Radio Astronomy

Jin Koda

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- Receiver
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  - Heterodyne receiver
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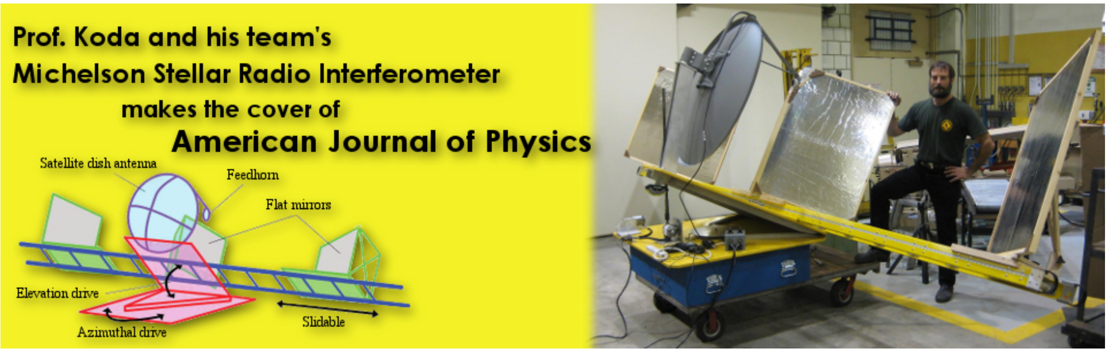


Radio Signal from the Sky

How much energy would we collect?

Suppose we observe a 10 Jy calibrator with the CARMA array for 1 year, 24 hours/day.

$$E = \frac{1}{2} S \eta A \Delta \nu t$$

- The factor of ½ arises because we are sensitive to 1 polarization
- S = source flux density = 10 Jy = 10×10^{-26} watts m^{-2} Hz^{-1}
- η = aperture efficiency ~ 0.60
- A = geometrical collecting area = $6 \times 85 m^2 + 9 \times 29 m^2 = 771\ m^2$
- Δν = instantaneous bandwidth = 3GHz = 3×10^9 Hz
- t = 1 year = 3×10^7 sec

Result: E = 2×10^{-6} joules

1 calorie = 4.2 joules heats 1 cm^3 (20 drops?) of water by 1 C.

➔ Must observe for 100,000 years to heat 1 drop of water by 1 C.

Antenna

Radio telescopes

Telescope/Antenna

Antenna response function

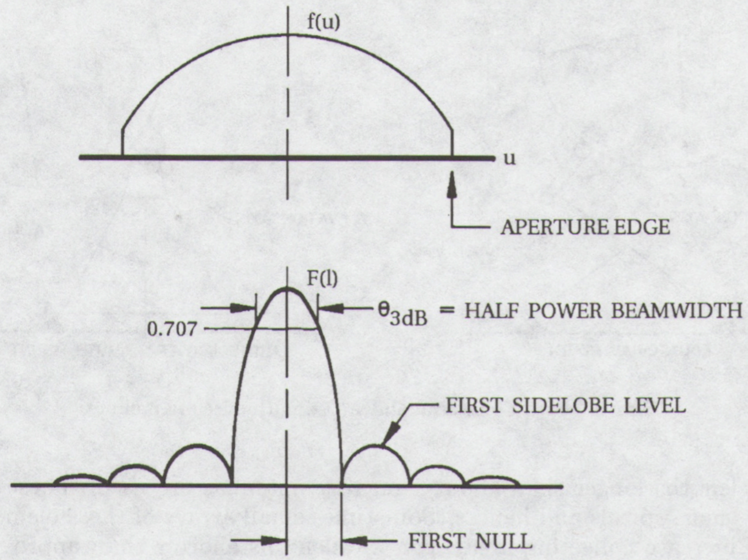


Figure 3–3. The Fourier transform relationship between an antenna aperture distribution and its far-field radiation pattern. The form of the aperture distribution, $f(u)$, and the radiation pattern, $F(l)$, are shown for a one-dimensional example. In general both quantities are complex. Only the amplitudes are shown here.

- Resolution $\sim \frac{\lambda}{D}$
- Main beam & sidelobes
  - depends on surface accuracy
  - sidelobes could be very high

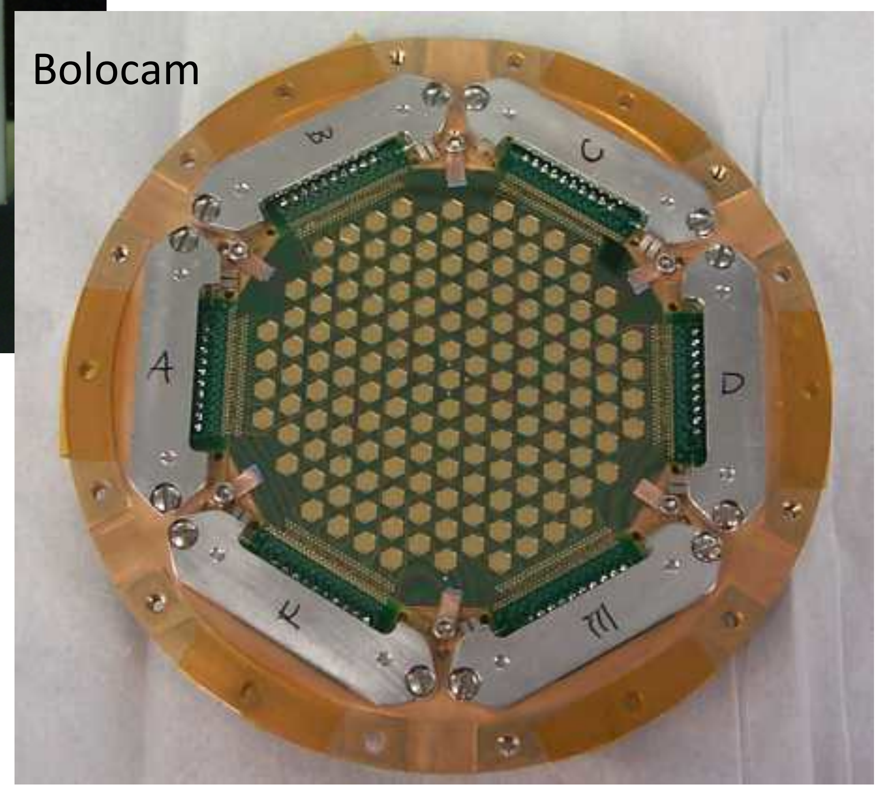
Receiver

detectors for radio astronomy

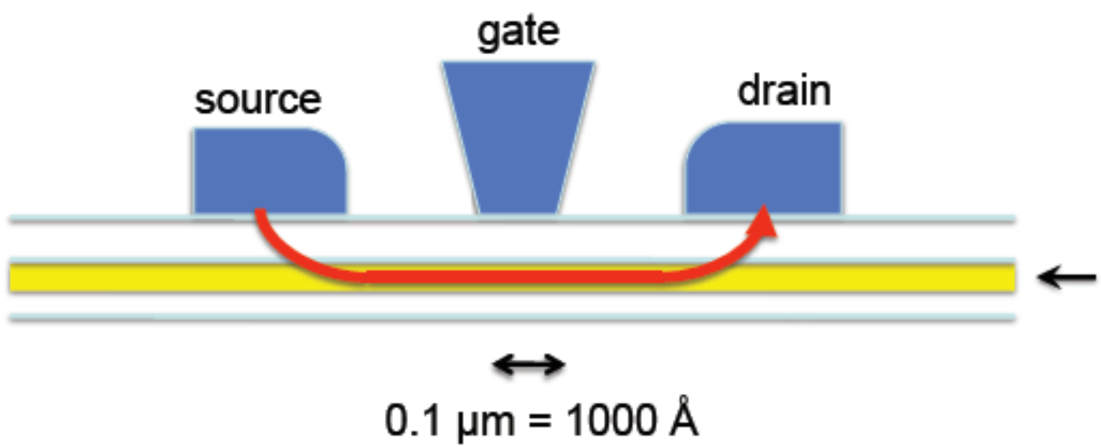
1. bolometers
   - absorbed photon increases temperature, changes resistance
   - phase of incoming signal is lost – unsuitable for aperture synthesis
   - operate at ~0.3 K
2. HEMT (High Electron Mobility Transistor) amplifiers
   - preferred below 50 GHz, good up to 115 GHz
   - operate at ~20 K
3. SIS mixers
   - mixes incoming signal with local oscillator to convert it to a lower frequency where it is amplified (by HEMT)
   - preferred for 100+ GHz
   - operate at ~4 K

Detect signal as electromagnetic wave, not as photon

Bolometer



High Electron Mobility Transistor (HEMT) amplifier



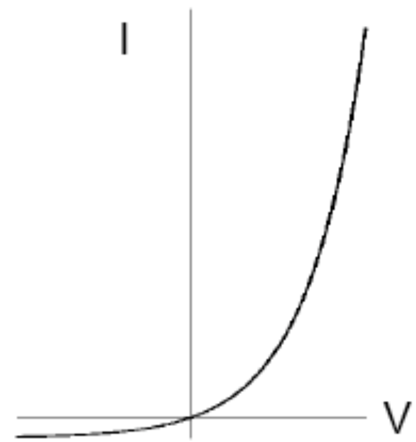
in HEMT (but not in FET), current travels through very pure layer -> no scattering by impurities

- gate voltage controls width of channel, modulates current from source to drain
- to operate at 100 GHz, charge carriers must transit under the gate in ~ 1/10 x 1/100 GHz ~ 10^{-12} sec
- must travel 0.1 um in 10^{-12} sec ~ 100 km s^{-1}

heterodyne receiver

- converts incoming signal to a lower frequency where it can be amplified
- how? 'mix' the incoming signal with a strong 'local oscillator' in a nonlinear device to generate an 'intermediate frequency (IF)'
- essentially the local oscillator is a clock that samples the incoming signal periodically
- example of nonlinear device: a diode

current/voltage relationship for a diode



mixer – a nonlinear device

- linear device (superposition principle):
  $\omega_1, \omega_2 \rightarrow$ $\boxed{\text{linear device}}$ $\rightarrow \omega_1, \omega_2$
- nonlinear device:
  $\omega_1, \omega_2 \rightarrow$ $\boxed{\text{nonlinear device}}$ $\rightarrow \omega_1, \omega_2, \omega_1+\omega_2, \omega_1-\omega_2, 2\omega_1+\omega_2, \ldots$
- diode is an example of a nonlinear device:

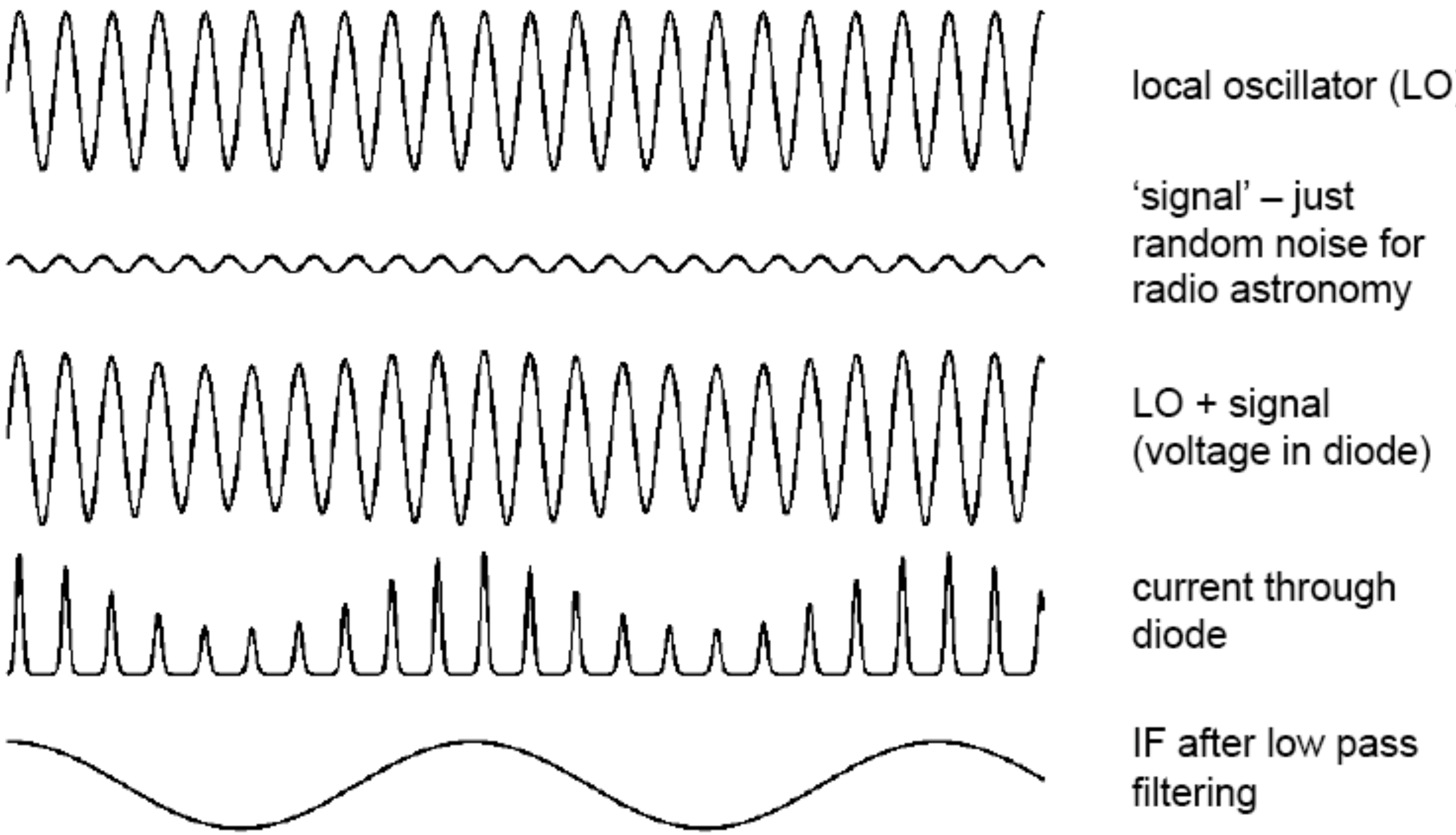
  $I = I_0(e^{\alpha V} - 1) \sim I_0(\alpha V + \tfrac{1}{2}\alpha^2 V^2 + ..)$

  $V = A\cos\omega_1 t + B\cos\omega_2 t$

  $V^2 = A^2\cos^2\omega_1 t + B^2\cos^2\omega_2 t + 2AB\cos\omega_1 t\cos\omega_2 t + \ldots$
  $= \ldots + AB\cos(\omega_1 + \omega_2)t + AB\cos(\omega_1 - \omega_2)t + \ldots$

- note: *amplitude* at frequency ω_1-ω_2 is *linearly* related to amplitudes A and B

waveforms in a heterodyne receiver



Why Large Telescope?

Why Large Telescope?

