Capacitated UAV Vehicle Routing Problem

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Abstract - **In recent years, many e-commerce companies like Amazon, Google, FedEx, etc., are shifting towards unmanned vehicle delivery services. As much as there are some advantages of using unmanned vehicles such as it saves time and cost and it is environmentally friendly, there are several constraints of this service such as limited battery supply, limited weight package delivery, etc. In this paper, we will solve the vehicle routing problem of UAVs using Savings algorithm.**

Index Terms – Vehicle Routing Problem, unmanned vehicles, limited weight.

INTRODUCTION

Logistics as a science has its origins in the military area; the transportation of weapons, food, and men at the service was coordinated through it. With the passage of time, the concept began to be implemented in the business world, and for a long time, the logistics function was regarded as a routine, operational, and required activity for transporting goods from seller to buyer (Ballou, 2004). Later, beginning in the 1950s, the world experienced a cycle of expansion and steady demand increase, causing production and sales capacity to exceed the companies' ability to distribute products. As a result of the low compliance throughout those years, delivering orders on time became a difficulty. Then, in 1980, the notion of response time was developed, which is a combination of physical distribution and material management; experts understood that the faster the company's response time to the client, the higher its profitability. As concepts evolved, so did methods, and businesses sought to become more efficient. As a result, they extended their logistics activities1 and discovered that one of the most expensive items is transportation, which accounts for between 10% and 20% of the ultimate cost of a product or service (Toth andVigo, 2002) The basic modes of transportation include mode selection, route design, vehicle programming, and shipment consolidation, albeit they are expressed in a variety of ways (Ballou, 2004). The problem is usually referred to as the vehicle routing difficulty when it comes to route design (VRP). Both companies that own the trans-port service as part of their processes and companies that provide the services seek to optimise resources within the route selection process, because a good selection saves time, resources such as fuel, fleet maintenance, salaries, and improvements in service indicators as a promise of product delivery, among other things. The VRP can be considered as the natural extension of the TSP, in the sense that unlike the TSP, in the VRP we consider that the vehicles, or the agents in charge of providing a service to the nodes, have a limited capacity; therefore, most likely the entire route cannot be made through a single route, with a single vehicle that leaves and returns to the storage, traveling all the nodes, but to respect the restriction of the limited capacity of the vehicles so. In general, several routes are required, or what is the same, the solution of the VRP will be a set of Hamiltonian cycles that sstart from the deposit and such that each node is traveled only once.

Problems related to the distribution of goods between warehouses and customers are generally considered as vehicle routing problems (VRPs). The VRP was first proposed by Dantzig and Ramser in 1959 to model how a fleet of homogeneous trucks could serve the demand for oil from a number of gas stations from a central hub with a minimum travel distance. Five years later, Clarke and Wright added more practical restrictions to VRPs in which the delivery of goods to each customer must occur within a set of bounds. This type of problem model became known as the VRP with time windows (VRPTW), which is one of the most widely studied topics in the field of operations research. However, current VRP models differ significantly from those introduced by Dantzig and Ramser and Clarke and Wright, because they aim to incorporate real-world complexities. In the recent years, many e-commerce companies like Amazon, Google, FedEx etc., are shifting towards unmanned vehicle delivery services.

RELATED WORK

There are several advantages of using unmanned vehicles such as it saves time and cost, it is environmentally friendly. However, there are several constraints of this service such as limited battery supply, limited weight package delivery, etc. These delivery drones travel along the delivery trucks. In paper [1], the author has discussed hybrid genetic algorithm to solve vehicle routing problem. By implementing m-TSP algorithm for vehicles and a Parallel Machine Scheduling problem for drone, they have implemented solution to find the path to all the customers in an optimized way. DHULKEFL, TERZİOĞLU, DURDU

[2] has discussed a methodology to resolve the UAVs routing problem using classic algorithm "Dijkstra". They had considered certain obstacles in routes such as building of different dimensions.

The Vehicle-Routing Problem (VRP) is a common name for problems involving the construction of a set of routes for a fleet of vehicles. The vehicles start their routes at a depot, call at customers, to whom they deliver goods, and return to the depot. The objective function for the vehicle-routing problem is to minimize costs by finding optimal routes, which are usually the shortest routes. The classic VRP (also known as Capacitated Vehicle Routing Problem – CVRP) is defined on a graph $G = (V, E)$, where $V = \{v0, v1,\ldots,vn\}$ is a set of vertices and $E=\{(vi, vj):i_ij,vi,vj\ \}$ is a set of edges. Vertex v0 represents a depot, and the other vertex represents customers. A cost function, Cij, is associated with each edge of E. Each customer has a non-negative demand, di. A fleet of m identical vehicles of capacity Q are based at the depot. VRP consists of designing a set of, at most, m delivery or collection routes, such that:

i. Each route starts and ends at the depot.
ii. Each customer is called at exactly once and by only one vehicle.
iii. The total demand on each route does not exceed Q.
iv. The total routing cost is minimized. For the CVRP, the cost function, Cij, represents edge distances, and therefore the optimal solution is a set of routes with the shortest length.

