CQF FINAL PROJECT

Portfolio Construction using Black-Litterman Model and Factors

CERTIFICATE IN QUANTITATIVE FINANCE

CQF

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Abstract

This project applies the Black–Litterman (BL) framework to the construction of a global multi-asset portfolio comprising country-level equity indices, fixed income assets, and commodities. The analysis integrates a factor-based covariance model with equilibrium expected returns inferred via reverse optimization and investor views expressed in relative-return form.

The study systematically examines how Bayesian updating alters expected returns, efficient frontiers, portfolio allocations, and risk decomposition. Posterior estimates are evaluated across multiple portfolio construction paradigms, including mean–variance optimization, maximum Sharpe ratio portfolios, minimum variance portfolios, and benchmark-relative allocations, all subject to realistic long-only and weight constraints. Investor preferences are further explored through three distinct risk-aversion profiles—Trustee, Market, and Kelly—highlighting how allocations and risk contributions evolve as tolerance for volatility increases.

A central empirical focus is placed on sensitivity and robustness. Portfolio outcomes are analyzed under alternative specifications of prior uncertainty (τ), view confidence (Ω), and covariance estimation methods, including factor-based, Ledoit–Wolf shrinkage, and EWMA estimators. The interaction between subjective views and estimation uncertainty is shown to materially influence allocation stability and risk concentration, while preserving qualitative investment themes.

Empirical results demonstrate that incorporating views within the Black–Litterman framework moderates equilibrium expected returns, leading to a posterior efficient frontier that lies below the prior frontier. This reflects deliberate shrinkage toward economically plausible expectations rather than mechanical return maximization. Posterior portfolios exhibit reduced concentration in U.S. equities and increased exposure to emerging market equities, corporate credit, and real assets, consistent with a growth-oriented macroeconomic scenario.

Overall, the findings highlight the practical value of the Black–Litterman framework as a disciplined portfolio construction methodology. By explicitly modeling uncertainty and anchoring beliefs to a reference portfolio, the framework produces stable, interpretable allocations that balance market consensus with investor judgment while mitigating estimation risk.

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1. Introduction

Portfolio construction in multi-asset settings is fundamentally challenged by the estimation of expected returns. While asset return covariances can often be estimated with reasonable stability, expected returns are notoriously noisy and highly sensitive to sampling error. Traditional mean–variance optimization amplifies this instability, frequently producing extreme and unintuitive portfolio allocations that are difficult to justify economically.

The Black–Litterman (BL) framework addresses this limitation by combining a market-implied equilibrium return vector with subjective investor views in a Bayesian setting. Rather than relying on direct forecasts of absolute returns, the model anchors expectations to a reference portfolio and incorporates views in a controlled and transparent manner. As a result, Black–Litterman portfolios tend to be more stable, diversified, and economically interpretable than those produced by unconstrained optimization.

This project applies the Black–Litterman framework to the construction of a global multi-asset portfolio comprising country-level equity indices, fixed income assets, and commodities. Compared to regionally aggregated equity exposures, the use of country-level indices introduces greater heterogeneity in volatility, correlations, and factor sensitivities, allowing macroeconomic and structural views to propagate more evenly across the portfolio.

A central focus of the analysis is the interaction between equilibrium priors, subjective views, and portfolio risk. The views employed in this study are inspired in part by a growth-oriented scenario drawn from BlackRock's capital market assumptions, including themes related to the diffusion of artificial intelligence and productivity enhancements. These views are not adopted mechanically; rather, they are selectively adapted and augmented to ensure economic coherence and to facilitate meaningful comparative analysis within the Black–Litterman framework. The objective is not to maximize expected returns, but to examine how the incorporation of such views reshapes expected returns, twists the efficient frontier, and influences portfolio allocations under realistic assumptions and uncertainty.

The main contributions of this project are threefold. First, it demonstrates the use of a factor-based covariance model within a global multi-asset Black–Litterman setting. Second, it introduces heterogeneous prior assumptions across asset classes, particularly for commodities, reflecting their distinct economic roles. Third, it provides a comprehensive empirical comparison of equilibrium and posterior portfolios, illustrating how investor views affect expected returns, diversification, and risk allocation.

The remainder of the report is organized as follows. Section 2 describes the data and asset universe. Section 3 outlines the methodological framework, including the construction of priors, views, and posterior estimates. Section 4 presents the empirical results and portfolio implications. Section 5 discusses robustness and limitations, and Section 6 concludes.

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2. Data and Asset Universe

This section describes the asset universe, data sources, and sample period used in the empirical analysis. The portfolio is constructed as a global multi-asset investment universe comprising equities, fixed income, and commodities. The choice of assets is motivated by the need to capture heterogeneous sources of risk and return while maintaining sufficient historical depth for robust estimation.

2.1 Asset Universe

Equities

Equity exposure is represented using country-level indices rather than aggregated regional benchmarks. This choice allows for greater differentiation in volatility, macroeconomic sensitivity, and factor exposure across markets. Country-level equities also enable macroeconomic views to propagate more evenly across the portfolio, reducing excessive concentration in a single regional equity allocation.

The equity universe includes the following markets:

- **United States (SPY):** Large-cap U.S. equities, representing the global technology and innovation leader.
- **Europe (VGK):** Developed European equities, capturing exposure to mature industrial and consumer sectors.
- **Japan (EWJ):** Japanese equities, reflecting corporate governance reform and export-oriented growth.
- **China (FXI):** Chinese large-cap equities, representing policy-driven growth and global manufacturing exposure.
- **India (INDY):** Indian large-cap equities, capturing structural growth and productivity catch-up.
- **Brazil (EWZ):** Brazilian equities, providing exposure to commodity-driven growth and emerging market cyclicality.

All equity series are sourced from exchange-traded funds (ETFs) available on Yahoo Finance, which provide transparent and liquid proxies for the underlying equity markets.

Fixed Income

Fixed income exposure is constructed to clearly separate interest rate risk from credit risk. Rather than using a broad aggregate bond index, two distinct instruments are employed:

- **U.S. Treasury Bonds (IEF):** A government bond proxy representing pure duration and interest rate exposure.
- **Investment-Grade Corporate Credit (LQD):** U.S. corporate bonds providing exposure to credit spreads and corporate balance sheet strength.

This separation allows interest rate and credit risks to be modeled independently in both the factor model and the Black–Litterman framework.

Commodities

- **Broad Commodities (DBC):** A diversified commodity index capturing exposure to energy, metals, and agricultural commodities. Commodities are included primarily for diversification, inflation protection, and sensitivity to global investment cycles, rather than long-term growth.

2.2 Data Sources and Sample Period

Asset price data are obtained from Yahoo Finance and converted to monthly total returns using adjusted closing prices. Fixed income yields and macroeconomic series are sourced from the Federal Reserve Economic Data (FRED) database, while equity factor data are obtained from the Fama–French data library.

The analysis is conducted at a monthly frequency to balance statistical robustness with economic interpretability. The sample period begins in 2009, ensuring at least 15 years of data availability across all assets while avoiding structural breaks associated with earlier market regimes.

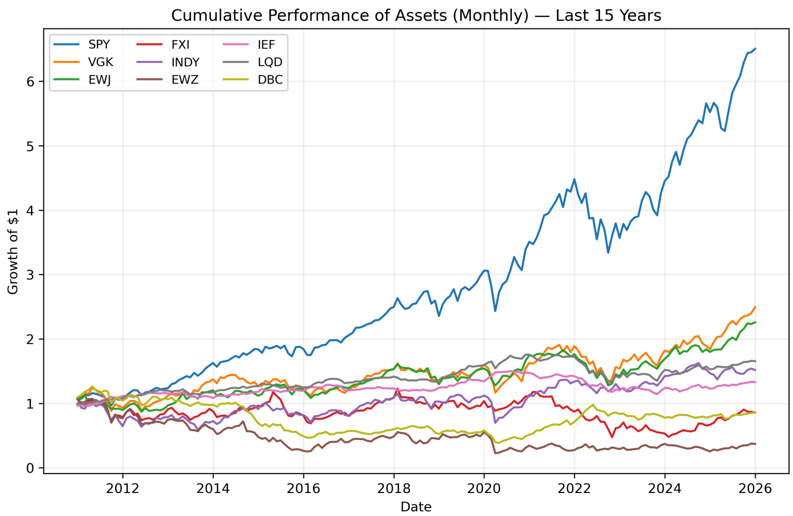

Figure 2.1: *Cumulative Returns of Portfolio Assets (Monthly).*

Figure 2.1 shows the cumulative performance of all assets in the investment universe from the start of the sample period. The figure highlights substantial heterogeneity in long-term returns, volatility, and drawdown behavior across asset classes and regions, motivating the use of a diversified multi-asset framework and structured portfolio construction approach.

