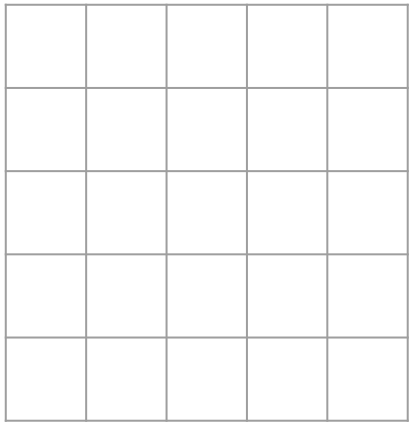
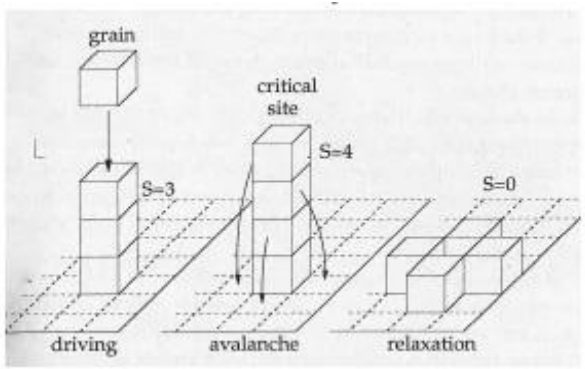
Sandpile model: Study of power laws

Model overview



NxN 2D-lattice

- “Sand” is randomly dropped on a 2D lattice
- We can place sand particle one over another till the critical value (z_c)
- Any sand particle crossing the boundary of NxN lattice is lost
- If at any (x,y) point z_c=4, then the cell collapses and transfer the sand to the adjacent cells (this is known as avalanche, toppling)
- We have to wait till all the avalanche settles down, then drop another



Basic idea of code

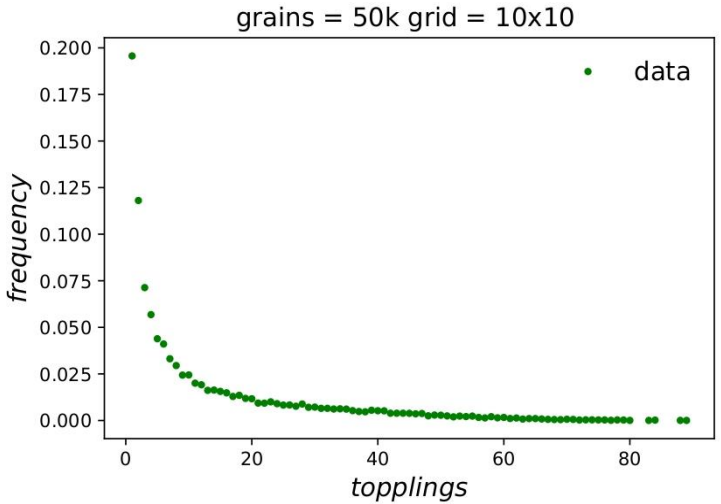
Consider a lattice (x,y) and a function z(x,y) which represents the number of grains in the cells.

Starting with a flat surface z(x,y) = 0 for all x and y:

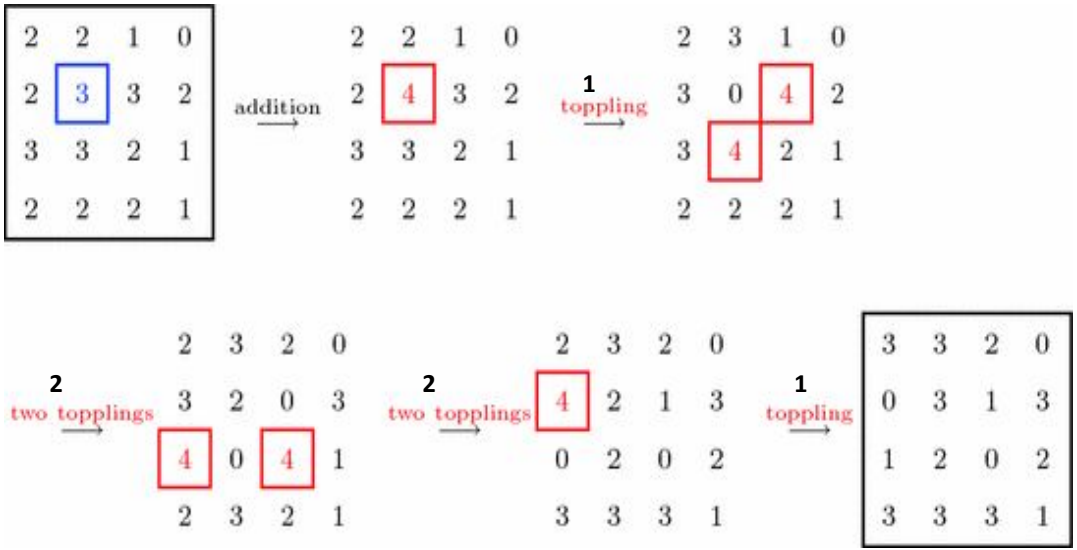
Add a grain of sand: $z(x,y) \longrightarrow z(x,y) + 1$
if $z(x,y) > z_c$ then an avalanche occurs

