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GATE AIR - 13

M.Tech in Data Science From IIT Guwahati

**Expertise in Machine Learning, Deep Learning, Artificial** 

**Intelligence, Probability and Statistics** 





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# Problems of Week 1,2 (August)

**Note**: These Problems of the Day (POTD) will be the cornerstone of your entire GATE Data Science preparation. Keep in mind, that the goal is to focus on the learning experience these problems offer, rather than just finding the answers.

Start engaging with them now, and you'll notice a significant difference in your exam—trust me.

Happy learning!

The probability that a student passes only in History is 2/5. The probability that the student passes only in Geography is 1/4. The probability that the student passes in both History and Geography is 1/8. What is the probability that the student fails in both subjects?

A.9/40

B.19/40

C.21/40

D.35/40

A factory produces defective widgets daily, following a Poisson distribution with a mean of 3. Additionally, a secondary process randomly increases the defects, following a Poisson distribution with a mean of 1. What is the probability that the total number of defective widgets over 2 days does not exceed 3?

A series system consists of four components with the time to failure following an exponential distribution. The mean time to failure for each component is given respectively as follows (in hours): 10, 4, 5, 2. In a series system, all components must function for the system to work.

Determine the maximum decrease in the probability that the system survives for a time of 3 hours if any one of the components is replaced with a component having a mean time to failure of 10 hours

Consider a naive Bayes classifier with four categorical input variables,  $X_1, X_2, X_3$ , and  $X_4$ , and one categorical output variable, Y. Each input variable  $X_i$  can take on 3 possible values (0, 1, or 2), and the output variable Y can take on 3 possible values (0, 1, or 2). The prior probabilities of the output classes are given as follows:

- P(Y=0)=0.2
- P(Y=1)=0.5
- P(Y=2)=0.3

Given this information, how many parameters must be estimated to train this naive Bayes classifier?

In a given city, 45.4% of men and 42.0% of women eat breakfast. Suppose random samples of 300 women and 300 men are chosen. Calculate the probability that more women than men eat breakfast. Use the given z-values and their corresponding probabilities to assist in your calculation:

- At Z = 0.16, the probability is 0.5636.
- At Z = 0.26, the probability is 0.6026.
- At Z = 0.50, the probability is 0.6915.
- At Z = 0.70, the probability is 0.7580.
- At Z = 0.84, the probability is 0.7995.
- At Z = 0.90, the probability is 0.8159.
- At Z = 1.00, the probability is 0.8413.

# POTD #6[MSQ]

#### Which of the following statements are not completely correct?

- 1. If  $X_i$   $(i=1,2,\ldots,n)$  are independent and identically distributed random variables,  $E(\overline{X})=\mu$  and  $\mathrm{Var}(\overline{X})=rac{\sigma^2}{n}.$
- 2. If  $X_i$   $(i=1,2,\ldots,n)$  are independent and identically distributed (i.i.d.) random variables with mean  $\mu$  and variance  $\sigma^2$ ,  $\overline{X}$  follows a normal distribution with mean  $\mu$  and variance  $\frac{\sigma^2}{n}$ .
- 3. If  $X_i$   $(i=1,2,\ldots,n)$  are independent and identically normally distributed random variables with mean  $\mu$  and variance  $\sigma^2$ ,  $\overline{X}$  follows a normal distribution with mean  $\mu$  and variance  $\frac{\sigma^2}{n}$ .
- 4. If  $X_1, X_2, \ldots, X_n$  are samples from identical and independent distributions, then  $\frac{(n-1)s^2}{\sigma^2}$  follows a chi-square distribution with n-1 degrees of freedom. Here,  $s^2$  is the sample variance.

Suppose there are 30 different types of collectible stickers, and each time you obtain a sticker, it is equally likely to be any one of the 30 types. You decide to buy a set of 15 stickers. Compute the expected number of different types of stickers that will be contained in your set of 15 stickers.

In a binary Naive Bayes classifier with three continuous features, where for each class, the features are normally distributed, how many parameters need to be estimated from the data?

- A) 7
- B) 8
- C) 13
- D) Infinite parameters are needed for continuous features

Given the following kernel function and the set of points plotted on the 2D plane, determine the equation of the maximum margin decision boundary:

$$K(ec{u},ec{v})=2|ec{u}||ec{v}|$$

where  $\vec{u}$  and  $\vec{v}$  are any two vectors representing points on the 2D plane, and  $|\vec{u}|$  and  $|\vec{v}|$  represent their magnitudes (Euclidean distance from the origin).

The points are distributed as follows on the graph:

- Positive class points: (-1,-1),(0,0),(1,1),(2,2)
- Negative class points: (4,4), (-3,-3), (-4,-4)

Using the provided kernel and the graph, determine the equation of the maximum margin decision boundary that separates the positive and negative classes.

A) 
$$x_1^2 + x_2^2 = 12.5$$

B) 
$$\frac{x_1^2}{9} + \frac{x_2^2}{4} = 1$$

C) 
$$x_1^2 + 4x_2^2 = 25$$

D) 
$$x_1 + x_2 = 5$$

# Free answers and solutions link-

https://unacademy.com/class/potd-week12/DATRGU06

https://unacademy.com/class/potd-week12part/W7KZURHN

Discuss your Doubts here - https://t.me/ManojGateDA