Codebook- Team Know_no_algo

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Contents

1 Format

1.1 Format c++

```
#include <bits/stdc++.h>  #define long long long ↩
   int
using namespace std;  #define Max 100010
#define mp make_pair  #define pb push_back
#define INF 1e16  #define INF2 1e9+9
#define pi 3.141592653589  #define x first
#define y second
long cons;
long check(long a){
 if(a>=cons)a%=cons;
 return a;
}
long check2(long a){
 a%=cons ;
 if(a<0) a+=cons ;
 return a;
}
long GCD(long a,long b){
 if(b==0)
  return a;
 return GCD(b,a%b);
}
long exp(long a,long n){
 if(n==0) return 1;
 if (n==1) return check(a);
 long b=exp(a,n/2);
 if(n%2==0) return check(b*b);
 return check(b*check(b*a));
}
int main(){
   ios::sync_with_stdio(false);cin.tie(0);
   cons=1000000007 ;
}
```

1.2 Format Java

```
import java.io.* ;
import java.util.* ;
import java.math.* ;
import java.text.* ;
import static java.lang.Math.min ;
import static java.lang.Math.max ;
public class Main{
 public static void main(String args[]) throws ↩
    IOException{
  Solver s = new Solver() ;
  s.init() ;
  s.Solve() ;
  s.Finish() ;
 }
}
class pair implements Comparable<pair>{
 long x,y ;
 pair(long x,long y){
  this.x = x ; this.y=y ;
 }
 public int compareTo(pair p){
  return (this.x<p.x ? -1 : (this.x>p.x ? 1 : (this↩
     .y<p.y ? -1 : (this.y>p.y ? 1 : 0)))) ;
 }
}
class Solver{
 void Solve() throws IOException{

 }
 void init(){
  op = new PrintWriter(System.out) ;
  ip = new Reader(System.in) ;
 }
 void Finish(){
  op.flush() ;
  op.close() ;
 }
 void p(Object o){
  op.print(o) ;
 }
 void pln(Object o){
  op.println(o) ;
 }
 PrintWriter op ;
 Reader ip ;
}
class Reader {
 BufferedReader reader;
 StringTokenizer tokenizer;
 Reader(InputStream input) {
  reader = new BufferedReader(
     new InputStreamReader(input) );
  tokenizer = new StringTokenizer("") ;
 }
 String s() throws IOException {
  while (!tokenizer.hasMoreTokens()){
   tokenizer = new StringTokenizer(
   reader.readLine()) ;
```

```
  }
  return tokenizer.nextToken();
 }
 int i() throws IOException {
  return Integer.parseInt(s()) ;
 }
 long l() throws IOException{
  return Long.parseLong(s()) ;
 }
 double d() throws IOException {
  return Double.parseDouble(s()) ;
 }
}
```

2 Strings

2.1 KMP

```
//Takes an array of characters and calculate
//lcp[i] where lcp[i] is the longest proper suffix ↩
   of the
//string c[0..i] such that it is also a prefix of ↩
   the string.
int[] kmp(char c[],int n){
 int lcp[] = new int[n] ;
 for(int i=1 ; i<n ; i++){
  int j = lcp[i-1] ;
  while(j!=0 && c[i]!=c[j]) j = lcp[j-1] ;
  if(c[i]==c[j]) j++ ;
  lcp[i]=j ;
 }
 return lcp ;
}
```

2.2 Manacher

```
//Given an array of characters in arr and the ↩
   length
//of the array as n it simply finds the longest ↩
   palindromic substring
//at each position with that position as the center↩
   of the palindrome.
// Array is 0-indexed
int[] Manacher(char c[],int n){
 int P[] = new int[n+1] ;
 int R=0,C=1 ;
 for(int i=1 ; i<=n ; i++){
  int rad=-1 ;
  if(i<=R)
   rad = min(P[2*C-i],(R-i)) ;
  else
   rad = 0 ;
  while((i+rad)<=n && (i-rad)>=1 && c[i-rad]==c[i+↩
     rad])
   rad++ ;
  P[i]=rad-1 ;
  if((i+rad)>R){
   C = i ;
   R = i+rad ;
  }
 }
 return P ;
}
```

2.3 Suffix_Array

```
int match(char t[],char s[],int pos,int n){
 for(int i=0 ; i<t.length ; i++)
  if(pos+i==n)
   return 1 ;
  else if(t[i]!=s[pos+i])
   return (t[i]<s[pos+i] ? -1 : 1) ;
 return 0 ;
}
int[] SufTrans(int P[][],int n){
 int suf[] = new int[n] ;
 for(int i=0 ; i<n ; i++) suf[P[19][i]] = i ;
 return suf ;
}
int LCP(int i,int j,int P[][],int n){
 if(i==j) return (n-i+1) ;
 int match=0 ;
 for(int k=19 ; i<n && j<n && k>=0 ; k--){
  if(P[k][i]==P[k][j]){
   match+=(1<<k) ;
   i+=(1<<k) ;
   j+=(1<<k) ;
  }
```

```
 }
 return match ;
}
int[][] suffix_array(char c[],int n){
 class Tuple implements Comparable<Tuple>{
  int idx ; pair p ;
  Tuple(int _idx,pair _p){
   idx = _idx ; p=_p ;
  }
  public int compareTo(Tuple _t){
   return p.compareTo(_t.p) ;
  }
 }
 int P[][] = new int[20][n] ;
 if(n!=1)
  for(int i=0 ; i<n ; i++) P[0][i] = (int) c[i] ;
 else
  P[0][0] = 0 ;
 for(int i=1,pow2=1 ; i<20 ; pow2<<=1,i++){
  Tuple L[] = new Tuple[n] ;
  for(int j=0 ; j<n ; j++){
   int y = ((j+pow2)<n ? P[i-1][j+pow2] : -1) ;
   L[j] = new Tuple(j,new pair(P[i-1][j],y)) ;
  }
  Arrays.sort(L) ;
  for(int j=0 ; j<n ; j++)
   if(j>0 && L[j].compareTo(L[j-1])==0)
    P[i][L[j].idx] = P[i][L[j-1].idx] ;
   else
    P[i][L[j].idx] = j ;
 }
 return P ;
}
```

2.4 Z algo

```
//Given an array of characters in c and
// length of array is n, find the z-array
//that is z[i]=longest prefix match of suffix
//at i and the original string
int[] Z_algo(char c[],int n){
 int Z[] = new int[n] ;
 int L=0,R=0 ;
 for(int i=1 ; i<n ; i++){
  if(i>R){
   L=i ; R=i ;
   while(R<n && c[R]==c[R-L]) R++ ;
   R-- ; Z[i] = (R-L+1) ;
  }else{
   int j = i-L ;
   if(Z[j]<(R-i+1))
    Z[i]=Z[j] ;
   else{
    L=i ;
    while(R<n && c[R]==c[R-L]) R++ ;
    R-- ; Z[i] = (R-L+1) ;
   }
  }
 }
 return Z ;
}
```