- Large collecting area
  - More photons
  - Fainter targets

- Higher resolution (at cm/mm-wavelengths)
  - Angular resolution: λ/D

Angular Resolution & Antenna response function

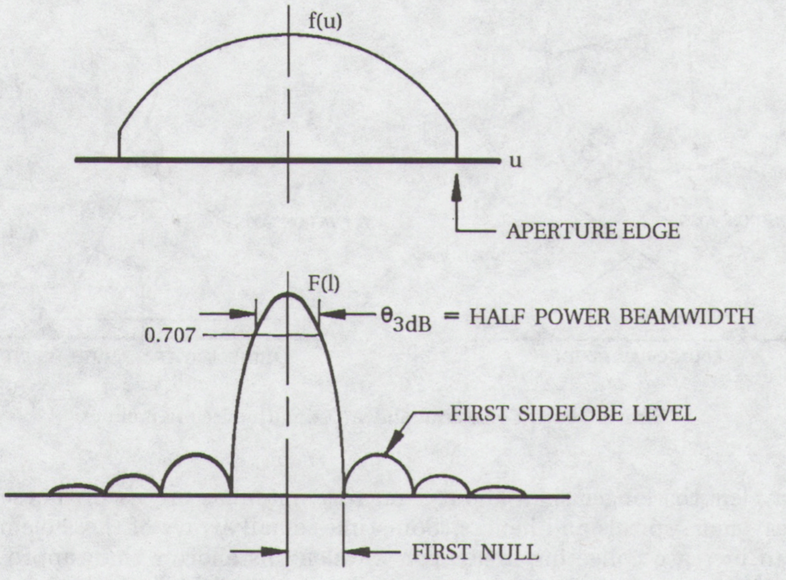
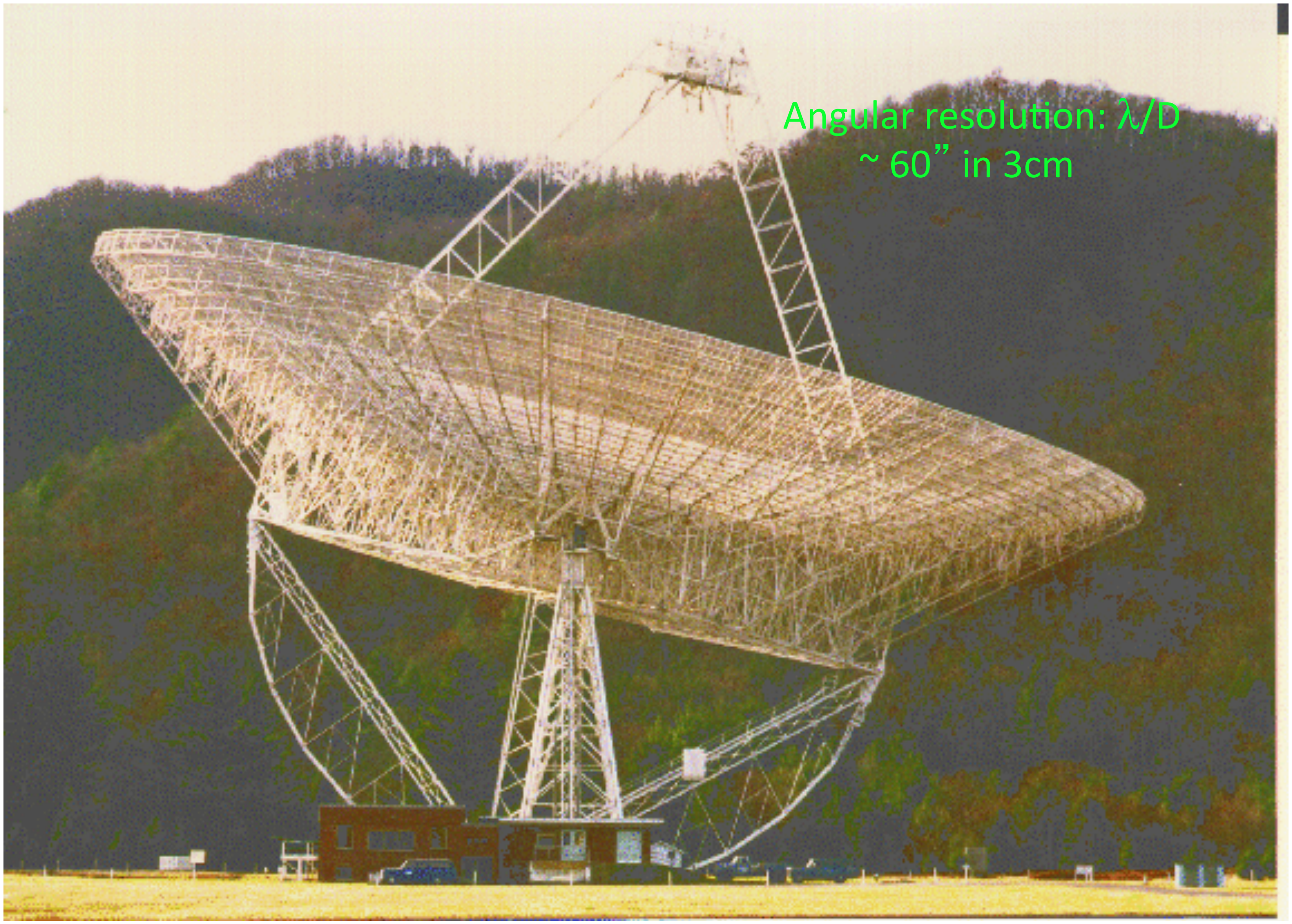


Figure 3–3. The Fourier transform relationship between an antenna aperture distribution and its far-field radiation pattern. The form of the aperture distribution, $f(u)$, and the radiation pattern, $F(l)$, are shown for a one-dimensional example. In general both quantities are complex. Only the amplitudes are shown here.

- Resolution ~ wavelength/antenna diameter
- Main beam & sidelobes

[Old] Green Bank Telescope (cm-wave)

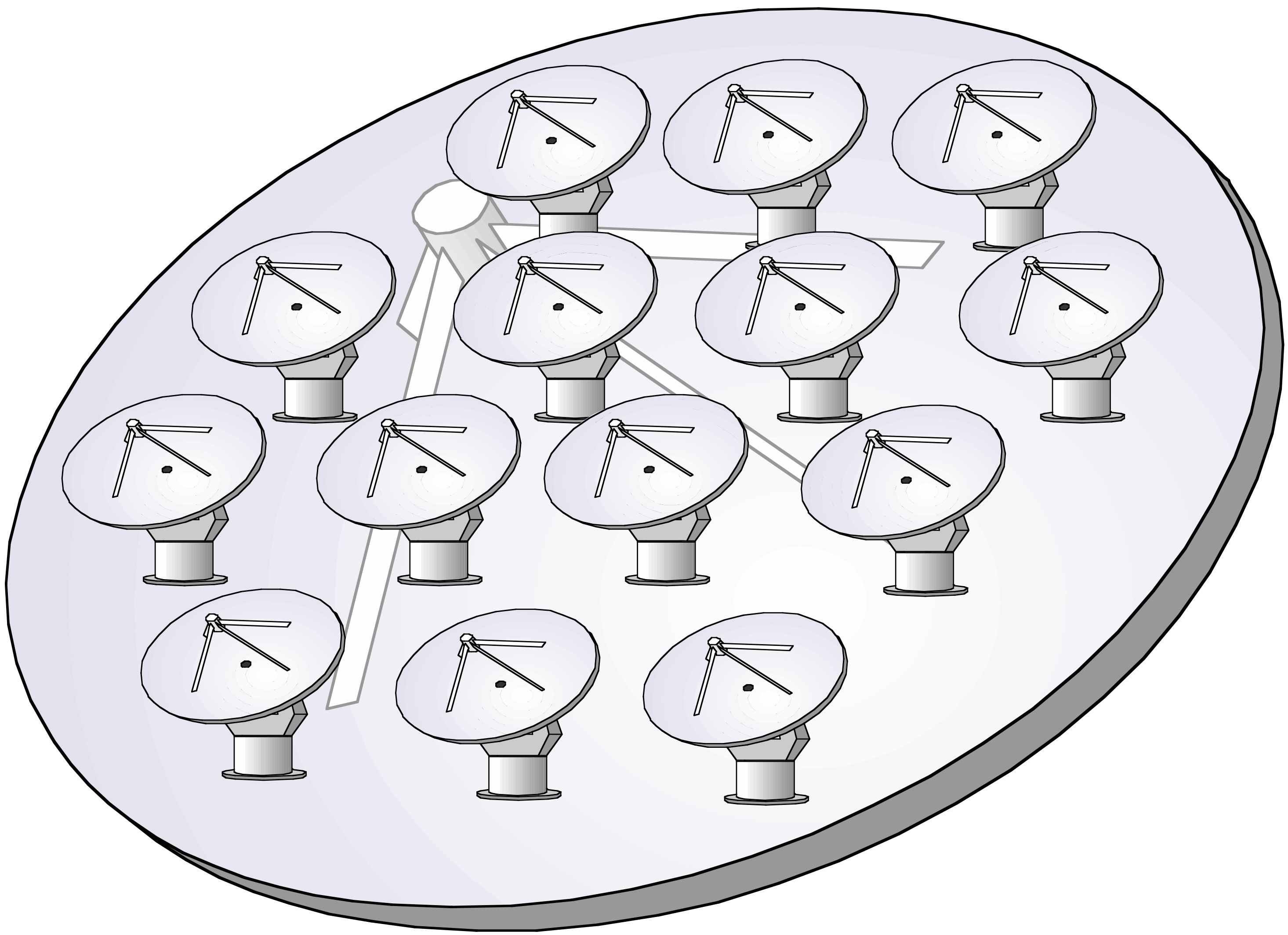


100m diameter

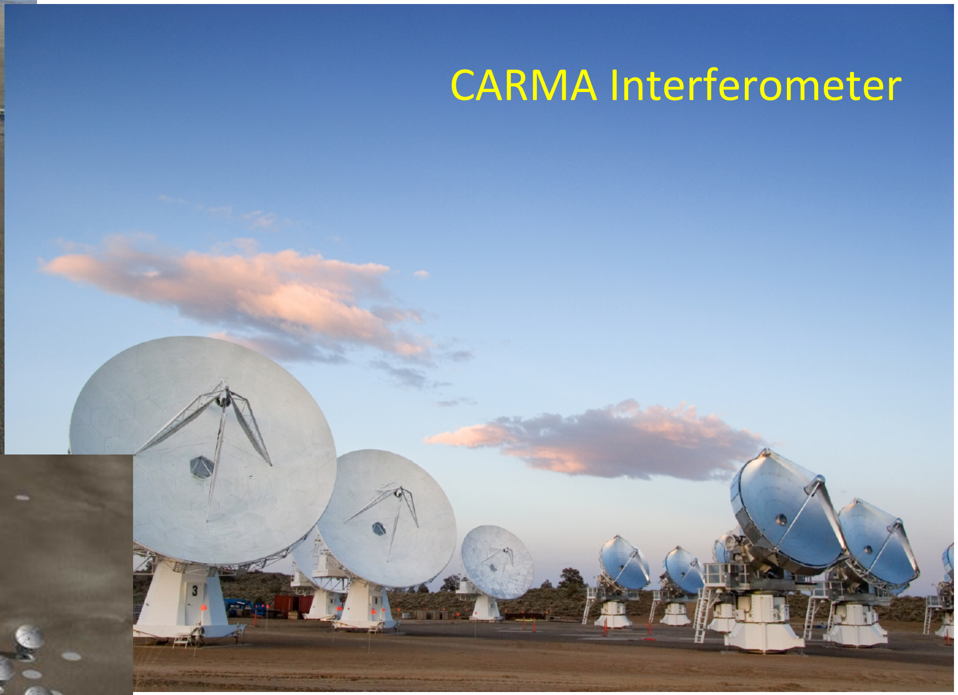
Collapsed

Interferometers

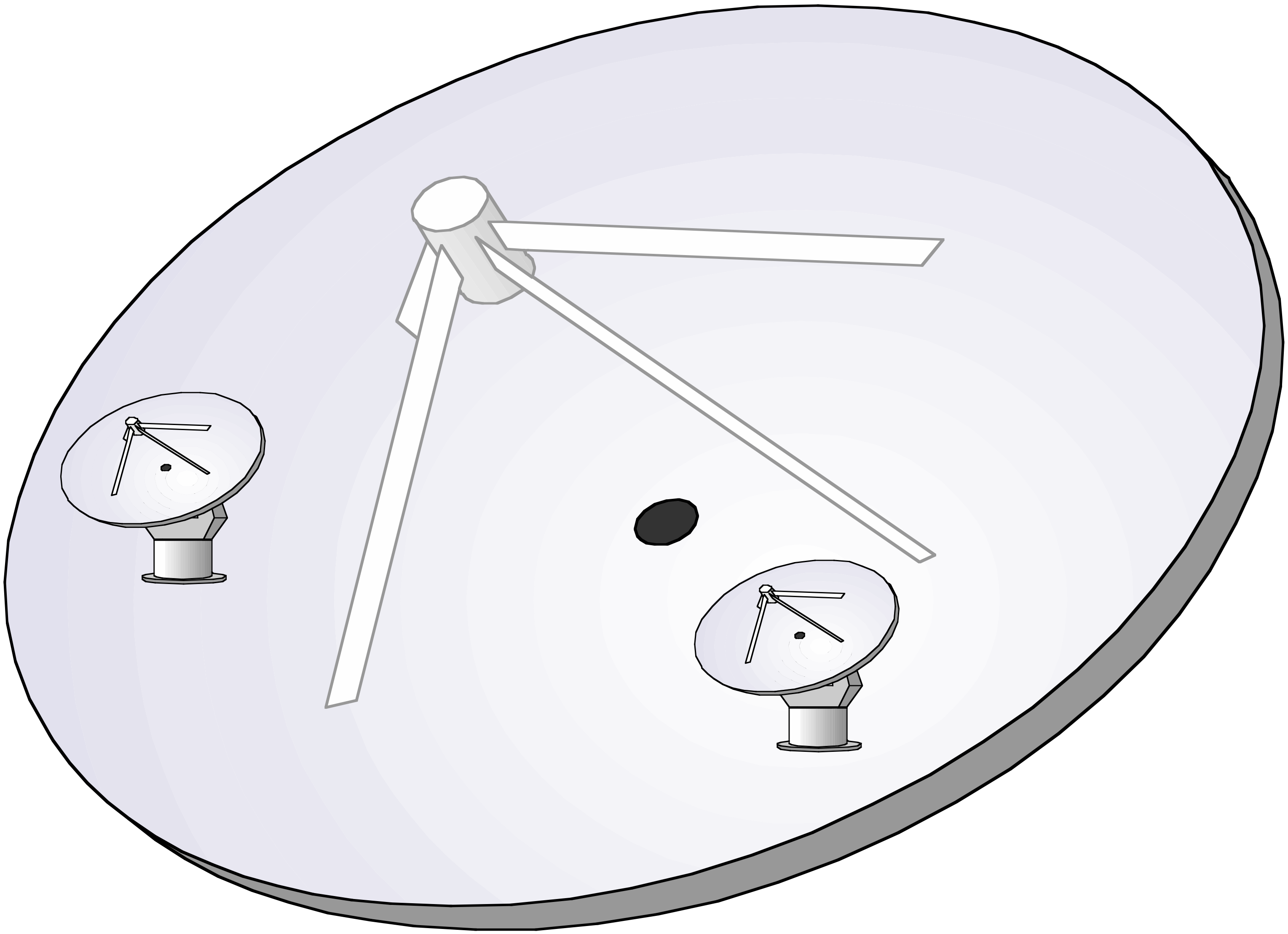
Interferometer (Many Elements)