VRP can be considered a generalization of the "Traveling-Salesman Problem". which is an NP-Hard problem and, therefore, cannot be solved optimally within a reasonable running time. Since CVRP was first introduced in 1959, a large number of algorithms for solving it, based on various heuristics and meta-heuristics, have been developed. Also, extensions to the basic VRP were developed as well, aiming to produce more realistic models, usually by adding more constraints to the original problem.

TRAVELING SALEMAN PROBLEM

The Traveling Salesman Problem (TSP) is a classical combinatorial optimization problem, which is simple to state but very difficult to solve. The problem is to find the shortest tour through a set of N vertices so that each vertex is visited exactly once. This problem is known to be NP-hard and cannot be solved exactly in polynomial time. Let us consider an example describing the Traveling salesman problem. We have a set of four cities A, B, C, and D. The distances between the cities are also given to us. Figure 1 illustrates the collection of the cities and their distances among each other. Here (4-1)! that is 3! route can be generated. The tour with A→B→C→D→A will be the optimal route for given problem.

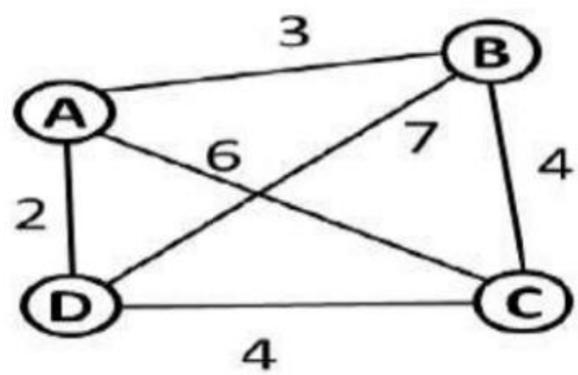

Figure 1: Traveling salesman problem

The popular solutions for TSP are:
The brute force approach: It Calculates and compares all possible solutions and determines the unique one.
The branch and bound method: The problem is divided into multiple sub problems. Where we solve each sub problem. A single problem may affect possible solutions of subsequent sub-problems.
Nearest neighbor method: visit the nearest destination and return to the source node after visiting all the nodes.

VEHICLE ROUTING PROBLEM

The vehicle routing problem (VRP) entails determining a set of routes for a fleet of vehicles departing from one or more warehouses in order to meet the demands of numerous geographically scattered clients as shown in figure 2. The VRP goal is to meet customer demand by maximizing some target, which is usually the overall cost of the routes, which is influenced by traffic congestion in large cities, cargo truck energy consumption, and other factors. Since the VRP problem is a generalization of the TSP, and because the TSP belongs to the NP-hard problem class, we can conclude that the VRP is also an NP-hard problem. The VRP model is divided into several categories based on the various factors that can be added or considered. The CVRP capability reflects the most basic version (for the acronym of capacitated vehicle routing problem).

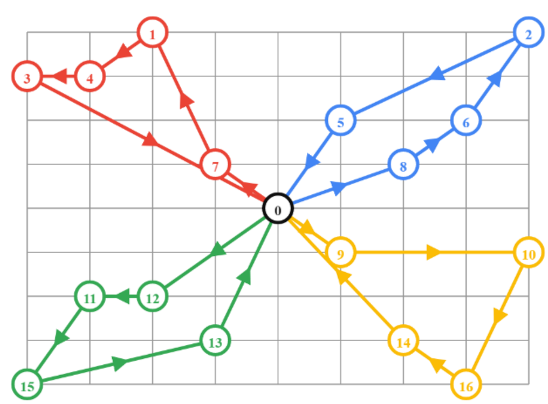

Figure 2: Vehicle Routing Problem

The CVRP is predicated on the following assumptions:
The vehicle fleet is homogenous, which means that all freight trucks have the same features:

i. The demand is predictable since it is known in advance, i.e., the quantity to be given to each client is known.
ii. Each truck will deliver the entire shipment to clients, preventing the distribution of fractional or partial loads that will be finished by another vehicle later.
iii. The load capacity of every vehicle in the fleet is the same.
iv. There is only one beginning location for the vehicles, which is referred to as a central warehouse.
v. Vehicles have capacity limits that are known ahead of time.