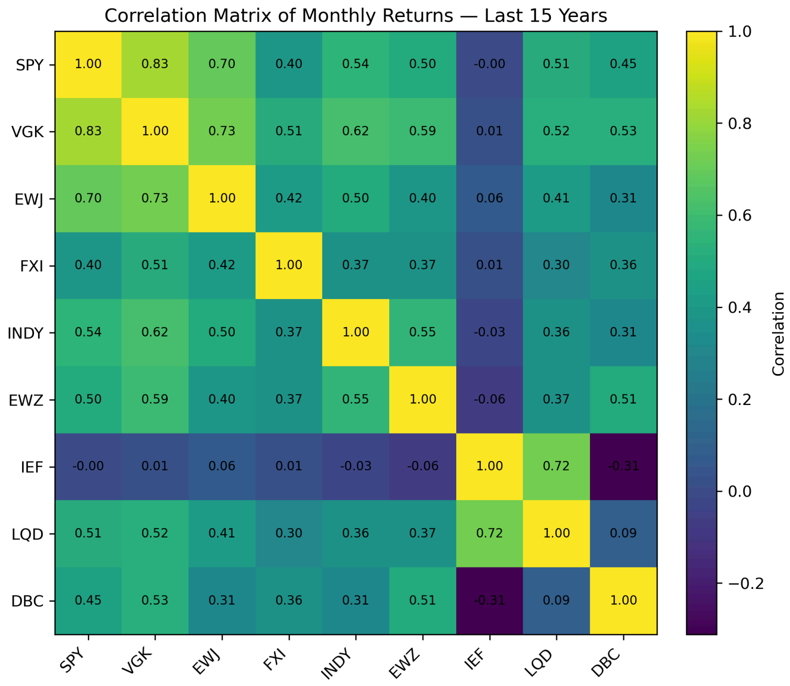

Figure 2.2: *Correlation Matrix of Asset Returns.*

Figure 2.2 presents the pairwise correlations of monthly asset returns over the sample period. While most assets exhibit positive correlation, meaningful differences in co-movement across regions and asset classes remain. This underscores the potential for diversification while also motivating the use of factor-based covariance estimation rather than reliance on raw sample correlations.

2.3 Strategic Reference Allocation

A strategic reference portfolio is used to infer equilibrium expected returns within the Black–Litterman framework. The reference allocation reflects a growth-oriented multi-asset mandate, with a dominant allocation to equities, complemented by fixed income and a modest allocation to commodities for diversification and inflation hedging.

At the asset-class level, the reference portfolio allocates approximately:

- **73% to equities**
- **23% to fixed income**
- **4% to commodities**

Within equities, weights are distributed according to broad market capitalization proxies, while fixed income exposure is split evenly between government bonds and corporate credit. This reference portfolio is not intended to be optimal, but rather to represent a reasonable long-term market consensus around which active views are expressed.

Further details on the construction of equilibrium returns and the role of the reference allocation are provided in Section 3.

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3. Methodology

This section describes the portfolio construction framework employed in the study. The approach combines a factor-based risk model with the Black–Litterman (BL) framework to generate stable and economically interpretable expected returns and portfolio allocations. The methodology proceeds in four steps: covariance estimation, equilibrium return inference, specification of investor views, and portfolio optimization.

3.1 Factor-Based Covariance Estimation

Asset return covariance is estimated using a multi-factor model rather than relying on the sample covariance matrix. This choice is motivated by the instability of sample covariances in multi-asset settings and the desire to obtain a more structured and interpretable representation of risk.

Each asset’s excess return is modeled as a linear function of systematic factors:

$$r_{i,t} = \alpha_i + {\beta_i}' f_t + \varepsilon_{i,t} \tag{1}$$

where f_t denotes a vector of factor returns and $\varepsilon_{i,t}$ captures asset-specific risk. The factor set includes equity style factors, interest rate changes, credit spread dynamics, and commodity-related risk drivers.

The asset covariance matrix is constructed as:

$$\Sigma = B \cdot Cov(f) \cdot B' + D \tag{2}$$

where B contains estimated factor loadings, $Cov(f)$ is the factor covariance matrix, and D is a diagonal matrix of idiosyncratic variances. To assess robustness, alternative covariance estimators—including shrinkage and exponentially weighted moving average (EWMA) estimators—are later considered in the portfolio optimization stage.

3.2 Equilibrium Expected Returns

The Black–Litterman framework requires a set of equilibrium expected returns that are consistent with a reference portfolio. These equilibrium returns are obtained via reverse optimization:

$$\Pi = \lambda \cdot \Sigma \cdot w_{ref} \tag{3}$$

where Π denotes the vector of implied excess returns, Σ is the asset covariance matrix, w_{ref} represents the strategic reference allocation, and λ is the representative risk aversion coefficient.

The reference portfolio reflects a growth-oriented multi-asset allocation and is not intended to be optimal. Rather, it serves as a neutral anchor representing long-term market consensus. The risk aversion parameter is inferred from long-term capital market assumptions to ensure consistency between expected returns and portfolio risk.

3.3 Prior Specification and Heterogeneous Assumptions

Expected return priors are constructed using long-term capital market assumptions for equities and fixed income assets. These assumptions provide stable estimates of long-run excess returns and volatilities consistent with institutional investment horizons.

Commodities are treated differently due to their lack of intrinsic cash flows and highly regime-dependent return behavior. Rather than relying on long-horizon historical averages, commodity expected returns are anchored at a small positive excess return with high uncertainty, while volatility is estimated over a shorter historical window. This specification reflects the role of commodities as diversifiers and inflation hedges rather than long-term growth assets.

This heterogeneous prior structure allows each asset class to be modeled in a manner consistent with its economic role while maintaining internal coherence within the Black–Litterman framework.

3.4 Black–Litterman Framework

Investor views are incorporated using the Black–Litterman Bayesian updating mechanism. Views are expressed in linear form:

$$P \cdot \mu = Q + \varepsilon \tag{4}$$

where μ is the vector of expected excess returns, P is the pick matrix linking views to assets, Q is the vector of view-implied excess returns, and ε represents view uncertainty.

The posterior expected returns are given by:

$$\mu^{BL} = \Pi + \tau\Sigma P'(P\tau\Sigma P' + \Omega)^{-1}(Q - P\Pi) \tag{5}$$

where τ is a scalar controlling the uncertainty of the prior and Ω is the covariance matrix of view errors.

Views are specified in relative return form to improve robustness and reduce sensitivity to absolute return forecasts. View uncertainty is calibrated using an Idzorek-style approach, whereby the variance of each view is proportional to the projected variance of the underlying assets scaled by subjective confidence levels. This ensures that no single view dominates the posterior unless supported by high confidence.

3.4.1 Specification of Investor Views

Investor views are formulated based on a medium-term macroeconomic scenario characterized by the diffusion of artificial intelligence, productivity catch-up in emerging markets, and a moderation in developed market growth leadership. Rather than forecasting absolute returns, all views are expressed in relative form to enhance robustness and reduce sensitivity to estimation error.

The views incorporate the following themes:

- stronger growth in selected emerging markets (India, Brazil, China),
- comparatively weaker growth in Europe,
- stable but higher-for-longer interest rates favoring credit over duration,

- continued relevance of commodities as diversification and inflation-sensitive assets.

Each view is expressed as a linear combination of asset returns, summarized in Table 3.1.

Table 3.1: Summary of Black–Litterman Views

View	Economic Interpretation	Mathematical Representation	Implied Excess Return (Q)
V1	Emerging market equities outperform U.S. equities	(INDY + FXI + EWZ) / 3 − SPY	+1.5%
V2	U.S. equities outperform European equities	SPY − VGK	+1.0%
V3	Japanese equities outperform European equities	EWJ − VGK	+0.75%
V4	Investment-grade credit outperforms government bonds	LQD − IEF	+0.75%
V5	Equities outperform bonds	Equity basket − Bond basket	+1.25%
V6	Commodities outperform government bonds	DBC − (IEF + LQD) / 2	+1.5%
V7	Brazilian equities outperform Chinese equities	EWZ − FXI	+0.25%

3.4.2 Calibration of View Confidence

Uncertainty associated with each view is explicitly modeled through the view error covariance matrix Ω. Rather than specifying Ω directly, confidence levels are assigned to each view and mapped to variance terms using an Idzorek-style approach.

For each view i, the variance is defined as:

$$\Omega_{ii} = \frac{1 - c_i}{c_i} \cdot \left(p_i' \tau \Sigma p_i\right), \tag{6}$$

where $c_i \in (0, 1)$ denotes the subjective confidence level assigned to the view. Higher confidence implies lower variance and therefore greater influence on posterior expected returns.

Confidence levels are chosen to reflect the relative strength of economic conviction behind each view, with higher confidence assigned to broad macro relationships (e.g., credit versus duration) and moderate confidence assigned to regional growth differentials. This calibration prevents any single view from dominating the posterior while allowing economically meaningful tilts to emerge.

Table 3.2: Confidence Levels Assigned to Investor Views

View	Description	Confidence Level
V1	Emerging markets vs U.S. equities	0.55
V2	U.S. vs European equities	0.70
V3	Japan vs Europe	0.60

View	Description	Confidence Level
V4	Credit vs government bonds	0.75
V5	Equities vs bonds	0.65
V6	Commodities vs bonds	0.55
V7	Brazil vs China	0.55

3.5 Portfolio Optimisation Problems

Once expected returns and covariances have been specified, portfolio construction is performed by solving a range of standard optimisation problems. Expected returns are taken either as the equilibrium prior vector Π or the Black–Litterman posterior vector μ^{BL}, while risk is measured using the covariance matrix Σ.

All optimisation problems impose the full-investment constraint $\mathbf{1}'w = 1$, and returns are expressed in excess of the risk-free rate.

3.5.1 Global Minimum Variance Portfolio

The global minimum variance (GMV) portfolio solves:

$$\min_{w} \quad w'\Sigma w \quad \text{s.t.} \quad \mathbf{1}'w = 1. \tag{7}$$

In the unconstrained case, the solution admits the closed-form expression:

$$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}. \tag{8}$$

The GMV portfolio serves as a benchmark that depends only on the covariance structure and is independent of expected returns.

