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GATE AIR - 13

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Linear Algebra Practice - IV

Q 1

For a matrix A and a vector b, any solution x to the equation $Ax = b$ (if it exists) can always be expressed as:

(a) A sum of a vector in the null space of A and a vector in the column space of A.

(b) A sum of a vector in the null space of A and a vector in the row space of A.

(c) A sum of a vector in the column space of A^T and a vector in the column space of A.

(d) A sum of a vector in the null space of A and a vector in the null space of A^T.

Q 2

The system $Ax = b$ is solvable if b is orthogonal to:

(a) The null space of A.

(b) The row space of A.

(c) The left null space of A.

(d) The column space of A.

Q 3

If A is a 4×3 matrix and the system $Ax = b$ is not solvable for some b, and the solutions are not unique when they exist, how many possible values can the rank of A take?

Q 4 [MSQ]

Let A and B be 4×4 matrices. Which of the following statements about the column space $C(AB)$ is always true?

(a) $C(AB) \subseteq C(A)$

(b) $C(AB) \supseteq C(B)$

(c) $C(AB) = C(A)$ if B is invertible

(d) $C(AB) \supseteq C(A)$

Q 5

If x_1 and x_2 are two solutions to $Ax = b$, $x_1 - x_2$ lies in:

(a) The column space of A

(b) The row space of A

(c) The null space of A

(d) The left null space of A

Q 6 [MSQ]

For a matrix A of size $m \times n$, suppose the system $Ax = b$ has a solution for every b. Which of the following statements are true?

(a) The column space of A is the whole space $\mathbb{R}^n$.

(b) The column space of A is the whole space $\mathbb{R}^m$.

(c) When A is reduced to row echelon form R, there is no zero row in R.

(d) A is full column rank.

(e) A is full row rank.

Q 7

For a matrix A of size $m \times n$, suppose the system $Ax = b$ has a unique solution for every b. What is the possible relationship between m and n?

(a) $m \geq n$

(b) $m \leq n$

(c) $m > n$

(d) $m < n$

(e) $m = n$

Q 8 [MSQ]

For a matrix A of size $m \times n$, suppose the system $Ax = b$ has a unique solution whenever it exists.

Which of the following statements are true?

(a) There is at least one free variable.

(b) The null space contains only the zero vector.

(c) Matrix A is full row rank.

(d) The row space is the whole of $\mathbb{R}^n$.

(e) The column space is the whole of $\mathbb{R}^m$.

Q 9 [MSQ]

Which of the following statements are true?

(a) In $\mathbb{R}^2$, any three vectors are linearly dependent.

(b) Any set of n vectors in $\mathbb{R}^m$ must be linearly dependent if $n > m$.

(c) The columns of every invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

(d) The number of vectors in every basis is equal to the dimension of the space.

Q 10 [MSQ]

After performing row operations (without row swapping), a matrix A is reduced to the row echelon form R given as:

$$R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following statements are true?

(a) Column 1 and column 2 of R are the basis vectors for the column space of A.

(b) Column 2 and column 3 of A are the basis vectors for the column space of A.

(c) Column 1 and column 3 of R are the basis vectors for the column space of A.

(d) Row 1 and row 2 of R are the basis vectors for the row space of A.

(e) Row 1 and row 2 of A are the basis vectors for the row space of A.

Q 11 [MSQ]

Which of the following statements is true?

(a) All bases for a vector space contain the same number of vectors.

(b) The null space of the identity matrix has dimension one.

(c) The column space of a 3×3 identity matrix has dimension 3.

(d) If the zero vector is in the row space of a matrix, the rows are linearly dependent.

Q 12

Describe the subspace of $\mathbb{R}^3$ (is it a line, a plane, or $\mathbb{R}^3$?) spanned by the following sets of vectors:

(a) The two vectors $(1, 1, -1)$ and $(-1, -1, 1)$.

(b) The three vectors $(0, 1, 1)$, $(1, 1, 0)$, and $(0, 0, 0)$.

(c) All vectors in $\mathbb{R}^3$ with whole number components.

(d) All vectors in $\mathbb{R}^3$ with positive components.

Q 13

Suppose $v_1, v_2, \ldots, v_6$ are six vectors in $\mathbb{R}^4$.

(a) Those vectors (do) (do not) (might not) span $\mathbb{R}^4$.

(b) Those vectors (are) (are not) (might be) linearly independent.

(c) Any four of those vectors (are) (are not) (might be) a basis for $\mathbb{R}^4$.

Q 14

Suppose A is a 5×4 matrix with rank 4. If the augmented matrix $[A\ b]$ is invertible, what is the nature of the solution to the system $Ax = b$?

(a) A unique solution

(b) No solution

(c) Infinitely many solutions

(d) Two solutions

Q 15 [MSQ]

Suppose a matrix A is converted to its row echelon form R after some row operations. Which of the following statements are true?

(a) The row space has a dimension equal to the number of nonzero rows in R.

