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GATE AIR - 13

M.Tech in Data Science From IIT Guwahati

Gate DSAI - Manoj Kumar

Linear Algebra Practice - III

Q 1 [MSQ]

Consider the equation $Ax = b$, where A is a real matrix, and select all correct statements based on the following scenarios:

(a) If $Ax = b$ is solvable for every b, A could be a 3×10 matrix with rank 3.

(b) If $Ax = b$ is solvable for every b, A could be a 10×3 matrix with rank 3.

(c) If $Ax = b$ has a unique solution for some b, A could be a 10×3 matrix with rank 3.

(d) If $Ax = b$ has a unique solution for some b, A could be a 3×10 matrix with rank 3.

Q 2 [MSQ]

For a matrix A of size $m \times n$, in which of the following cases is it **possible** for A to have **no free variables**?

(a) $m = n$.

(b) $m > n$.

(c) $n > m$.

(d) When $Ax = b$ has no solution.

Q 3 [MSQ]

Consider a system $Ax = b$, where $x_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ are two solutions. Based on this information, select all the correct statements:

(a) Matrix A has 3 rows.

(b) Matrix A has 3 columns.

(c) The rank of A can be 3.

(d) The rank of A can be 2.

Q 4 [MSQ]

Which of the following solutions for $Ax = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ are possible?

(a) $x = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

(b) $x = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} + \alpha_1 \begin{pmatrix} -1 \\ 5 \\ 17 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, for all $\alpha_1, \alpha_2 \in \mathbb{R}$.

(c) $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, for all $\alpha \in \mathbb{R}$.

(d) $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, for all $\alpha \in \mathbb{R}$.

(e) $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \alpha_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, for all $\alpha_1, \alpha_2 \in \mathbb{R}$.

Q 5 [MSQ]

The nullspace $N(A)$ of the real matrix A is spanned by the vector $v = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$. Based on this information, determine which of the following statements are correct:

(a) Matrix A can have 5 rows.

(b) Matrix A can have 5 columns.

(c) Matrix A has exactly rank 3.

(d) Matrix A has exactly rank 4.

Q 6

Consider the complete solution to $Ax = b$ given by:

$$x = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + d \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix},$$

for arbitrary constants c and d.

Based on this information, determine the answers to the following:

(a) What is the number of columns (n) of matrix A?

(b) What is the rank (r) of matrix A?

(c) What is the nullity of matrix A?

(d) What is the minimum number of rows (m) matrix A must have?

(e) Could A have more rows than columns?

Q 7 [MSQ]

Which of the following statements about matrix A are true?

(a) If column 4 of a 3×5 matrix A is all zeros, then column 4 is certainly a pivot column.

(b) If two rows in a 5×3 matrix A become completely zero after row reduction to row echelon form, then there are no free variables in A.

(c) If column 1 + column 3 + column 5 = 0 in a 4×5 matrix A with four pivot columns, then column 5 does not have a pivot.

(d) The null space of a 5×5 matrix A contains only $x = 0$ if A has exactly 5 pivot columns.

Q 8 [MSQ]

For which matrix size A, the dimension of the null space and the column space cannot be equal?

(a) 2×2

(b) 3×3

(c) 4×4

(d) Not equal for all possible matrix dimensions.

Q 9 [MSQ]

Consider the following system of equations $Ax = b$, where:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

Which of the following statements is/are **not correct**?

(a) A solution always exists for any vector b.

(b) The column space of matrix A is a subspace of $\mathbb{R}^2$.

(c) The null space of matrix A is a subspace of $\mathbb{R}^2$.

(d) There exists a vector b such that the system has infinitely many solutions.

(e) There exists a vector b such that the system has a unique solution.

Q 10 [MSQ]

Consider the system of equations $Ax = b$. Which of the following statements is/are correct?

(a) Any solution to $Ax = b$ is any linear combination of a particular solution x_p and a solution in the null space.

(b) The set of solutions to $Ax = b$ forms a subspace.

(c) If A has full column rank, then there will exist a solution x_n in the null space.

(d) For any square matrix A, if the solution is unique, then A must have a zero determinant.

Q 11 [MSQ]

All solutions to the system $Ax = b$ have the form:

$$x = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ for some scalar } c.$$

Which of the following statements is/are correct?

(a) Matrix A can have full column rank.

(b) The row rank of matrix A is 1.

(c) The vector b lies in the space spanned by the first column of matrix A.

(d) The vector b lies in the space spanned by all columns of matrix A.

(e) Matrix A must have 2 rows.

Q 12 [MSQ]

Consider a square matrix A and the system $Ax = b$. Which of the following statements are correct?

(a) For the same matrix A, there can be two different vectors b such that one results in a unique solution, while the other has no solution.

(b) For the same matrix A, there can be two different vectors b such that one results in a unique solution, while the other leads to infinitely many solutions.

(c) For the same matrix A, there can be two different vectors b such that one results in infinitely many solutions, while the other has no solution.

(d) If the vector b lies in the column space of A, the system $Ax = b$ will always have a unique solution.

Q 13 [MSQ]

Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 5 \\ 1 & 2 & 1 & 1 \end{bmatrix}.$$

(a) Determine the **basis for the null space** $N(A)$ of the given matrix A. Also, find the **rank** and **nullity** of A.

(b) For what value or values (if any) of α, does $Ax = \begin{bmatrix} 1 \\ 2\alpha \\ \alpha \end{bmatrix}$ have any solution x?

Q 14 [MSQ]

Consider the matrix $B = \begin{bmatrix} 1 & 1 & 5 \\ 2 & 0 & 4 \\ 3 & 1 & 9 \end{bmatrix}$. Which of the following options correctly represent the **basis vectors** for the column space of B?

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$

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Which of the following statements is/are TRUE?

Note: $\mathbb{R}$ denotes the set of real numbers.

Options:

(A) There exist $M \in \mathbb{R}^{3\times3}$, $\mathbf{p} \in \mathbb{R}^3$, and $\mathbf{q} \in \mathbb{R}^3$ such that $M\mathbf{x} = \mathbf{p}$ has a unique solution and $M\mathbf{x} = \mathbf{q}$ has infinite solutions.

(B) There exist $M \in \mathbb{R}^{3\times3}$, $\mathbf{p} \in \mathbb{R}^3$, and $\mathbf{q} \in \mathbb{R}^3$ such that $M\mathbf{x} = \mathbf{p}$ has no solutions and $M\mathbf{x} = \mathbf{q}$ has infinite solutions.

(C) There exist $M \in \mathbb{R}^{2\times3}$, $\mathbf{p} \in \mathbb{R}^2$, and $\mathbf{q} \in \mathbb{R}^2$ such that $M\mathbf{x} = \mathbf{p}$ has a unique solution and $M\mathbf{x} = \mathbf{q}$ has infinite solutions.

(D) There exist $M \in \mathbb{R}^{3\times2}$, $\mathbf{p} \in \mathbb{R}^3$, and $\mathbf{q} \in \mathbb{R}^3$ such that $M\mathbf{x} = \mathbf{p}$ has a unique solution and $M\mathbf{x} = \mathbf{q}$ has no solutions.

GATE CE 2020 [2 Marks]

[NAT]

Consider the system of equations:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

The value of x_3 (round off to the nearest integer) is ______.