$$z(x,y) \longrightarrow z(x,y) - 4$$
$$z(x \pm 1,y) \longrightarrow z(x \pm 1,y) + 1$$
$$z(x,y \pm 1) \longrightarrow z(x,y \pm 1) + 1$$

if $z(x,y \pm 1) = 4$ or $z(x \pm 1, y) = 4$
Update z

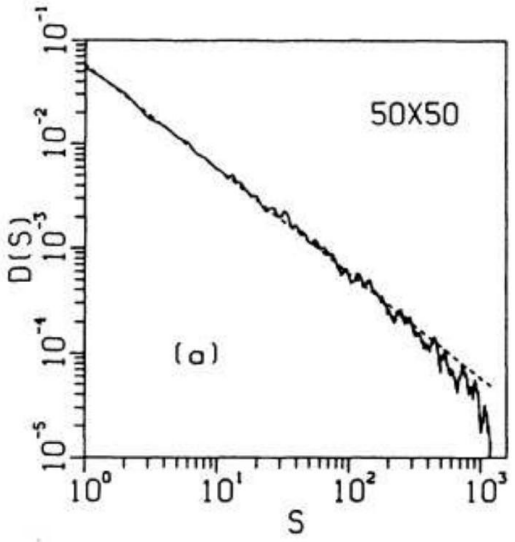


Toppling (example)



Sand added - 1
Total toppling - 6

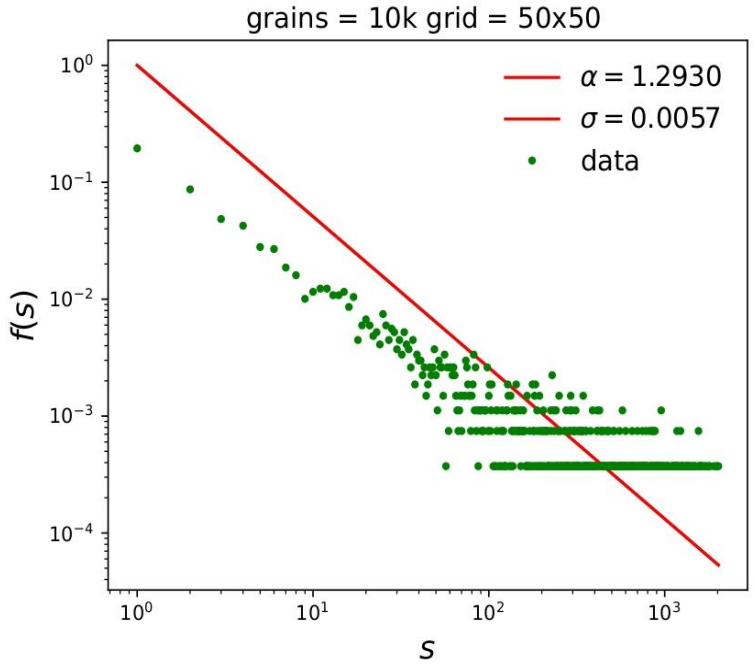
And we count the frequency of 6 toppings in the entire grain drop



Graphs for Avalanche Size

Value of alpha

No. of grains ↓	10x10	50x50	100x100
10k	1.4084	1.2930	2.2451
30k	1.4086	1.2640	1.2946
50k	1.4059	1.2595	1.2503
200k	1.4066	1.2563	1.2296