(b) The column space has a dimension equal to the number of nonzero rows in R.

(c) The null space has a dimension equal to the number of zero rows in R.

(d) The left null space has a dimension equal to the number of zero rows in R.

Q 16

If a 3×4 matrix has rank 3, what are the dimensions of its column space, row space, null space, and left null space?

Q 17

Find the dimensions of the four subspaces associated with the matrices:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}.$$

Q 18 [MSQ]

Which of the following statements is **impossible**?

a) The column space has basis $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and the null space has basis $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$.

b) The dimension of the null space equals 1 plus the dimension of the left null space.

c) The row space equals the column space, while the null space is not equal to the left null space.

Q 19,20

For the equation $A^T y = d$:

1. d is in which subspace of A for the system to be solvable?
2. The solution y is unique when which subspace of A contains only the zero vector?

Options:

(a) Row space of A

(b) Column space of A

(c) Null space of A

(d) Left null space of A

Q 21

If the first two rows of a matrix A are exchanged, which of the following subspaces of A remain the same?

(a) Row space of A

(b) Column space of A

(c) Null space of A

(d) Left null space of A

Q 22,23

Suppose A is the sum of two matrices of rank one: $A = uv^T + wz^T$.

1. Which vectors determine the **column space** of A?

   (a) u and v

   (b) u and w

   (c) v and z

   (d) w and z

2. Which vectors determine the **row space** of A?

   (a) u and v

   (b) u and w

   (c) v and z

   (d) w and z

Q 24,25

If $AB = 0$, where A and B are matrices, which of the following statements is/are correct?

1. The **columns of** B are in the:

   a) Column space of A

   b) Nullspace of A

   c) Row space of A

   d) Left nullspace of A

2. The **rows of** A are in the:

   a) Column space of B

   b) Nullspace of B

   c) Row space of B

   d) Left nullspace of B

Q 26 [MSQ]

If $Ax = b$ has a solution and $A^\top y = 0$, which of the following statements is true?

a) $y^\top b = 0$

b) $y^\top x = 0$

c) Both $y^\top b = 0$ and $y^\top x = 0$ are true

d) Neither $y^\top b = 0$ nor $y^\top x = 0$ is true

Q 27 [MSQ]

Given an $n \times n$ matrix A, where a_{ij} represents the element at the i-th row and j-th column of A, and $a_{ij} = i^2 + j^2$, which of the following statements are correct?

a) The column space of A is perpendicular to its null space.

b) The column space of A is perpendicular to its left null space.

c) The row space of A is perpendicular to its null space.

d) The row space of A is perpendicular to the null space of $A^\top$.

Q 28 [MSQ]

Given five vectors in $\mathbb{R}^7$, which of the following statements is/are correct regarding finding a basis for the space they span?

a) Place the vectors as **columns** in matrix A, convert A to its reduced row echelon form (RREF). The **pivot columns** of R form a basis for the space they span.

b) Place the vectors as **rows** in matrix A, convert A to its reduced row echelon form (RREF). The **nonzero rows** in R form a basis for the space they span.

c) Place the vectors as **rows** in matrix A, convert A to its reduced row echelon form (RREF). The **pivot columns** of R form a basis for the space they span.

d) Place the vectors as **rows** in matrix A, convert A to its reduced row echelon form (RREF). The rows in A corresponding to the **nonzero rows** in R form a basis for the space they span, assuming no row swaps are performed.

e) Place the vectors as **columns** in matrix A, convert A to its reduced row echelon form (RREF). The corresponding columns of A (corresponding to the pivot columns of R) form a basis for the space they span.

Q 29 [MSQ]

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 1 & 1 \\ 0 & 1 & 1 & 3 \end{pmatrix}.$$

Which of the following statements is/are correct?

a) $A\mathbf{x} = \mathbf{b}$ will not have a unique solution if it exists.

b) $A^\top \mathbf{y} = \mathbf{c}$ will not have a unique solution if it exists.

c) $A\mathbf{x} = \mathbf{b}$ may not have a solution.

d) $A^\top \mathbf{y} = \mathbf{c}$ may not have a solution.

Q 30

Suppose A is a 3×5 matrix, and the equation $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$ in $\mathbb{R}^3$. Answer the following questions:

a) What is the **column space** of A?

b) What is the **nullspace** of A, and what is its dimension?

c) What is the **rank** of A?

Q 31

Suppose S is a six-dimensional subspace of a nine-dimensional space $\mathbb{R}^9$. Which of the following statements are true?

(a) What are the possible dimensions of subspaces orthogonal to S?

(b) What are the possible dimensions of the orthogonal complement of S?

(c) What is the smallest possible size of a matrix A that has row space S?

(d) What is the smallest possible size of a matrix B that has nullspace $S^{\perp}$?

Q 32

Suppose you have a matrix $A = C^{-1}B$, where

$$B = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 2 & 2 \\ 4 & 2 & 2 \end{pmatrix}.$$

Compute the **first column** of A^{-1}.