The travel salesman problem is NP-Complete, and this is the ‘Generalization’ of it which means it is also NP-Complete. TSP has commanded so much attention because it’s so easy to describe yet so difficult to solve. In fact, TSP belongs to the class of combinatorial optimization problems known as NP-complete. This means that TSP is classified as NP-hard because it has no “quick” solution and the complexity of calculating the best route will increase when you add more destinations to the problem. So that’s why VRP is also NP-Complete. Both are decision problems and even though they are difficult to solve, it has been exactly 63 years since George Danzig has published the classic truck dispatching problem in 1959 in which he introduced the VRP. So, we can define VRP as ‘when the vehicle capacity limitation is concerned (i.e., the total customer demand requirement of each route of the vehicle cannot exceed the vehicle capacity) this becomes a VRP. Here, the goal is to find optimal routes for multiple vehicles visiting a set of locations. So, when there's only one vehicle, it reduces to the Traveling Salesperson Problem.

TSP vs VRP:

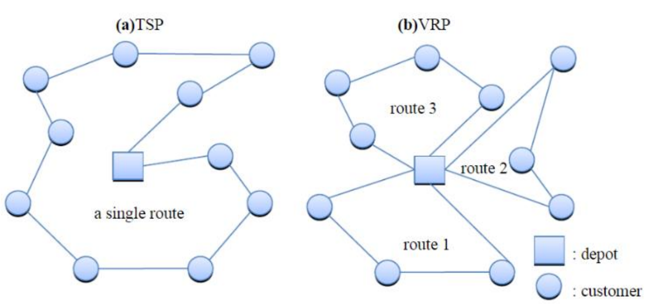

Figure 3: TSP VS VRP

The difference is illustrated in Figure3, in which the TSP is a single-route node-service-combination problem with no vehicle capacity limitation and a VRP is a multiple-route node-service-combination problem with vehicle capacity limitation.
Moreover, if the objective is simply to minimize total distance with single vehicle, it is a TSP. if the number of vehicles to be used is specified, it is a m-TSP. It can thus be seen that TSP's can be viewed as special cases of VRP's. By inference, VRP's can be expected to be more difficult than TSP's as far as optimum solutions are concerned. Indeed, although several versions of VRP's have been formulated as mathematical programming problems by various investigators, the largest vehicle routing problems of any complexity that have been solved exactly reportedly involved less than 30 delivery points by contrast, the heuristic approaches that we use i.e., savings algorithm can be used even with thousands of delivery points.

Variants of VRP:

i. Vehicle Routing Problem with Time Window (VRPTW):
It is where a delivery needs to be completed within a time window, as in you want the arrival time to be in that time window so these are known as Vehicle Routing Problem with Time Window (VRPTW). Routing solution when customers are promised delivery time-window (say 12:00-14:00)
ii. Capacitated Vehicle Routing Problem (CVRP):
The vehicle which can only carry maximum number of shipments or maximum volume so when you have products like that, here you use Capacitated Vehicle Routing Problem (CVRP). Maximum capacity of each vehicle/FE in terms of number of shipments.
iii. Capacitated Vehicle Routing Problem with Time Windows (CVRPTW):
when there are both constraints, customers with time slot as well as capacity constraint, you have the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW). Both constraints applied simultaneously.

SAVINGS ALGORITHM

One of the most well-known heuristics for VRP is the Clarke and Wright savings algorithm. It was based on [Clarke and Wright 1964], and it applies to situations where the number of cars isn't fixed (it's a decision variable), and it works for both directed and undirected problems. Let's say there are n demand sites in a certain area, each of which wants a certain amount of weight Qi I = 1, 2,..., n) of items delivered to it (goods are assumed indistinguishable but for their weight). The products in question are kept at D, a depot that also houses a fleet of cars. Maximum weight capacity and route time (or distance) limits are the same for all vehicles. All of their routes must begin and end at the depot, D. The goal is to find a set of delivery routes that minimize the overall distance covered by the entire fleet from the depot, D, to the various demand points. The weights Qi of the quantities needed are considered to be smaller than the vehicles' maximum weight capacity, and we demand that the entire quantity Qi demanded at a particular point I be delivered by a single vehicle (i.e., we do not allow for the possibility that one third, say, of Qi will be delivered by one vehicle and the remaining two thirds by another).