3.5.2 Mean–Variance Portfolio

The mean–variance portfolio maximises quadratic utility:

$$\max_{w} \quad w'\mu - \frac{\lambda}{2} w'\Sigma w \quad \text{s.t.} \quad \mathbf{1}'w = 1, \tag{9}$$

where μ denotes the vector of expected excess returns and $\lambda > 0$ is the risk-aversion parameter. This formulation is applied using both equilibrium returns Π and Black–Litterman posterior returns μ^{BL}.

3.5.3 Maximum Sharpe Ratio Portfolio

The maximum Sharpe (tangency) portfolio solves:

$$\max_{w} \quad \frac{w'\mu}{\sqrt{w'\Sigma w}} \quad \text{s.t.} \quad \mathbf{1}'w = 1. \tag{10}$$

In the unconstrained case, the solution is proportional to $\Sigma^{-1}\mu$ and is typically normalized to satisfy the budget constraint. Under realistic constraints, the problem is solved numerically.

3.5.4 Unconstrained Black–Litterman Benchmark

For reference, the unconstrained optimal portfolio implied directly by the Black–Litterman posterior returns is given by:

$$w^*_{BL} = \frac{1}{\lambda}\Sigma^{-1}\mu^{BL}. \tag{11}$$

This portfolio is not implementable in practice but provides a useful analytical benchmark for understanding the effect of posterior expected returns.

3.5.5 Risk-Aversion Profiles

Investor preferences are incorporated through the risk-aversion parameter λ in the mean–variance optimisation framework. To illustrate the impact of risk preferences on portfolio construction, three representative investor profiles are considered:

- **Trustee Investor:** highly risk-averse, prioritizing capital preservation
- **Market Investor:** moderate risk aversion, representative of a balanced institutional investor
- **Kelly Investor:** aggressive risk preference, emphasizing growth and higher volatility tolerance

Formally, portfolio weights are obtained by solving:

$$\max_{w} \quad w'\mu^{BL} - \frac{\lambda}{2}w'\Sigma w \quad \text{s.t.} \quad \mathbf{1}'w = 1, \tag{12}$$

for different values of λ. Higher values of λ penalize portfolio variance more heavily, resulting in more conservative allocations, while lower values of λ produce more aggressive portfolios with higher expected return and risk.

The resulting allocations for each investor profile are compared in the empirical results section to illustrate how risk preferences interact with Black–Litterman posterior expected returns.

3.6 Constrained Optimisation and Implementation

In practice, portfolio optimisation is subject to constraints reflecting regulatory, institutional, and implementability considerations. In this study, the following constraints are imposed:

- **Budget constraint:** $\mathbf{1}'w = 1$
- **Long-only constraint:** $w_i \geq 0$
- **Weight caps:** $w_i \leq w_i^{\max}$

Under these constraints, the optimisation problems become quadratic or conic programmes that do not generally admit closed-form solutions. Portfolio weights are therefore obtained using numerical optimisation techniques.

To evaluate the impact of expected return assumptions, covariance estimation, and optimization objectives, a set of portfolios is constructed by combining alternative objective functions with different covariance estimators. All portfolios are subject to long-only constraints, a full investment constraint, and asset-specific maximum weight limits reflecting market size and liquidity considerations.

Portfolio names follow a consistent convention. The prefix denotes the optimization objective, while the suffix identifies the covariance estimator employed. Three covariance estimators are considered: a factor-based covariance model, a Ledoit–Wolf shrinkage estimator, and an exponentially weighted moving average (EWMA) estimator.

The optimization objectives examined are summarized as follows:

- **Minimum Variance (Min_Var)**: minimizes total portfolio variance without reference to expected returns.
- **Mean–Variance with Black–Litterman Returns (MV_BL)**: maximizes a quadratic utility function using Black–Litterman posterior expected returns.
- **Maximum Sharpe Ratio (MaxSharpe_BL)**: maximizes expected excess return per unit of volatility using Black–Litterman posterior returns.
- **Minimum Parametric Value-at-Risk (Min_VaR_5%)**: minimizes a parametric Value-at-Risk measure at the 5% confidence level, incorporating both return and volatility.
- **Minimum Tracking Error (Min_TE)**: minimizes deviation from the strategic reference portfolio, subject to the covariance structure.
- **Regularized Mean–Variance (MV_BL_Reg)**: mean–variance optimization with an additional penalty that discourages large deviations from the reference portfolio, improving stability.

Each optimization objective is applied under alternative covariance estimators to assess robustness and sensitivity to risk modeling assumptions. These constrained portfolios form the basis of the empirical comparisons presented in the results section.

3.7 Interpretation and Limitations

The Black–Litterman framework does not aim to improve the efficient frontier in a purely mechanical sense. Instead, it produces posterior expected returns that reflect both market consensus and investor beliefs under uncertainty. As a result, posterior efficient frontiers may lie below their prior counterparts, reflecting more conservative and realistic return expectations.

Limitations of the approach include sensitivity to covariance estimation, the choice of view confidence parameters, and the tendency for correlations to increase during periods of market stress. These limitations are explored further in the empirical results and robustness analysis.

3.8 Factor Study and Systematic Backtesting

Before engaging in portfolio optimisation, the factor set is analysed as an object of study in its own right. This serves two purposes. First, it validates that the chosen factors exhibit distinct risk–return characteristics and are not redundant with the market factor. Second, it provides economic context for the factor-based covariance model used later in portfolio construction.

3.8.1 Factor Performance

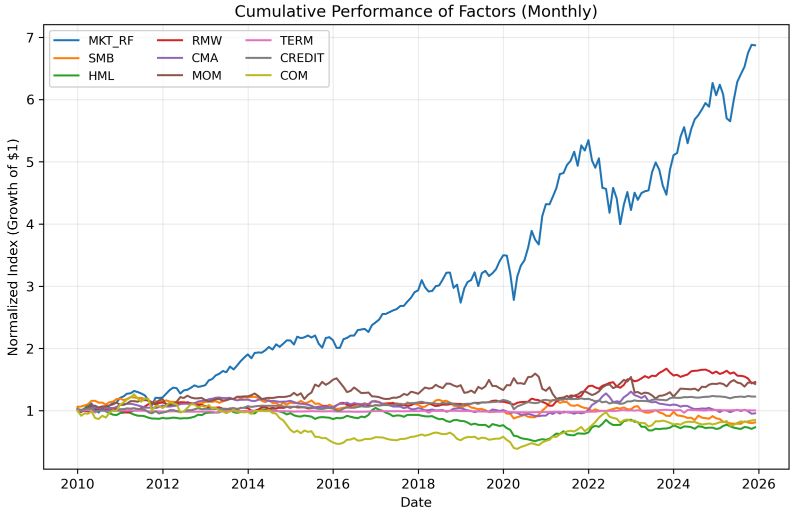

Figure 3.1: *Cumulative Performance of Factors (Monthly).*

Table 3.3: Factor Summary Statistics

Factor	Annual Return (%)	Volatility (%)	Sharpe Ratio
MKT_RF	6.47	15.61	0.41
SMB	-3.57	12.36	-0.29
HML	-1.69	11.02	-0.15
RMW	3.02	8.88	0.34
CMA	0.82	7.54	0.11
MOM	5.61	15.32	0.37
TERM	0.06	3.65	0.02
CREDIT	-1.89	6.98	-0.27
COM	-6.21	18.44	-0.34

Notes: All statistics are computed from monthly returns and annualized. Sharpe ratios are based on excess returns. Table 3.3 reports annualized return, volatility, and Sharpe ratios for the factor set over the sample period. Considerable heterogeneity is observed across factors in both risk and performance characteristics. The market excess return exhibits the highest volatility, while defensive macro factors such as TERM display substantially lower risk. Profitability (RMW) and momentum (MOM) achieve positive Sharpe ratios over the sample, whereas size (SMB), value (HML), credit, and commodity factors exhibit negative risk-adjusted performance. These results highlight the importance of diversification across factor types and reinforce the motivation for incorporating multiple factors within a structured portfolio construction framework rather than relying on any single factor premium.

The performance and volatility characteristics vary meaningfully across factors, supporting the use of a multi-factor structure rather than treating the investment universe as being driven solely by the equity market..

3.8.2 Systematic Backtesting of Factors vs Market Excess Returns

To evaluate the relationship between each factor and broad equity market risk, each factor is regressed on the market excess return:

$$f_t = \alpha + \beta MKT\_RF_t + \varepsilon_t. \tag{13}$$

Table 3.4: Regression of Factors on Market Excess Return (MKT_RF)

Factor	Alpha (ann, %)	Beta to MKT	R^2	t(Alpha)	t(Beta)
SMB	-3.81	0.22	0.12	-1.65	5.12
HML	-1.85	0.04	0.00	-0.64	0.74
RMW	3.29	-0.05	0.01	1.87	-1.60
CMA	0.82	-0.06	0.02	0.44	-1.82
MOM	6.20	-0.24	0.09	2.05	-4.29
TERM	0.05	-0.00	0.00	0.08	-0.01
CREDIT	-1.97	0.26	0.53	-2.12	14.63
COM	-6.83	0.55	0.23	-1.79	7.64

Note: Alpha is annualized from monthly regression intercepts. All regressions use monthly data. Table 3.4 reports regressions of each factor on the market excess return (MKT_RF). Betas quantify the degree of market dependence, while alpha represents the factor's average return unexplained by the market. Most factors exhibit low market dependence (low R^2), consistent with the interpretation that they capture distinct sources of systematic variation. CREDIT and COM show relatively higher market-related variation over the sample (higher R^2), while MOM displays a negative market beta in this sample, indicating that momentum behaved as a partial hedge against market returns in certain periods. Rolling regressions are then used to assess time variation in exposures.

