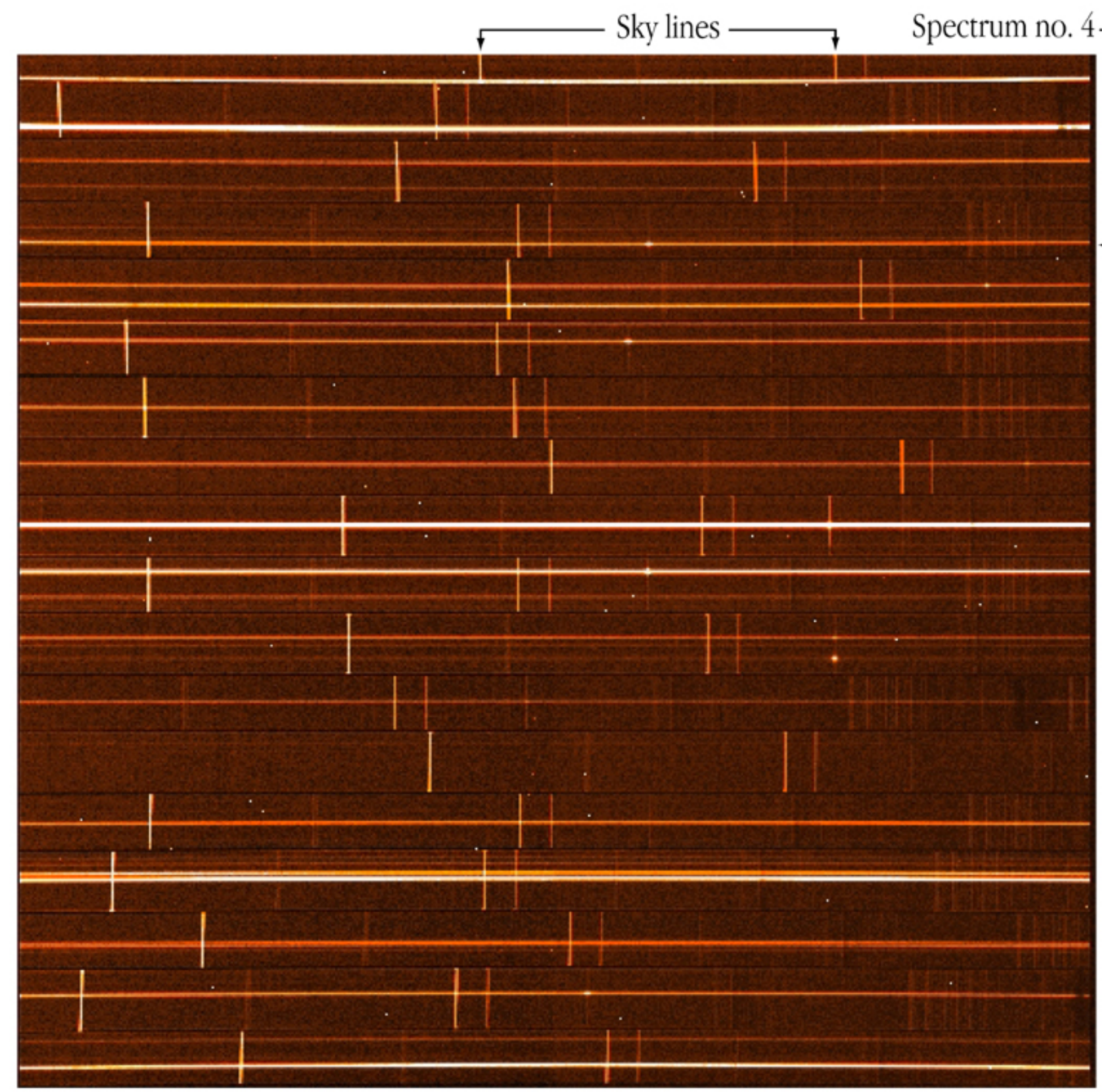
Multi-object spectrographs

make a mask with multiple slits, one per target



Open Cluster NGC 330 in SMC - VLT UT1 + FORS1 (MOS-mode)

ESO PR Photo 38c/98 (7 October 1998)

