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GATE AIR - 13

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Expertise in Machine Learning, Deep Learning, Artificial Intelligence, Probability and Statistics

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Problems of Week 1 (September)

Note : These Problems of the Day (POTD) will be the cornerstone of your entire GATE Data Science preparation. Keep in mind, that the goal is to focus on the learning experience these problems offer, rather than just finding the answers.

Start engaging with them now, and you'll notice a significant difference in your exam—trust me.

Happy learning!

POTD #22

A certain component is critical to the operation of an electrical system and must be replaced immediately upon failure. The mean lifetime of this type of component is 100 hours, with a standard deviation of 30 hours. The system requires continual operation for the next 2000 hours. How many of these components must be in stock to ensure that the probability of the system remaining in continual operation for the next 2000 hours is at least 90%? Assume that the lifetime of the components follows a normal distribution, and use the given Z-values: $Z(0.90) = 1.28$ and $Z(0.01) = -2.33$.

POTD #23

The time it takes a central processing unit (CPU) to process a certain type of job is normally distributed. A term $\hat{\sigma}_1^2$ is defined as:

$$\hat{\sigma}_1^2 = \frac{1}{15}\sum_{n=1}^{15} x_n^2$$

where x_n represents the observed processing times for each job in a sample of 15 jobs. The expected value of $\hat{\sigma}_1^2$ is known to be $\frac{42}{5}$.

What is the probability that the variance of this sample will exceed 12?

POTD #24

Suppose we are attempting to locate a target in three-dimensional space, and the three coordinate errors (in meters) of the point chosen are independent normal random variables with mean 0 and standard deviation 2. The probability that the distance between the point chosen and the target exceeds 3 meters can be expressed as $P\left(\chi_3^2 > x\right)$, where χ_3^2 represents a chi-square random variable with 3 degrees of freedom. What is the value of x?

POTD #25

For a square matrix A, the system $Ax = b$ is transformed into an upper triangular system $Ux = c$ by applying a series of row operations to both A and b. Based on this transformation, which of the following statements are correct?

1. A^{-1} exists if and only if c is in the column space of U.
2. A^{-1} may not exist even if b is in the column space of A.
3. A^{-1} will exist if at least one pivot in U is non-zero.
4. The column space of A and the column space of A^T are always identical.
5. The dimensions of the column space of A and the column space of A^T are always equal.

POTD #26

Consider a square matrix A and the system $Ax = b$. Which of the following statements are correct?

1. For the same matrix A, there can be two different vectors b such that one results in a unique solution, while the other has no solution.
2. For the same matrix A, there can be two different vectors b such that one results in a unique solution, while the other leads to infinitely many solutions.
3. For the same matrix A, there can be two different vectors b such that one results in infinitely many solutions, while the other has no solution.
4. If the vector b lies in the column space of A, the system $Ax = b$ will always have a unique solution.

POTD #27

Given the following dataset with two features X_1 and X_2, and a corresponding label Y:

X_1	X_2	Y
2	3	1
1	2	0
3	1	1

Apply the gradient descent algorithm to minimize the Mean Squared Error (MSE), starting with the initial weight vector $\mathbf{w} = [0, 0]$. The prediction for each instance is given by $\hat{y}_i = \mathbf{w}^T \mathbf{x}_i$, where $\mathbf{w}^T$ is the transpose of the weight vector $\mathbf{w}$ and $\mathbf{x}_i = [x_{i1}, x_{i2}]$ is the input vector for the i -th data point.

If a learning rate of 0.5 is used, what is the first update to the weight vector $\mathbf{w}$ after the first iteration of the gradient descent algorithm?

Book Problems to Practice

Sam will read either one chapter of his probability book or one chapter of his history book. If the number of misprints in a chapter of his probability book is Poisson distributed with mean 2 and if the number of misprints in his history chapter is Poisson distributed with mean 5, then assuming Sam is equally likely to choose either book, what is the expected number of misprints that Sam will come across?

Suppose that $p(x, y)$, the joint probability mass function of X and Y, is given by

$$p(1,1) = 0.5, \quad p(1,2) = 0.1, \quad p(2,1) = 0.1, \quad p(2,2) = 0.3$$

Calculate the conditional probability mass function of X given that $Y = 1$.

Book Problems to Practice

The joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{2} y e^{-xy}, & 0 < x < \infty,\ 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

What is $E[e^{X/2} \mid Y = 1]$?

A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety?

Book Problems to Practice

The average salary of newly graduated students with bachelor's degrees in chemical engineering is \$53,600, with a standard deviation of \$3,200. Approximate the probability that the average salary of a sample of 12 recently graduated chemical engineers exceeds \$55,000.

When we attempt to locate a target in two-dimensional space, suppose that the coordinate errors are independent normal random variables with mean 0 and standard deviation 2. Find the probability that the distance between the point chosen and the target exceeds 3.

If X and Y are independent Poisson random variables with respective means λ_1 and λ_2, calculate the conditional expected value of X given that $X + Y = n$.

Book Problems to Practice

There are n components. On a rainy day, component i will function with probability p_i; on a nonrainy day, component i will function with probability q_i, for $i = 1, \ldots, n$. It will rain tomorrow with probability α. Calculate the conditional expected number of components that function tomorrow, given that it rains.

Suppose the joint density of X and Y is given by

$$f(x, y) = \begin{cases} 4y(x-y)e^{-(x+y)}, & 0 < x < \infty,\ 0 \le y \le x \\ 0, & \text{otherwise} \end{cases}$$

Compute $E[X \mid Y = y]$.

Book Problems to Practice

The following table uses data concerning the percentages of teenage male and female full-time workers whose annual salaries fall in different salary groupings. Suppose random samples of 1,000 men and 1,000 women were chosen. Use the table to approximate the probability that

(a) at least half of the women earned less than $20,000

(b) more than half of the men earned $20,000 or more

(c) more than half of the women and more than half of the men earned $20,000 or more

(d) 250 or fewer of the women earned at least $25,000

(e) at least 200 of the men earned $50,000 or more

(f) more women than men earned between $20,000 and $24,999

Earnings Range	Percentage of Women	Percentage of Men
$4,999 or less	2.8	1.8
$5,000 to $9,999	10.4	4.7
$10,000 to $19,999	41.0	23.1
$20,000 to $24,999	16.5	13.4
$25,000 to $49,999	26.3	42.1
$50,000 and over	3.0	14.9

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