2.5 Hashing

```
vector<long> hashed1[10*Max];
vector<long> hashed2[10*Max];
long p1=2350490027,p2=1628175011;
long p3=2911165193,p4=1040332871;
2350490027,2125898167,1628175011,1749241873,
1593209441,1524872353,1040332871,2911165193,
1387346491,2776808933
void calc_hashed(int ind,vector<long> &hashed,long ↩
   prime){
 long val=1;
 int x=neighbour[ind].size();
 hashed.resize(x);
 for(int i=0;i<x;i++){
  if(i==0)
   hashed[i]=neighbour[ind][i];
  else
   hashed[i]=check(hashed[i-1]+neighbour[ind][i]*↩
      val) ;
  val=check(val*prime);
 }
}
```

2.6 Trie

```
struct node{
 int ind;
 node *arr[26] ;
```

```
};
node* getnode(int ind){
 node *temp=new node() ;
 temp->ind=ind ;
 for(int i=0;i<26;i++)
  temp->arr[i]=NULL ;
 return temp ;
}
void insert(node *root,string &s,int pos){
 int x=(s.length()) ;
 for(int i=0;i<x;i++){
  int ch=s[i]-97 ;
  if(root->arr[ch]==NULL)
   root->arr[ch]=getnode(pos) ;
  root=root->arr[ch] ;
 }
}
```

3 Trees

3.1 Centroid Tree

```
vector<int> graph[3*Max];
int size[3*Max];
bool usable[3*Max];
int centroid_parent[3*Max];
void calc_size(int i,int pa){
 size[i]=1;
 for(auto itr:graph[i]){
  if(itr!=pa && usable[itr]){
   calc_size(itr,i);
   size[i]+=size[itr];
  }
 }
}
int getCentroid(int i,int len,int pa){
 for(auto itr:graph[i]){
  if(itr!=pa && usable[itr]){
   if(size[itr]>(len/2))
    return getCentroid(itr,len,i);
  }
 }
 return i;
}
void build_centroid(int i,int coun){
 queue<pair<int,int> > q;
 q.push({i,-1});
 while(!q.empty()){
  auto itr=q.front();
  q.pop();
  calc_size(itr.x,-1);
  int centroid=getCentroid(itr.x,size[itr.x],-1);
  centroid_parent[centroid]=itr.y;
  for(auto itr2:graph[centroid]){
   if(usable[itr2]){
    q.push({itr2,centroid});
   }
  }
  usable[centroid]=false;
 }
}
```

3.2 Heavy Light Decomposition

```
int chainNo[Max];
int pos_in_chain[Max];
int parent_in_chain[Max];
int parent[Max];
int chain_count=0;
int total_in_chain[Max];
int pos_count=0;
vector<int> graph[Max];
int arr[Max];
int subtree_count[Max];
int max_in_subtree[Max];
int height[Max];
vector<vector<pair<int,int> > > vec;
int max_elem,max_count;
void simple_dfs(int i){
 subtree_count[i]=1;
 int max_val=0;
 int ind=-1;
 for(auto itr:graph[i]){
  height[itr]=1+height[i];
  simple_dfs(itr);
  subtree_count[i]+=subtree_count[itr];
  if(max_val<subtree_count[itr]){
   max_val=subtree_count[itr];
   ind=itr;
  }
 }
 max_in_subtree[i]=ind;
}
```

```
void dfs(int i){
 if(pos_count==0)
  parent_in_chain[chain_count]=i;
 chainNo[i]=chain_count;
 pos_in_chain[i]=++pos_count;
 total_in_chain[chain_count]++;
 if(max_in_subtree[i]!=-1){
  dfs(max_in_subtree[i]);
 }
 for(auto itr:graph[i]){
  if(itr!=max_in_subtree[i]){
   chain_count++;
   pos_count=0;
   dfs(itr);
  }
 }
}
int pos;int chain;int val;
void update(int s,int e,int n){
 if(pos>e || pos<s)
  return;
 vec[chain][n]={val,1};
 if(s==e)
  return;
 int mid=(s+e)>>1;
 update(s,mid,2*n);
 update(mid+1,e,2*n+1);
 if(vec[chain][2*n].x<vec[chain][2*n+1].x)
  vec[chain][n]=vec[chain][2*n+1];
 else if(vec[chain][2*n].x>vec[chain][2*n+1].x)
  vec[chain][n]=vec[chain][2*n];
 else{
  vec[chain][n]={vec[chain][2*n].x,vec[chain][2*n].↩
     y+vec[chain][2*n+1].y};
 }
}
int qs;int qe;
void query_tree(int s,int e,int n){
 if(s>qe || qs>e)
  return;
 if(s>=qs && e<=qe){
  if(vec[chain][n].x>max_elem){
   max_elem=vec[chain][n].x;
   max_count=vec[chain][n].y;
  }
  else if(vec[chain][n].x==max_elem){
   max_count+=vec[chain][n].y;
  }
  return;
 }
 if(vec[chain][n].x <max_elem)
  return;
 int mid=(s+e)>>1;
 query_tree(s,mid,2*n);
 query_tree(mid+1,e,2*n+1);
}
void query(int i){
 if(i==-1)
  return;
 qs=1;qe=pos_in_chain[i];chain=chainNo[i];
 query_tree(1,total_in_chain[chainNo[i]],1);
 i=parent[parent_in_chain[chainNo[i]]];
 query(i);
}
```

3.3 Heavy Light Trick

```
void dfs(int i,int pa){
 int coun=1 ;
 for(auto itr:a[i]){
  if(itr.x!=pa){
   prod[itr.x]=check(prod[i]*itr.y) ;
   dfs(itr.x,i);
   coun+=siz[itr.x] ;
  }
 }
 siz[i]=coun ;
}
long ans=0 ;
void add(int i,int pa,int x){
 coun[mapped_prod[i]]+=x ;
 for(auto itr:a[i])
  if(itr.x!=pa && !big[itr.x])
   add(itr.x,i,x) ;
}
void solve(int i,int pa){
 long temp=check(multi*inv[i]);
 int xx=m[temp];
 ans+=coun[xx];
 for(auto itr:a[i])
  if(itr.x!=pa && !big[itr.x])
   solve(itr.x,i) ;
}
void dfs2(int i,int pa,bool keep){
 int mx=-1,bigc=-1;
 for(auto itr:a[i]){
  if(itr.x!=pa){
   if(siz[itr.x]>mx)
   mx=siz[itr.x],bigc=itr.x;
  }
 }
```

```
 for(auto itr:a[i]){
  if(itr.x!=pa && itr.x!=bigc)
   dfs2(itr.x,i,0);
 }
 if(bigc!=-1){
  dfs2(bigc,i,1);
  big[bigc]=true;
 }
 multi=check(p*check(prod[i]*prod[i]));
 long temp=check(p*prod[i]);
 ans+=coun[m[temp]];
 coun[mapped_prod[i]]++;
 for(auto itr:a[i])
  if(itr.x!=pa && !big[itr.x]){
   solve(itr.x,i);
   add(itr.x,i,1);
  }
 if(bigc!=-1)
  big[bigc]=false;
 if(keep==0)
  add(i,pa,-1);
}
```