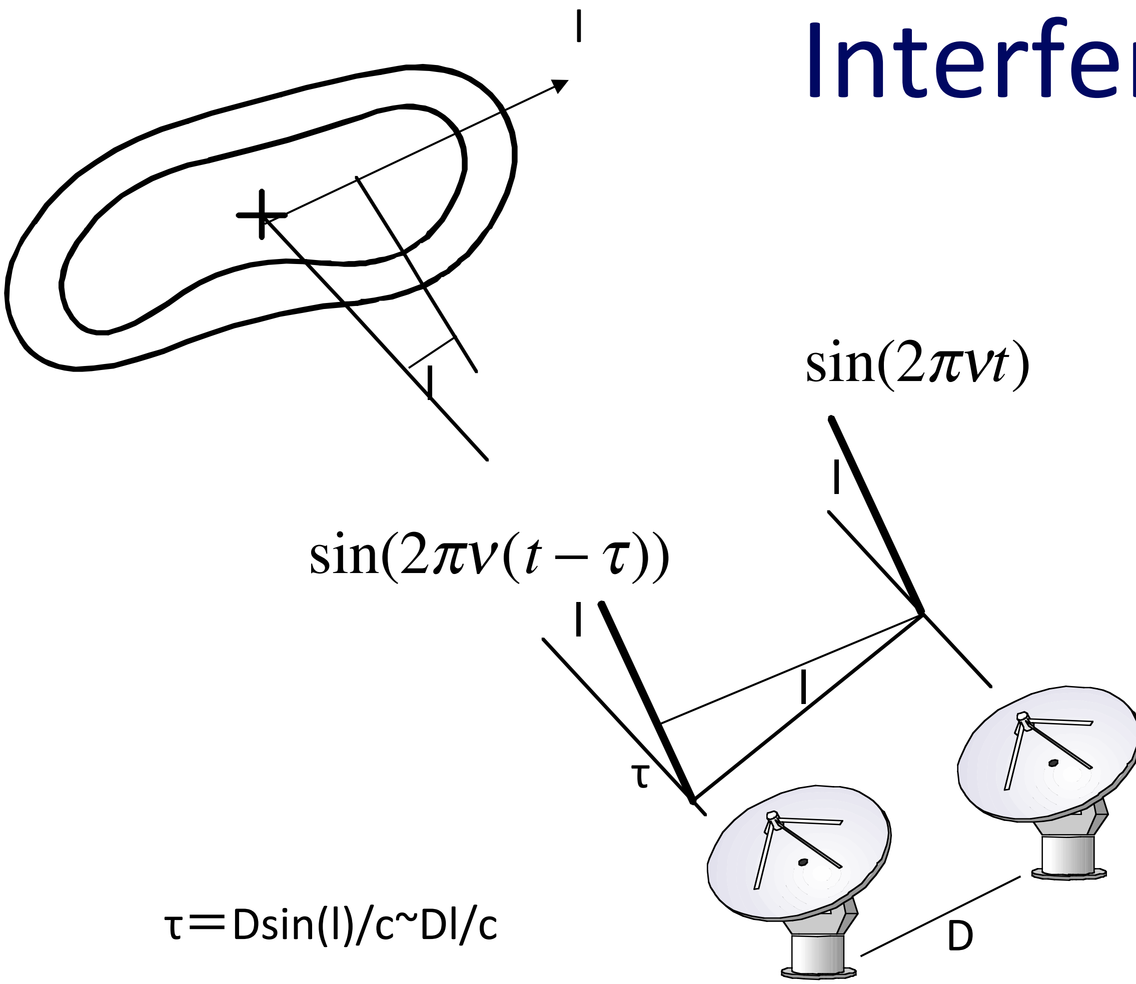
Radio Interferometers

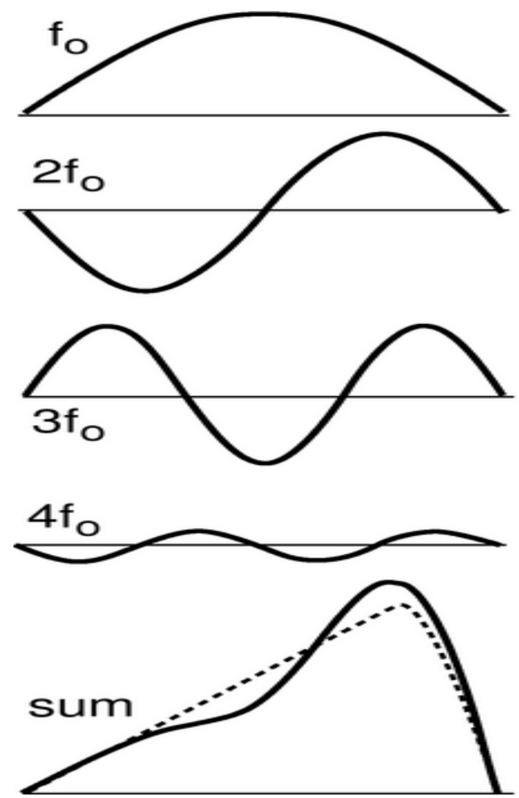


Interferometer (2 Elements)



Interferometer

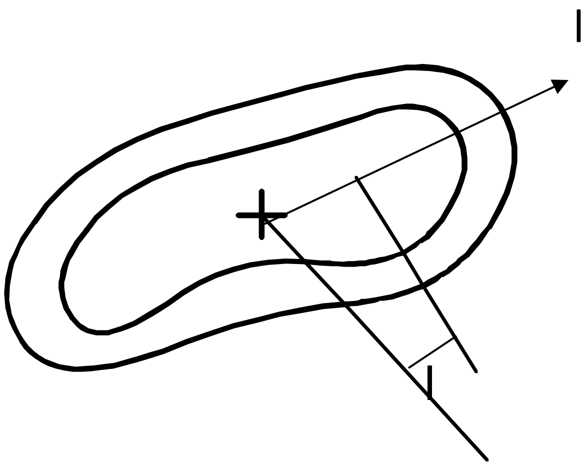


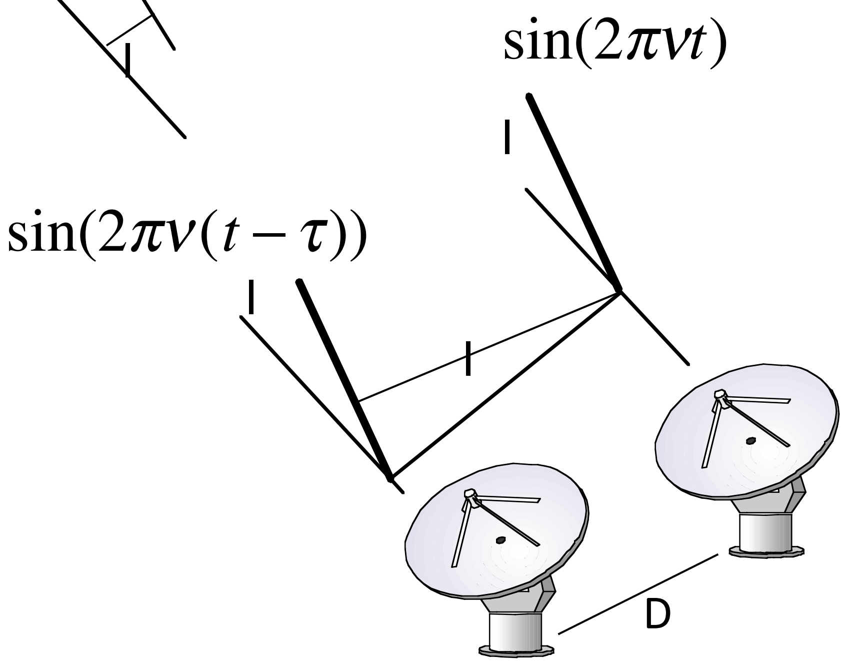


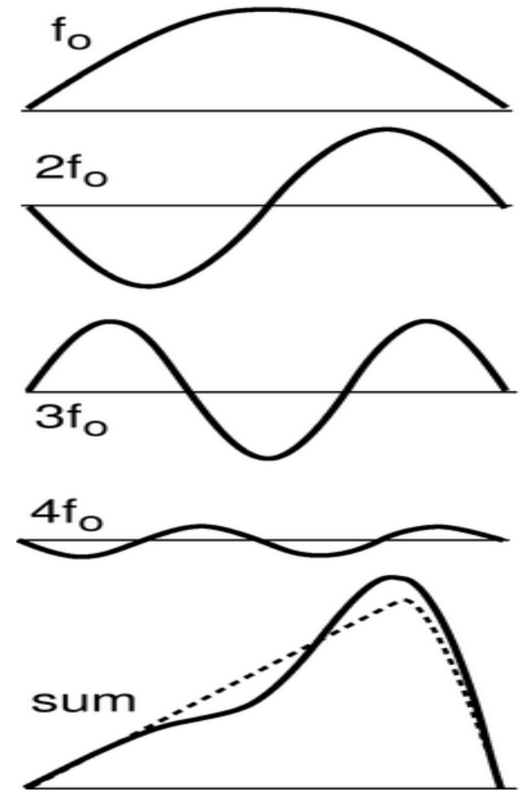
$$
\begin{aligned}
F &= \sin(2\pi\nu t)\sin(2\pi\nu(t-\tau)) \\
&\approx \cos(2\pi\nu\tau) = \cos\left(\frac{2\pi D l}{\lambda}\right)
\end{aligned}
$$

A Fourier component

Interferometer



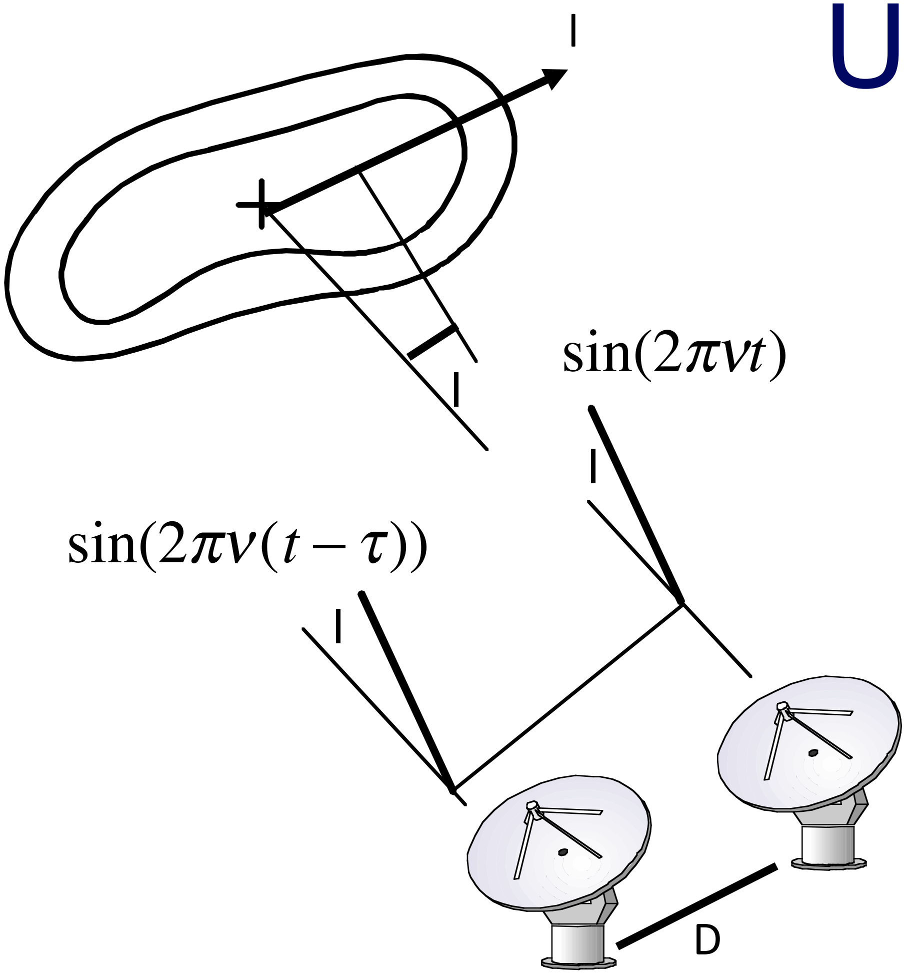


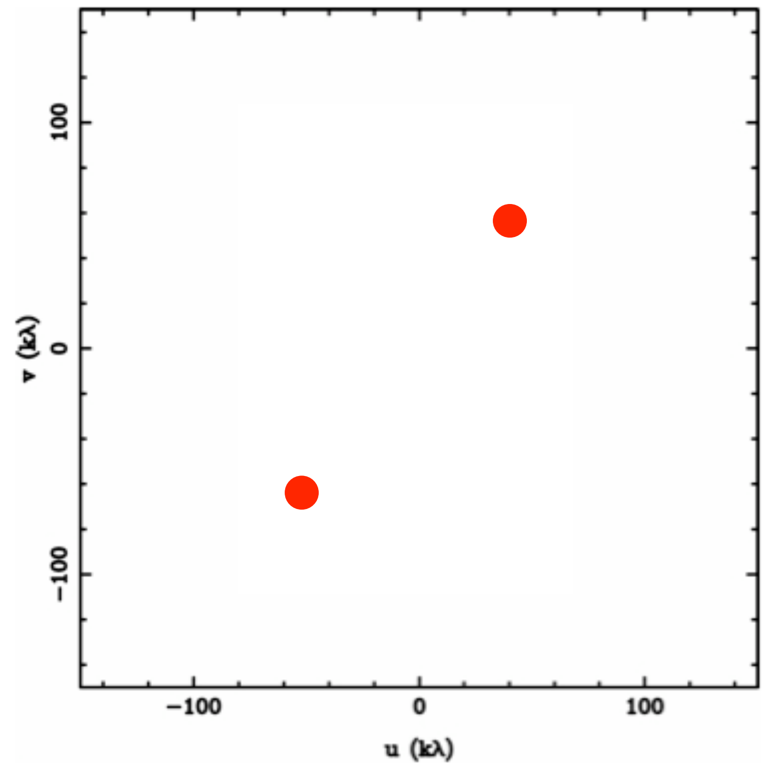


$$F = \sin(2\pi\nu t)\sin(2\pi\nu(t-\tau))$$
$$\approx \cos(2\pi\nu\tau) = \cos(\frac{2\pi Dl}{\lambda})$$

$$\mathrm{Im}age = \sum_D A_D \cos(\frac{2\pi Dl}{\lambda})$$

UV-coverage



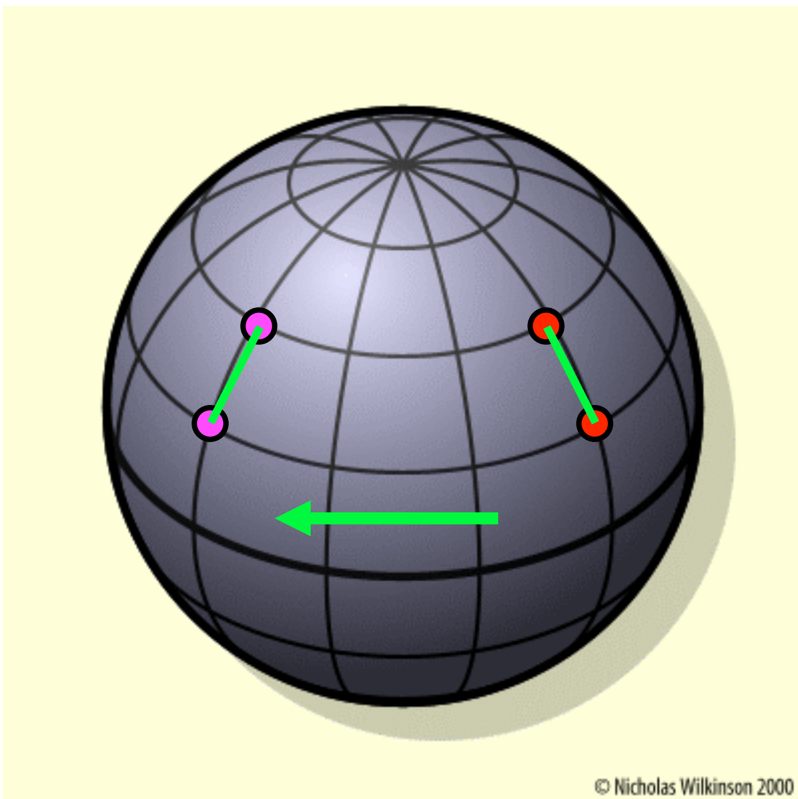


$$F = \sin(2\pi\nu t)\sin(2\pi\nu(t-\tau)) \approx \cos(2\pi\nu\tau) = \cos\left(\frac{2\pi Dl}{\lambda}\right)$$

