DPP - Daily Practice Problems

Date : | **Start Time :** | **End Time :**

MATHEMATICS CM07

SYLLABUS : Sequences and Series

Max. Marks : 74 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 20 Questions divided into 5 sections.
  Section I has **6** MCQs with ONLY 1 Correct Option, **3** marks for each correct answer and **–1** for each incorrect answer.
  Section II has **4** MCQs with ONE or MORE THAN ONE Correct options.
  For each question, marks will be awarded in one of the following categories:
  Full marks: **+4** If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
  Partial marks: **+1** For darkening a bubble corresponding to each correct option provided NO INCORRECT option is darkened.
  Zero marks: If none of the bubbles is darkened.
  Negative marks: **–2** In all other cases.
  Section III has **4** Single Digit Integer Answer Type Questions, **3** marks for each Correct Answer and 0 marks in all other cases.
  Section IV has Comprehension Type Questions having **4** MCQs with ONLY ONE corect option, 3 marks for each Correct Answer and **0** marks in all other cases.
  Section V has **2** Matching Type Questions, **2** mark for the correct matching of each row and 0 marks in all other cases.
- You have to evaluate your Response Grids yourself with the help of Solutions.

Section I - Straight Objective Type

This section contains 6 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

1. If $a_1, a_2, a_3, \ldots$ are in H.P. and $f(k) = \left(\sum_{r=1}^{n} a_r\right) - a_k$, then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)} \cdots \frac{a_n}{f(n)}$ are in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

2. If a, b, c, d are non–zero real numbers such that $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right) \le \left(ab+bc+cd\right)^2$, then a, b, c, d are in

(a) AP (b) GP
(c) HP (d) None of these

RESPONSE GRID	1. ⓐⓑⓒⓓ 2. ⓐⓑⓒⓓ

Space for Rough Work

3. If $a > 0$, $b > 0$, $c > 0$ and the minimum value of $a\left(b^2+c^2\right)+b\left(c^2+a^2\right)+c\left(a^2+b^2\right)$ is λabc, then find the value of λ

(a) 2 (b) 1

(c) 6 (d) 3

4. If S_r denotes the sum of the first r terms of an AP, and the value of $=$ pr $+ q$ then find the value of $p+q$

(a) –1 (b) 1

(c) 3 (d) None of these

5. If $H_1, H_2, \ldots. H_n$ are n harmonic means between a and $b(\neq a)$, then find the value of $\dfrac{H_1+a}{H_1-a}+\dfrac{H_n+b}{H_n-b}$

(a) $n+1$ (b) $n-1$

(c) $2n$ (d) $2n+3$

6. If $a_1, a_2, \ldots\ldots\ldots, a_n$ are in H.P., then the expression $a_1a_2+a_2a_3+\ldots\ldots\ldots.+a_{n-1}a_n$ is equal to

(a) $n(a_1-a_n)$ (b) $(n-1)(a_1-a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

Section II - Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONE OR MORE** is/are correct.

7. If a, b, c are in AP and a^2, b^2, c^2 are in HP, then

(a) $a=b=c$ (b) $a, b, -\frac{1}{2}c$ are in GP

(c) a, b, c are in GP (d) $-\frac{1}{2}a, b, c$ are in GP

8. Sum to n terms of the series $S_n=\dfrac{1}{(1+x)(1+2x)}+\dfrac{1}{(1+2x)(1+3x)}+\dfrac{1}{(1+3x)(1+4x)}+\ldots$ is

(a) $S_{10}=\dfrac{10}{(1+x)(1+11x)}$

(b) $S_{10}=\dfrac{10}{(1+2x)(1+11x)}$

(c) $S_{16}=\dfrac{16}{(1+x)(1+17x)}$

(d) $S_{18}=\dfrac{18}{(1+x)(1+17x)}$

9. For $0<\phi<\pi/2$, if

$$x=\sum_{n=0}^{\infty}\cos^{2n}\phi,\ y=\sum_{n=0}^{\infty}\sin^{2n}\phi,\ z=\sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi \text{ then:}$$

(a) xyz = xz + y (b) xyz = xy + z

(c) xyz = x + y + z (d) xyz = yz + x

10. Given that α, γ are roots of the equation $Ax^2-4x+1=0$ and β, δ the roots of the equation $Bx^2-6x+1=0$, and $\alpha, \beta, \alpha, \gamma$ and δ are in HP, then