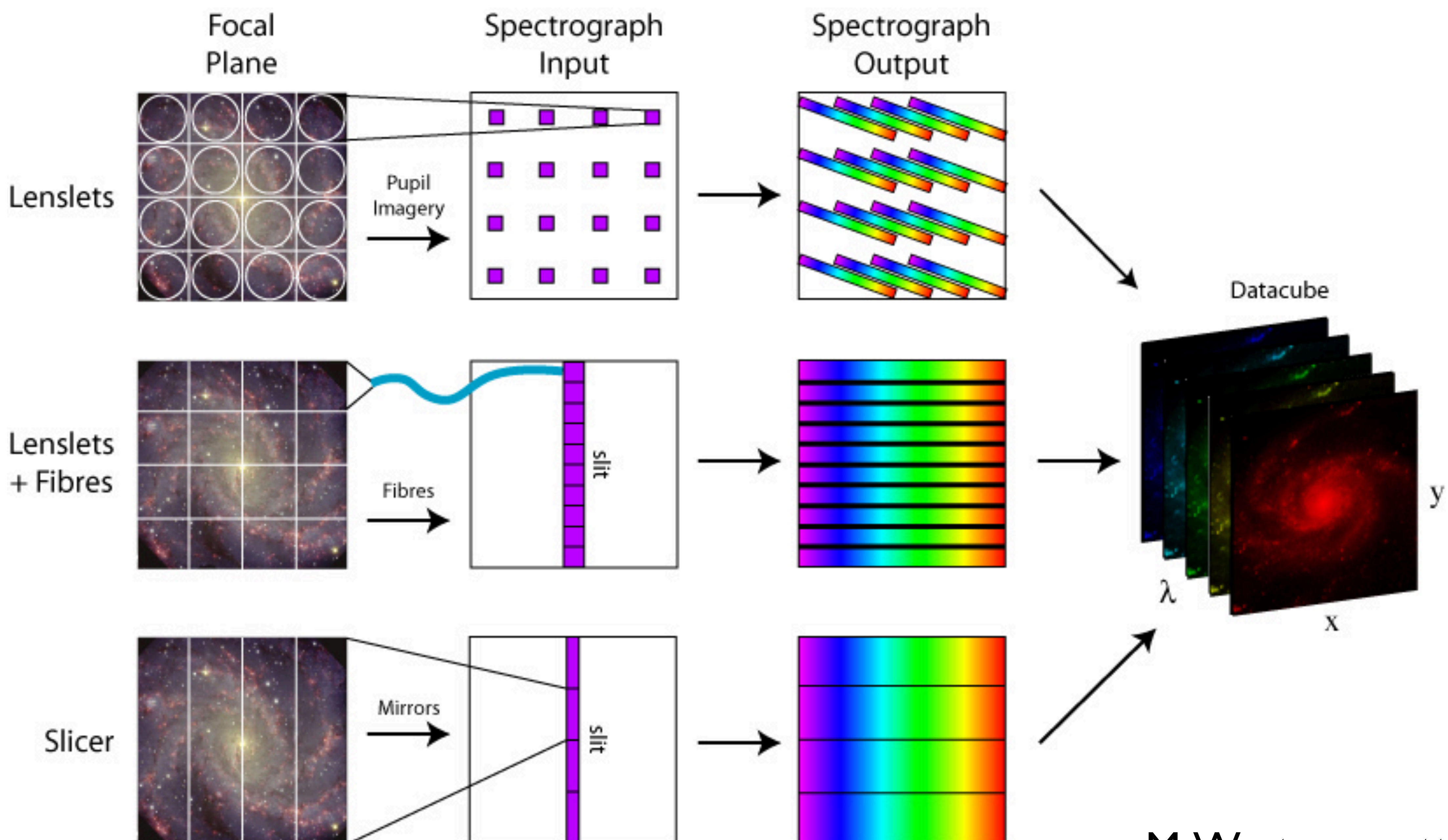


Spectra of Stars in Open Cluster NGC 330 in SMC - VLT UT1 + FORS1 (MOS-mode)

ESO PR Photo 38d/98 (7 October 1998)

Integral-Field Units

divide image into "spaxels" (spectroscopic pixels)



M. Westmoquette