The phrases "supply" and "quantity supplied" can obviously be changed for "demand" and "quantity required," making the depot a collection point. As a result, the VRP is equally applicable to the collection of solid waste from a specified set of points as it is to parcel delivery to a specific set of sites. in some VRP applications, the maximum weight or maximum route-time constraints may be loosened. Both, though, frequently play a part. For example, in the newspaper delivery situation just discussed, one constraint was that all deliveries to newsstands had to be accomplished within an hour of press time, in addition to the maximum quantity of newspapers that a vehicle could carry. When neither constraint holds, the VRP becomes a travelling salesman problem: a 1-TSP [from if the goal is to reduce overall distance; an m-TSP if the number of cars to be employed is stated. As a result, TSPs can be thought of as special examples of VRPs.

Clarke and Wright's "savings" method is by far the most well-known approach to the VRP problem. Its core concept is pretty straightforward. Consider a depot with D demand points and n demand points. Assume that the first solution to the VRP consists of dispatching one vehicle to each of the n demand sites utilising n vehicles. This solution's overall tour length is plainly 2 d. (D, i). If we now employ a single vehicle to serve two points, say I and j, in a single trip, the total distance travelled is:

$$s(i,j) = d(D,i) + d(D,j) - d(i,j)$$

The "savings" coming from combining points I and j into a single tour are denoted by the quantity s(i, j). Combining I and j in a single trip gets increasingly favourable as s(i, j) grows greater. I and j, on the other hand, cannot be joined if the resulting tour violates one or more of the VRP's requirements.
The algorithm steps can be defined as:

Step 1.
Compute the savings s(i, j) = d(D, i) + d(D, j) - d(i, j) for every pair (i, j) of demand points.

Step 2.
Rank and list the savings s(i, j) in decreasing order of magnitude. The "savings list" is created as a result of this. Begin processing the savings list with the topmost element (the greatest s(i, j)). Include link (i, j) in a route for the savings s(i, j) under consideration if no route restrictions will be violated by including (i, j) in a route, and if:

i. Either neither i nor j have been assigned to a route yet, in which case a new route with both I and j is created.

Step 3.

ii. Or exactly one of the two points (i or j) has already been included in an existing route and that point is not interior to that route (a point is interior to a route if it is not adjacent to the depot D in the order of traversal of points), in which case the link (i, j) is added to that same route.
iii. Or, if both i and j have already been included in two separate current routes and neither point is located within its own route, the two routes are merged.

Step 4.
Return to Step 3 and process the next entry in the savings list s(i, j) if the list has not been exhausted; otherwise, stop: the VRP solution is made up of the routes produced in Step 3. (Any points that were not assigned to a route during Step 3 must be supplied by a vehicle route that starts at depot D and goes to the unassigned point before returning to D.)

MODELING OF PROBLEM

Let,
N = number of nodes (1 - depot, 2, ..., n - hospitals)
d_{ij} = distance from point i to point j
D_i = demand of client i
C = capacity of each drone
K = No. of vehicles
x_{ij} = 1 if a drone goes from node i to node j (binary)
x_{ij} = 0 otherwise
f_{ij} = number of units in a drone going from node i to node j

Objective Function:

Minimize $i \in N \; j \in N \; d_{ij} \, x_{ij}$

Subject to
$i \in V \; x_{ij}=1$
$j \in V \; x_{ij}=1$
$i \in V \; x_{i0}=K$
$i \in V \; x_{0j}=K$
$0 \leq f_{ij} \leq C x_i$
$0 \leq D_k \leq 150$
$X_{ij} \in \{0,1\}$
$K>=0$

The objective is to find optimal routes for multiple vehicles visiting a set of locations. But what does it mean by "optimal routes" for a VRP? One answer is the routes with the least total distance. Therefore, the distance is being minimized here. As, xij is a binary variable, so if the node is in route it will be 1 else 0.
Constraints 1 and 2 state that exactly one arc enters and exactly one leaves each vertex associated with a customer, respectively.
Constraints 3 and 4 say that the number of vehicles leaving the depot is the same as the number entering.

Constraints 5 is the capacity cut constraints, which impose that the demand on each route must not exceed the vehicle capacity.It is assumed that the weights of the quantities demanded are less than the maximum weight capacity of the vehicles and we require that the whole quantity demanded at a given point is delivered by a single vehicle (i.e., we do not allow for the possibility that one third, say, of weight will be delivered by one vehicle and the remaining two thirds by another).
Constraints 6 is for the distance of the overall route that we have given in our input, that it should not exceed 150.
Finally, the last constraints are the integrality constraint.

IMPLEMENTATION

We have used Google OR tools for implementation and considered the delivery of medical supplies from one hospital to other, for this purpose we have generated an input distance matrix as given in table 4 where source: FedEx ship center is our depot and other rows and columns are hospitals. The distance from one point to other is mentioned.