(a) A = 3, B = 8 (b) A = 8, B = 3

(c) A = 3, B = –8 (d) A + B = 11

RESPONSE GRID	
	3. ⓐⓑⓒⓓ 4. ⓐⓑⓒⓓ 5. ⓐⓑⓒⓓ 6. ⓐⓑⓒⓓ 7. ⓐⓑⓒⓓ
	8. ⓐⓑⓒⓓ 9. ⓐⓑⓒⓓ 10. ⓐⓑⓒⓓ

Space for Rough Work

Section III - Integer Type

This section contains 4 questions. The answer to each of the questions is a single digit integer ranging from 0 to 9.

11. Let $a_1, a_2, \ldots, a_{10}$ be in AP and $h_1, h_2, \ldots, h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then find the value of a_4h_7

12. If a, b, c are in $G.P.$, x and y be the $A.M.$s between a, b and b, c respectively, then $\left(\frac{a}{x}+\frac{c}{y}\right)\left(\frac{b}{x}+\frac{b}{y}\right)$ is equal to.

13. If $(1+x)(1+x^2)(1+x^4)\ldots(1+x^{128}) = \sum_{r=0}^{n} x^r$, then unit digit of n is

14. Sum to n terms of the series $\frac{1}{5!}+\frac{1!}{6!}+\frac{2!}{7!}+\frac{3!}{8!}+$

$\ldots = \frac{1}{a}\left[\frac{1}{b!}-\frac{(n+c)!}{(n+d)!}\right]$ then $(a + b - c - d)$ is

Section IV - Comprehension Type

Based upon the given paragraphs, 4 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

PARAGRAPH-1

If $a_1, a_2, \ldots, a_n$ are in A.P., then $\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$ are in H.P. and vice–versa.

If $a_1, a_2, \ldots, a_n$ are in A. P. with common difference d, then for any b (> 0), the number $b^{a_1}, b^{a_2}, b^{a_3} \ldots, b^{a_n}$ are in G.P. with common ratio b^d.

If $a_1, a_2, \ldots a_n$ are positive and in G.P. with common ratio r, then for any base b (b > 0), $\log_b a_1, \log_b a_2, \ldots \log_b a_n$ are in A.P. with common difference $\log_b r$.

15. If a, b, c are in H.P., then $e^{(-a)^{-1}}, e^{(-b)^{-1}}, e^{(-c)^{-1}}$ are in

(a) A.P. (b) GP
(c) H.P. (d) None of these.

16. If x, y, z are respectively the p^{th}, q^{th} and the r^{th} terms of an A.P., as well as of a G.P., then the value of $(x^{y-z}), (y^{z-x}), (z^{x-y})$ is

(a) 1 (b) –1
(c) 0 (d) 2

PARAGRAPH-2

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots$

17. The sum $V_1 + V_2 + \ldots + V_n$ is

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$

(b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$

(d) $\frac{1}{3}(2n^3-2n+3)$

18. T_r is always

(a) an odd number (b) an even number
(c) a prime number (d) a composite number

RESPONSE GRID	
	11. ⓪①②③④⑤⑥⑦⑧⑨ 12. ⓪①②③④⑤⑥⑦⑧⑨
	13. ⓪①②③④⑤⑥⑦⑧⑨ 14. ⓪①②③④⑤⑥⑦⑧⑨
	15. ⓐⓑⓒⓓ 16. ⓐⓑⓒⓓ 17. ⓐⓑⓒⓓ 18. ⓐⓑⓒⓓ

Space for Rough Work

Section V - Matrix-Match Type

This section contains 2 questions. It contains statements given in two columns, which have to be matched. Statements in column I are labelled as A, B, C and D whereas statements in column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A-p, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following:

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

19. Match the columns

	Column I		Column II
(A)	The sum of the first n natural number is one – fifth of the sum of their squares, then n is	(p)	4
(B)	The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{3})x + 8 + 2\sqrt{3} = 0$ is	(q)	2
(C)	If x, y, z are in HP, $(z > y > x)$. The value of $\dfrac{(\log(x+z)+\log(x-2y+z))}{\log(z-x)}$ is	(r)	1
(D)	The nth term of GP is 128 and the sum to its n terms is 255. If its common ratio is 2, the its first term is	(s)	7

