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GATE AIR - 13

M.Tech in Data Science From IIT Guwahati

Expertise in Machine Learning, Deep Learning, Artificial Intelligence, Probability and Statistics

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Problems of Week 3 (September)

Note : These Problems of the Day (POTD) will be the cornerstone of your entire GATE Data Science preparation. Keep in mind, that the goal is to focus on the learning experience these problems offer, rather than just finding the answers.

Start engaging with them now, and you'll notice a significant difference in your exam—trust me.

Happy learning!

POTD #34

Suppose that a random vector (X, Y) has a uniform probability density over a circle of radius 4 centered at the origin $(0, 0)$. Let Z represent the distance of the point (X, Y) from the center of the circle. What is the probability that Z lies between 1 and 2?

POTD #35

Given the bivariate random vector (X, Y) with the joint probability density function:

$$f_{X,Y}(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise,} \end{cases}$$

Find the probability $P\left(0 < X < \frac{1}{2} \mid Y = \frac{3}{4}\right)$.

POTD #36

Suppose that a bivariate random vector (X, Y) has the joint probability density function:

$$f(x,y) = \begin{cases} 1 & \text{if } -y < x < y \text{ and } 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Based on this, which of the following statements is true for the random variables X and Y?

1. Correlated and dependent
2. Correlated and independent
3. Uncorrelated and independent
4. Uncorrelated and dependent

POTD #37

Given five vectors in $\mathbb{R}^7$, which of the following statements is/are correct regarding finding a basis for the space they span?

1. Place the vectors as columns in matrix A, convert A to its reduced row echelon form (RREF). The pivot columns of R form a basis for the space they span.
2. Place the vectors as rows in matrix A, convert A to its reduced row echelon form (RREF). The nonzero rows in R form a basis for the space they span.
3. Place the vectors as rows in matrix A, convert A to its reduced row echelon form (RREF). The pivot columns of R form a basis for the space they span.
4. Place the vectors as rows in matrix A, convert A to its reduced row echelon form (RREF). The rows in A corresponding to the nonzero rows in R form a basis for the space they span, assuming no row swaps are performed.
5. Place the vectors as columns in matrix A, convert A to its reduced row echelon form (RREF). The corresponding columns of A (corresponding to the pivot columns of R) form a basis for the space they span.

POTD #38

Which of the following statements are correct? (Multiple correct answers)

1. A and A^T have the same number of pivots.
2. A and A^T have the same left null space.
3. If the row space equals the column space, then $A^T = A$.
4. If $A^T = -A$, then the row space of A equals the column space.
5. If matrices A and B share the same four fundamental subspaces, then A is a multiple of B.

POTD #39

Consider a linear regression model of the form:

$$y = w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_p x_p$$

where y is the prediction, and $\mathbf{w} = [w_0, w_1, w_2, \ldots, w_p]$ is the parameter vector, with each w_i corresponding to the independent variables $x_1, x_2, \ldots, x_p$. Now, all the independent variables $x_1, x_2, \ldots, x_p$ for each data point are multiplied by 5.

How will the new prediction y_{new} change when linear regression is performed on this modified dataset?

Options:

A. y (No change in prediction)

B. $5y$

C. $y/5$

D. None of the above

Similar Problems

Q.36 Consider the following joint distribution of random variables X and Y:

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 \le x \le 2, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Which one or more of the following statements is/are correct?

(A) X and Y are mutually uncorrelated.

(B) X and Y are mutually independent.

(C) The mean of X is 1.

(D) The mean of Y is 0.5.

Similar Problems

The system of linear equations in real (x, y) given by

$$(xy)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0\ 0)$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

Options:

- (a) $x = 2, y = -2$
- (b) $x = -1, y = 4$
- (c) $x = 1, y = 1$
- (d) $x = 4, y = -2$

Similar Problems

Consider a system of linear equations $Ax = b$, where

$$\begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This system of equations admits ____.

Options:

- (a) a unique solution for x
- (b) infinitely many solutions for x
- (c) no solutions for x
- (d) exactly two solutions for x

Similar Problems

A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $Ax = 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

Options:

- (a) The given set of equations will have a unique solution.
- (b) The given set of equations will be satisfied by a zero vector of appropriate size.
- (c) The given set of equations will have infinitely many solutions.
- (d) The given set of equations will have many but a finite number of solutions.