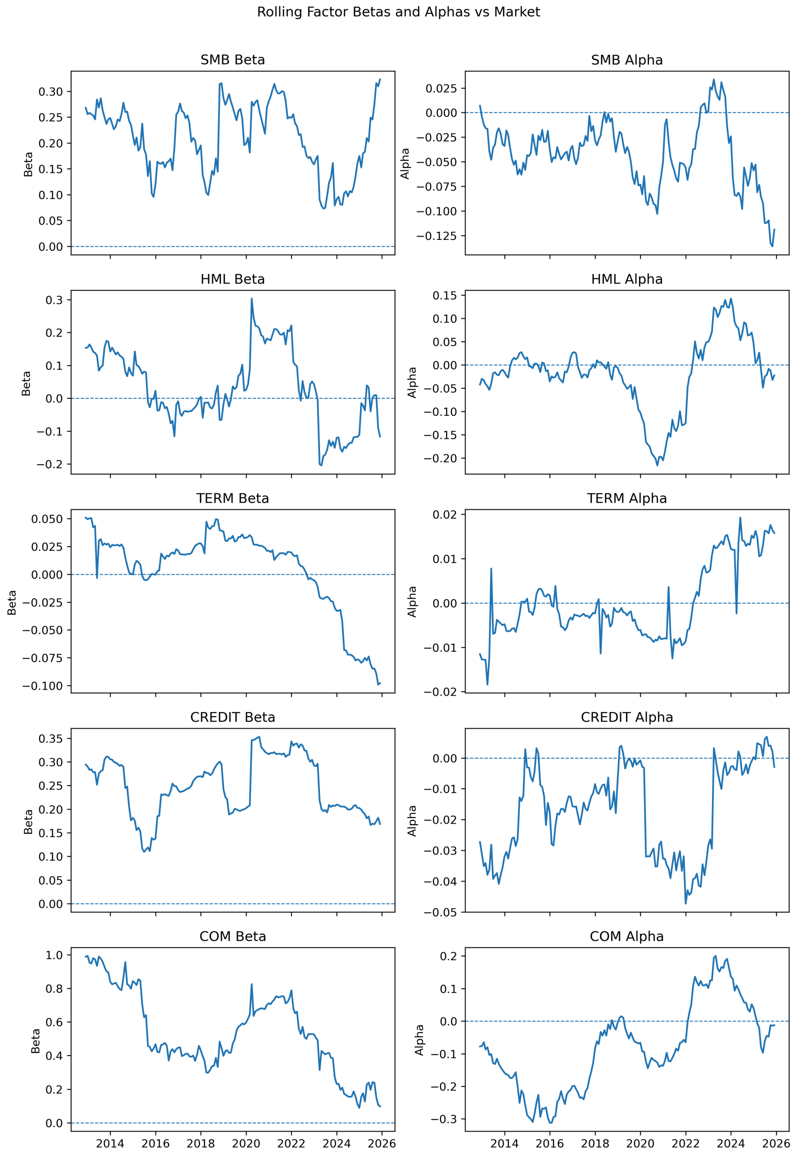

Figure 3.2: *Rolling Alphas (Annualized) and Betas of Selected Factors vs Market.*

The results highlight that factor exposures and abnormal returns can vary over time, consistent with the view that factor premia and macro relationships are regime dependent. This motivates later robustness checks and underscores the limitations of assuming constant correlations and factor loadings.

3.8.3 Portfolio Exposures to Factors

Finally, the optimised portfolios are regressed against the same factor set to assess systematic exposures and determine whether portfolio performance is primarily explained by factor premia:

$$r_{p,t}^{excess} = \alpha_p + \beta_p' f_t + \varepsilon_t. \tag{14}$$

These portfolio factor exposures are reported in the empirical results section and used to interpret the allocation shifts induced by the Black–Litterman framework.

Table 3.5: Factor Regression of BL Mean–Variance Portfolio

Factor	Beta
Market (MKT_RF)	0.76

Factor	Beta
Size (SMB)	-0.11
Value (HML)	0.01
Momentum (MOM)	-0.07
Term	-0.04
Credit	0.32
Commodity	0.06

Table 3.6: Portfolio Regression Summary Statistics

Metric	Value
Alpha (monthly)	-0.19%
Alpha (annualized)	-2.25%
R^2	0.91
Annualized Return	8.20%
Annualized Volatility	13.88%
Sharpe Ratio	0.59

Table 3.5 reports factor exposures of the Black–Litterman mean–variance portfolio. Portfolio returns are strongly explained by systematic risk factors, with an R^2 of approximately 0.91. The portfolio exhibits a market beta below one, reflecting reduced dependence on broad equity market movements, alongside meaningful exposure to credit risk and modest sensitivity to commodity-related factors.

The estimated alpha is small and statistically weak, indicating that portfolio performance is largely attributable to intentional factor tilts rather than unexplained excess returns. This result supports the internal consistency of the factor-based covariance model and confirms that the Black–Litterman framework produces portfolios whose behavior aligns with their underlying economic assumptions.

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4. Empirical Results

This section presents the empirical results of the Black–Litterman portfolio construction framework. The analysis compares equilibrium priors with posterior estimates incorporating investor views, focusing on expected returns, efficient frontiers, portfolio allocations, and risk characteristics. All results are based on monthly data and annualized for interpretability.

4.1 Prior and Posterior Expected Returns

Table 4.1 reports equilibrium (prior) expected excess returns and Black–Litterman posterior expected returns for all assets in the investment universe.

Table 4.1: Prior vs Posterior Expected Excess Returns

Asset	Π (Equilibrium, %)	μ^{BL} (Posterior, %)	Δ (Posterior − Prior, %)
SPY	4.52	2.64	-1.88
VGK	4.98	1.95	-3.03
EWJ	3.26	2.17	-1.09
FXI	3.50	2.74	-0.76
INDY	3.95	2.61	-1.34
EWZ	5.77	3.48	-2.29
IEF	-0.04	0.18	+0.22
LQD	1.24	0.93	-0.31
DBC	2.90	1.63	-1.27

Table 4.1 compares equilibrium expected excess returns implied by the market portfolio with Black–Litterman posterior expected returns after incorporating investor views. Posterior returns are uniformly more conservative than equilibrium priors, reflecting the effect of uncertainty and the Bayesian shrinkage mechanism inherent in the Black–Litterman framework.

Assets favored in the view specification—such as emerging market equities and commodities—retain relatively higher posterior expected returns, although at reduced magnitudes compared to equilibrium levels. In contrast, developed market equities, particularly Europe, experience larger downward adjustments, consistent with views indicating weaker relative growth. Fixed income assets exhibit modest posterior returns, with investment-grade credit outperforming government bonds in line with the credit-versus-duration view.

Overall, the posterior return vector reflects the directional influence of the stated views while avoiding extreme deviations from market-implied expectations.

Incorporating investor views redistributes expected returns rather than increasing them uniformly. Posterior expected returns tilt toward emerging market equities— particularly India, Brazil, and China—reflecting productivity catch-up, infrastructure investment, and AI-driven growth dynamics. U.S. equities retain positive expected returns but lose relative dominance, while European equities are comparatively weaker due to structural and regulatory constraints.

Fixed income posterior returns favor investment-grade credit over government bonds, consistent with expectations of stable growth and contained default risk. Commodities receive modest positive expected returns despite neutral long-term priors, driven primarily by scenario-based views and diversification benefits rather than structural growth assumptions.

4.2 Efficient Frontier Comparison

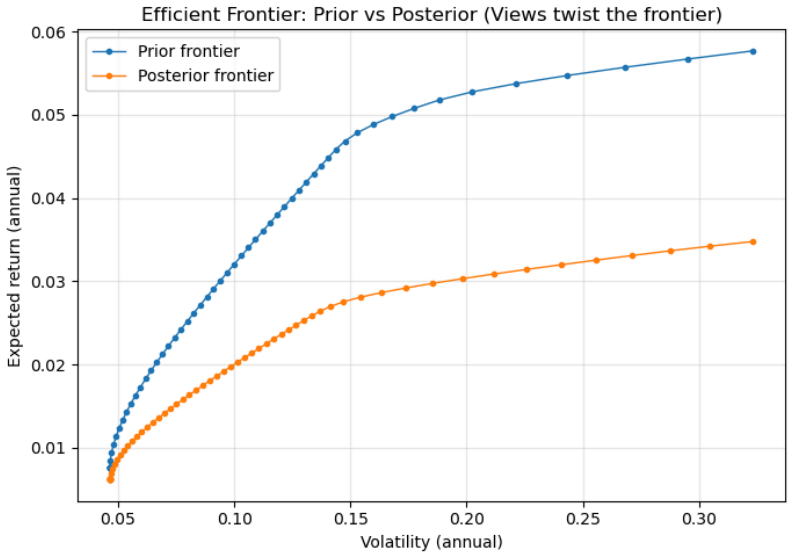

Figure 4.1: *Prior vs Posterior Efficient Frontiers.*

The figure 4.1 compares efficient frontiers constructed using equilibrium expected returns (Π) and Black–Litterman posterior expected returns (μ^{BL}). The posterior efficient frontier lies below the prior frontier across most levels of portfolio volatility. This behavior reflects the Bayesian shrinkage mechanism inherent in the Black–Litterman framework.