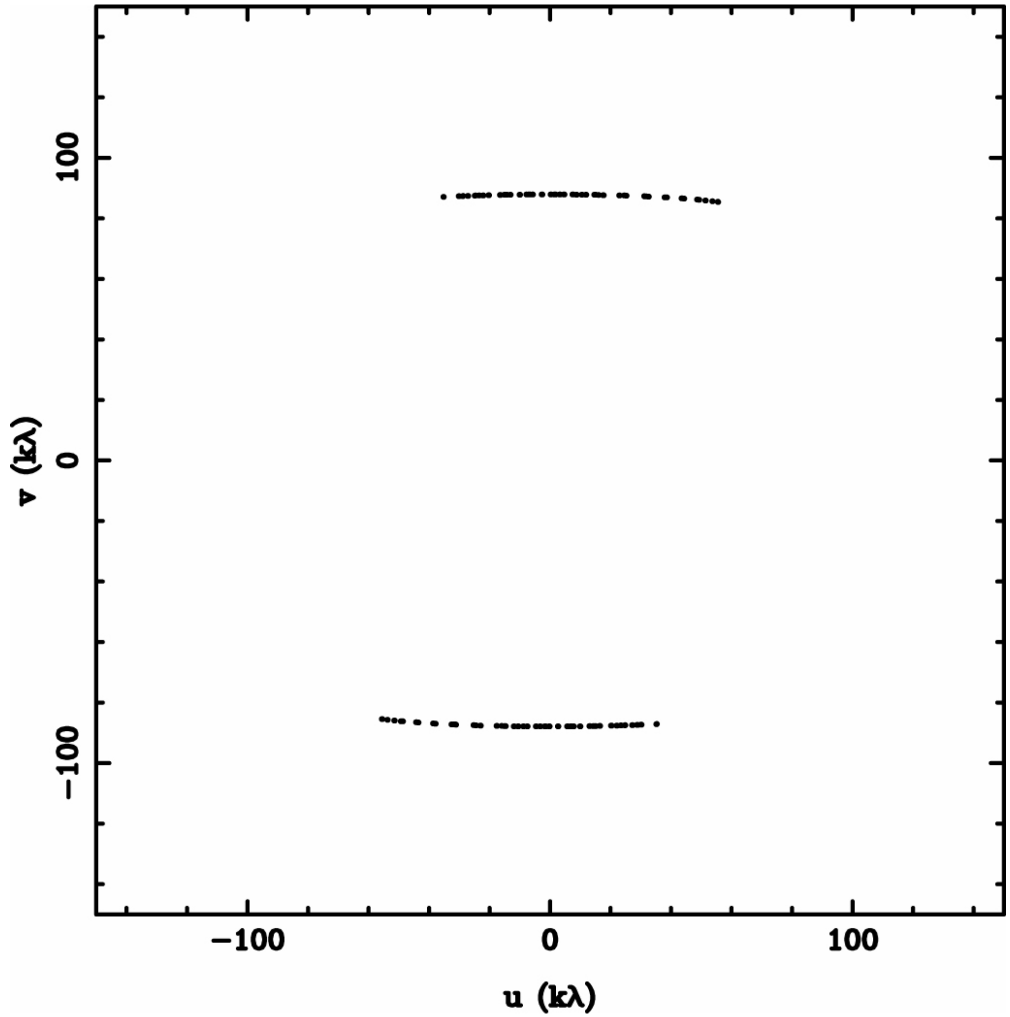
$$\mathrm{Im}age = \sum_D A_D \cos\left(\frac{2\pi Dl}{\lambda}\right)$$

Synthesis imaging (2 antennas)

Earth (rotating)



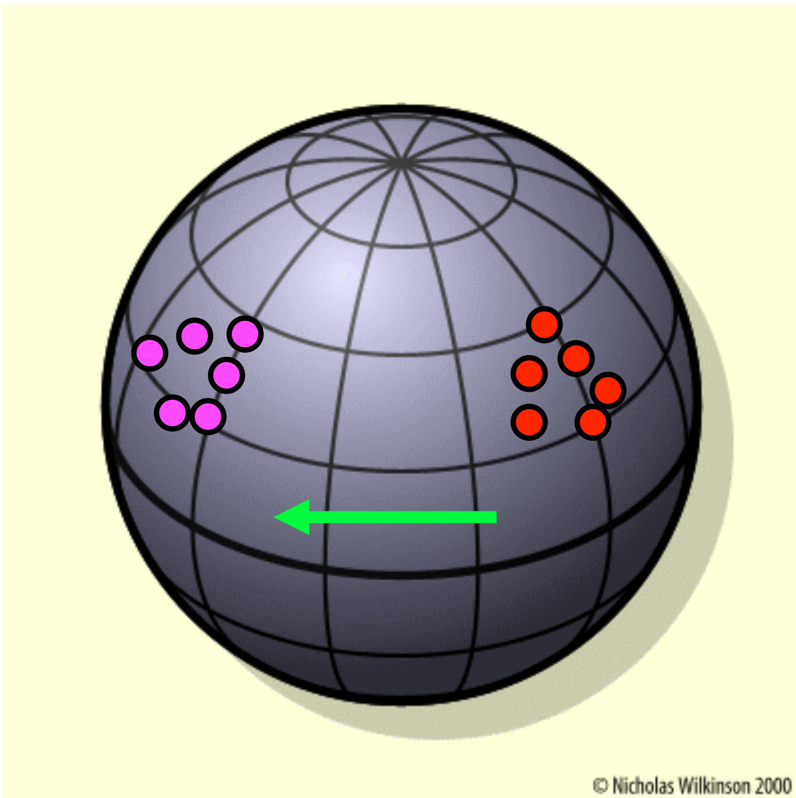
Data coverage in Fourier space



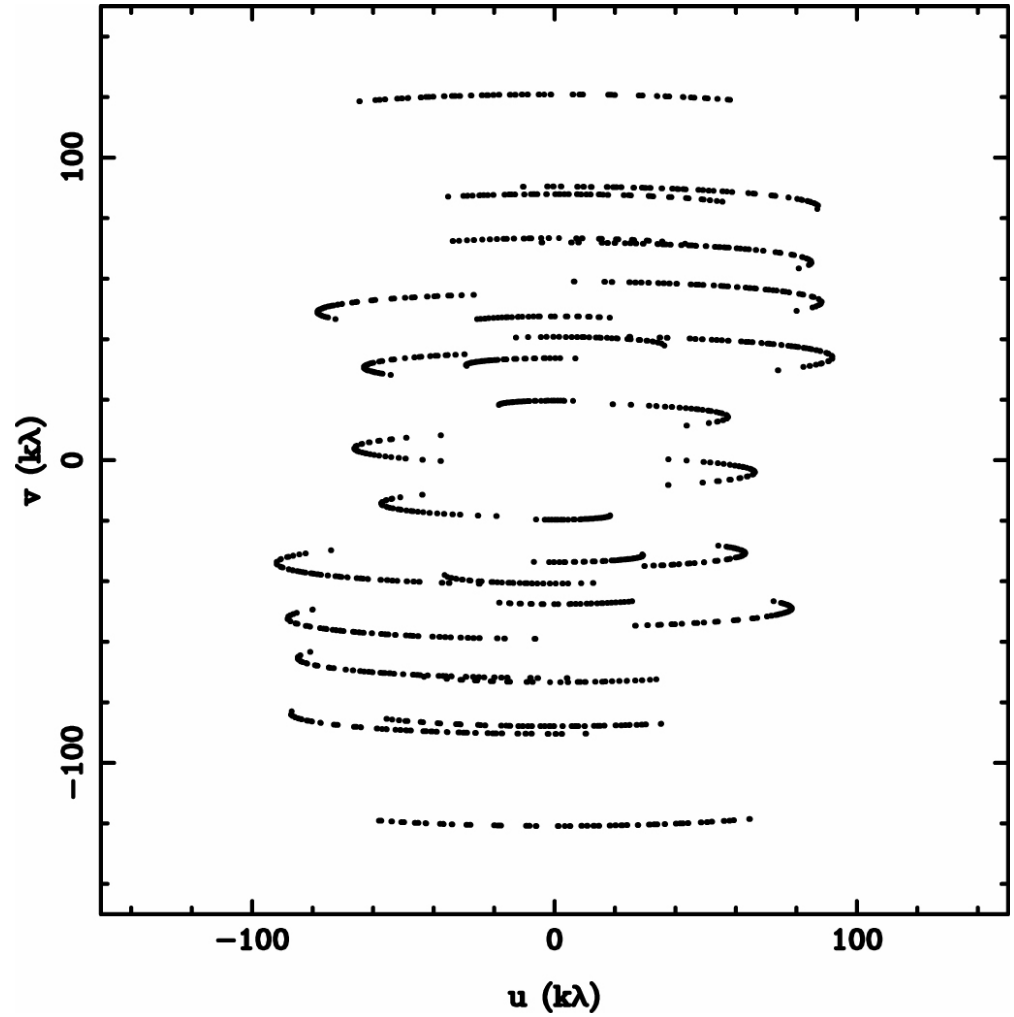
Look down the earth
from your favorite target

Synthesis imaging (6 antennas)

Earth (rotating)



Data coverage in Fourier space

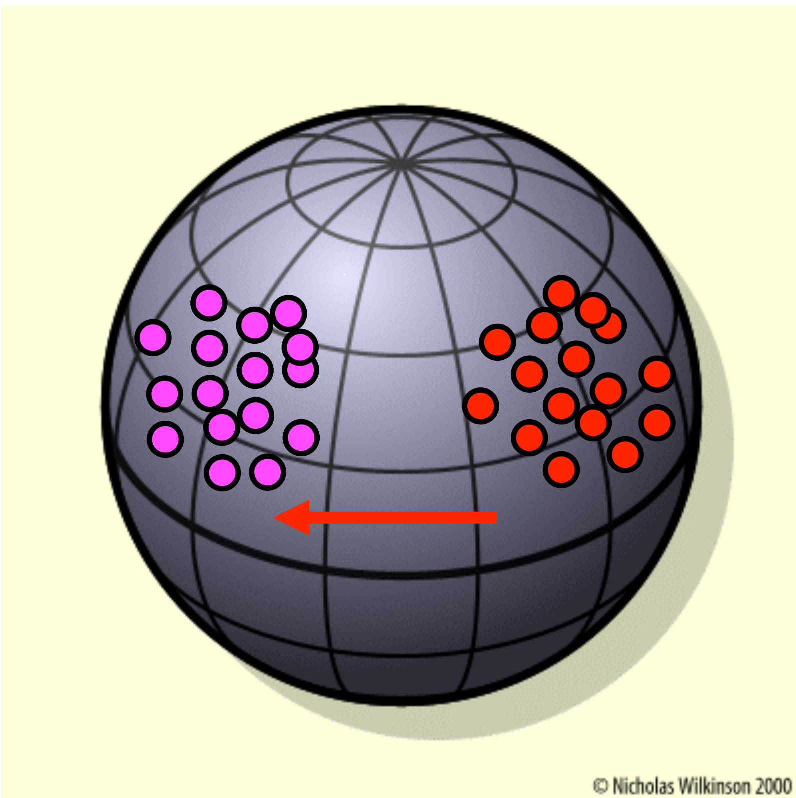


# of ant pairs = N(N+1)/2

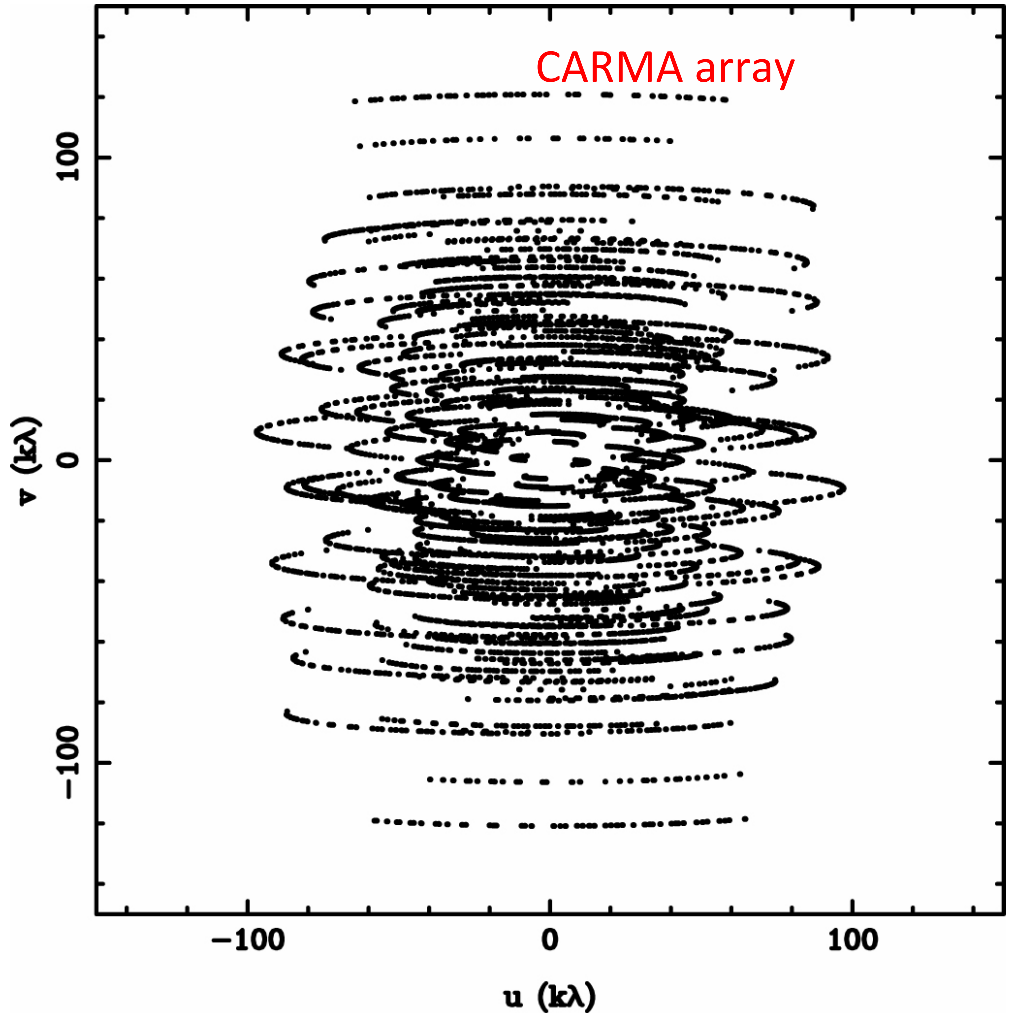
N=6 ⟶ 15 pairs

Synthesis imaging (15 antennas)