3.4 LCA

```
int pa[21][3*Max], level[3*Max];
int lca(int u,int v){
 if(level[u]>level[v])return lca(v,u);
 for(long i=19;i>=0 && level[v]!=level[u];i--){
  if(level[v]>=level[u]+(1<<i))
   v=pa[i][v];
 }
 if(u==v)return u;
 for(long i=19;i>=0;i--){
  if(pa[i][u]!=pa[i][v]){
   u=pa[i][u];v=pa[i][v];
  }
 }
 return pa[0][u];
}
```

3.5 LCA Tree

```
vpi auxTree[N];
int parent[N];
ll parWgt[N];
int conAuxTree(set<int, disComp> &nodes) {
    vi originalNodes(nodes.begin(), nodes.end());
    for (int i=0; i<originalNodes.size()-1; i++) {
        nodes.insert(LCA(originalNodes[i], ↩
           originalNodes[i+1]));
    }
    int root = *nodes.begin();
    parent[root] = 0;
    int cur = root;
    auto sit = next(nodes.begin());
    while (sit != nodes.end()) {
        while (!isAnc(cur, *sit)) {
            assert(cur);
            cur = parent[cur];
        }
        parent[*sit] = cur;
        parWgt[*sit] = rootDis[*sit] - rootDis[cur↩
           ];
        auxTree[cur].push_back({*sit, parWgt[*sit↩
           ]});
        cur = *sit;
        ++sit;
    }
    return root;
}
```

4 Graph and Matching, Flows

4.1 AP and Bridges

```
// Finds bridges and cut vertices
// Receives:
// N: number of vertices
// l: adjacency list
// Gives:
// vis, seen, par (used to find cut vertices)
// ap - 1 if it is a cut vertex, 0 otherwise
// brid - vector of pairs containing the bridges
typedef pair<int, int> PII;
int N;
vector <int> l[MAX];
vector <PII> brid;
```

```
int vis[MAX], seen[MAX], par[MAX], ap[MAX];
int cnt, root;
void dfs(int x){
 if(vis[x] != -1)
  return;
 vis[x] = seen[x] = cnt++ ;
 int adj = 0;
 for(int i = 0; i < (int)l[x].size(); i++){
  int v = l[x][i];
  if(par[x] == v) continue ;
  if(vis[v] == -1){
   adj++;
   par[v] = x;
   dfs(v);
   seen[x] = min(seen[x], seen[v]);
   if(seen[v] >= vis[x] && x != root)
    ap[x] = 1 ;
   if(seen[v] == vis[v])
    brid.push_back(make_pair(v, x));
  }
  else{
   seen[x] = min(seen[x],vis[v]);
   seen[v] = min(seen[x],seen[v]);
  }
 }
 if(x == root) ap[x] = (adj>1);
}
void bridges(){
 brid.clear();
 for(int i = 0; i < N; i++){
  vis[i] = seen[i] = par[i] = -1;
  ap[i] = 0;
 }
 cnt = 0;
 for(int i = 0; i < N; i++)
  if(vis[i] == -1){
   root = i;
   dfs(i);
  }
}
```

4.2 Euler Walk

```
vector<pair<int,int> > graph[202];
bool visited[202];
vector<int> odd;
bool used_edges[41000];
stack<int> s;
int tot_edges;
int counter[202];

void dfs(int i)
{
  visited[i]=true;

  int len=graph[i].size();
  if(len&1)
    odd.pb(i);

  for(auto itr:graph[i])
    if(!visited[itr.x])
      dfs(itr.x);
}
void euler_tour(int i)
{
    visited[i]=true;
    s.push(i);

    int x=graph[i].size();
    while(counter[i]<x)
    {
      auto itr=graph[i][counter[i]];
      counter[i]++;

      if(!used_edges[itr.y])
      {
        used_edges[itr.y]=true;
        if(itr.y<=tot_edges)
          cout<<i<<" "<<itr.x<<"\n";
        euler_tour(itr.x);
      }
    }
    s.pop();
}
```

4.3 Bipartite Matching

```
vector<pair<int,pair<int,bool> > > graph[1000];
vector<bool> edge_use,visited;
vector<int> parent,edge_number;

void dfs(int i){
 visited[i]=true;
 for(auto itr:graph[i]){
  if(visited[itr.x])
   continue;
```

```
  if(edge_use[itr.y.x] && itr.y.y){
   parent[itr.x]=i;
   edge_number[itr.x]=itr.y.x;
   dfs(itr.x);
  }
  else if(!edge_use[itr.y.x] && (!itr.y.y)){
   parent[itr.x]=i;
   edge_number[itr.x]=itr.y.x;
   dfs(itr.x);
  }
 }
}

void edge_reverse(int t){
 if(t==0)
  return;
 edge_use[edge_number[t]]=!edge_use[edge_number[t↩
    ]];
 edge_reverse(parent[t]);
}
// |l|,|r| are the number of vertices in the left ↩
   side and right side respectively.
int s=0;
int t=(|l|+|r|+1) ;
for(int i=1;i<=h;i++){
 ++edge_count;
 graph[0].pb({i,{edge_count,true}});
 graph[i].pb({0,{edge_count,false}});
}
int matching=0;
edge_use.resize(edge_count+1);
visited.resize(t+1);
edge_number.resize(t+1);
parent.resize(t+1);
for(int i=0;i<=edge_count;i++)
  edge_use[i]=true;
while(true){
 for(int i=0;i<=t;i++){
  visited[i]=false;
  edge_number[i]=-1;
  parent[i]=-1;
 }
 dfs(s);
 if(!visited[t])
  break;
 edge_reverse(t);
 matching++;
}
```

4.4 Dinic- Maximum Flow $O(EV^2)$

```
const int N = 20005 ;
const int E = N*1005 ;
int t, n, m;
int par[N];
/* START DINIC */
int nodes, edges;
int eu[E], ev[E], ef[E], ec[E];
int dist[N], q[N], ed[N];
vector<int> adj[N];
void init(int n) {
 ::nodes = n;
 ::edges = 0;
 for (int i = 0; i < nodes; ++i)
  adj[i].clear();
}
int newedge(int u, int v, int flow, int cap) {
 eu[edges] = u;
 ev[edges] = v;
 ef[edges] = flow;
 ec[edges] = cap;
 return edges++;
}
void addedge(int u, int v, int cap) {
 int uv = newedge(u, v, 0, cap);
 int vu = newedge(v, u, 0, 0);
 adj[u].push_back(uv);
 adj[v].push_back(vu);
}
bool bfs(int src, int snk) {
 memset(dist, -1, sizeof(int) * nodes);
 int h = 0, t = 0;
 dist[src] = 0;
 q[t++] = src;
 while (h != t && dist[snk] == -1) {
  int u = q[h++]; if (h == N) h = 0;
  for (int e : adj[u]) {
   int v = ev[e];
   if (dist[v] < 0 && ef[e] < ec[e]) {
    dist[v] = dist[u] + 1;
    q[t++] = v;
    if (t == N) t = 0;
   }
  }
 }
 return ~dist[snk];
}
bool dfs(int u, int snk, int flow) {
 if (flow <= 0) return 0;
 if (u == snk) return flow;
```

```
 for (int& i = ed[u]; i < (int) adj[u].size(); ++i)↩
     {
  int e = adj[u][i];
  int v = ev[e];
  if (dist[u] + 1 == dist[v]) {
   int fl = min(flow, ec[e] - ef[e]);
   int df = dfs(v, snk, fl);
   if (df == 0) continue;
   ef[e] += df;
   ef[e^1] -= df;
   return df;
  }
 }
 return 0;
}
int dinic(int src, int snk) {
 int mf = 0;
 while (bfs(src, snk)) {
  memset(ed, 0, sizeof(int) * nodes);
  int df;
  while (df = dfs(src, snk, INT_MAX))
   mf += df;
 }
 return mf;
}
int main() {
 scanf("%d", &t);
 while (t--) {
  scanf("%d%d", &n, &m);
  int source = n + m;
  int sink = source + 1;
  init(sink + 1);
  for (int i = 2; i <= n; ++i) {
   scanf("%d", &par[i]);
   addedge(par[i] - 1, i - 1, INT_MAX);
  }
  for (int i = 1; i <= n; ++i) {
   addedge(i - 1, sink, 1);
  }
  for (int i = 0; i < m; ++i) {
   addedge(source, i + n, 1);
   int len;
   scanf("%d", &len);
   for (int j = 0; j < len; ++j) {
    int ai;
    scanf("%d", &ai);
    addedge(i + n, ai - 1, 1);
   }
  }
  printf("%d\n", dinic(source, sink));
 }
}
```