Equilibrium returns represent an internally consistent market consensus that assumes perfect aggregation of information and ignores parameter uncertainty. In contrast, the posterior incorporates explicit uncertainty through the prior scaling parameter and the view error covariance matrix, resulting in more conservative expected return estimates.

The downward shift of the posterior frontier should not be interpreted as inferior performance. Rather, it indicates improved robustness and reduced sensitivity to estimation error. The Black–Litterman framework trades mechanical frontier maximization for economically plausible and stable portfolio outcomes.

4.3 Portfolio Allocations

Table 4.2: Prior vs Posterior Portfolio Weights

Asset	Prior (%)	BL Mean–Variance (%)	Min Var (Factor) (%)	Min Var (Ledoit–Wolf) (%)
SPY	53.20	62.46	0.00	2.06

Asset	Prior (%)	BL Mean–Variance (%)	Min Var (Factor) (%)	Min Var (Ledoit–Wolf) (%)
VGK	10.60	0.00	0.00	0.00
EWJ	4.40	9.74	1.68	6.84
FXI	2.00	14.27	0.00	0.00
INDY	2.30	8.77	0.01	0.00
EWZ	0.50	4.76	0.00	0.00
IEF	11.50	0.00	56.32	60.78
LQD	11.50	0.00	29.30	14.47
DBC	4.00	0.00	12.69	15.84

Table 4.2 summarizes the strategic reference allocation and posterior portfolio weights obtained under alternative optimization objectives. The Black–Litterman mean–variance portfolio displays a strong tilt toward equities, particularly U.S. and emerging market exposures, reflecting the relative return views embedded in the posterior expected returns.

In contrast, global minimum variance portfolios allocate predominantly to fixed income and commodities, emphasizing risk minimization rather than return generation. Differences between the factor-based and Ledoit–Wolf minimum variance allocations highlight the sensitivity of risk-minimizing portfolios to the choice of covariance estimator, while preserving a common preference for lower-volatility assets.

Table 4.3: Backtest Performance of Posterior Portfolios

Metric	BL Mean–Variance	Min Var (Factor)	Min Var (Ledoit–Wolf)
Annual Return	9.62%	2.71%	2.86%
Annual Volatility	13.89%	5.51%	5.34%
Sharpe Ratio	0.69	0.49	0.54
Maximum Drawdown	-23.54%	-16.50%	-14.12%

Table 4.3 reports the historical performance of the posterior portfolios. As expected, the mean–variance portfolio achieves higher return and Sharpe ratio at the cost of greater volatility and drawdown, whereas minimum variance portfolios deliver lower returns with improved downside protection.

Overall, posterior allocations differ meaningfully from the strategic reference portfolio. Exposure shifts toward selected emerging markets and away from Europe reflect relative growth expectations, while moderate allocations to corporate credit and commodities support diversification and inflation-sensitive positioning. These results illustrate how Black–Litterman posterior returns, when combined with different optimization objectives, produce economically interpretable and risk-consistent portfolio outcomes.

4.4 Risk Contributions and Diversification

Figure 4.2 presents the decomposition of total portfolio risk into asset-level contributions for selected posterior portfolios. Risk contributions are computed ex ante using the portfolio covariance matrix and quantify the share of total volatility attributable to each asset.

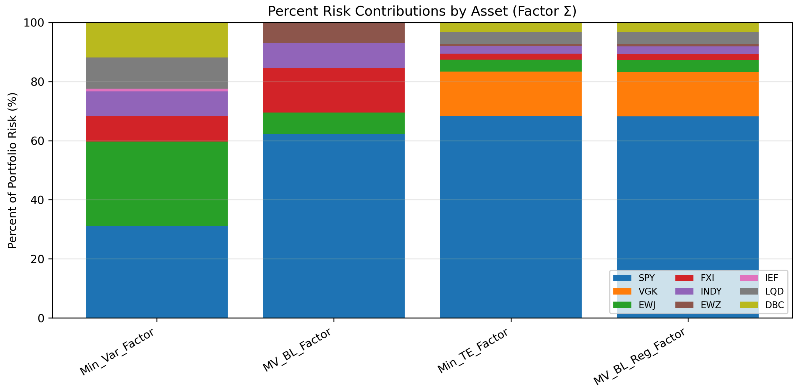

Figure 4.2: *Percent Risk Contributions by Asset — Posterior Portfolios.*

The Black–Litterman mean–variance portfolio exhibits a concentration of risk in equity assets, particularly U.S. and emerging market equities, reflecting its return-seeking objective. In contrast, minimum variance and benchmark-aware portfolios display a more balanced distribution of risk across fixed income and diversifying assets, including commodities.

Importantly, diversification improvements arise not from mechanically lower correlations, but from allocating capital toward assets with differentiated risk profiles. Government bonds contribute disproportionately to volatility reduction, while investment-grade credit and commodities provide incremental diversification. These patterns are consistent with the portfolios' optimization objectives and highlight the role of Black–Litterman posterior returns in shaping economically plausible risk allocations.

Table 4.4: Effective Number of Bets (ENB) Across Posterior Portfolios

Portfolio	Effective Number of Bets
Min Var (Factor)	3.08
MV BL (Factor)	4.04
Min Tracking Error (Factor)	2.95
MV BL Reg (Factor)	3.07

Consistent with the risk contribution analysis, minimum variance and regularized portfolios exhibit higher effective numbers of bets, indicating reduced risk concentration and improved diversification.

4.5 Allocation Behavior Across Risk Levels

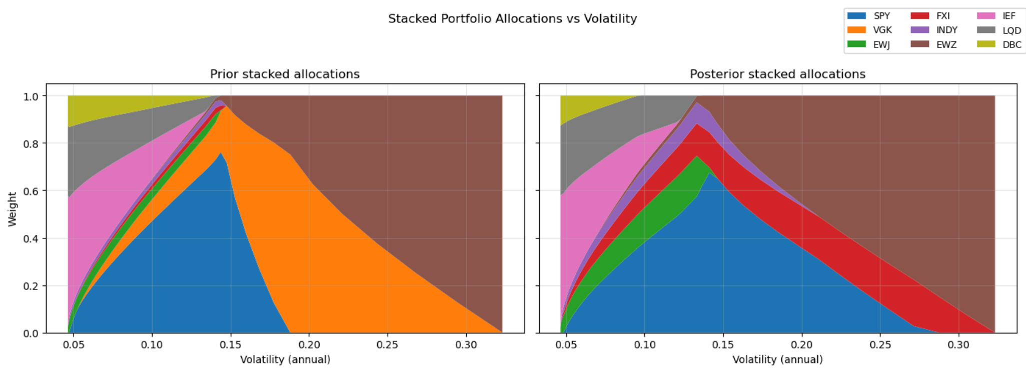

Figure 4.3: *Asset Allocations as a Function of Target Volatility.*

Figures 4.3 illustrates how portfolio composition evolves along the efficient frontier under equilibrium and Black–Litterman posterior expected returns, respectively. Volatility serves as a proxy for investor risk tolerance, with higher levels corresponding to more aggressive portfolios.

Under equilibrium returns, allocations shift rapidly toward a small number of equity markets as volatility increases, leading to pronounced concentration at higher risk levels. In contrast, posterior allocations exhibit smoother transitions and maintain broader diversification across regions and asset classes over a wider range of volatilities.

This difference reflects the effect of Bayesian shrinkage in the Black–Litterman framework, which moderates extreme expected return differentials and results in more stable allocation paths. As a consequence, posterior portfolios avoid abrupt concentration and better preserve diversification as investor risk tolerance varies.

4.6 Risk-Aversion Profiles (Trustee / Market / Kelly)

To illustrate the role of investor preferences in portfolio construction, Black–Litterman posterior expected returns are optimized under three representative levels of risk aversion. A high risk-aversion (Trustee) profile places a strong penalty on variance, resulting in conservative allocations dominated by fixed income assets. A low risk-aversion (Kelly) profile emphasizes expected return, leading to more aggressive equity exposure. The Market profile represents an intermediate case.

Table 4.5: Portfolio Weights under Risk-Aversion Profiles (%)

Asset	Trustee (High λ)	Market (Mid λ)	Kelly (Low λ)
SPY	0.00	20.59	40.00
VGK	0.00	0.00	0.00
EWJ	7.98	11.15	23.35
FXI	1.50	6.10	16.70
INDY	2.08	4.75	13.26
EWZ	0.00	0.32	6.69
IEF	40.00	29.00	0.00
LQD	36.58	22.61	0.00
DBC	11.86	5.48	0.00

Table 4.6: Portfolio Performance Metrics by Risk-Aversion Profile

Profile	Excess Return (%)	Volatility (%)	Sharpe	ENB	Tracking Error
Trustee (High λ)	0.88	5.03	0.17	3.74	0.08
Market (Mid λ)	1.44	7.11	0.20	4.72	0.05
Kelly (Low λ)	2.60	13.52	0.19	4.04	0.05

As risk aversion decreases, portfolio volatility and expected excess returns increase, reflecting a systematic trade-off between risk and return. While the Kelly portfolio achieves the highest expected return, it also exhibits substantially higher volatility. The Market profile delivers a more balanced outcome, achieving improved diversification and risk-adjusted performance relative to the Trustee portfolio.