20. Match the columns

	Column I		Column II
(A)	The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy $G^2 + 3H = 48$ then G^2 is equal to	(p)	308
(B)	$S_n = n^3 - (n-1)^3 + (n-2)^3 - \ldots + (-1)^{n-1} \cdot 1^3$ Then find $\dfrac{S_{39}}{100}$	(q)	32
(C)	If the first two terms of a harmonic progression be ½ and 1/3, then the harmonic mean of the first four terms is	(r)	$\dfrac{240}{77}$
(D)	Find the number of numbers lying between 100 and 500 that are divisible by 7 but not by 21.	(s)	38

RESPONSE GRID	
	19. A - (p)(q)(r)(s); **B -** (p)(q)(r)(s); **C -** (p)(q)(r)(s); **D -** (p)(q)(r)(s)
	20. A - (p)(q)(r)(s); **B -** (p)(q)(r)(s); **C -** (p)(q)(r)(s); **D -** (p)(q)(r)(s)

DAILY PRACTICE PROBLEM DPP CM07 - MATHEMATICS

Total Questions	20	Total Marks	74
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	26	Qualifying Score	37

$$\textbf{Net Score} = \sum_{i=1}^{V}\left[(\text{correct}_i \times MM_i) - (In_i - NM_i)\right]$$

Space for Rough Work

1. **(c)** We have

$$f(k)=\left(\sum_{r=1}^{n}a_r\right)-a_k=S_n-a_k$$

$$\Rightarrow \frac{f(k)}{a_k}=\frac{S_n}{a_k}-1\forall k=1,2,\ldots,n$$

Given $a_1, a_2, \ldots, a_n$ are in H.P.

$\Rightarrow \frac{1}{a_1},\frac{1}{a_2},\ldots\ldots\ldots..,\frac{1}{a_n}$ are in A.P.

$\Rightarrow \frac{S_n}{a_1}-1,\frac{S_n}{a_2}-1\ldots\ldots\ldots,\frac{S_n}{a_n}-1$ are in A.P.

$\Rightarrow \frac{f(1)}{a_1},\frac{f(2)}{a_2},\ldots\ldots\ldots,\frac{f(n)}{a_n}$ are in A.P.

2. **(b)** On simplification,

$$\left(b^2-ac\right)^2+\left(c^2-bd\right)^2+\left(ad-bc\right)^2\le 0,$$

which is possible iff; each of

$$\left(b^2-ac\right)=\left(c^2-bd\right)=\left(ad-bc\right)=0$$

$$\Rightarrow b^2=ac,c^2=bd,ad=bc\Rightarrow \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

3. **(c)** We know that AM $\geq$ GM

$$\frac{ab^2+ac^2+bc^2+ba^2+ca^2+cb^2}{6}\ge\left(a^6b^6c^6\right)^{1/6}$$

$$\Rightarrow a\left(b^2+c^2\right)+b\left(c^2+a^2\right)+c\left(a^2+b^2\right)\ge 6abc$$

4. **(c)** $\frac{S_{3r}-S_{r-1}}{S_{2r}-S_{2r-1}}$

$$=\frac{\frac{3r}{2}\left[2a+(3r-1)d\right]-\frac{(r-1)}{2}\left[2a+(r-2)d\right]}{\frac{2r}{2}\left[2a+(2r-1)d\right]-\frac{(2r-1)}{2}\left[2a+(2r-2)d\right]}$$

$$\Rightarrow \frac{2a(2r+1)+d\left(8r^2-2\right)}{2a+d(4r-2)}$$

$$=\frac{(2r+1)\left[2a+2(2r-1)d\right]}{\left[2a+2(2r-1)d\right]}$$

$=(2x+1)=(pr+q)$

so $p=2$ and $q=1$

$p+q=2+1=3$

5. **(c)** As a, H_1, H_2, ..., H_n, b are in HP.