4.5 Minimum Cost Bipartite Matching $O(V^3)$

```
// Min cost bipartite matching via shortest ↩
   augmenting path
// This is an O(n^3) implementation of a shortest ↩
   augmenting path
// algorithm for finding min cost perfect matchings↩
    in dense
// graphs. In practice, it solves 1000x1000 ↩
   problems in around 1 second.
//  cost[i][j] = cost for pairing left node i with ↩
   right node j
//  Lmate[i] = index of right node that left node i↩
    pairs with
//  Rmate[j] = index of left node that right node j↩
    pairs with
// The values in cost[i][j] may be positive or ↩
   negative.To perform
// maximization, simply negate the cost[][] matrix.
typedef vector<long> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;
long MinCostMatching(const VVD &cost, VI &Lmate, VI↩
    &Rmate) {
 int n = int(cost.size());
 // construct dual feasible solution
 VD u(n); VD v(n) ;
 for (int i = 0; i < n; i++) {
  u[i] = cost[i][0];
  for (int j = 1; j < n; j++) u[i] = min(u[i], cost↩
     [i][j]) ;
 }
 for (int j = 0; j < n; j++) {
  v[j] = cost[0][j] - u[0];
  for (int i = 1; i < n; i++) v[j] = min(v[j], cost↩
     [i][j] - u[i]) ;
 } // construct primal solution satisfying ↩
    complementary slackness
 Lmate = VI(n, -1);
 Rmate = VI(n, -1);
 int mated = 0;
 for (int i = 0; i < n; i++) {
  for (int j = 0; j < n; j++) {
   if (Rmate[j] != -1) continue;
   if ((cost[i][j] - u[i] - v[j])==0){
    Lmate[i] = j;
    Rmate[j] = i;
    mated++;
    break;
   }
```

```
  }
 }
 VD dist(n); VI dad(n); VI seen(n);
 // repeat until primal solution is feasible
 while (mated < n) {   // find an unmatched left ↩
    node
  int s = 0;
  while (Lmate[s] != -1) s++;    // initialize ↩
     Dijkstra
  fill(dad.begin(), dad.end(), -1);
  fill(seen.begin(), seen.end(), 0);
  for (int k = 0; k < n; k++)
   dist[k] = cost[s][k] - u[s] - v[k];
  int j = 0;
  while (true){     // find closest
   j = -1;
   for (int k = 0; k < n; k++) {
    if (seen[k]) continue ;
    if (j == -1 || dist[k] < dist[j]) j = k;
   }
   seen[j] = 1 ;     // termination condition
   if (Rmate[j] == -1) break ;    // relax ↩
      neighbors
   const int i = Rmate[j] ;
   for (int k = 0; k < n; k++) {
    if (seen[k]) continue;
    const long new_dist = dist[j] + cost[i][k] - u[↩
       i] - v[k];
    if (dist[k] > new_dist) {
     dist[k] = new_dist;
     dad[k] = j;
    }
   }
  }   // update dual variables
  for (int k = 0; k < n; k++) {
   if (k == j || !seen[k]) continue;
   const int i = Rmate[k];
   v[k] += dist[k] - dist[j];
   u[i] -= dist[k] - dist[j];
  }
  u[s] += dist[j] ;    // augment along path
  while (dad[j] >= 0) {
   const int d = dad[j];
   Rmate[j] = Rmate[d];
   Lmate[Rmate[j]] = j;
   j = d;
  }
  Rmate[j] = s; Lmate[s] = j;
  mated++;
 }
 long value = 0;
 for (int i = 0; i < n; i++)
  value += cost[i][Lmate[i]];
  return value;
}
VVD cost;
cost.resize(n+m-1);
VI Lmate,Rmate;
MinCostMatching(cost,Lmate,Rmate)
```

4.6 Minimum Cost Maximum Flow

```
struct Edge {
    int u, v;
    long long cap, cost;
    Edge(int _u, int _v, long long _cap, long long ↩
       _cost) {
        u = _u; v = _v; cap = _cap; cost = _cost;
    }
};
struct MinimumCostMaximumFlow{
    int n, s, t;
    long long flow, cost;
    vector<vector<int> > graph;
    vector<Edge> e;
    vector<long long> dist;
    vector<int> parent;
    MinimumCostMaximumFlow(int _n){
        // 0-based indexing
        n = _n;
        graph.assign(n, vector<int> ());
    }
    void add(int u, int v, long long cap, long long↩
        cost, bool directed = true){
        graph[u].push_back(e.size());
        e.push_back(Edge(u, v, cap, cost));
        graph[v].push_back(e.size());
        e.push_back(Edge(v, u, 0, -cost));
        if(!directed)
            add(v, u, cap, cost, true);
    }
    pair<long long, long long> getMinCostFlow(int ↩
       _s, int _t){
        s = _s; t = _t;
        flow = 0, cost = 0;
        while(SPFA()){
            flow += sendFlow(t, 1LL<<62);
        }
        return make_pair(flow, cost);
    }
    bool SPFA(){
        parent.assign(n, -1);
```

```
        dist.assign(n, 1LL<<62);        dist[s] = 0;
        vector<int> queuetime(n, 0);    queuetime[s] = 1;
        vector<bool> inqueue(n, 0);     inqueue[s] = true;
        queue<int> q;                   q.push(s);
        bool negativecycle = false;
        while(!q.empty() && !negativecycle){
            int u = q.front(); q.pop(); inqueue[u] = false;
            for(int i = 0; i < graph[u].size(); i++){
                int eIdx = graph[u][i];
                int v = e[eIdx].v, w = e[eIdx].cost, cap = e[eIdx].cap;
                if(dist[u] + w < dist[v] && cap > 0){
                    dist[v] = dist[u] + w;
                    parent[v] = eIdx;
                    if(!inqueue[v]){
                        q.push(v);
                        queuetime[v]++;
                        inqueue[v] = true;
                        if(queuetime[v] == n+2){
                            negativecycle = true;
                            break;
                        }
                    }
                }
            }
        }
        return dist[t] != (1LL<<62);
    }
    long long sendFlow(int v, long long curFlow){
        if(parent[v] == -1)
            return curFlow;
        int eIdx = parent[v];
        int u = e[eIdx].u, w = e[eIdx].cost;
        long long f = sendFlow(u, min(curFlow, e[eIdx].cap));
        cost += f*w;
        e[eIdx].cap -= f;
        e[eIdx^1].cap += f;
        return f;
    }
};
int source=2*n+1;
int sink=2*n+2;
MinimumCostMaximumFlow mcmf(id+10);
mcmf.add(source,i,1,k);
cout<<mcmf.getMinCostFlow(source,sink).second<<endl;
```