Earth (rotating)



Data coverage in Fourier space

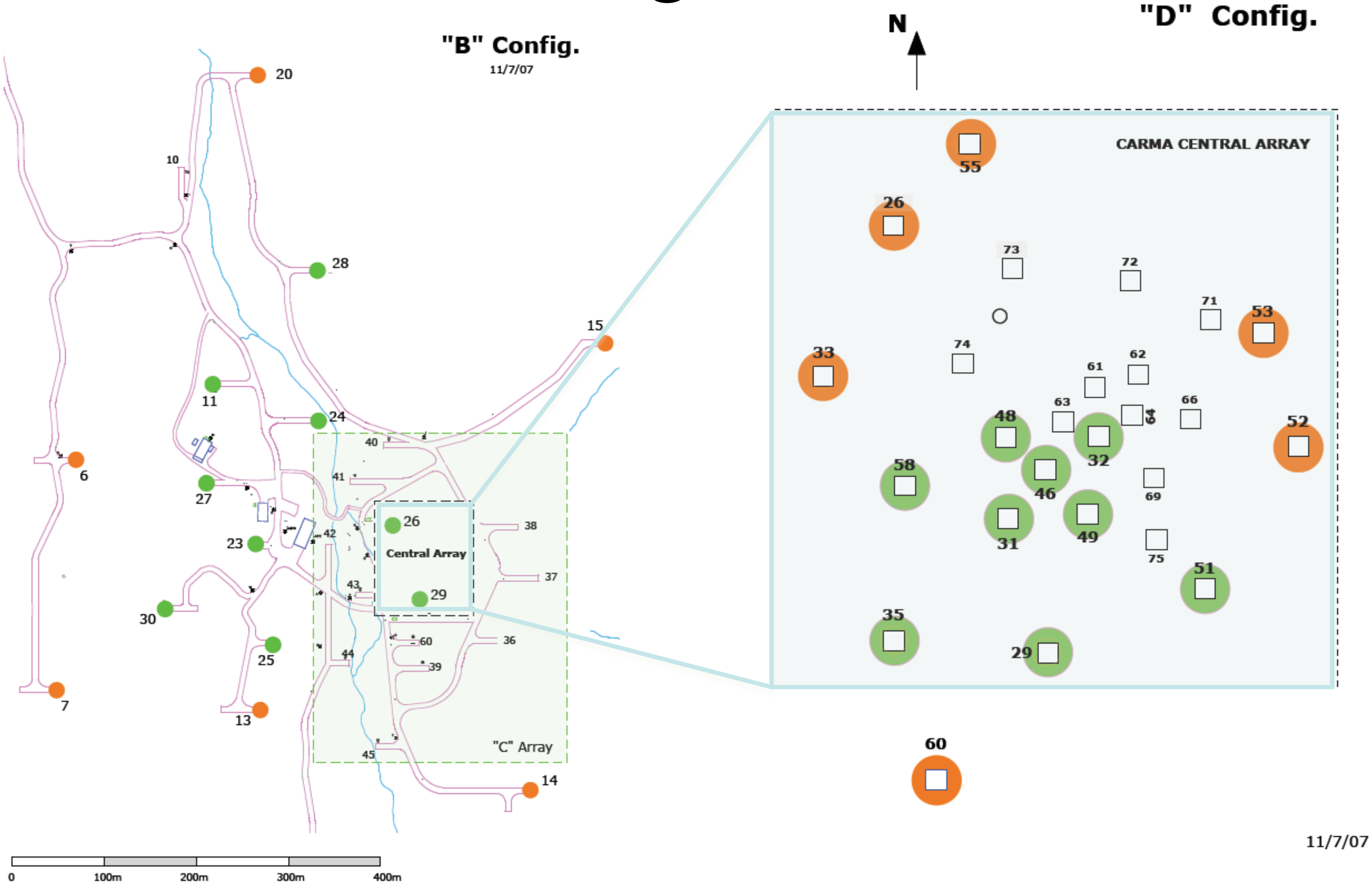


# of ant pairs = N(N+1)/2

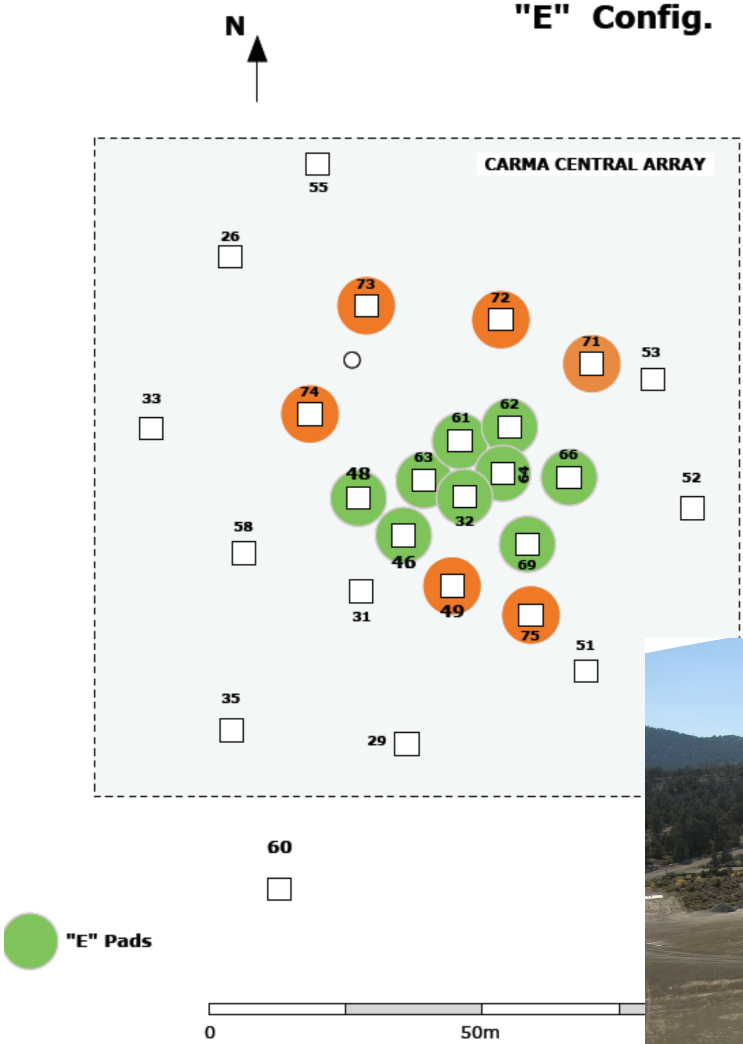
N=15 ⟶ 105 pairs!

