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GATE AIR - 13

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Linear Algebra Practice - V

Q 1

Let A be a 7×5 matrix of rank 4.

(i) Give the **size** and **rank** of the following projection matrices:

1. P_1: projection onto $\mathcal{C}(A)$
2. P_2: projection onto $\mathcal{C}(A^\top)$
3. P_3: projection onto $\mathcal{N}(A)$
4. P_4: projection onto $\mathcal{N}(A^\top)$

(ii) Give a sum or product of two of these P matrices that must equal 0 (the zero matrix).

(iii) Give a sum or product of two of these P matrices that must equal I (the identity matrix).

Q 2

Let A be a matrix, and suppose the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for some $\mathbf{b}$. To find the closest vector $\mathbf{p}$ to $\mathbf{b}$ in the column space of A, we project $\mathbf{b}$ onto the column space of A, resulting in the system $A\hat{\mathbf{x}} = \mathbf{p}$.

Which of the following subspaces does $\mathbf{b} - A\hat{\mathbf{x}}$ belong to?

a. Column space of A

b. Row space of A

c. Null space of A

d. Left null space of A

Q 3

Let A be a 4×3 matrix obtained by removing the last column of the 4×4 identity matrix. Let

$$\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

(i) What is the shape of the projection matrix P that projects vectors onto the column space of A?

(ii) What is the projection of $\mathbf{b}$ onto the column space of A?

Q 4

Given **b** = (0, 2, 4) and **A** with columns (0, 1, 2) and (1, 2, 0):

1. Find the projection of **b** onto the column space of **A**.
2. Compute the projection matrix **P** and check if **P** = **I**.

Q 5

(a) If **P** is a 2×2 projection matrix onto the line through $(1, 1)$, then $I - P$ is the projection matrix onto:

1. A line
2. A plane
3. The entire $\mathbb{R}^3$ space

(b) If **P** is a 3×3 projection matrix onto the line through $(1, 1, 1)$, then $I - P$ is the projection matrix onto:

1. A line
2. A plane
3. The entire $\mathbb{R}^3$ space

Q 6

Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$.

Which of the following statements are correct?

(a) The orthogonal projection of vector $\mathbf{b}$ onto the column space of A is $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$.

(b) If P is the projection matrix, then $(I - P)^2 = I - 2P$.

(c) If P is the projection matrix onto the column space of A, then $I - P$ projects onto the null space of A.

(d) The projection matrix given by $A(A^T A)^{-1} A^T$ exists only if A has linearly independent columns.

Q 7

Given an $m \times n$ matrix A such that $A^T Ax = 0$, which of the following statements are correct?

(a) Ax is in the null space of A.

(b) Ax is in the column space of A.

(c) Ax is always a zero vector.

(d) The rank of matrix A is n.

GATE DA 2024

Let $\mathbb{R}$ be the set of real numbers, U be a subspace of $\mathbb{R}^3$, and $M \in \mathbb{R}^{3\times 3}$ be the matrix corresponding to the projection onto the subspace U.

Which of the following statements is/are TRUE?

a. If U is a 1-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.

b. If U is a 2-dimensional subspace of $\mathbb{R}^3$, then the null space of M is a 1-dimensional subspace.

c. $M^2 = M$.

d. $M^3 = M$.