Similar Problems

Consider the following system of linear equations.

$$x_1 + 2x_2 = b_1;$$

$$2x_1 + 4x_2 = b_2;$$

$$3x_1 + 7x_2 = b_3;$$

$$3x_1 + 9x_2 = b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

Options:

- (a) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
- (b) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$
- (c) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$
- (d) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

Similar Problems

The set of equations

$$x + y + z = 1$$

$$ax - ay + 3z = 5$$

$$5x - 3y + az = 6$$

has infinite solutions if $a =$

Options:

- (a) -3
- (b) 3
- (c) 4
- (d) -4

Similar Problems

Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation $Ax = 0$ has infinitely many solutions is ____.

Similar Problems

Let $c_1, \ldots, c_n$ be scalars, not all zero, such that

$$\sum_{i=1}^{n} a_i x_i = 0$$

where a_i are column vectors in $\mathbb{R}^n$.

Consider the set of linear equations $Ax = b$, where $A = [a_1, \ldots, a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has:

Options:

- (a) a unique solution at $x = J_n$, where J_n denotes an n-dimensional vector of all 1s.
- (b) no solution.
- (c) infinitely many solutions.
- (d) finitely many solutions.

Similar Problems

Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution.
II. If $m > n$, then none of these systems has a solution.
III. If $m = n$, then there exists a system which has a solution.

Which one of the following is **CORRECT**?

Options:

- (a) I, II, and III are true
- (b) Only II and III are true
- (c) Only III is true
- (d) None of them is true

Similar Problems

Consider the following linear system:

$$x + 2y - 3z = a$$

$$2x + 3y + 3z = b$$

$$5x + 9y - 6z = c$$

This system is consistent if a, b, and c satisfy the equation:

Options:

- (a) $7a - b - c = 0$
- (b) $3a + b - c = 0$
- (c) $3a - b + c = 0$
- (d) $7a - b + c = 0$

Similar Problems

Consider a system of linear equations:

$$x - 2y + 3z = -1$$

$$x - 3y + 4z = 1$$

$$-2x + 4y - 6z = k$$

The value of k for which the system has infinitely many solutions is ______.

Similar Problems

For what value of P the following set of equations will have no solutions:

$$2x + 3y = 5$$

$$3x + Py = 10$$

Similar Problems

The system of linear equations:

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix}$$

has:

Options:

- (a) a unique solution
- (b) infinitely many solutions
- (c) no solution
- (d) exactly two solutions

Similar Problems

Given a system of equations:

$$x + 2y + 2z = b_1$$

$$5x + y + 3z = b_2$$

Which of the following is true regarding its solutions:

Options:

- (a) The system has a unique solution for any given b_1 and b_2
- (b) The system will have infinitely many solutions for any given b_1 and b_2
- (c) Whether or not a solution exists depends on the given b_1 and b_2
- (d) The system would have no solution for any value of b_1 and b_2

Similar Problems

Consider the following system of equations:

$$2x_1 + x_2 + x_3 = 0$$

$$x_2 - x_3 = 0$$

$$x_1 + x_2 = 0$$

The system has:

Options:

- (a) A unique solution
- (b) No solution
- (c) Infinite number of solutions
- (d) Five solutions

Similar Problems

You are given a matrix A of dimensions 5×7 where each column represents a vector in $\mathbb{R}^5$. Without fully computing the reduced row echelon form (RREF), can you determine how many vectors are required to form a basis for the space spanned by the columns of A?

A) At least 3, since the matrix is rectangular

B) Exactly 5, as the matrix has 5 rows

C) The number of pivot columns in the matrix A

D) Exactly 7, as the matrix has 7 columns

Similar Problems

If $A = uv^T$ is a 2 by 2 matrix of rank 1, redraw Figure 3.5 to show clearly the Four Fundamental Subspaces. If B produces those same four subspaces, what is the exact relation of B to A?

If a, b, c are given with $a \neq 0$, how would you choose d so that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has rank 1? Find a basis for the row space and null space. Show they are perpendicular!

$A^T y = d$ is solvable when d is in which of the four subspaces? The solution y is unique when the ____ contains only the zero vector.

Similar Problems

True or false (with a reason or a counterexample):

(a) A and A^T have the same number of pivots.

(b) A and A^T have the same left nullspace.

(c) If the row space equals the column space then $A^T = A$.

(d) If $A^T = -A$, then the row space of A equals the column space.

True or false (with a reason or a counterexample):

(a) If $m = n$ then the row space of A equals the column space.

(b) The matrices A and $-A$ share the same four subspaces.

(c) If A and B share the same four subspaces then A is a multiple of B.

Free Solutions Link-

https://unacademy.com/class/potd-week-3-september/BG3BJQH6

Discuss your Doubts here - https://t.me/ManojGateDA