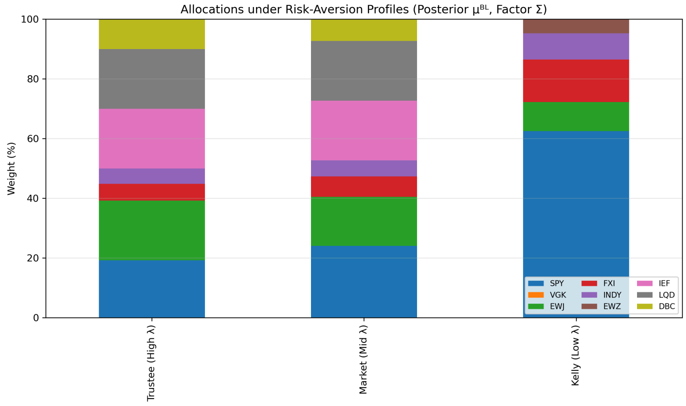

Figure 4.4: *Portfolio Allocations under Risk-Aversion Profiles.*

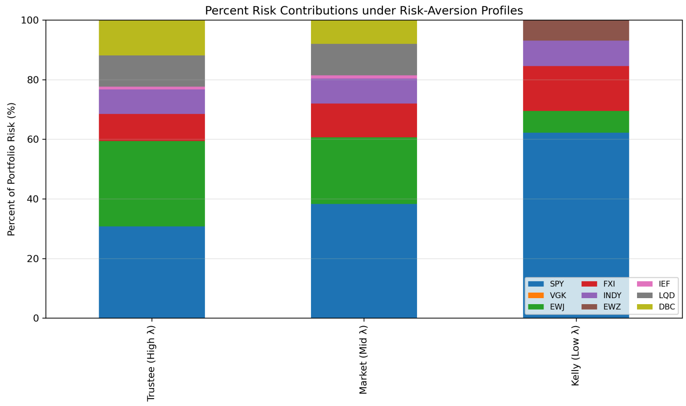

Figure 4.5: *Risk Contributions under Risk-Aversion Profiles.*

Risk contribution analysis shows that higher risk-aversion portfolios concentrate risk primarily in fixed income assets, whereas lower risk-aversion portfolios shift risk toward equities and growth-oriented assets. These results demonstrate how investor risk preferences interact with Black–Litterman posterior beliefs to generate economically interpretable portfolio outcomes.

4.7 Summary of Empirical Findings

The empirical analysis demonstrates how the Black–Litterman framework reshapes expected returns, portfolio allocations, and risk characteristics in a coherent and economically interpretable manner. Posterior expected returns are systematically more conservative than equilibrium priors, reflecting Bayesian shrinkage and explicit treatment of estimation uncertainty, while preserving the directional influence of investor views.

Efficient frontier analysis shows that posterior frontiers lie below prior frontiers across most volatility levels, not as a sign of inferior performance, but as an intended consequence of more robust expectation formation. Portfolio allocations derived from posterior returns differ materially from the strategic reference portfolio and vary significantly across optimization objectives, highlighting the importance of aligning portfolio construction with investor preferences.

Risk contribution analysis reveals that diversification improvements arise primarily from reallocating capital toward assets with differentiated risk profiles rather than from mechanically lower correlations. Minimum variance and regularized portfolios exhibit more balanced risk distributions and higher effective numbers of bets, while return-seeking portfolios concentrate risk in growth-oriented assets.

Finally, risk-aversion profiles illustrate how a common set of posterior beliefs can produce markedly different portfolios depending on investor tolerance for risk. Taken together, these results underscore the flexibility of the Black–Litterman framework as a bridge between market equilibrium, subjective views, and practical portfolio construction.

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5. Robustness and Limitations

This section evaluates the robustness of the Black–Litterman portfolio results to key modeling choices and highlights inherent limitations of the approach. Given the well-known sensitivity of optimized portfolios to expected returns, covariance estimates, and subjective views, robustness analysis is essential for interpreting the empirical findings.

All sensitivity tests are conducted under explicit long-only and asset-level weight constraints, reflecting realistic implementation limits and preventing economically implausible concentration.

5.1 Sensitivity to Prior Uncertainty (τ)

The parameter τ controls the uncertainty assigned to the equilibrium prior expected returns. Smaller values of τ place greater weight on the market-implied equilibrium, while larger values increase the relative influence of investor views.

Across a broad range of τ values (0.01 to 0.10), posterior allocations remain qualitatively stable. While the magnitude of tilts toward emerging market equities and away from U.S. equities varies with τ, the identity of the assets receiving overweight and underweight positions is largely preserved. This indicates that the posterior results are not driven by an extreme or finely tuned choice of prior uncertainty.

Importantly, binding asset-level weight constraints play a stabilizing role: as τ increases, allocations tend to reach prescribed maximum weights rather than diverging further. This behavior reflects realistic portfolio construction limits and prevents excessive sensitivity to subjective inputs.

5.2 Sensitivity to View Confidence (Ω)

View confidence is incorporated through the view error covariance matrix Ω. Scaling Ω up or down uniformly corresponds to decreasing or increasing confidence in the specified views.

As view confidence increases (lower Ω scaling), posterior allocations move further away from the equilibrium prior toward assets favored by the macroeconomic scenario, particularly emerging market equities. Conversely, higher Ω scaling dampens the impact of views, producing allocations closer to the reference portfolio.

Crucially, even under high-confidence scenarios, posterior portfolios do not exhibit extreme concentration. The combination of relative-return views, explicit uncertainty calibration, and binding weight constraints ensures that allocations remain well-diversified. This robustness highlights a key practical advantage of the Black–Litterman framework over unconstrained mean–variance optimization.

5.3 Joint Sensitivity to τ and Ω

Joint variation of τ and Ω confirms that posterior allocations respond smoothly to changes in modeling assumptions. Allocation shifts occur along intuitive dimensions— such as equity versus fixed income and developed versus emerging markets—without producing abrupt or unstable portfolio behavior.

Overall, the sensitivity analysis suggests that the empirical results are driven by the economic content of the views and the covariance structure rather than by arbitrary parameter choices.

Table 5.1: Representative Sensitivity of Posterior Allocations to τ and Ω

τ	Ω Scale	SPY	FXI	INDY	EWZ	IEF	LQD	DBC
0.01	0.5	65.00	7.53	7.60	5.96	0.00	0.00	0.00
0.05	1.0	62.46	14.27	8.77	4.76	0.00	0.00	0.00
0.10	2.0	62.46	14.27	8.77	4.76	0.00	0.00	0.00

5.4 Alternative Covariance Estimation

Portfolio outcomes are also sensitive to the choice of covariance estimator. To assess this sensitivity, portfolio optimizations are repeated using three alternative covariance specifications: a factor-based covariance matrix, a Ledoit–Wolf shrinkage estimator, and an exponentially weighted moving average (EWMA) estimator. All optimizations are conducted under identical long-only and asset-level maximum weight constraints to ensure comparability.

Table 5.2 summarizes representative portfolio outcomes across covariance estimators for selected optimization objectives.

Table 5.2: MV-BL Portfolio Outcomes under Alternative Covariance Estimators

Covariance	Excess Return (%)	Volatility (%)	ENB	Tracking Error
Factor	2.64	13.71	2.34	0.04
Ledoit–Wolf	2.65	13.79	2.26	0.04
EWMA	2.78	11.35	3.16	0.05

While absolute allocations and risk metrics vary across covariance models, the qualitative conclusions remain robust. Growth-oriented portfolios consistently tilt toward emerging market equities under the Black–Litterman posterior, while U.S. equity concentration remains constrained by both views and weight limits. Minimum variance and tail-risk–oriented portfolios allocate more heavily to fixed income and commodities, reflecting their volatility-reduction objectives.

Differences across covariance estimators primarily affect the degree of diversification rather than the direction of allocations. In particular, EWMA-based portfolios tend to exhibit higher effective numbers of bets, reflecting greater sensitivity to recent return dynamics. Overall, these results indicate that the empirical findings are not artifacts of a specific covariance estimation technique, but are driven by the interaction of posterior expected returns, optimization objectives, and realistic portfolio constraints.

5.5 Correlation Regimes and Market Stress

A key limitation of any mean–variance–based framework is the implicit assumption of stable correlations. Empirical evidence shows that asset correlations tend to increase during periods of market stress, thereby reducing the effectiveness of diversification when it is most needed.

The Black–Litterman framework does not eliminate this structural limitation. While the posterior allocations derived in this study are more balanced and robust under normal market conditions, diversification benefits may deteriorate during systemic risk events. In such environments, assets that appear weakly correlated in tranquil periods can become highly correlated as liquidity constraints and risk aversion increase across markets.

Accordingly, Black–Litterman portfolios should be interpreted as strategic or medium-term allocations rather than as tactical risk-off hedges. Stress testing, scenario analysis, and complementary risk management tools remain essential when deploying such portfolios in practice.

5.6 Model Risk and Practical Considerations

Several sources of model risk must be acknowledged. First, expected returns remain the most uncertain inputs in portfolio construction, even within the Black–Litterman framework. Although Bayesian shrinkage improves stability, posterior expected returns still depend on assumptions regarding prior uncertainty, view specification, and confidence calibration.