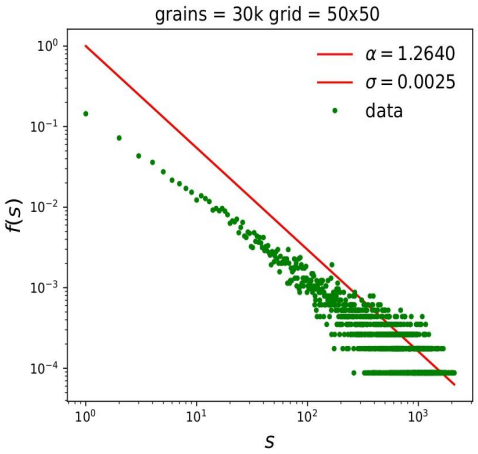


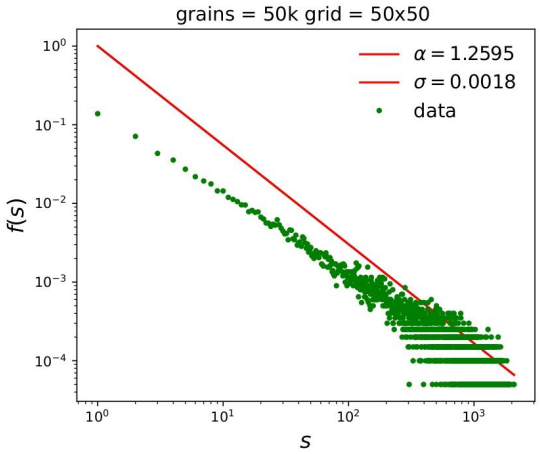
Power law: $Y = bx^{\alpha}$

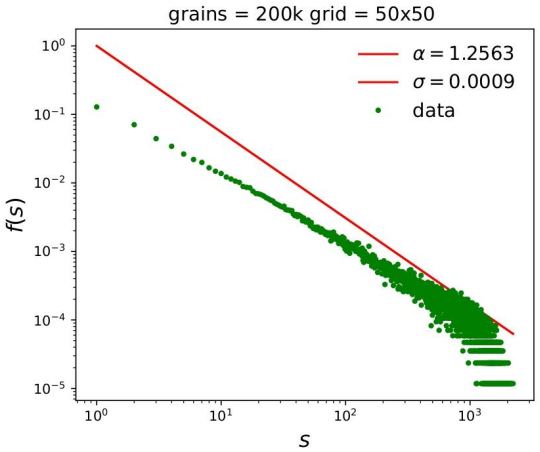
frequency $\propto$ (toppling)$^{\alpha}$

toppling ---> s

Result







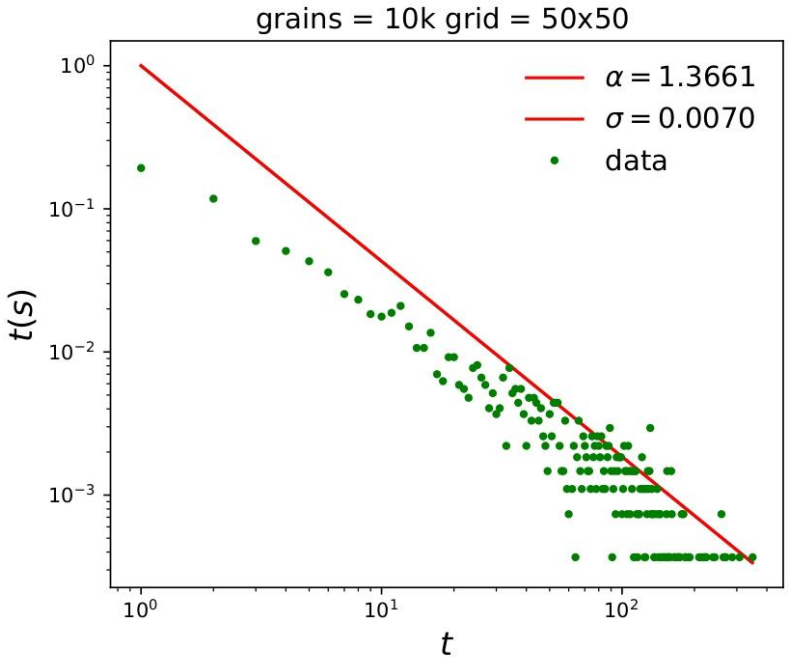
- Value of alpha for avalanche size from BTW review letter: **α = 0.98**
- Value of alpha from my code: **α = 1.2682**