$\frac{1}{a},\frac{1}{H_1},\frac{1}{H_2},\ldots,\frac{1}{H_n},\frac{1}{b}$ are in AP

Let d be the common difference of the AP, then

$$\frac{1}{b}=\frac{1}{a}+(n+1)d\Rightarrow d=\frac{1}{n+1}\frac{a-b}{ab}$$

Thus, $\frac{1}{H_1}=\frac{1}{a}+d$ and $\frac{1}{H_n}=\frac{1}{b}-d$

$$\Rightarrow \frac{a}{H_1}=1+ad \text{ and } \frac{b}{H_n}=1-bd$$

Now, $$\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}=\frac{1+\frac{a}{H_1}}{1-\frac{a}{H_1}}+\frac{1+\frac{b}{H_n}}{1-\frac{b}{H_n}}$$

$$=\frac{1+1+ad}{1-1-ad}+\frac{1+1-ba}{1-1+bd}$$

$$=\frac{2+ad}{-ad}+\frac{2-bd}{bd}=\frac{2a-abd-2b-abd}{abd}$$

$$=\frac{2\left[(a-b)-abd\right]}{abd}$$

$$=\frac{2\left[(n+1)dab-abd\right]}{abd}=2n$$

6. **(d)** $\frac{1}{a_2}-\frac{1}{a_1}=\frac{1}{a_3}-\frac{1}{a_2}=\ldots\ldots\ldots=\frac{1}{a_n}-\frac{1}{a_{n-1}}=d$ (say)

Then $a_1a_2=\frac{a_1-a_2}{d}$, $a_2a_3=\frac{a_2-a_3}{d}$,

$\ldots\ldots\ldots, a_{n-1}a_n=\frac{a_{n-1}-a_n}{d}$

$\therefore \quad a_1a_2+a_2a_3+\ldots\ldots\ldots+a_{n-1}a_n$

$$=\frac{a_1-a_2}{d}+\frac{a_2-a_3}{d}+\ldots.+\frac{a_{n-1}-a_n}{d}$$

$$=\frac{1}{d}[a_1-a_2+a_2-a_3+\ldots.+a_{n-1}-a_n]$$

$$=\frac{a_1-a_n}{d}$$

Also, $\frac{1}{a_n}=\frac{1}{a_1}+(n-1)d$

$$\Rightarrow \frac{a_1-a_n}{a_1a_n}=(n-1)d\Rightarrow \frac{a_1-a_n}{d}=(n-1)a_1a_n$$

Which is the required result.

7. **(a, b, d)** Since three numbers in AP so $2b = a + c$ and

$$b^2 = \frac{2a^2c^2}{a^2 + c^2}$$

Eliminating b, we get $\left(\frac{a+c}{2}\right)^2 = \frac{2a^2c^2}{a^2+c^2}$

$$\Rightarrow \left(a^2+c^2\right)^2 + 2ac\left(a^2+c^2\right) - 8a^2c^2 = 0$$

$$\Rightarrow \left(a^2+c^2+4ac\right)\left(a^2+c^2-2ac\right) = 0$$

$$\Rightarrow \left[(a+c)^2 + 2ac\right](a-c)^2 = 0$$

$$\Rightarrow 4\left(b^2 + \frac{1}{2}ac\right)(a-c)^2 = 0$$

$$\Rightarrow a - c = 0 \quad or \quad b^2 = -\frac{1}{2}ac$$

If $a = c$, we get $a = b = c$

If $b^2 = -\frac{1}{2}ac$, then either $a, b, -\frac{1}{2}c$ are in GP

or $-\frac{1}{2}a, b, c$ are in GP

8. **(a, c)** If t_r denotes the rth term of the series, then

$$xt_r = \frac{x}{(1+rx)(1+(r+1)x)}$$

$$= \frac{1}{1+rx} - \frac{1}{1+(r+1)x}$$

$$\Rightarrow x\sum_{r=1}^{n} t_r = \sum_{r=1}^{n}\left[\frac{1}{1+rx} - \frac{1}{1+(r+1)x}\right]$$

$$= \frac{1}{1+x} - \frac{1}{1+(n+1)x} = \frac{nx}{(1+x)(1+(n+1)x)}$$

$$\Rightarrow S_n = \sum_{r=1}^{n} t_r = \frac{n}{(1+x)\left[1+(n+1)x\right]}$$

9. **(b, c)** We have for $0 < \phi < \frac{\pi}{2}$

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi = 1 + \cos^2\phi + \cos^4\phi + ...\infty$$

