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GATE AIR - 13

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Linear Algebra Practice - I

Q 1 [MSQ]

Let A be a matrix with m rows and n columns, and consider the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x}$ is a vector of dimensions $n \times 1$. Which of the following statements about $A\mathbf{x}$ and $\mathbf{b}$ are correct?

(A) $A\mathbf{x}$ is a vector of dimension $n \times 1$ and is a linear combination of the columns of A.

(B) $A\mathbf{x}$ is a vector of dimension $m \times 1$ and is a linear combination of the rows of A.

(C) $A\mathbf{x}$ is a vector of dimension $m \times 1$ and is a linear combination of the columns of A.

(D) The system $A\mathbf{x} = \mathbf{b}$ has a solution only if $\mathbf{b}$ can be expressed as a linear combination of the columns of A.

Q 2 [MSQ]

Let A be a matrix, and consider the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} \in \mathbb{R}^3$. Which of the following could be the dimensions of A such that a solution exists for every $\mathbf{b} \in \mathbb{R}^3$?

(A) A is 2×3

(B) A is 3×2

(C) A is 3×3

(D) A is 3×5

Q 3 [MCQ]

Consider the system of linear equations $A\mathbf{x} = \mathbf{b}$, where A is a 13×15 matrix. For the solution to exist for **every** $\mathbf{b}$, which of the following must be true?

(a) In A, all columns must be linearly independent.

(b) In A, there must be exactly 13 columns of A that are independent.

(c) In A, there must be at least 13 columns of A that are independent.

(d) This is not possible.

Q 4 [MCQ]

Consider the system of linear equations $Ax = b$, where A is a 15×13 matrix. For the solution to exist for every b:

(a) 13 rows of A are independent.

(b) 15 rows of A are independent.

(c) At least 13 rows of A are independent.

(d) This is not possible.

Q 5 [MCQ]

Let A and B be two matrices of the same dimensions, each having exactly **one linearly independent column.** What is the maximum possible number of linearly independent columns in the matrix $A + B$?

(A) 1

(B) 2

(C) 3

(D) 4

Q 6 [MSQ]

Consider the system $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} \neq \mathbf{0}$ and $\mathbf{b} = \mathbf{0}$. Which of the following statements about A are true?

a. The columns of A can be linearly independent.

b. The columns of A are necessarily linearly dependent.

c. If Gaussian elimination reduces A to an upper triangular matrix U, and $U\mathbf{x} = \mathbf{c}$, then $\mathbf{c}$ will be a non-zero vector.

d. If A is a square matrix, its rows are necessarily linearly independent.

Q 7 [MSQ]

Consider the matrix A and the equation $A\mathbf{x} = \mathbf{b}$, where:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \\ -2 & -4 \end{bmatrix}$$

Which of the following statements are **correct** regarding the system $A\mathbf{x} = \mathbf{b}$?

(A) The system does not have a solution for any vector $\mathbf{b}$.

(B) For matrix A, all columns are on the same line, and all rows are on the same line.

(C) For a vector $\mathbf{b}$ that is in the opposite direction to the first column of A, a solution exists.

(D) All rows and columns of matrix A lie on the same line.

Q 8 [MSQ]

For a square matrix A, the system $A\mathbf{x} = \mathbf{b}$ is transformed into an upper triangular system $U\mathbf{x} = \mathbf{c}$ by applying a series of row operations to both A and $\mathbf{b}$. Based on this transformation, which of the following statements are correct?

(A) A^{-1} exists if and only if $\mathbf{c}$ can be expressed as a linear combination of the columns of U.

(B) A^{-1} may not exist even if $\mathbf{b}$ can be expressed as a linear combination of the columns of A.

(C) A^{-1} will exist if at least one pivot in U is non-zero.

(D) A^{-1} will exist if no pivot in U is zero.

Q 9 [MSQ]

Which of the following statements about matrices is true?

(a) For an $m \times n$ matrix with n linearly independent columns, it must be true that $m \geq n$.

(b) A square matrix with at least one row of zeros is not invertible.

(c) A square matrix in which one row is a linear combination of other rows is invertible.

(d) If the columns of a square matrix A are linearly independent, then A is invertible.