4.7 General Unweighted Maximum Matching (Edmonds' algorithm)

```
// Unweighted general matching.
// Vertices are numbered from 1 to V.
// G is an adjlist.
// G[x][0] contains the number of neighbours of x.
// The neigbours are then stored in G[x][1] .. G[x][G[x][0]].
// Mate[x] will contain the matching node for x.
// V and E are the number of edges and vertices.
// Slow Version (2x on random graphs) of Gabow's implementation
// of Edmonds' algorithm (O(V^3)).
const int MAXV = 250;
int G[MAXV][MAXV];
int VLabel[MAXV];
int  Queue[MAXV];
int   Mate[MAXV];
int   Save[MAXV];
int   Used[MAXV];
int    Up, Down;
int          V;

void ReMatch(int x, int y)
{
  int m = Mate[x]; Mate[x] = y;
  if (Mate[m] == x)
    {
      if (VLabel[x] <= V)
        {
          Mate[m] = VLabel[x];
          ReMatch(VLabel[x], m);
        }
      else
        {
          int a = 1 + (VLabel[x] - V - 1) / V;
          int b = 1 + (VLabel[x] - V - 1) % V;
          ReMatch(a, b); ReMatch(b, a);
        }
    }
}

void Traverse(int x)
{
  for (int i = 1; i <= V; i++) Save[i] = Mate[i];
  ReMatch(x, x);
  for (int i = 1; i <= V; i++)
    {
```

```
            if (Mate[i] != Save[i]) Used[i]++;
            Mate[i] = Save[i];
        }
}

void ReLabel(int x, int y)
{
    for (int i = 1; i <= V; i++) Used[i] = 0;
    Traverse(x); Traverse(y);
    for (int i = 1; i <= V; i++)
        {
            if (Used[i] == 1 && VLabel[i] < 0)
                {
                    VLabel[i] = V + x + (y - 1) * V;
                    Queue[Up++] = i;
                }
        }
}

// Call this after constructing G
void Solve()
{
    for (int i = 1; i <= V; i++)
        if (Mate[i] == 0)
            {
                for (int j = 1; j <= V; j++) VLabel[j] = ↩
                    -1;
                VLabel[i] = 0; Down = 1; Up = 1; Queue[Up↩
                    ++] = i;
                while (Down != Up)
                    {
                        int x = Queue[Down++];
                        for (int p = 1; p <= G[x][0]; p++)
                            {
                                int y = G[x][p];
                                if (Mate[y] == 0 && i != y)
                                    {
                                        Mate[y] = x; ReMatch(x, y);
                                        Down = Up; break;
                                    }
                                if (VLabel[y] >= 0)
                                    {
                                        ReLabel(x, y);
                                        continue;
                                    }
                                if (VLabel[Mate[y]] < 0)
                                    {
                                        VLabel[Mate[y]] = x;
                                        Queue[Up++] = Mate[y];
                                    }
                            }
                    }
            }
}

// Call this after Solve(). Returns number of edges↩
    in matching (half the number of matched ↩
   vertices)
int Size()
{
    int Count = 0;
    for (int i = 1; i <= V; i++)
        if (Mate[i] > i) Count++;
    return Count;
}
```

4.8 König's Theorem (Text)

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover. To exhibit the vertex cover:

1. Find a maximum matching
2. Change each edge **used** in the matching into a directed edge from **right to left**
3. Change each edge **not used** in the matching into a directed edge from **left to right**
4. Compute the set T of all vertices reachable from unmatched vertices on the left (including themselves)
5. The vertex cover consists of all vertices on the right that are **in** T, and all vertices on the left that are **not in** T

4.9 Minimum Edge Cover (Text)

If a minimum edge cover contains C edges, and a maximum matching contains M edges, then $C + M = |V|$. To obtain the edge cover, start with a maximum matching, and then, for every vertex not matched, just select some edge incident upon it and add it to the edge set.

5 Data Structures

5.1 Persistent Segment Tree

```
struct node{
 int coun;
 node *l,*r ;
 node(int coun,node *l,node *r):
  coun(coun),l(l),r(r){}
  node *inser(int l,int r,int pos) ;
};
node* node::inser(int l,int r,int pos){
 if(l<=pos && pos<=r){
  if(l==r){
   return new node(this->coun+1,NULL,NULL);
  }
  int mid=(l+r)>>1;
  return new node(this->coun+1,this->l->inser(l,mid,pos),this->r->inser(mid+1,r,pos));
 }
 return this;
}
int query(node *lef,node *rig,int cc,int s,int e){
 if(s==e)
   return s;
 int co=rig->l->coun-lef->l->coun;
 int mid=(s+e)>>1;
 if(co>=cc)
  return query(lef->l,rig->l,cc,s,mid);
 return query(lef->r,rig->r,cc-co,mid+1,e);
}
node *null=new node(0,NULL,NULL);
node *root[100100];
map<int,int> m;
int mm[100100];
int arr[100100];
int main(){
 ios::sync_with_stdio(false);cin.tie(0);
 int n,mmm;cin>>n>>mmm;
 null->l=null->r=null;
 root[0]=null;
 for(int i=1;i<=n;i++) cin>>arr[i],m[arr[i]]=1 ;
 int maxy=-1;
 for(auto itr:m){
  m[itr.x]=++maxy;
  mm[maxy]=itr.x;
 }
 for(int i=1;i<=n;i++)
  root[i]=root[i-1]->inser(0,maxy,m[arr[i]]);
 while(mmm-->0){
  int i,j,k;cin>>i>>j>>k;
  cout<<mm[query(root[i-1],root[j],k,0,maxy)]<<"\n";
 }
 return 0;
}
```