$$\frac{1}{1-\cos^2\phi} = \frac{1}{\sin^2\phi} \qquad ....(1)$$

[Using sum of infinite G.P. $\cos^2\alpha$ being < 1]

$$y = \sum_{n=0}^{\infty} \sin^{2n}\phi = 1 + \sin^2\phi + \sin^4\phi + ...\infty$$

$$= \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi} \qquad ....(2)$$

$$z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$$

$$= 1 + \cos^2\phi\sin^2\phi + \cos^4\phi\sin^4\phi + ...\infty$$

$$= \frac{1}{1-\cos^2\phi\sin^2\phi} \qquad ....(3)$$

Substituting the values of $\cos^2\phi$ and $\sin^2\phi$ in (3), from (1) and (2), we get

$$z = \frac{1}{1-\frac{1}{x}.\frac{1}{y}} \Rightarrow z = \frac{xy}{xy-1}$$

$\Rightarrow xyz - z = xy \Rightarrow xyz = xy + z.$

Also, $x + y + z = \frac{1}{\cos^2\phi} + \frac{1}{\sin^2\phi} + \frac{1}{1-\cos^2\phi\sin^2\phi}$

$$\frac{[\sin^2\phi(1-\cos^2\phi\sin^2\phi) + \cos^2\phi(1-\cos^2\phi\sin^2\phi) + \cos^2\phi\sin^2\phi]}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)}$$

$$= \frac{(\sin^2\phi + \cos^2\phi)\,(1-\cos^2\phi\sin^2\phi) + \cos^2\phi\sin^2\phi}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)}$$

$$= \frac{1}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)} = xyz$$

Thus (b) and (c) both are correct.

10. **(a, d)** α, β, γ and δ are in HP

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\delta}$ are in AP

Let d be the common difference of this AP.

Now, a, γ are roots of $Ax^2 - 4x + 1 = 0$

$$\therefore \frac{\alpha+\gamma}{\alpha\gamma} = \frac{\frac{4}{A}}{\frac{1}{A}} = 4$$

or $\frac{1}{\alpha} + \frac{1}{\gamma} = 4 ie, \frac{1}{\alpha} + \frac{1}{\alpha} + 2d = 4$

or $\frac{1}{\alpha} + d = 2$ ...(i)

β, δ are roots of $Bx^2 - 6x + 1 = 0$

$$\therefore \frac{\beta+\delta}{\beta\delta} = \frac{1}{\beta} + \frac{1}{\delta} = \frac{6/B}{1/B} = 6$$

or $\frac{1}{\alpha} + d + \frac{1}{\alpha} + 3d = 6$

or $\frac{1}{\alpha} + 2d = 3$ ...(ii)

From Eqs. (i) and (ii), on solving, we get

$\frac{1}{\alpha}=1, d=1 \therefore \frac{1}{\alpha}=1, \frac{1}{\beta}=2, \frac{1}{\gamma}=3, \frac{1}{\delta}=4$

Since, $\frac{1}{\alpha\gamma}=A \Rightarrow A=3$

Also, $\frac{1}{\beta\delta}=B, \Rightarrow B=8$

Hence, A = 3 and B = 8.

11. **(6)** Given that $a_{10}=3 \Rightarrow a_1+9d=3$

$\Rightarrow$ $2+9d = 3$ [$a_1=2$]

$\Rightarrow$ $d=\frac{1}{9}$

$\therefore$ $a_4=a_1+3d=2+\frac{3}{9}=\frac{7}{3}$

$h_{10}=3$

$\Rightarrow$ $\frac{1}{h_{10}}=\frac{1}{3}$

$\Rightarrow$ Common difference of corresponding AP is $D=-\frac{1}{54}$

$\therefore$ $\frac{1}{h_7}=\frac{1}{h_1}+6D=\frac{1}{2}-\frac{1}{9}=\frac{7}{18}$