Graphs for Avalanche Time

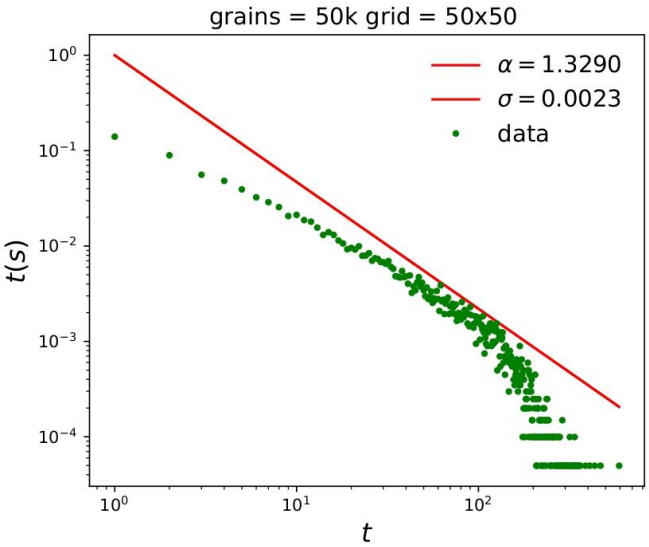
Value of alpha

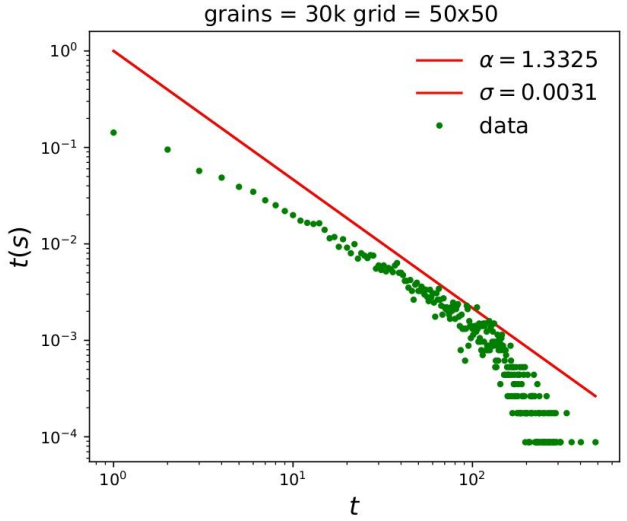
No. of grains ↓	10x10	50x50	100x100
10k	1.4817	1.3661	2.2275
30k	1.4760	1.3325	1.3714
50k	1.4768	1.3290	1.3198

Power law: $Y = bx^{\alpha}$



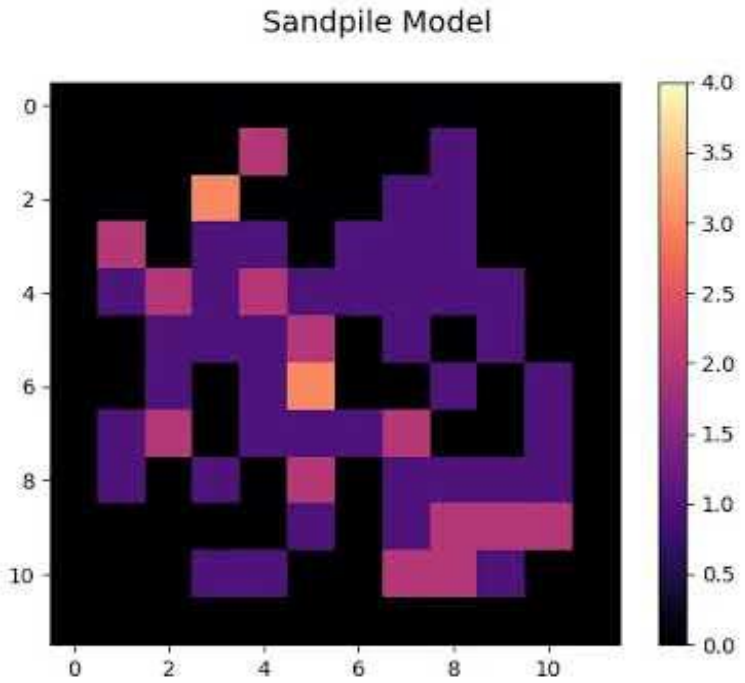
Result





- Value of alpha for avalanche time from BTW review letter: **α = 0.42**
- Value of alpha from my code: **α = 1.3425**

Simulation video



Reference: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.59.381

Thank You

Instructor: Dr. MK Verma
Department of Physics, IIT Kanpur

Saroj Anjesh, 170634
Department of Physics, IIT Kanpur