Configuration Change

B-configuration

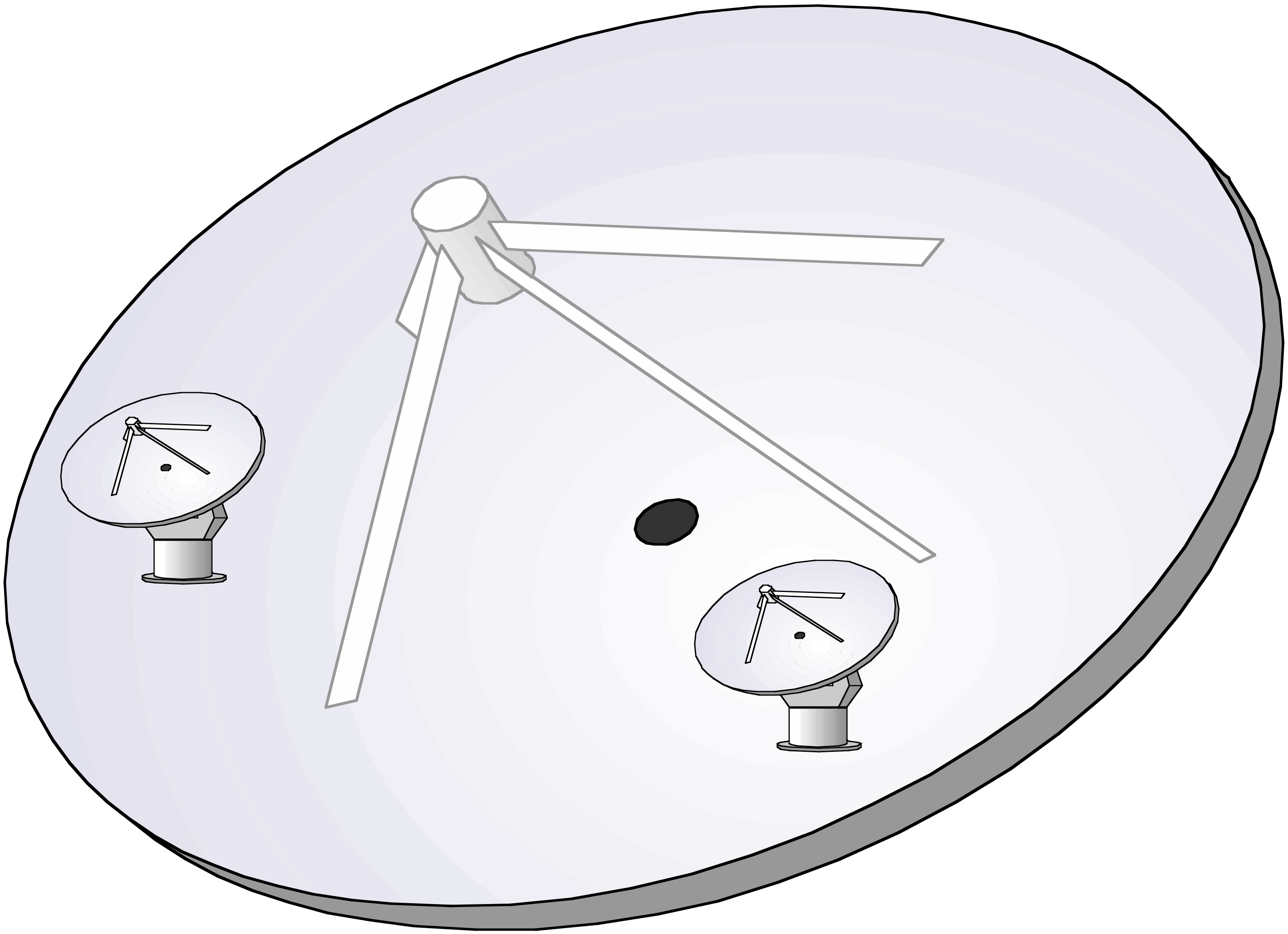


E-configuration

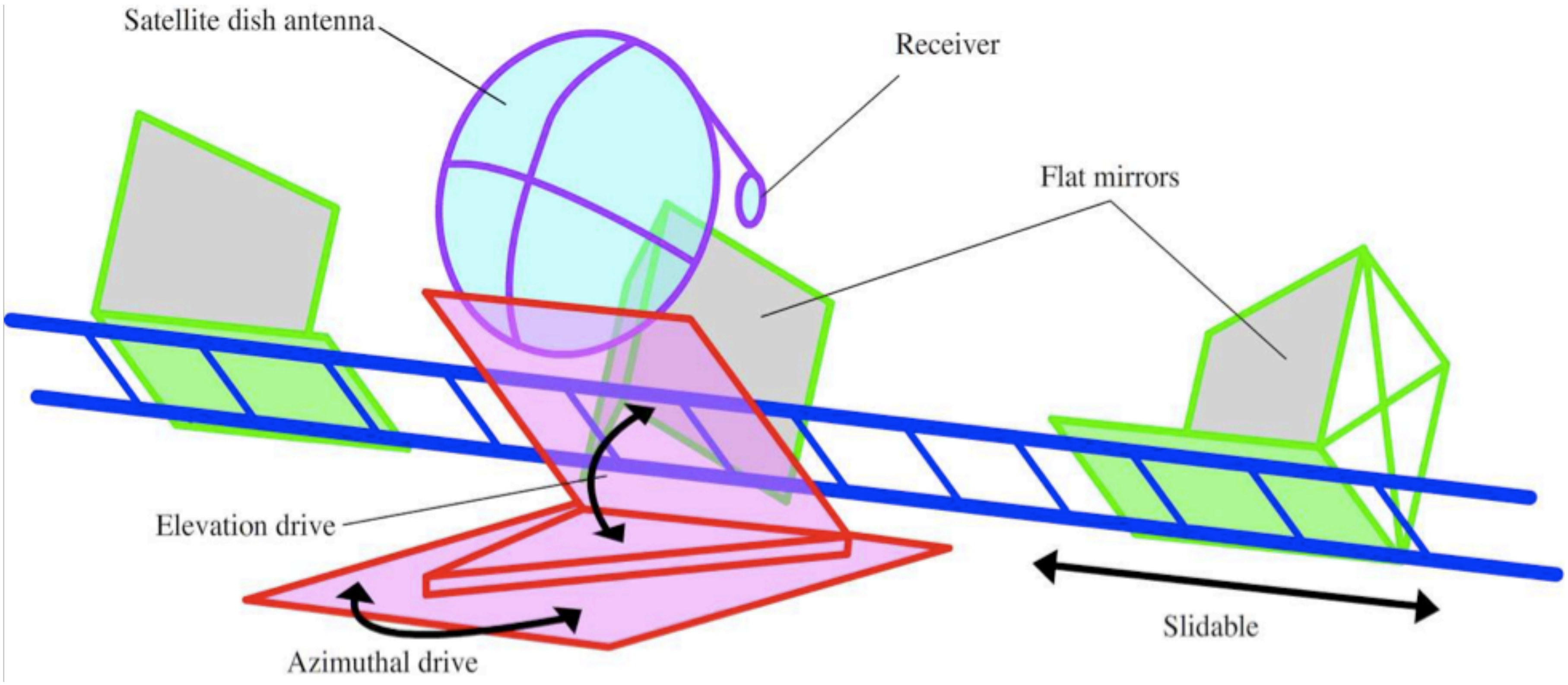


Stony Brook Radio Interferometer

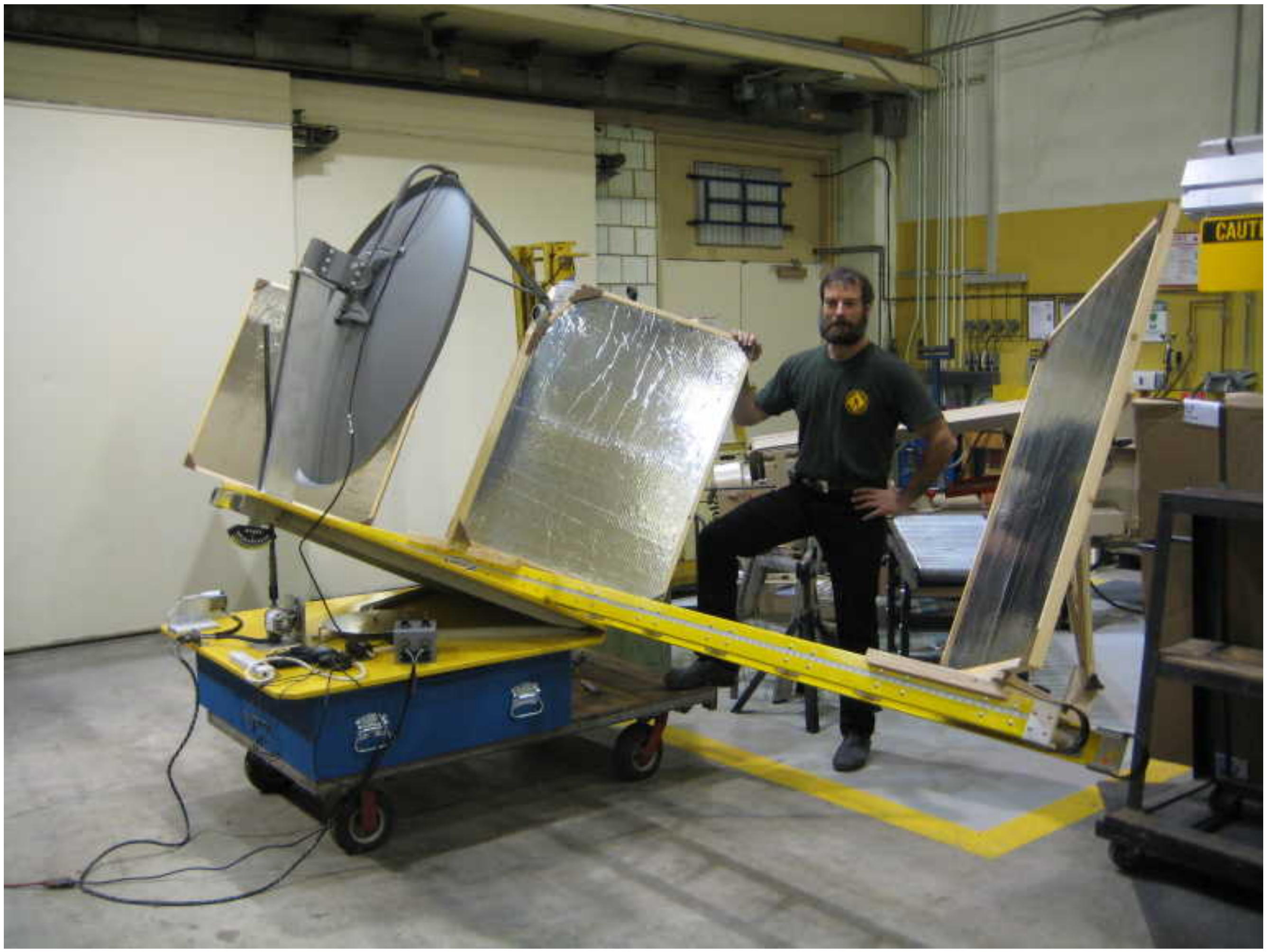
Interferometer (2 Elements)