$\Rightarrow$ $h_7=\frac{18}{7}$

so, $a_4h_7=\frac{7}{3}\times\frac{18}{7}=6$

12. **(4)** Given $b^2=ac, x=\frac{a+b}{2}, y=\frac{b+c}{2}$

Now, $\frac{a}{x}+\frac{c}{y}=\frac{2a}{a+b}+\frac{2c}{b+c}$

$=\frac{2(ab+ac+ac+bc)}{ab+ac+b^2+bc}=2 \qquad [\because b^2=ac]$

Again $\frac{b}{x}+\frac{b}{y}=2b\left[\frac{1}{a+b}+\frac{1}{b+c}\right]$

$=\frac{2b(b+c+a+b)}{ab+ac+b^2+bc}=2$

$\therefore$ $\left(\frac{a}{x}+\frac{c}{y}\right)\left(\frac{b}{x}+\frac{b}{y}\right)=4.$

13. **(5)** $(1+x)(1+x^2)(1+x^4)\ldots(1+x^{128})$

$=1+x+x^2+\ldots+x^n$

$\Rightarrow$ $(1-x)\{(1+x)(1+x^2)(1+x^4)\ldots(1+x^{128})\}$

$=1-x^{n+1}$

$\Rightarrow$ $(1-x^2)\{(1+x^2)\ldots(1+x^{128})\}=1-x^{n+1}$

$\Rightarrow$ $(1-x^4)(1+x^4)\ldots(1+x^{128})=1-x^{n+1}$

$\Rightarrow$ $1-x^{256}=1-x^{n+1}$

$\therefore$ $n+1=256$ or $n=255$

14. **(4)** We have $t_r=\frac{(r-1)!}{(r+4)!}$

And $t_{r+1}=\frac{r!}{(r+5)!}$

Now, $rt_r-(r+5)t_{r+1}=\frac{r!}{(r+4)!}-\frac{r!}{(r+4)!}=0$

$\Rightarrow$ $rt_r-(r+1)t_{r+1}=4t_{r+1}$

$\Rightarrow$ $4\sum_{r=1}^{n-1}t_{r+1}=\sum_{r=1}^{n-1}\left[rt_r-(r+1)t_{r+1}\right]$

$\Rightarrow$ $4(t_2+t_3+\ldots+t_n)=1t_1-nt_n$

$\Rightarrow$ $4(t_1+t_2+\ldots+t_n)=5t_1-nt_n$

$=5\left(\frac{0!}{5!}\right)-\frac{n(n-1)!}{(n+4)!}=\frac{1}{4!}-\frac{n!}{(n+4)!}$

$\Rightarrow$ $t_1+t_2+\ldots+t_n=\frac{1}{4}\left[\frac{1}{4!}-\frac{n!}{(n+4)!}\right]$

So a = 4, b = 4, c = 0 and d = 4 and a + b–c–d = 4

15. **(b)** Here a, b, c, are in H.P.

$\Rightarrow$ a^{-1}, b^{-1}, c^{-1} are in A.P.

$\Rightarrow$ $e^{(-a)^{-1}}, e^{(-b)^{-1}}, e^{(-c)^{-1}}$ are in G.P.

16. **(a)** Since x, y, z are respectively the p^{th}, q^{th} and the r^{th} terms of a G.P so $\ell n\, x$, $\ell n\, y$, $\ell n\, z$ are in A.P. with common difference ℓnt. Here t is the common ratio

Also, x, y, z are in A.P.(say with common difference d.)

Hence, $x-y=(p-q)d$ etc.

and $\ell n\, x-\ell n\, y=(p-q)\ell n\, t$.

Let $S=(x^{y-z}), (y^{z-x}), (z^{x-y})$

so that $\ell n\, S=(y-z)\ell n\, x+(z-x)\ell n\, y$
$+(x-y)\ell n\, z$

$=(q-r)d\ell n\, x+(r-p)d\ell n\, y+(p-q)d\ell n\, z$

$=d[p(\ell n\, z-\ell n\, y)+q(\ell n\, x-\ell n\, z)$
$+r(\ell n\, y-\ell n\, x)]$

$=d\ell nt\,[p(r-q)+q(p-r)+r(q-p)]=0$

$\Rightarrow$ $S=1$

17. **(b)** $V_1+V_2+.....+V_n=\sum_{r=1}^{n}V_r=\sum_{r=1}^{n}\left(r^3-\frac{r^2}{2}+\frac{r}{2}\right)$

$=\sum n^3-\frac{\sum n^2}{2}+\frac{\sum n}{2}$

$$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \frac{n(n+1)}{4}\left[n(n+1) - \frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)(3n^2+n+2)}{12}$$

18. **(d)** $T_r = V_{r+1} - V_r - 2$

$$= \left[(r+1)^3 - \frac{(r+1)^2}{2} + \frac{r+1}{2}\right] - \left[r^3 - \frac{r^2}{2} + \frac{r}{2}\right] - 2$$

$= 3r^2 + 2r + 1$

$T_r = (r+1)(3r-1)$

For each r, T_r has two different factors other than 1 and itself.