5.2 BIT- Point Update + Range Sum

```
// Binary indexed tree supporting binary search.
struct BIT {
    int n;
    vector<int> bit;
    // BIT can be thought of as having entries f[1], ..., f[n]
    // which are 0-initialized
    BIT(int n):n(n), bit(n+1) {}
    // returns f[1] + ... + f[idx-1]
    // precondition idx <= n+1
    int read(int idx) {
        idx--;
        int res = 0;
        while (idx > 0) {
            res += bit[idx];
            idx -= idx & -idx;
        }
        return res;
    }
    // returns f[idx1] + ... + f[idx2-1]
    // precondition idx1 <= idx2 <= n+1
    int read2(int idx1, int idx2) {
        return read(idx2) - read(idx1);
    }
    // adds val to f[idx]
    // precondition 1 <= idx <= n (there is no element 0!)
    void update(int idx, int val) {
        while (idx <= n) {
            bit[idx] += val;
            idx += idx & -idx;
        }
    }
    // returns smallest positive idx such that read(idx) >= target
    int lower_bound(int target) {
        if (target <= 0) return 1;
        int pwr = 1; while (2*pwr <= n) pwr*=2;
```

```
        int idx = 0; int tot = 0;
        for (; pwr; pwr >>= 1) {
            if (idx+pwr > n) continue;
            if (tot + bit[idx+pwr] < target) {
                tot += bit[idx+=pwr];
            }
        }
        return idx+2;
    }
    // returns smallest positive idx such that read↩
       (idx) > target
    int upper_bound(int target) {
        if (target < 0) return 1;
        int pwr = 1; while (2*pwr <= n) pwr*=2;
        int idx = 0; int tot = 0;
        for (; pwr; pwr >>= 1) {
            if (idx+pwr > n) continue;
            if (tot + bit[idx+pwr] <= target) {
                tot += bit[idx+=pwr];
            }
        }
        return idx+2;
    }
};
```

5.3 BIT- Range Update + Range Sum

```
// BIT with range updates, inspired by Petr ↩
   Mitrichev
struct BIT {
    int n;
    vector<int> slope;
    vector<int> intercept;
    // BIT can be thought of as having entries f↩
       [1], ..., f[n]
    // which are 0-initialized
    BIT(int n): n(n), slope(n+1), intercept(n+1) {}
    // returns f[1] + ... + f[idx-1]
    // precondition idx <= n+1
    int query(int idx) {
        int m = 0, b = 0;
        for (int i = idx-1; i > 0; i -= i&-i) {
            m += slope[i];
            b += intercept[i];
        }
        return m*idx + b;
    }
    // adds amt to f[i] for i in [idx1, idx2)
    // precondition 1 <= idx1 <= idx2 <= n+1 (you ↩
       can't update element 0)
    void update(int idx1, int idx2, int amt) {
        for (int i = idx1; i <= n; i += i&-i) {
            slope[i] += amt;
            intercept[i] -= idx1*amt;
        }
        for (int i = idx2; i <= n; i += i&-i) {
            slope[i] -= amt;
            intercept[i] += idx2*amt;
        }
    }
};
update(ft, p, v):
  for (; p <= N; p += p&(-p))
    ft[p] += v

# Add v to A[a...b]
update(a, b, v):
  update(B1, a, v)
  update(B1, b + 1, -v)
  update(B2, a, v * (a-1))
  update(B2, b + 1, -v * b)

query(ft, b):
  sum = 0
  for(; b > 0; b -= b&(-b))
    sum += ft[b]
  return sum

# Return sum A[1...b]
query(b):
  return query(B1, b) * b - query(B2, b)

# Return sum A[a...b]
query(a, b):
return query(b) - query(a-1)
```

5.4 BIT- 2D

```
void update(int x , int y , int val){
    while (x <= max_x){
        updatey(x , y , val);
        // this function should update array tree[x↩
           ]
        x += (x & -x);
    }
}
```

```
void updatey(int x , int y , int val){
    while (y <= max_y){
        tree[x][y] += val;
        y += (y & -y);
    }
}
void update(int x , int y , int val){
    int y1;
    while (x <= max_x){
        y1 = y;
        while (y1 <= max_y){
            tree[x][y1] += val;
            y1 += (y1 & -y1);
        }
        x += (x & -x);
    }
}

int getSum(int BIT[][N+1], int x, int y)
{
    int sum = 0;

    for(; x > 0; x -= x&-x)
    {
        // This loop sum through all the 1D BIT
        // inside the array of 1D BIT = BIT[x]
        for(; y > 0; y -= y&-y)
        {
            sum += BIT[x][y];
        }
    }
    return sum;
}
```

6 Math

6.1 Convex Hull

```
struct point{
 int x, y;
 point(int _x = 0, int _y = 0){
  x = _x, y = _y;
 }
 friend bool operator < (point a, point b){
  return (a.x == b.x) ? (a.y < b.y) : (a.x < b.x);
 }
};
point pt[2*Max], hull[2*Max];
//Here idx is the new length of the hull
int idx=0,cur;
inline long area(point a, point b, point c){
 return (b.x - a.x) * 1LL * (c.y - a.y) - (b.y - a.↩
    y) * 1LL * (c.x - a.x);
}
inline long dist(point a, point b){
 return (a.x - b.x) * 1LL * (a.x - b.x) + (a.y - b.↩
    y) * 1LL * (a.y - b.y);
}
inline bool is_right(point a, point b){
 int dx = (b.x - a.x);
 int dy = (b.y - a.y);
 return (dx > 0) || (dx == 0 && dy > 0);
}
inline bool compare(point b, point c){
 long det = area(pt[1], b, c);
 if(det == 0){

  if(is_right(pt[1], b) != is_right(pt[1], c))
   return is_right(pt[1], b);
  return (dist(pt[1], b) < dist(pt[1], c));
 }
 return (det > 0);
}
void convexHull(){
 int min_x = pt[1].x, min_y = pt[1].y, min_idx = 1;
 for(int i = 2; i <= cur; i++){
  if(pt[i].y < min_y || (pt[i].y == min_y && pt[i].↩
     x < min_x)){
   min_x = pt[i].x;
   min_y = pt[i].y;
   min_idx = i;
  }
 }
 swap(pt[1], pt[min_idx]);
 sort(pt + 2, pt + 1 + cur, compare);
 idx = 2;
 hull[1] = pt[1], hull[2] = pt[2];
 for(int i = 3 ; i <= cur ; i++){
  while(idx>=2 && (area(hull[idx - 1], hull[idx], ↩
     pt[i]) <= 0)) idx-- ;
  hull[++idx] = pt[i] ;
 }
}
```

6.2 FFT

```
//"root" is the primitive root such that
// root^n=1 modulo mod, where n = 2^k and
// root_i is the inverse of root
// FFT function takes an array L and a boolean
// parameter invert which tells whether to take
// inverse fourier transform or not,gives a new
// array which is the fourier/inverse transform of ↩
   L,in
// the modulo field mod, the size of the array is
// n, which is a perfect power of 2.
// Note the transform is NOT inplace
long[] FFT(long L[],boolean invert){
 int n = L.length ;
 L = Arrays.copyOf(L,n) ;
 for(int i = 1,j = 0 ; i<n ; i++) {
        int bit = n>>1 ;
        for(; j>=bit ; bit>>=1) j-=bit ;
        j+=bit ;
        if(i<j){
         long tmp = L[i] ;
         L[i]=L[j] ; L[j]=tmp ;
        }
    }
 for(int m=2 ; m<=n ; m<<=1){
  long wlen = invert ? root_i : root ;
  for(long i=m ; i<n ; i<<=1)
   wlen = (wlen*wlen)%mod ;
  long w=1 ;
  for(int i=0 ; i<m/2 ; i++){
   for(int k=i ; k<n ; k+=m){
    long u = L[k] ;
    long v = (w*L[k+m/2])%mod ;
    L[k]=(u+v)%mod ;
    L[k+m/2]=(u-v+mod)%mod ;
   }
   w = (w*wlen)%mod ;
  }
 }
 if(invert){
  long ninv = Mod.d(1,n) ;
   for(int i=0 ; i<n ; i++)
   L[i]=(L[i]*ninv)%mod ;
 }
 return L ;
}
```

