Inference Rules for Functional Dependencies (proof not needed)

Inference Rule (IR):

- The Armstrong's axioms are the basic inference rule.
- Armstrong's axioms are used to conclude functional dependencies on a relational database.
- The inference rule is a type of assertion. It can apply to a set of FD(functional dependency) to derive other FD.
- Using the inference rule, we can derive additional functional dependency from the initial set.

The Functional dependency has 6 types of inference rule:

Reflexive Rule (IR_1)

In the reflexive rule, if Y is a subset of X, then X determines Y.

If $X \supseteq Y$ then $X \rightarrow Y$

Example:

1. X = {a, b, c, d, e}
2. Y = {a, b, c}

2. Augmentation Rule (IR_2)

The augmentation is also called as a partial dependency. In augmentation, if X determines Y, then XZ determines YZ for any Z.

If $X \rightarrow Y$ then $XZ \rightarrow YZ$

Example:

For R(ABCD), **if** $A \rightarrow B$ then $AC \rightarrow BC$

3. Transitive Rule (IR_3)

In the transitive rule, if X determines Y and Y determine Z, then X must also determine Z.

If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

4. Union Rule (IR_4)

Union rule says, if X determines Y and X determines Z, then X must also determine Y and Z.

If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$

Proof:

1. $X \rightarrow Y$ (given)
2. $X \rightarrow Z$ (given)
3. $X \rightarrow XY$ (using IR_2 on 1 by augmentation with X. Where $XX = X$)
4. $XY \rightarrow YZ$ (using IR_2 on 2 by augmentation with Y)
5. $X \rightarrow YZ$ (using IR_3 on 3 and 4)

5. Decomposition Rule (IR_5)

Decomposition rule is also known as project rule. It is the reverse of union rule.

This Rule says, if X determines Y and Z, then X determines Y and X determines Z separately.

If $X \rightarrow YZ$ then $X \rightarrow Y$ and $X \rightarrow Z$

Proof:

1. $X \rightarrow YZ$ (given)
2. $YZ \rightarrow Y$ (using IR_1 Rule)
3. $X \rightarrow Y$ (using IR_3 on 1 and 2)

6. Pseudo transitive Rule (IR_6)

In Pseudo transitive Rule, if X determines Y and YZ determines W, then XZ determines W.

If $X \rightarrow Y$ and $YZ \rightarrow W$ then $XZ \rightarrow W$

Proof:

1. $X \rightarrow Y$ (given)
2. $WY \rightarrow Z$ (given)
3. $WX \rightarrow WY$ (using IR_2 on 1 by augmenting with W)
4. $WX \rightarrow Z$ (using IR_3 on 3 and 2)

7. Composition Rule

If $X \rightarrow Y$ and $A \rightarrow B$ then $XA \rightarrow YB$