Second, factor models and covariance estimators rely on historical relationships that may not persist across future regimes. Structural shifts in macroeconomic conditions, monetary policy, or market microstructure can alter both factor premia and asset correlations. Third, the formulation of investor views and confidence levels introduces an unavoidable degree of subjectivity, which must be managed through discipline and economic justification.

Despite these limitations, the Black–Litterman framework offers a transparent and coherent methodology for incorporating subjective beliefs into portfolio construction. By anchoring expectations to a reference portfolio and explicitly modeling uncertainty, the framework mitigates many of the instabilities and extreme allocations associated with traditional mean–variance optimization.

5.7 Summary

The robustness analysis demonstrates that the empirical results are not driven by arbitrary parameter choices or fragile modeling assumptions. While portfolio allocations vary with prior uncertainty, view confidence, and covariance estimation techniques, the qualitative conclusions remain stable across a wide range of specifications.

Overall, the findings reinforce the value of the Black–Litterman framework as a practical tool for constructing diversified portfolios under uncertainty. At the same time, they highlight the importance of informed judgment, economic intuition, and complementary risk management practices when applying the framework in real-world investment settings.

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6. Conclusion

This project applied the Black–Litterman framework to the construction of a global multi-asset portfolio comprising country-level equity indices, fixed income assets, and commodities. By combining a factor-based covariance model with equilibrium expected returns and explicitly stated relative views, the analysis demonstrated how investor beliefs can be incorporated in a disciplined manner while controlling estimation risk.

Empirically, the introduction of Black–Litterman posterior returns resulted in a systematic moderation of expected excess returns across all assets. Equilibrium expected returns ranging from approximately 3–6% annually were reduced to a narrower posterior range of ~ 1.5–3.5%, reflecting Bayesian shrinkage and explicit uncertainty modeling. This moderation translated into a posterior efficient frontier that lay below the prior frontier across most volatility levels, emphasizing robustness over optimistic return forecasts.

Portfolio allocations exhibited economically meaningful shifts. Relative to the strategic reference allocation, U.S. equity exposure declined from over 50% in the prior to approximately 20–40% under moderate risk aversion, while allocations to emerging market equities—particularly India, China, and Brazil—increased materially. These shifts are consistent with growth and productivity catch-up dynamics emphasized in contemporary capital market outlooks, including AI-driven investment themes. European equities remained persistently underweighted, reflecting weaker growth expectations and regulatory constraints.

Across optimization objectives, posterior portfolios demonstrated improved diversification characteristics. Effective numbers of bets increased from approximately 3 in minimum variance portfolios to over 4 in Black–Litterman mean–variance portfolios, while risk contributions became more evenly distributed across asset classes. Fixed income assets continued to provide volatility dampening, with investment-grade credit favored over government bonds, and commodities maintained a persistent allocation driven by diversification and inflation sensitivity rather than return maximization.

Robustness analysis confirmed that these qualitative conclusions were stable across alternative assumptions regarding prior uncertainty, view confidence, and covariance estimation. While absolute allocations varied with parameter choices, the direction of allocation shifts and diversification outcomes remained consistent. This finding aligns with industry practice, where Black–Litterman is used as a strategic allocation tool rather than a source of aggressive tactical bets.

Overall, the results reinforce the value of the Black–Litterman framework as a practical portfolio construction methodology. Rather than seeking mechanical improvements in mean–variance efficiency, the framework provides a transparent mechanism to balance market consensus with investor beliefs, producing stable and economically interpretable portfolios under uncertainty. Future extensions could explore dynamic or regime-based views, alternative factor structures, or out-of-sample performance evaluation to further assess real-world applicability.

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Appendix

Appendix A: Correlation Matrix Factors and Ledoit-Wolf

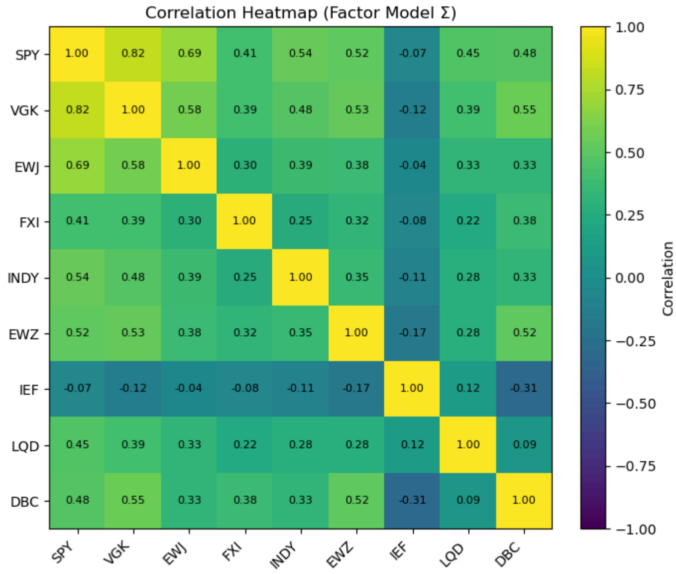

Figure A.1: *Correlation Matrix Heatmap Factors.*

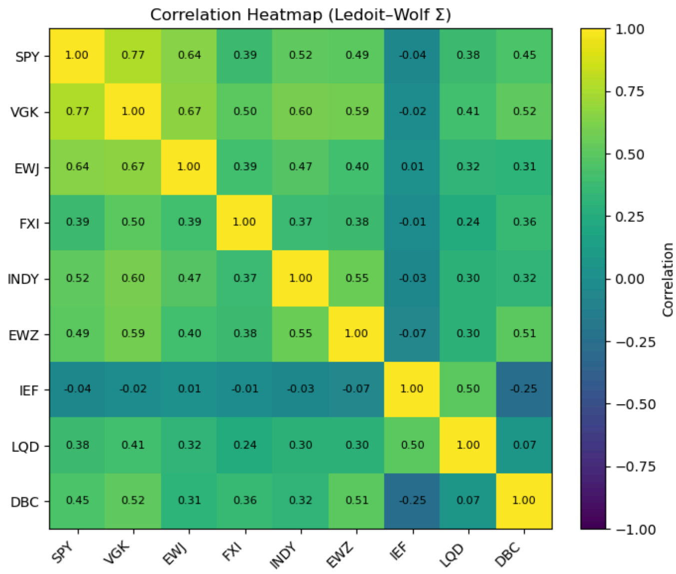

Figure A.2: *Correlation Matrix Heatmap Ledoit-Wolf.*

Appendix B: Allocation Paths Along the Efficient Frontier

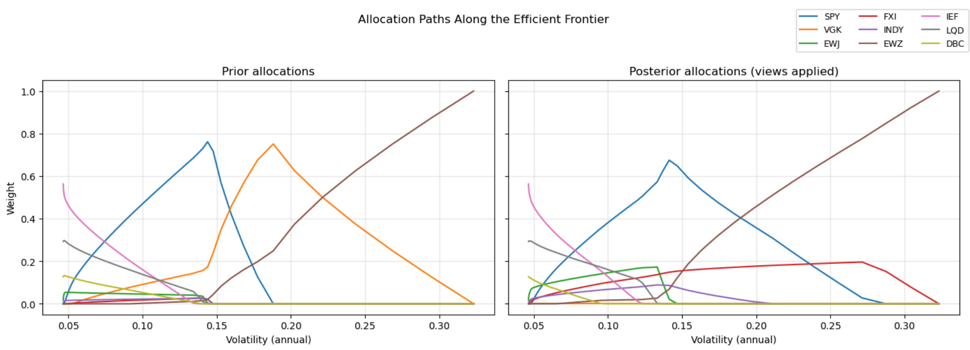

Figure B.1: *Allocation Paths Along the Efficient Frontier.*

Appendix C: Factor Attribution and Portfolio Regression Diagnostics

This appendix reports full ordinary least squares (OLS) regression results for selected portfolios regressed on the factor set used in the covariance model. The regressions serve as diagnostic tools to assess whether portfolio performance is explained by systematic factor exposures and to validate the internal consistency of the portfolio construction framework.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.907
Model:                            OLS   Adj. R-squared:                  0.903
Method:                 Least Squares   F-statistic:                     255.5
Date:                Tue, 20 Jan 2026   Prob (F-statistic):           3.33e-91
Time:                        04:27:43   Log-Likelihood:                 573.55
No. Observations:                 192   AIC:                            -1131.
Df Residuals:                     184   BIC:                            -1105.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0019      0.001     -1.959      0.052      -0.004    1.31e-05
x1             0.7648      0.034     22.441      0.000       0.698       0.832
x2            -0.1109      0.037     -2.958      0.003      -0.185      -0.037
x3             0.0090      0.033      0.274      0.784      -0.056       0.074
x4            -0.0715      0.029     -2.495      0.013      -0.128      -0.015
x5            -0.0359      0.128     -0.280      0.780      -0.288       0.217
x6             0.3189      0.096      3.330      0.001       0.130       0.508
x7             0.0645      0.023      2.773      0.006       0.019       0.110
==============================================================================
Omnibus:                        0.948   Durbin-Watson:                   2.097
Prob(Omnibus):                  0.623   Jarque-Bera (JB):                0.609
...
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure C.1: *BL Mean–Variance Portfolio.*

