

# Manoj Kumar

GATE AIR - 13

M.Tech in Data Science From IIT Guwahati

Expertise in Machine Learning, Deep Learning, Artificial Intelligence, Probability and Statistics



Telegram: @Manoj\_Gate\_DSAI



**CLICK**



Unacademy



# Problems of Week 3 (August)

**Note** : These Problems of the Day (POTD) will be the cornerstone of your entire GATE Data Science preparation. Keep in mind, that the goal is to focus on the learning experience these problems offer, rather than just finding the answers.

**Start engaging with them now, and you'll notice a significant difference in your exam—trust me.**

Happy learning!

## POTD #10

In August 2013, a poll reported that 52% of respondents approved of President Obama's job performance, with a margin of error of  $\pm 4\%$  at a 95% confidence level.

Estimate the minimum number of people surveyed to achieve this margin of error.

- At  $Z = -0.025$ , the probability is 0.49
- At  $Z = 0.025$ , the probability is 0.51
- At  $Z = -0.05$ , the probability is 0.48
- At  $Z = 0.05$ , the probability is 0.52
- At  $Z = 0.95$ , the probability is 0.8289
- At  $Z = 1.96$ , the probability is 0.975
- At  $Z = -1.96$ , the probability is 0.025

## POTD #11

You are conducting a hypothesis test to check if the mean of a signal,  $\mu$ , is different from the expected value of 8. The standard deviation of the signal is 2, and you plan to perform a two-tailed test at a significance level of  $\alpha = 0.05$ .

You suspect the true mean might actually be 9.2, and for this value, the rejection of the null hypothesis  $H_0 : \mu = 8$  will occur only due to signal values greater than 8. To ensure your test has a 75% probability of rejecting the null hypothesis when the true mean is 9.2, you need to determine the minimum sample size.

- At  $Z = -1.96$ , the probability is 0.025
- At  $Z = -1.645$ , the probability is 0.05
- At  $Z = -0.674$ , the probability is 0.25
- At  $Z = 0$ , the probability is 0.5
- At  $Z = 0.674$ , the probability is 0.75
- At  $Z = 1.645$ , the probability is 0.95
- At  $Z = 1.96$ , the probability is 0.975

## POTD #12

A researcher is conducting a hypothesis test on the mean of a population with a known population standard deviation  $\sigma = 5$ . The null hypothesis states that the population mean is  $\mu = 50$ . The researcher performs a two-tailed test at a significance level of  $\alpha = 0.05$ , using a sample of size  $n = 100$ . The observed sample mean is  $\bar{X} = 52$ .

Which of the following statements are **true**? (Select all that apply)

- A) Changing the significance level from  $\alpha = 0.05$  to  $\alpha = 0.01$  will reduce the probability of making a Type 2 error.
- B) A p-value of 0.03 is enough evidence to reject the null hypothesis at the  $\alpha = 0.05$  significance level.
- C) If the actual population mean is 51, the probability of making a Type 2 error is zero.
- D) Decreasing the sample size might lead to a higher p-value.
- E) Decreasing the sample size may lead to the rejection of the null hypothesis.

### POTD #13

Consider that  $X$  and  $Y$  are independent continuous random variables with uniform probability distributions. The random variable  $X$  is uniformly distributed between 2 and 3, and  $Y$  is uniformly distributed between 1 and 4.

Determine the probability that, for every possible value of  $X$ , the corresponding value of  $Y$  is less than or equal to the square of  $X$ .

## POTD #14

Consider a trained logistic regression model with a weight vector  $W$  and a test accuracy of  $A$  on a given dataset. Assume there is no bias term in the model.

What will happen if the weight vector  $W$  is divided by 2?

- A) The probability of predicting each class will reduce, and the test accuracy will decrease.
- B) The probability of predicting each class will remain the same, and the test accuracy will remain the same.
- C) The probability of predicting each class will reduce, but the test accuracy will remain the same.
- D) The probability of predicting each class will remain the same, but the test accuracy will decrease.

## POTD #15

Suppose that a bivariate random vector  $(X, Y)$  has the following probability density function:

$$f(x, y) = e^{-x-y}, \quad x > 0, y > 0$$

$$f(x, y) = 0 \text{ otherwise.}$$

Select the correct statements for the random variables  $X$  and  $Y$ :

1.  $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$
2.  $\text{Var}(X - Y) < \text{Var}(X) + \text{Var}(Y)$
3.  $\text{Var}(X - Y) > \text{Var}(X) + \text{Var}(Y)$
4.  $E(X - Y) = E(X) + E(Y)$



## Similar Gate Problems

If  $X$  is a random variable with **uniform probability density function** in the interval  $[-2, 10]$ . For  $Y = 2X - 6$ , the conditional probability  $P(Y \leq 7 \mid X \geq 5)$  (rounded off to 3 decimal places) is \_\_\_\_.

POTD by Manoj Kumar

## Similar Gate Problems

Consider that  $X$  and  $Y$  are independent continuous valued random variables with uniform probability density functions given by  $X \sim U[2, 3]$  and  $Y \sim U[1, 4]$ . Then  $P(Y \leq X)$  is equal to \_\_\_\_\_ (rounded off to 2 decimal places).

POTD by Manoj Kumar

# Free answers and solutions link-

<https://unacademy.com/class/potd-week-3-august/N8L8IP1S>

Discuss your Doubts here - <https://t.me/ManojGateDA>