Manoj Kumar

GATE AIR - 13

M.Tech in Data Science From IIT Guwahati

Gate DSAI - Manoj Kumar

Linear Algebra Practice - II

Q 1 [MSQ]

Which of the following statements is true?

(a) Every subspace contains the zero vector.

(b) A subspace containing two vectors $\mathbf{v}$ and $\mathbf{w}$ may not contain all linear combinations of $\mathbf{v}$ and $\mathbf{w}$.

(c) The system $A\mathbf{x} = \mathbf{b}$ is solvable, and if we add $\mathbf{b}$ as an extra column to A, the column space of A does not get larger.

(d) The system $A\mathbf{x} = \mathbf{b}$ is solvable, and if we add $\mathbf{b}$ as an extra column to A, the column space of A becomes larger.

Q 2 [MSQ]

Which of the following subsets of $\mathbb{R}^3$ are subspaces?

(a) The set of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that $b_1 + 2b_2 + 3 = 0$.

(b) The set of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that $b_1 + b_2 + b_3 = 0$.

(c) The set of all vectors that are linear combinations of $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$.

(d) The set of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that $b_1 \leq b_3$.

(e) The set of vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ such that $b_1 b_2 b_3 = 0$.

Q 3 [MSQ]

Which of the following sets of vectors are vector subspaces of $\mathbb{R}^3$?

(a) All vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T$ such that $10x + y + 2014z = 0$.

(b) All vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T$ such that $x + y + z \leq 2014$.

(c) All vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T$ such that $x + y + z = 0$ and $x + 2y + 3z = 0$.

(d) All vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T$ such that $x + y + z = 0$ or $x + 2y + 3z = 0$.

(e) All vectors $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}^T$ such that $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Q 4 [MSQ]

Let

$$A = \begin{bmatrix} 0 & 0 \\ 6 & 9 \\ 2 & 3 \end{bmatrix}.$$

Which of the following statements are correct?

(A) The column space of A is a point.

(B) The column space of A is a line.

(C) The row space of A is a vector subspace of $\mathbb{R}^3$.

(D) The row space of A is a vector subspace of $\mathbb{R}^2$.

Q 5 [MSQ]

Which of the following statements about a matrix A and a vector $\mathbf{b}$ in $\mathbb{R}^n$ are true?

(a) The vectors $\mathbf{b}$ that are not in the column space $C(A)$ form a subspace.

(b) If $C(A)$ contains only the zero vector, then A is the zero matrix.

(c) The column space of $2A$ equals the column space of A.

(d) The column space of A^T is the set of all linear combinations of the columns of A.

Q 6 [MSQ]

Let A, B, and C be matrices such that $AB = C$. Consider the following statements:

(A) If the columns of B are dependent, the columns of C can be linearly independent.

(B) If A is 5×3 and B is 3×5, then $AB = I$ (where I is the identity matrix) is impossible.

(C) For any two square matrices A and B, if $AB = I$, then $BA = I$.

(D) For A and B to satisfy $AB = I$, both A and B must be square matrices.

Q 7

1. Use Gaussian elimination to transform the given matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ -1 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

into its row echelon form.

Q 8

1. Use Gaussian elimination to transform the given matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 \\ -1 & 2 & -2 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

into its row echelon form.

2. Using the result from part (1), write $A = LU$, where L is a lower triangular matrix and U is an upper triangular matrix.

Q 9 [MSQ]

Consider the matrix

$$A = \begin{pmatrix} 2 & 3 & 5 \\ 2 & 4 & 5 \\ -2 & 0 & -5 \end{pmatrix}$$

and the general right-hand side

$$b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

a) Factor the 3×3 matrix A into LU, where L is lower triangular, and U is upper triangular.

b) Describe the column space of A exactly through a condition on b.

Q 10

Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & 7 & 10 \end{pmatrix}.$$

a) Find the $A = LU$ factorization of the matrix A, where L is lower triangular and U is upper triangular.

b) Solve the system $Ax = \begin{pmatrix} 3 \\ 10 \\ 20 \end{pmatrix}$.

Q 11 [MSQ]

Which of the following statements about an $n \times n$ matrix A are correct regarding the invertibility of A?

(a) A is invertible if and only if elimination produces n pivots.

(b) A is invertible if there is a non-trivial solution $\mathbf{x}$ to $A\mathbf{x} = 0$.

(c) If A is invertible, then elimination on A can proceed to completion without requiring row permutations.

(d) If A is invertible, then A^T (the transpose of A) is also invertible.

Q 12 [MSQ]

Which of the following permutation matrices P, when multiplied by a vector $\mathbf{X}$ of size 4×1 as $P\mathbf{X}$, reverses the order of the vector?

(a) P is formed by swapping row 1 with row 2 and row 3 with row 4 of the identity matrix.

(b) P is formed by swapping row 1 with row 3 and row 2 with row 4 of the identity matrix.

(c) P is formed by swapping row 1 with row 4 and row 2 with row 3 of the identity matrix.

(d) P is formed by swapping row 1 with row 3 while keeping the other rows unchanged in the identity matrix.

Q 13 [MSQ]

Which of the following statements are true?

(a) The column space of A and the column space of A^T (the transpose of A) are always identical.

(b) The dimensions of the column space of A and the column space of A^T are always equal.

(c) If $A^T = -A$ (i.e., A is skew-symmetric), then the row space of A equals the column space of A.

(d) The column space of the zero matrix is a line.

Similar Gate Problems

GATE DA 2024 - 2M

Select all choices that are subspaces of $\mathbb{R}^3$.

Note: $\mathbb{R}$ denotes the set of real numbers.

(A)

$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$$

(B)

$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : \mathbf{x} = \alpha^2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$$

(C)

$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 = 0,\ 4x_1 - 2x_2 + 3x_3 = 0 \right\}$$

(D)

$$\left\{ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 : 5x_1 + 2x_3 + 4 = 0 \right\}$$

[MCQ] [GATE-CS-2022: 2 Marks]

Consider solving the following system of simultaneous equations using LU decomposition:

$$x_1 + x_2 - 2x_3 = 4$$

$$x_1 + 3x_2 - x_3 = 7$$

$$2x_1 + x_2 - 5x_3 = 7$$

The matrices L and U are denoted as:

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}.$$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

Options: (a) $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$

(b) $L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$

(c) $L_{32} = 2, U_{33} = 2, x_1 = -1$

(d) $L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$

[NAT] [GATE-CS-2015: 1 Mark]

In the LU decomposition of the matrix

$$A = \begin{pmatrix} 2 & 2 \\ 4 & 9 \end{pmatrix}$$

if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is _____.

[MCQ] [GATE-EE-2011: 2 Marks]

The matrix

$$A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

is decomposed into a product of a lower triangular matrix $[L]$ and an upper triangular matrix $[U]$. The properly decomposed $[L]$ and $[U]$ matrices respectively are:

(a)

$$L = \begin{pmatrix} 1 & 0 \\ 4 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$$

(b)

$$L = \begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(c)

$$L = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$$

(d)

$$L = \begin{pmatrix} 2 & 0 \\ 4 & -3 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix}$$