

### 5.1

$P \vee Q$  and  $\neg P \rightarrow Q$

Valid -  $P$  or  $Q$  means either  $P$  or  $Q$  are true or  $P$  and  $Q$  are both true. If we know also that not  $P$ , then in order for  $P$  or  $Q$  to be true, at least  $Q$  must be true, therefore it is valid.

### 5.3

$\neg (P \vee Q) \rightarrow \neg P$

Not Valid - In order for not  $P$  or  $Q$  to be true, you can have either not  $P$ , not  $Q$  or not  $P$  and not  $Q$ . Although it is reasonable to infer not  $P$ , it is not guaranteed.

### 5.6

$P \wedge Q$  and  $\neg P \rightarrow R$

Valid - This is valid, because every time both  $P \wedge Q$  and  $\neg P$  are true,  $R$  is also true. But because  $P \wedge Q$  and  $\neg P$  are contradictory, there is no row of the truth table which makes both of them true. But by the definition of TT validity it is valid.

### 5.8

Either  $a$  is left of  $b$  or  $a$  is right of  $b$ .  $a$  is back of  $b$  or not left of  $b$  or both.  $b$  is in front of  $a$  or  $a$  is not right of  $b$  or both.  $c$  is in the same column as  $a$  and  $c$  is in the same row as  $b$ . In any case,  $a$  is at the back of  $b$ .

### 5.20

Since he is proving a contradiction he is arguing only against watching the Wes Craven slasher movie, he is not proving why we should watch the romantic comedy. He is using proof by contradiction and proof by cases. I would say to him: you can close your eyes when the blood shows up and besides, how can you go to med school if you can't stand the sight of blood? Even if we watched the romantic comedy, we would end late and you wouldn't get much sleep anyway. If you want a good night's sleep, you shouldn't be out in the first place.

### 5.26

Consider an arbitrary prime number  $k$ . We want to show that  $\sqrt[n]{n}$  is irrational.

Suppose that  $\sqrt[n]{n}$  is actually rational. Then there are two numbers  $x$  and  $y$ , not divisible by  $n$ , so  $\sqrt[n]{n} = x/y$ . We have  $n = x^2/y^2$ , and so  $y^2 = nx^2$ . So  $y^2$  is divisible by  $n$ . This means that  $y$  is divisible by  $n$ . But then  $x^2$  is divisible by  $n$ , and so  $x$  is divisible by  $n$ . This contradiction. Suggesting that our assumption that  $\sqrt[n]{n}$  is rational must have been wrong.