6.3 Convex Hull Trick

```
mylist hull(mylist pts){
 int n = pts.size() ;
 if(n<2) return pts ;
 Collections.sort(pts,new Comparator<pair>(){
  public int compare(pair p1,pair p2){
   if(p1.x!=p2.x) return Double.compare(p1.x,p2.x) ↩
      ;
   return Double.compare(p2.y,p1.y) ;
  }
 }) ;
 mylist h = new mylist() ;
 h.add(pts.get(0)) ; h.add(pts.get(1)) ;
 int idx=1 ;
 for(int i=2 ; i<n ; i++){
  pair p = pts.get(i) ;
  while(idx>0){
   if(isOriented(h.get(idx-1),h.get(idx),p))
    break ;
   else
    h.remove(idx--) ;
  }
  h.add(p) ;
  idx++ ;
 }
 while(idx>0 && h.get(idx).x==h.get(idx-1).x) h.↩
    remove(idx--) ;
 Collections.reverse(h) ;
 return h ;
}
public boolean isOriented(pair p1,pair p2,pair p3){
 double val = ((p2.y-p1.y)*(p3.x-p2.x))-((p2.x-p1.x↩
    )*(p3.y-p2.y)) ;
 return val>=0 ;
}
```

6.4 Miscellaneous Geometry

```
// C++ routines for computational geometry.

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
```

```cpp
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
// if the projection doesn't lie on the segment, returns closest vertex
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}
```

```cpp
// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// determine if c and d are on same side of line passing through a and b
bool OnSameSide(PT a, PT b, PT c, PT d) {
  return cross(c-a, c-b) * cross(d-a, d-b) > 0;
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
```

```
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-↩
     b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex ↩
   polygon (by William
// Randolph Franklin); returns 1 for strictly ↩
   interior points, 0 for
// strictly exterior points, and 0 or 1 for the ↩
   remaining points.
// Note that it is possible to convert this into an↩
    *exact* test using
// integer arithmetic by taking care of the ↩
   division appropriately
// (making sure to deal with signs properly) and ↩
   then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i↩
         ].y) / (p[j].y - p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a ↩
   polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.↩
       size()], q), q) < EPS)
      return true;
    return false;
}

// compute intersection of line through points a ↩
   and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c,↩
    double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a ↩
   with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, ↩
   double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ↩
     ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (↩
   possibly nonconvex)
// polygon, assuming that the coordinates are ↩
   listed in a clockwise or
// counterclockwise fashion.  Note that the ↩
   centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i↩
       ].y);
  }
  return c / scale;
}
```

```
// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}
```

6.5 Gaussian elimination for square matrices of full rank; finds inverses and determinants

```
// Gauss-Jordan elimination with full pivoting.
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//           A MUST BE INVERTIBLE!
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
    if (fabs(a[pj][pk]) < EPS) { return 0; }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}
```

7 Number Theory Reference

7.1 Fast factorization (Pollard rho) and primality testing (Rabin–Miller)

```
typedef long long unsigned int llui;
typedef long long int lli;
typedef long double float64;

llui mul_mod(llui a, llui b, llui m){
   llui y = (llui)((float64)a*(float64)b/m+(float64↩
      )1/2);
   y = y * m;
   llui x = a * b;
   llui r = x - y;
   if ( (lli)r < 0 ){
      r = r + m; y = y - 1;
   }
   return r;
}

llui C,a,b;
llui gcd(){
   llui c;
   if(a>b){
      c = a; a = b; b = c;
   }
   while(1){
      if(a == 1LL) return 1LL;
      if(a == 0 || a == b) return b;
      c = a; a = b%a;
      b = c;
   }
}

llui f(llui a, llui b){
   llui tmp;
   tmp = mul_mod(a,a,b);
   tmp+=C; tmp%=b;
   return tmp;
}

llui pollard(llui n){
   if(!(n&1)) return 2;
   C=0;
   llui iteracoes = 0;
   while(iteracoes <= 1000){
      llui x,y,d;
      x = y = 2; d = 1;
      while(d == 1){
         x = f(x,n);
         y = f(f(y,n),n);
         llui m = (x>y)?(x-y):(y-x);
         a = m; b = n; d = gcd();
      }
      if(d != n)
         return d;
      iteracoes++; C = rand();
   }
```

```
   return 1;
}

llui pot(llui a, llui b, llui c){
   if(b == 0) return 1;
   if(b == 1) return a%c;
   llui resp = pot(a,b>>1,c);
   resp = mul_mod(resp,resp,c);
   if(b&1)
      resp = mul_mod(resp,a,c);
   return resp;
}

// Rabin-Miller primality testing algorithm
bool isPrime(llui n){
   llui d = n-1;
   llui s = 0;
   if(n <=3 || n == 5) return true;
   if(!(n&1)) return false;
   while(!(d&1)){ s++; d>>=1; }
   for(llui i = 0;i<32;i++){
      llui a = rand();
      a <<=32;
      a+=rand();
      a%=(n-3); a+=2;
      llui x = pot(a,d,n);
      if(x == 1 || x == n-1) continue;
      for(llui j = 1;j<= s-1;j++){
         x = mul_mod(x,x,n);
         if(x == 1) return false;
         if(x == n-1)break;
      }
      if(x != n-1) return false;
   }
   return true;
}
map<llui,int> factors;
// Precondition: factors is an empty map, n is a ↩
   positive integer
// Postcondition: factors[p] is the exponent of p ↩
   in prime factorization of n
void fact(llui n){
   if(!isPrime(n)){
      llui fac = pollard(n);
      fact(n/fac); fact(fac);
   }else{
      map<llui,int>::iterator it;
      it = factors.find(n);
      if(it != factors.end()){
         (*it).second++;
      }else{
         factors[n] = 1;
      }
```

```
  }
}
```