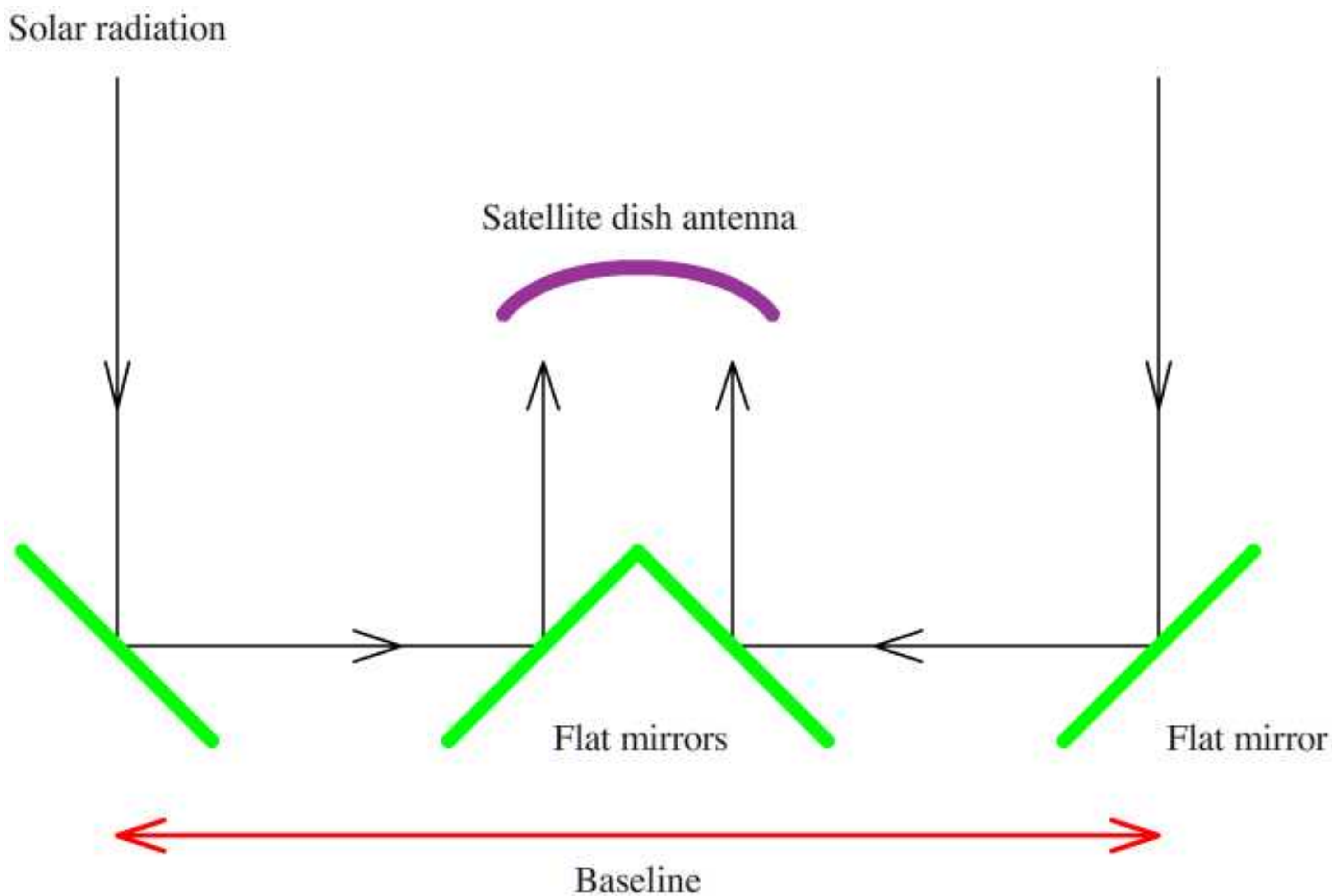


Stony Brook Radio Interferometer



Stony Brook Radio Interferometer

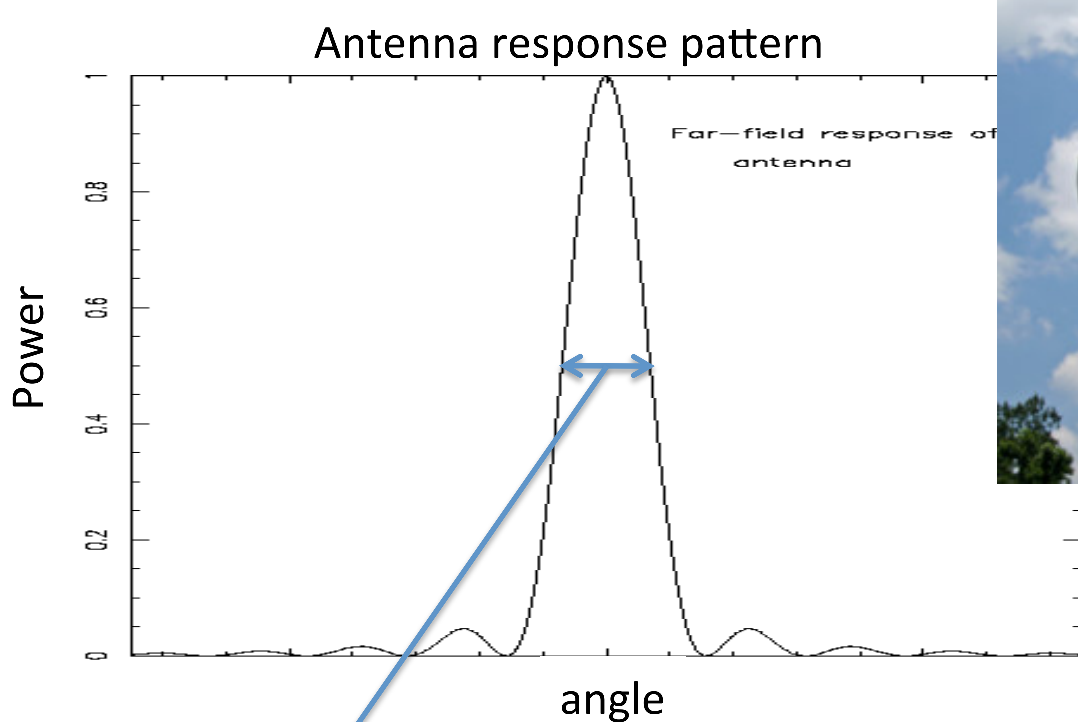



Solar radiation
Satellite dish antenna
Flat mirrors
Flat mirror
Baseline

Components

Telescope Structure

Antenna



Angular resolution

$$\theta_{beam} = \frac{\lambda}{B}$$

wavelength

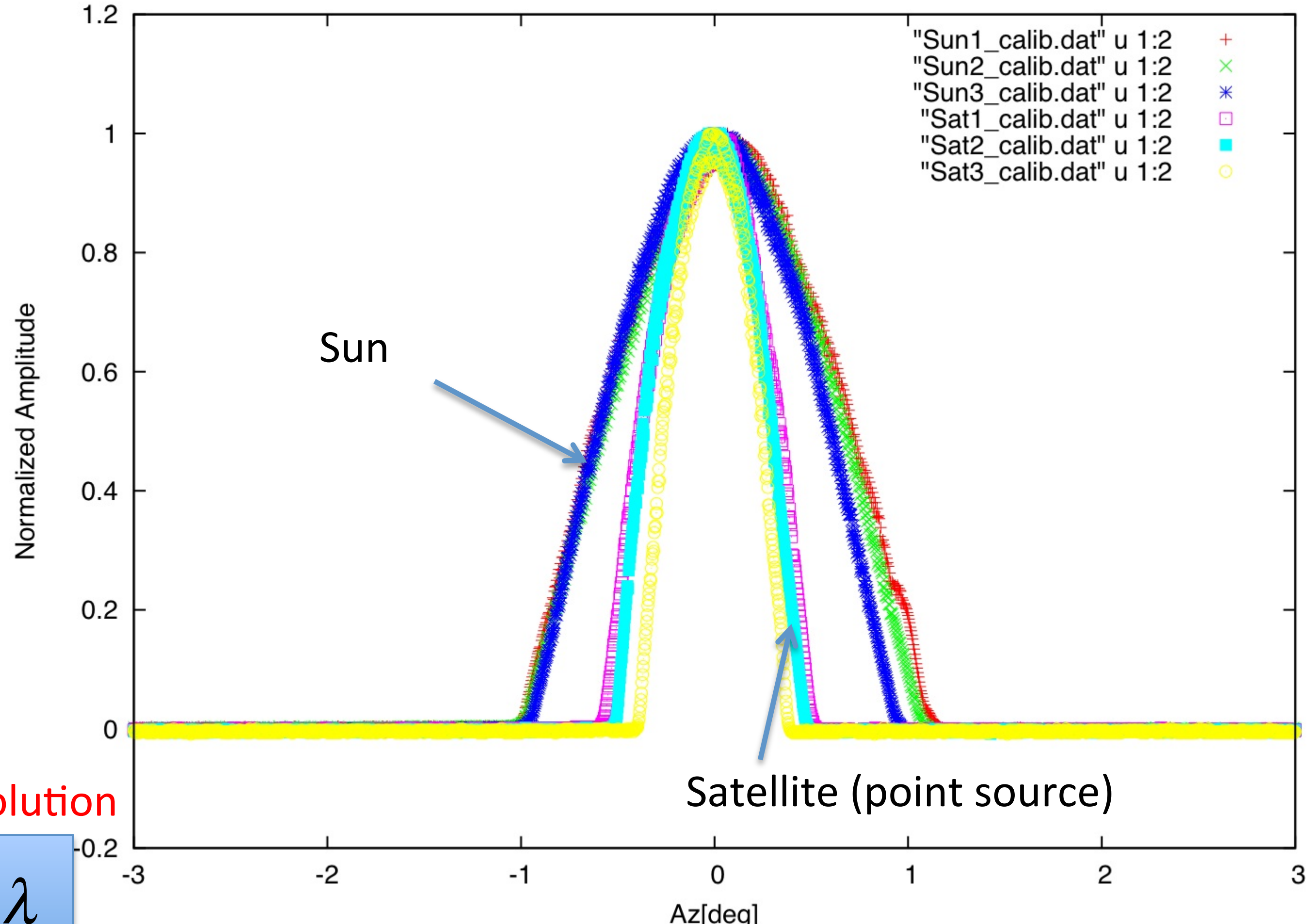
$$\lambda = 2.6cm$$

Antenna diameter

$$B = 1m$$

$$\theta = 1.45\deg$$

Antenna Response Pattern



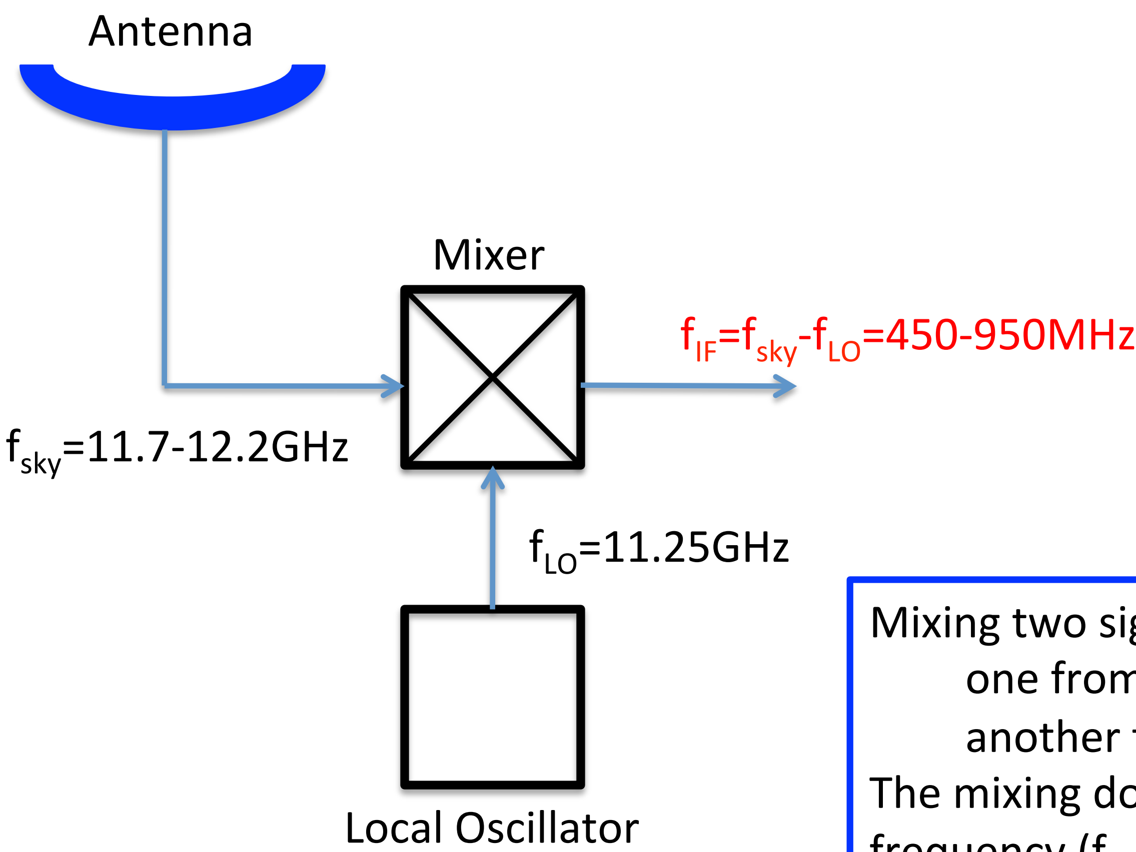
Angular resolution

$$\theta_{beam} = \frac{\lambda}{B}$$

Satellite = a point source

→ Profile = antenna response pattern

Heterodyne Receiver I



Mixing two signals:

one from the sky

another from the local oscillator.

The mixing down-converts the high sky frequency (f_{sky}) to a lower frequency ($f_{IF}=f_{sky}-f_{LO}$). The low frequency is easier to handle.

$$\sin(2\pi f_1 t)\sin(2\pi f_2 t) = \frac{1}{2}\cos[2\pi\underbrace{(f_1 - f_2)}_{\text{Down-converted frequency}}t] - \frac{1}{2}\cos[2\pi(f_1+f_2)t]$$

Heterodyne Receiver II

Non-linear response of an amplifier (inside the receiver)

$$F(v) = \underline{\alpha_1 v + \alpha_2 v^2} + \alpha_3 v^3 + \ldots$$

Let's think only about the first two terms

$$v_{out} = F(\underline{A_1 \sin \omega_1 t + A_2 \sin \omega_2 t})$$

Signals from the sky and local oscillator

$$v_{out} = \alpha_1 (A_1 \sin \omega_1 t + A_2 \sin \omega_2 t) + \alpha_2 (A_1 \sin \omega_1 t + A_2 \sin \omega_2 t)^2 + \ldots$$

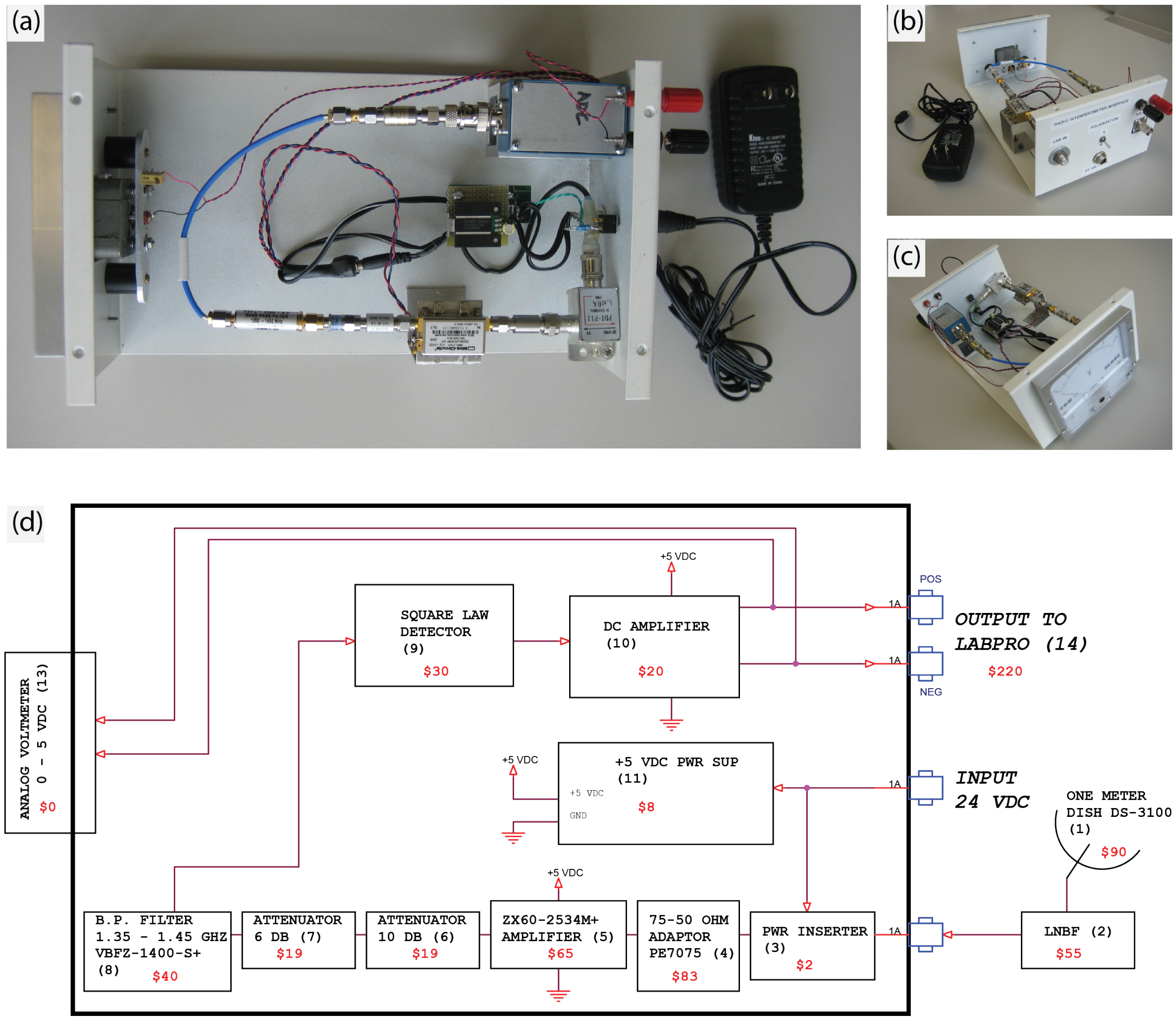
Trigonometric identities

$$\begin{aligned} v_{out} = {} & \alpha_1 (A_1 \sin \omega_1 t + A_2 \sin \omega_2 t) \\ & + \alpha_2 \Big(\frac{A_1^2}{2}[1 - \sin 2\omega_1 t] + A_1 A_2 [\cos(\omega_1 t - \omega_2 t) - \cos(\omega_1 t + \omega_2 t)] \\ & + \frac{A_2^2}{2}[1 - \sin 2\omega_2 t]\Big) + \ldots \end{aligned}$$

Filter out unnecessary frequencies

$$v_{out} = \underline{\alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2) t} - \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2) t + \ldots$$

Detector



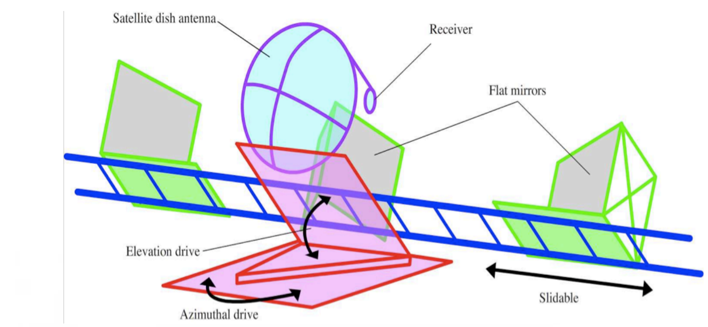
(a)
(b)
(c)
(d)
ANALOG VOLTMETER 0 - 5 VDC (13)
$0
SQUARE LAW DETECTOR (9)
$30
+5 VDC
DC AMPLIFIER (10)
$20
POS
1A
1A
NEG
OUTPUT TO LABPRO (14)
$220
+5 VDC
+5 VDC PWR SUP (11)
+5 VDC
GND
$8
1A
INPUT 24 VDC
ONE METER DISH DS-3100 (1)
$90
+5 VDC
B.P. FILTER 1.35 - 1.45 GHZ VBFZ-1400-S+ (8)
$40
ATTENUATOR 6 DB (7)
$19
ATTENUATOR 10 DB (6)
$19
ZX60-2534M+ AMPLIFIER (5)
$65
75-50 OHM ADAPTOR PE7075 (4)
$83
PWR INSERTER (3)
$2
1A
LNBF (2)
$55

A/D Converter

Analog signal → Digital signal → Computer

Mirrors and Skin Depth

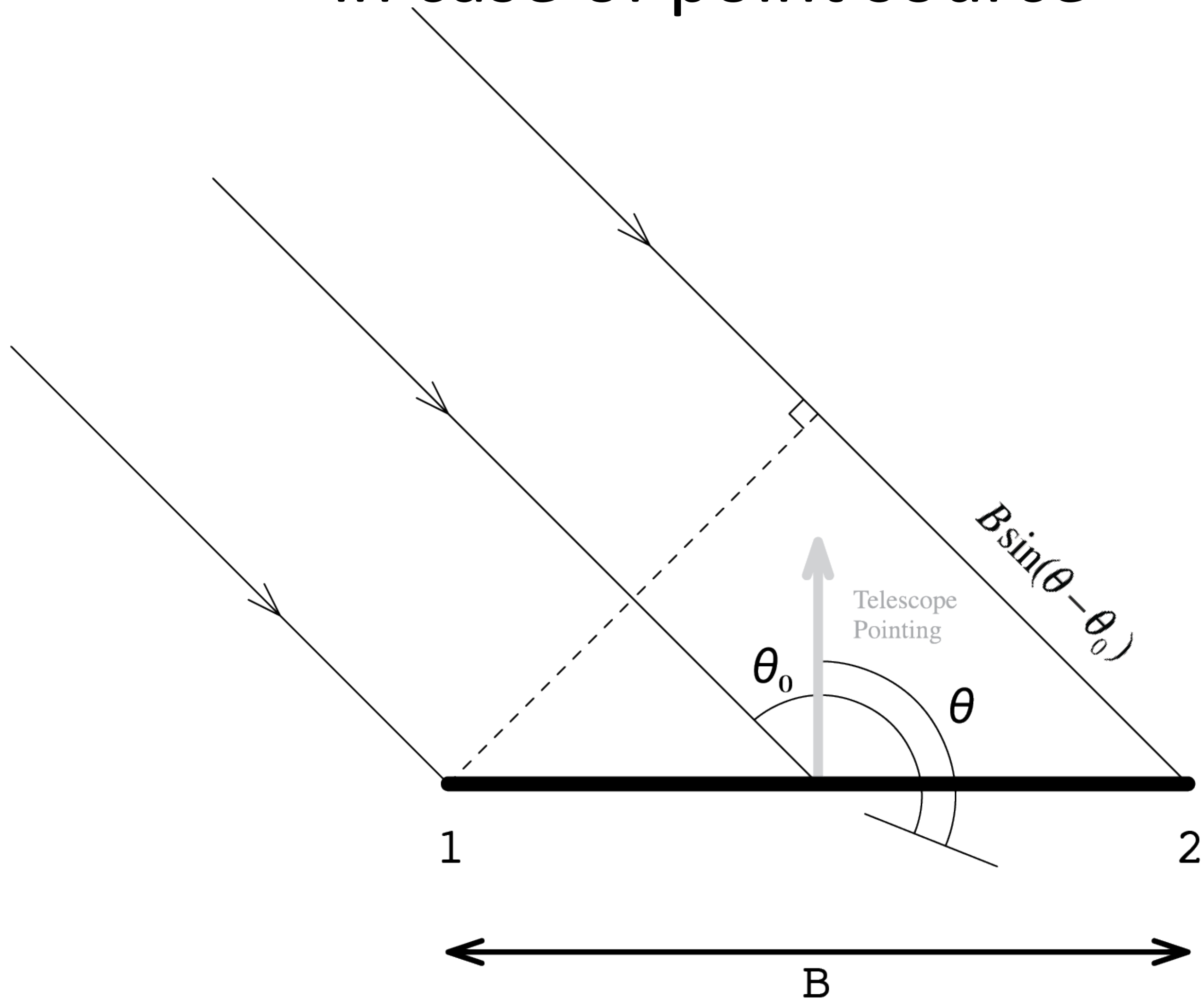
- Material that reflects 2.6cm radio signals
  - Aluminum: skin depth ~ 0.8micron
- Necessary surface accuracy
  - Lambda/10 ~ 3mm



Kitchen aluminum foil is good enough.