$\therefore$ T is always a composite number.

19. **(A) → (*s*), (B) → (*p*), (C) → (*q*), (D) → (*r*)**

(A) Given, $\sum n = \frac{1}{5}\left(\sum n^2\right)$

or $\quad \frac{n(n+1)}{2} = \frac{1}{5}\frac{n(n+1)(2n+1)}{6}$

$\Rightarrow$ $2n+1 = 15$

$\Rightarrow$ $2n = 14$

$\Rightarrow$ $n = 7$

(B) Let α and β be the roots of the given equation,

$\alpha + \beta = \frac{4+\sqrt{3}}{5+\sqrt{2}}$ and $\alpha\beta = \frac{8+2\sqrt{3}}{5+\sqrt{2}}$

Hence, required harmonic mean

$$= \frac{2\alpha\beta}{\alpha+\beta} = \frac{2\left(\frac{8+2\sqrt{3}}{5+\sqrt{2}}\right)}{\frac{4+\sqrt{3}}{5+\sqrt{2}}} = 4$$

(C) x, y, z are in HP.

$$y = \frac{2xz}{x+z}$$

$\Rightarrow$ $x - 2y + z = x + z - \frac{4xz}{x+z}$

$$= \frac{(x+z)^2 - 4xz}{x+z} = \frac{(z-x)^2}{x+z}$$

$\Rightarrow$ $(x+z)(x-2y+z) = (z-x)^2$

$\Rightarrow$ $\log(x+z) + \log(x-2y+z) = 2\log(z-x)$

(D) $\frac{128r-a}{r-1} = 255$

$\Rightarrow$ $\frac{256-a}{2-1} = 255$ $\qquad$ [since $r = 2$]

$\Rightarrow$ $256 - a = 225$

$\Rightarrow$ $a = 1$

20. **(A) → (*q*); (B) → (*p*); (C) → (*r*); (D) → (*s*)**

(A) $a + b = 12$

$$ab + \frac{6ab}{a+b} = 48$$

$$ab + \frac{ab}{2} = 48$$

$\therefore ab = 32$

(B) As $n = 39$ is odd, the value of the given expression

$= 1^3 - 2^3 + 3^3 - \ldots + n^3$

$= \left(1^3 + 2^3 + 3^3 + \ldots + n^3\right) - 2\left\{2^3 + 4^3 + \ldots + (n-1)^3\right\}$

$$= \left\{\frac{n(n+1)}{2}\right\}^2 - 16\left\{1^3 + 2^3 + \ldots + \left(\frac{n-1}{2}\right)^3\right\}$$

$$= \frac{n^2(n+1)^2}{4} - 16\cdot\left\{\frac{\frac{n-1}{2}\cdot\frac{n+1}{2}}{2}\right\}^2$$

$$= \frac{(n+1)^2\cdot(2n-1)}{4}$$

On putting the value we get $\frac{S_{39}}{100} = \frac{30800}{100} = 308$

(C) HM of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ is

$$\frac{4}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}} = \frac{240}{77}$$

(D) The numbers between 100 and 500 that are divisible by 7 are 105, 112, 119, 126, 133, 140, 147, ..., 483, 490, 497.

Let such numbers be n.

Then, $497 = 105 + (n-1) \times 7$ or $n = 57$

So there are 57 number of numbers lying between 100 and 500 that are divisible by 7

The number between 100 and 500 that are divisible by 21 are 105, 126, 147, ..., 483.

Let such number be m.

Then, $483 = 105 + (m-1) \times 21$ or $n = 19$

So there are 19 number of numbers lying between 100 and 500 that are divisible by 21

Hence, the required number $= n - m = 57 - 19 = 38$