7.2 Modular arithmetic and linear Diophantine solver

```
// This is a collection of useful code for solving ↩
   problems that
// involve modular linear equations.  Note that all↩
    of the
// algorithms described here work on nonnegative ↩
   integers.

typedef vector<int> VI;
typedef pair<int,int> PII;

// return a % b (positive value)
int mod(int a, int b) {
  return ((a%b)+b)%b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
  int tmp;
  while(b){a%=b; tmp=a; a=b; b=tmp;}
  return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
  return a/gcd(a,b)*b;
}

// returns d = gcd(a,b); finds x,y such that d = ax↩
    + by
int extended_euclid(int a, int b, int &x, int &y) {
  int xx = y = 0;
  int yy = x = 1;
  while (b) {
    int q = a/b;
    int t = b; b = a%b; a = t;
    t = xx; xx = x-q*xx; x = t;
    t = yy; yy = y-q*yy; y = t;
  }
  return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int↩
    n) {
  int x, y;
  VI solutions;
  int d = extended_euclid(a, n, x, y);
  if (!(b%d)) {
    x = mod (x*(b/d), n);
    for (int i = 0; i < d; i++)
      solutions.push_back(mod(x + i*(n/d), n));
  }
  return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 ↩
   on failure
int mod_inverse(int a, int n) {
  int x, y;
  int d = extended_euclid(a, n, x, y);
  if (d > 1) return -1;
  return mod(x,n);
}

// Chinese remainder theorem (special case): find z↩
    such that
// z % x = a, z % y = b.  Here, z is unique modulo ↩
   M = lcm(x,y).
// Return (z,M).  On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, ↩
   int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i.  Note that the ↩
   solution is
// unique modulo M = lcm_i (x[i]).  Return (z,M).  ↩
   On
// failure, M = -1.  Note that we do not require ↩
   the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI↩
    &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.first, ret.↩
       second, x[i], a[i]);
    if (ret.second == -1) break;
  }
  return ret;
}
```

```
// computes x and y such that ax + by = c; on failure, x = y =-1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
  int d = gcd(a,b);
  if (c%d) {
    x = y = -1;
  } else {
    x = c/d * mod_inverse(a/d, b/d);
    y = (c-a*x)/b;
  }
}
```

7.3 Polynomial Coefficients (Text)

$(x_1 + x_2 + ... + x_k)^n = \sum_{c_1+c_2+...+c_k=n} \frac{n!}{c_1!c_2!...c_k!} x_1^{c_1} x_2^{c_2} ... x_k^{c_k}$

7.4 Möbius Function (Text)

$$\mu(n) = \begin{cases} 0 & n \text{ not squarefree} \\ 1 & n \text{ squarefree w/ even no. of prime factors} \\ 1 & n \text{ squarefree w/ odd no. of prime factors} \end{cases}$$

Note that $\mu(a)\mu(b) = \mu(ab)$ for a, b relatively prime Also $\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{otherwise} \end{cases}$

Möbius Inversion If $g(n) = \sum_{d|n} f(d)$ for all $n \geq 1$, then $f(n) = \sum_{d|n} \mu(d) g(n/d)$ for all $n \geq 1$.

7.5 Burnside's Lemma (Text)

The number of orbits of a set X under the group action G equals the average number of elements of X fixed by the elements of G.

Here's an example. Consider a square of $2n$ times $2n$ cells. How many ways are there to color it into X colors, up to rotations and/or reflections? Here, the group has only 8 elements (rotations by 0, 90, 180 and 270 degrees, reflections over two diagonals, over a vertical line and over a horizontal line). Every coloring stays itself after rotating by 0 degrees, so that rotation has X^{4n^2} fixed points. Rotation by 180 degrees and reflections over a horizonal/vertical line split all cells in pairs that must be of the same color for a coloring to be unaffected by such rotation/reflection, thus there exist X^{2n^2} such colorings for each of them. Rotations by 90 and 270 degrees split cells in groups of four, thus yielding X^{n^2} fixed colorings. Reflections over diagonals split cells into $2n$ groups of 1 (the diagonal itself) and $2n^2 - n$ groups of 2 (all remaining cells), thus yielding $X^{2n^2-n+2n} = X^{2n^2+n}$ unaffected colorings. So, the answer is $(X^{4n^2} + 3X^{2n^2} + 2X^{n^2} + 2X^{2n^2+n})/8$.

8 Miscellaneous

8.1 2-SAT

```
// 2-SAT solver based on Kosaraju's algorithm.
// Variables are 0-based. Positive variables are stored in vertices 2n, corresponding negative variables in 2n+1
// TODO: This is quite slow (3x-4x slower than Gabow's algorithm)
struct TwoSat {
 int n;
 vector<vector<int> > adj, radj, scc;
 vector<int> sid, vis, val;
 stack<int> stk;
 int scnt;

 // n: number of variables, including negations
 TwoSat(int n): n(n), adj(n), radj(n), sid(n), vis(n), val(n, -1) {}

 // adds an implication
 void impl(int x, int y) { adj[x].push_back(y); radj[y].push_back(x); }
 // adds a disjunction
 void vee(int x, int y) { impl(x^1, y); impl(y^1, x); }
 // forces variables to be equal
```

```
void eq(int x, int y) { impl(x, y); impl(y, x); ↩
    impl(x^1, y^1); impl(y^1, x^1); }
// forces variable to be true
void tru(int x) { impl(x^1, x); }

void dfs1(int x) {
 if (vis[x]++) return;
 for (int i = 0; i < adj[x].size(); i++) {
  dfs1(adj[x][i]);
 }
 stk.push(x);
}

void dfs2(int x) {
 if (!vis[x]) return; vis[x] = 0;
 sid[x] = scnt; scc.back().push_back(x);
 for (int i = 0; i < radj[x].size(); i++) {
  dfs2(radj[x][i]);
 }
}

// returns true if satisfiable, false otherwise
// on completion, val[x] is the assigned value of ↩
    variable x
// note, val[x] = 0 implies val[x^1] = 1
bool two_sat() {
 scnt = 0;
 for (int i = 0; i < n; i++) {
  dfs1(i);
 }
 while (!stk.empty()) {
  int v = stk.top(); stk.pop();
  if (vis[v]) {
   scc.push_back(vector<int>());
   dfs2(v);
   scnt++;
  }
 }
 for (int i = 0; i < n; i += 2) {
  if (sid[i] == sid[i+1]) return false;
 }
 vector<int> must(scnt);
 for (int i = 0; i < scnt; i++) {
  for (int j = 0; j < scc[i].size(); j++) {
   val[scc[i][j]] = must[i];
   must[sid[scc[i][j]^1]] = !must[i];
  }
 }
 return true;
}
};
```

8.2 Stable Marriage Problem (Gale–Shapley algorithm)

```
// Gale-Shapley algorithm for the stable marriage ↩
    problem.
// madj[i][j] is the jth highest ranked woman for ↩
    man i.
// fpref[i][j] is the rank woman i assigns to man j↩
    .
// Returns a pair of vectors (mpart, fpart), where ↩
    mpart[i] gives the partner of man i, and fpart ↩
    is analogous
pair<vector<int>, vector<int> > stable_marriage(↩
    vector<vector<int> >& madj, vector<vector<int> ↩
    >& fpref) {
 int n = madj.size();
 vector<int> mpart(n, -1), fpart(n, -1);
 vector<int> midx(n);
 queue<int> mfree;
 for (int i = 0; i < n; i++) {
  mfree.push(i);
 }
 while (!mfree.empty()) {
  int m = mfree.front(); mfree.pop();
  int f = madj[m][midx[m]++];
  if (fpart[f] == -1) {
   mpart[m] = f; fpart[f] = m;
  } else if (fpref[f][m] < fpref[f][fpart[f]]) {
   mpart[fpart[f]] = -1; mfree.push(fpart[f]);
   mpart[m] = f; fpart[f] = m;
  } else {
   mfree.push(m);
  }
 }
 return make_pair(mpart, fpart);
}
```

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