Equations

Mixing Signals from Two Mirrors
in case of point source



$E_1(\theta) = E(\theta_0)\cos[2\pi\nu t]$

Signal from Mirror 1

$E_2(\theta) = E(\theta_0)\cos[2\pi\nu(t-\tau)]$

Signal from Mirror 2

Total Power
in case of point source

Total power – we observe this!

$$P(\theta)=\left\langle E_{tot}^2(\theta)\right\rangle=\left\langle (E_1(\theta)+E_2(\theta))^2\right\rangle$$

$$=\left\langle E^2(\theta_0)\left[\cos^2(2\pi\nu t)+\cos^2(2\pi\nu(t-\tau))+2\cos(2\pi\nu t)\cos(2\pi\nu(t-\tau))\right]\right\rangle$$

$$=\left\langle E^2(\theta_0)\left[\frac{1+\cancel{\cos(4\pi\nu t)}}{2}+\frac{1+\cancel{\cos(4\pi\nu(t-\tau))}}{2}+\cancel{\cos(2\pi\nu(2t-\tau))}+\cos(2\pi\nu\tau)\right]\right\rangle$$

Time average→ 0

$$=\left\langle E^2(\theta_0)[1+\cos(2\pi\nu\tau)]\right\rangle$$

$$\tau=\frac{B\sin(\theta-\theta_0)}{c} \qquad B_\lambda=\frac{B}{\lambda} \qquad \sin(\theta-\theta_0)\approx\theta-\theta_0$$

$$\boxed{P(\theta)\approx\left\langle E^2(\theta_0)\left[1+\cos[2\pi B_\lambda(\theta-\theta_0)]\right]\right\rangle}$$

Total Power

In case of general astronomical objects

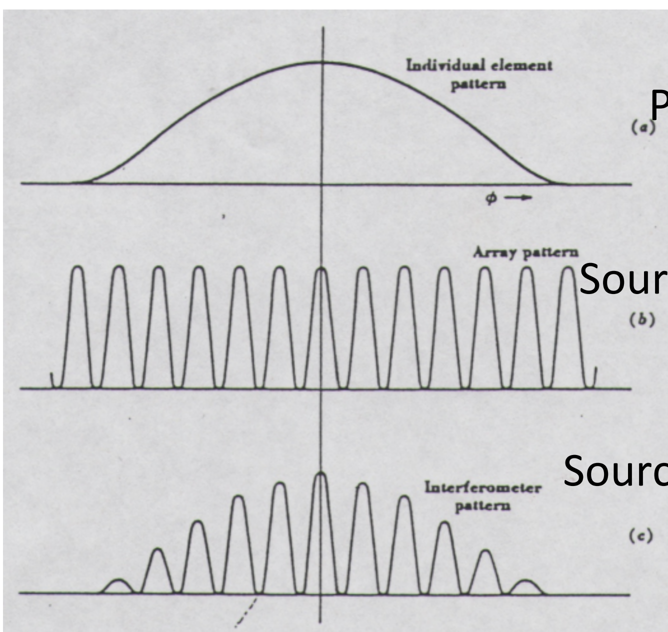
$$P(\theta) \approx \int \underbrace{\varepsilon(\theta_0)}_{\text{Energy density}} \left[1+\cos[2\pi B_\lambda(\theta-\theta_0)]\right] d\theta_0$$

Point source (broadcast satellite)

$$\varepsilon(\theta_0) = \varepsilon_0 \delta(\theta_0 - \theta_c)$$

Disk (Sun)

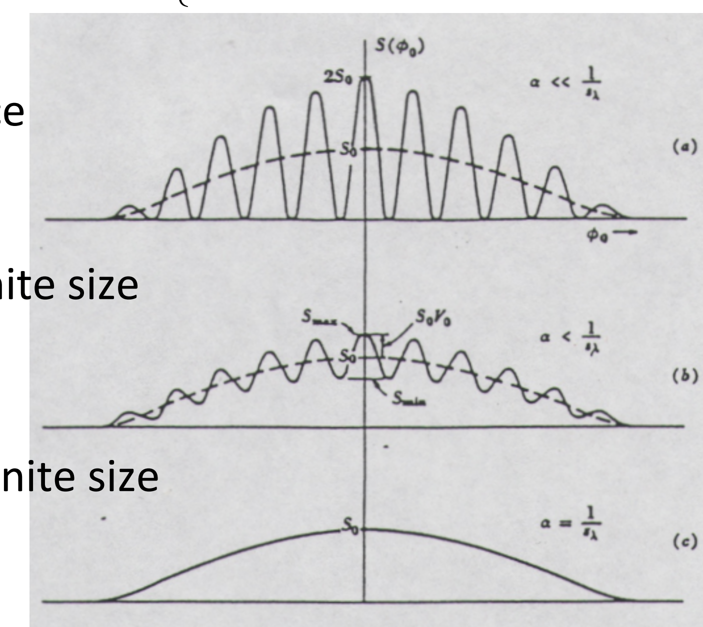
$$\varepsilon(\theta^0) = \begin{cases} 0 & (\text{else}) \\ \varepsilon^0 & (-\alpha/2 < \theta < \alpha/2) \end{cases}$$



Point source

Source with finite size

Source with infinite size



Visibility

Get info from P(θ).

$$P(\theta) \approx \int \varepsilon(\theta_0)\left[1+\cos[2\pi B_\lambda(\theta-\theta_0)]\right]d\theta_0$$

$$\equiv S_0\left[1+V(\theta,B_\lambda)\right]$$

$$\boxed{\begin{aligned} S_0 &\equiv \int \varepsilon(\theta_0)d\theta_0 \\ \varepsilon'(\theta_0) &\equiv \varepsilon(\theta_0)/S_0 \end{aligned}}$$

$$V(\theta,B_\lambda) \equiv \int \varepsilon'(\theta_0)\cos[2\pi B_\lambda(\theta-\theta_0)]d\theta_0$$

$$= \cos[2\pi B_\lambda\theta]\int \varepsilon'(\theta_0)\cos[2\pi B_\lambda\theta_0]d\theta_0$$

$$+\sin[2\pi B_\lambda\theta]\int \varepsilon'(\theta_0)\sin[2\pi B_\lambda\theta_0]d\theta_0$$

$$\equiv \underline{V_0(B_\lambda)}\cos[2\pi B_\lambda(\theta-\Delta\theta)]$$

"visibility".

Visibility = F. T. of Energy Distribution

What is visibility?

By definition:

$$V_0(B_\lambda)\cos[2\pi B_\lambda \Delta\theta] = \int \varepsilon'(\theta_0)\cos(2\pi B_\lambda \theta_0)d\theta_0 \qquad (1)$$

$$V_0(B_\lambda)\sin[2\pi B_\lambda \Delta\theta] = \int \varepsilon'(\theta_0)\sin(2\pi B_\lambda \theta_0)d\theta_0 \qquad (2)$$

(1)+i(2)

$$\boxed{V_0(B_\lambda) = \underbrace{\int \varepsilon'(\theta_0)\exp(i2\pi B_\lambda \theta_0)d\theta_0}_{\text{Fourier transformation of the energy distribution}} \times \underbrace{\exp[-i2\pi B_\lambda \Delta\theta]}_{\text{Phase shift}}}$$

Fourier transformation of the energy distribution   Phase shift

Reminder: we observe the power P(θ),

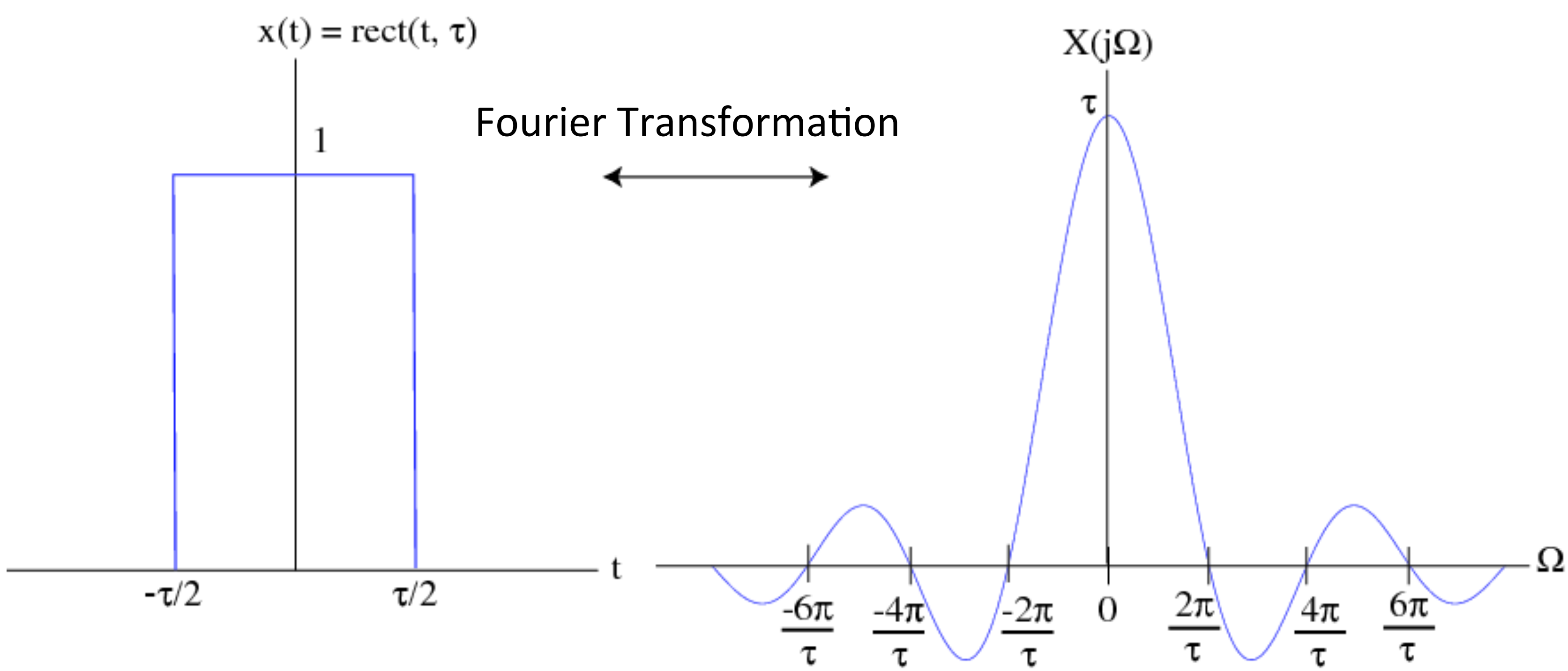
$$P(\theta) \equiv S_0\left[1 + \underline{V_0(B_\lambda)}\cos[2\pi B_\lambda(\theta - \Delta\theta)]\right]$$

Visibility = F.T. of energy distribution across the target = amplitude of sinusoidal power change.

Sun ~ box shape in 2-d slice

The Sun in the sky

Visibilities

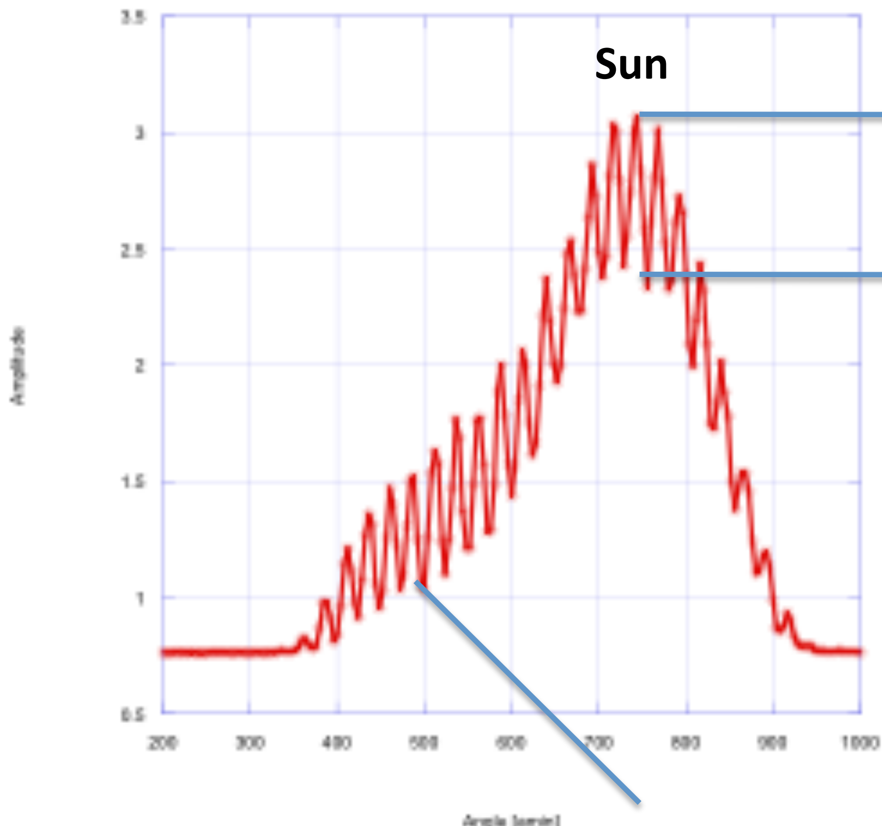


This width gives us the diameter of the Sun

-- do calculations yourself --

How to Measure Visibilities

$$P(\theta) = S_0\left[1 + V_0(B_\lambda)\cos[2\pi B_\lambda(\theta - \Delta\theta)]\right]$$



$$P_{\max} = S_0\left[1 + V_0(B_\lambda)\right]$$

$$P_{\min} = S_0\left[1 - V_0(B_\lambda)\right]$$

$$\downarrow$$

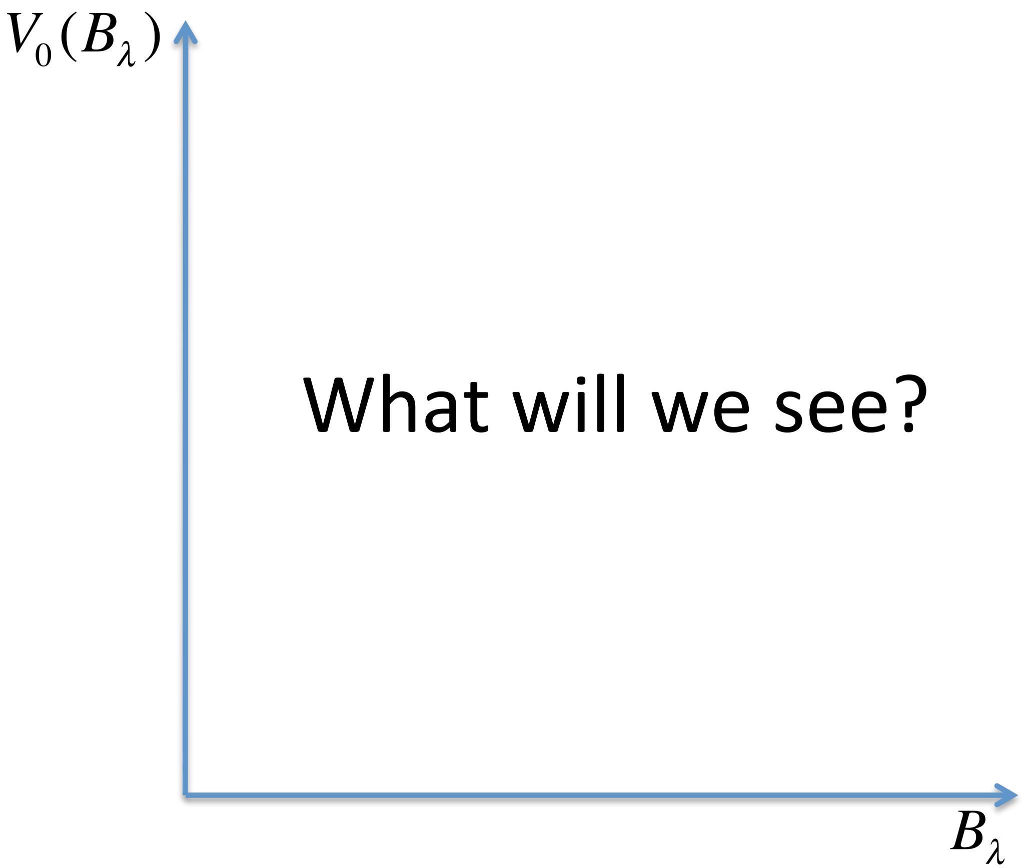
$$V_0(B_\lambda) = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}}$$

One Fourier component

Satellite
which happened to be next to the Sun
at the time of measurement.

Changing B (baseline length)
→Many Fourier components

What Do We Expect?



F.T. of Sun's disk = *sinc* function

Work on equations and figure this out. Try the lab measurements and see if you see what you expect.