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.802
Model:                            OLS   Adj. R-squared:                  0.794
Method:                 Least Squares   F-statistic:                     106.4
Date:                Tue, 20 Jan 2026   Prob (F-statistic):           3.07e-61
Time:                        05:07:09   Log-Likelihood:                 597.54
No. Observations:                 192   AIC:                            -1179.
Df Residuals:                     184   BIC:                            -1153.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0009      0.001     -1.070      0.286      -0.003       0.001
x1             0.3729      0.030     12.396      0.000       0.314       0.432
x2            -0.0625      0.033     -1.890      0.060      -0.128       0.003
x3            -0.0618      0.029     -2.136      0.034      -0.119      -0.005
x4            -0.0678      0.025     -2.680      0.008      -0.118      -0.018
x5            -0.3992      0.113     -3.533      0.001      -0.622      -0.176
x6             0.2665      0.085      3.154      0.002       0.100       0.433
x7             0.0842      0.021      4.105      0.000       0.044       0.125
==============================================================================
Omnibus:                        9.562   Durbin-Watson:                   2.089
Prob(Omnibus):                  0.008   Jarque-Bera (JB):               19.727
...
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure C.2: *Minimum Variance Portfolio.*

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.954
Model:                            OLS   Adj. R-squared:                  0.952
Method:                 Least Squares   F-statistic:                     547.5
Date:                Tue, 20 Jan 2026   Prob (F-statistic):          1.48e-119
Time:                        05:08:16   Log-Likelihood:                 684.56
No. Observations:                 192   AIC:                            -1353.
Df Residuals:                     184   BIC:                            -1327.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0012      0.001     -2.297      0.023      -0.002      -0.000
x1             0.6600      0.019     34.525      0.000       0.622       0.698
x2            -0.1183      0.021     -5.628      0.000      -0.160      -0.077
x3            -0.0029      0.018     -0.159      0.874      -0.039       0.033
x4            -0.0389      0.016     -2.422      0.016      -0.071      -0.007
x5            -0.2308      0.072     -3.215      0.002      -0.373      -0.089
x6             0.2047      0.054      3.811      0.000       0.099       0.311
x7             0.0456      0.013      3.497      0.001       0.020       0.071
==============================================================================
Omnibus:                        2.889   Durbin-Watson:                   1.899
Prob(Omnibus):                  0.236   Jarque-Bera (JB):                3.070
...
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure C.3: *Minimum Tracking Error Portfolio.*

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.910
Model:                            OLS   Adj. R-squared:                  0.907
Method:                 Least Squares   F-statistic:                     265.9
Date:                Tue, 20 Jan 2026   Prob (F-statistic):           1.20e-92
Time:                        05:08:21   Log-Likelihood:                 576.89
No. Observations:                 192   AIC:                            -1138.
Df Residuals:                     184   BIC:                            -1112.
Df Model:                           7
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0019      0.001     -1.972      0.050      -0.004    1.06e-06
x1             0.7699      0.033     22.986      0.000       0.704       0.836
x2            -0.1134      0.037     -3.079      0.002      -0.186      -0.041
x3             0.0075      0.032      0.232      0.817      -0.056       0.071
x4            -0.0748      0.028     -2.654      0.009      -0.130      -0.019
x5            -0.0433      0.126     -0.344      0.731      -0.291       0.205
x6             0.3014      0.094      3.203      0.002       0.116       0.487
x7             0.0673      0.023      2.944      0.004       0.022       0.112
==============================================================================
Omnibus:                        1.891   Durbin-Watson:                   2.087
Prob(Omnibus):                  0.388   Jarque-Bera (JB):                1.636
...
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

Figure C.4: *BL Mean–Variance (Ledoit–Wolf Covariance).*

Appendix D: Joint Sensitivity to τ and Ω

Table D.1: Full Sensitivity of Posterior Portfolio Allocations to τ and Ω (%)

τ	Ω Scale	SPY	VGK	EWJ	FXI	INDY	EWZ	IEF	LQD	DBC
0.01	0.10	35.60	0.00	15.16	20.00	15.08	7.89	0.00	6.27	0.00
0.01	0.50	65.00	0.00	13.91	7.53	7.60	5.96	0.00	0.00	0.00
0.01	1.00	65.00	7.68	9.33	3.74	6.65	7.59	0.00	0.00	0.00
0.01	2.00	65.00	24.58	0.00	0.00	2.58	7.84	0.00	0.00	0.00
0.05	0.10	0.00	0.00	0.94	20.00	20.00	13.99	15.07	20.00	10.00
0.05	0.50	35.60	0.00	15.16	20.00	15.08	7.89	0.00	6.27	0.00
0.05	1.00	62.46	0.00	9.74	14.27	8.77	4.76	0.00	0.00	0.00
0.05	2.00	65.00	0.00	13.23	8.82	7.59	5.36	0.00	0.00	0.00
0.10	0.10	0.00	0.00	0.00	20.00	20.00	13.86	16.14	20.00	10.00
0.10	0.50	8.44	0.00	12.33	20.00	19.18	10.05	0.00	20.00	10.00

τ	Ω Scale	SPY	VGK	EWJ	FXI	INDY	EWZ	IEF	LQD	DBC
0.10	1.00	35.60	0.00	15.16	20.00	15.08	7.89	0.00	6.27	0.00
0.10	2.00	62.46	0.00	9.74	14.27	8.77	4.76	0.00	0.00	0.00

Reported values represent posterior portfolio weights under alternative combinations of the prior uncertainty parameter τ and view confidence scaling Ω. All portfolios are subject to long-only and asset-level maximum weight constraints. The table illustrates that allocation changes occur smoothly and remain economically plausible across a wide range of parameter choices.

Appendix E: Sensitivity to Covariance Estimation

Table E.1: Portfolio Performance under Alternative Covariance Estimators

Portfolio	Ret (%)	Vol (%)	ENB	TE
Min_Var_Factor	1.61	8.06	4.58	0.04
MV_BL_Factor	2.64	13.71	2.34	0.04
MaxSharpe_BL_Factor	2.78	15.14	2.45	0.06
Min_VaR_5%_BL_Factor	1.49	8.44	4.66	0.04
Min_TE_Factor	2.03	10.97	2.02	0.00
MV_BL_Reg_Factor	2.03	10.98	2.03	0.00
Min_Var_LedoitWolf	1.61	8.33	4.30	0.04
MV_BL_LedoitWolf	2.65	13.79	2.26	0.04
MaxSharpe_BL_LedoitWolf	2.78	15.05	2.41	0.06
Min_VaR_5%_BL_LedoitWolf	1.67	9.37	5.77	0.06
Min_Var_EWMA	1.64	7.31	5.88	0.05
MV_BL_EWMA	2.78	11.35	3.16	0.05
MaxSharpe_BL_EWMA	2.78	12.57	2.18	0.05
Min_VaR_5%_BL_EWMA	1.65	7.76	5.73	0.06

Table E.2: Portfolio Weights under Alternative Covariance Estimators (%)

Portfolio	SPY	VGK	EWJ	FXI	INDY	EWZ	IEF	LQD	DBC
Min_Var_Factor	19.38	0.00	20.00	5.38	5.24	0.00	20.00	20.00	10.00
MV_BL_Factor	62.46	0.00	9.74	14.27	8.77	4.76	0.00	0.00	0.00
MaxSharpe_BL_Factor	65.00	0.00	0.00	20.00	0.00	15.00	0.00	0.00	0.00
Min_VaR_5%_BL_Factor	0.00	17.04	20.00	3.97	9.00	0.00	20.00	20.00	10.00
Min_TE_Factor	53.20	10.60	4.40	2.00	2.30	0.50	11.50	11.50	4.00
MV_BL_Reg_Factor	53.22	10.54	4.42	2.07	2.34	0.56	11.43	11.46	3.97
Min_Var_LedoitWolf	23.14	0.00	20.00	3.89	2.97	0.00	20.00	20.00	10.00
MV_BL_LedoitWolf	63.95	0.00	9.57	15.13	6.40	4.94	0.00	0.00	0.00

Portfolio	SPY	VGK	EWJ	FXI	INDY	EWZ	IEF	LQD	DBC
MaxSharpe_BL_LedoitWolf	65.00	0.00	0.00	20.00	0.00	15.00	0.00	0.00	0.00
Min_VaR_5%_BL_LedoitWolf	0.00	0.00	20.00	10.66	13.28	6.06	20.00	20.00	10.00
Min_TE_LedoitWolf	53.20	10.60	4.40	2.00	2.30	0.50	11.50	11.50	4.00
MV_BL_Reg_LedoitWolf	53.22	10.54	4.42	2.06	2.33	0.56	11.42	11.46	3.98
Min_Var_EWMA	9.74	0.00	14.62	5.64	20.00	0.00	20.00	20.00	10.00
MV_BL_EWMA	47.12	0.00	0.00	17.88	20.00	15.00	0.00	0.00	0.00
MaxSharpe_BL_EWMA	65.00	0.00	0.00	20.00	0.00	15.00	0.00	0.00	0.00
Min_VaR_5%_BL_EWMA	0.00	0.00	20.00	11.11	14.93	3.96	20.00	20.00	10.00
Min_TE_EWMA	53.20	10.60	4.40	2.00	2.30	0.50	11.50	11.50	4.00
MV_BL_Reg_EWMA	53.21	10.56	4.39	2.07	2.39	0.61	11.36	11.41	4.01

Notes: All portfolios are subject to identical long-only and asset-level maximum weight constraints as defined in Section 5.1. Numerical values close to zero arise from solver tolerances and are reported as 0.00 for clarity.