	Source	Portage District General Hospital	Morris General Hospital	Bethesda Regional Health Centre	Ste. Anne Hospital	Beausejour Hospital	Grace Hospital	Seven Oaks General Hospital	Concordia Hospital	Cymbalisty Residential Care Home	Misericordia Health Centre	Robert Steen Hospital	St. Boniface Hospital: Emergency Room	HSC Women's Hospital
Source: FedEx Ship Centre	0	84	72	72	56	75	9	11	14	4	8	12	10	6
Portage District General Hospital	84	0	132	157	141	148	75	99	110	79	84	89	87	85
Morris General Hospital	72	132	0	69	79	126	66	77	74	87	123	88	80	100
Bethesda Regional Health Centre	72	157	69	0	20	70	92	76	67	71	67	64	63	67
Ste. Anne Hospital	56	141	79	20	0	53	60	61	47	55	51	48	51	51
Beausejour Hospital	75	148	126	70	53	0	80	60	79	66	68	67	60	62
Grace Hospital	9	75	66	92	60	80	0	18	18	5	10	14	13	11
Seven Oaks General Hospital	11	99	77	76	61	60	18	0	12	12	10	14	11	7
Concordia Hospital	14	110	74	67	47	79	18	12	0	14	10	14	8	9
Cymbalisty Residential Care Home	4	79	87	71	55	66	5	12	14	0	5	8	8	7
Misericordia Health Centre	8	84	123	67	51	68	10	10	10	5	0	5	4	3
Robert Steen Hospital	12	89	88	64	48	67	14	14	14	8	5	0	5	7
St. Boniface Hospital: Emergency Room	10	87	80	63	47	60	13	11	8	8	4	5	0	4
HSC Women's Hospital	6	85	100	67	51	62	11	7	9	7	3	7	4	0

Table 4: Input distance Matrix

For the capacity constraints, we have assumed some values as shown below in table 5. These are the requirements from the hospitals which needs to be delivered along with the maximum capacity constraint of 25KG.

Capacity constraints	Drone can carry upto - 25kgs
Demands	
Portage District General Hospital	8
Morris General Hospital	11
Bethesda Regional Health Centre	5
Ste. Anne Hospital	6
Beausejour Hospital	7
Grace Hospital	9
Seven Oaks General Hospital	4
Concordia Hospital	14
Cymbalisty Residential Care Home	2
Misericordia Health Centre	1
Robert Steen Hospital	3
St. Boniface Hospital: Emergency Room	10
HSC Women's Hospital	20

Table 5: Capacity Constraints

The input distance matrix given to the code as shown in figure 11 below. The distance calculated in table 4 are giving as an input to create the data model for distance matrix in the code.

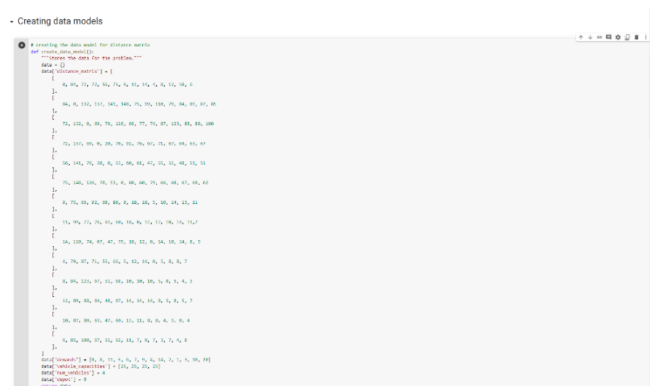

Figure 11: Input code

Whereas the output received from code is shown in figure 12 where the routes for all vehicles are displayed. Every vehicle will start from a depot and return to it after giving the delivery. The routes are optimized based on the distance.

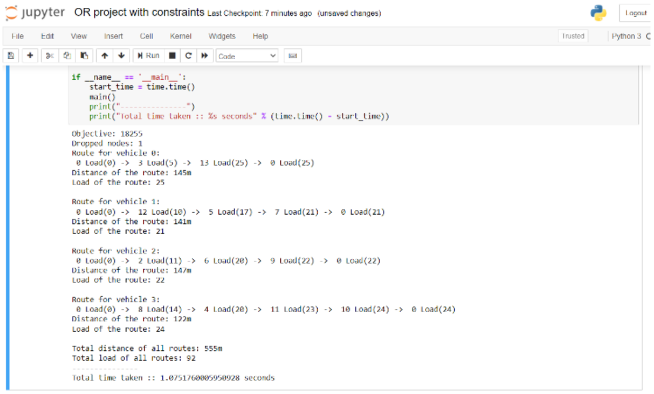

Figure 12: Output received

The output Network graph is:

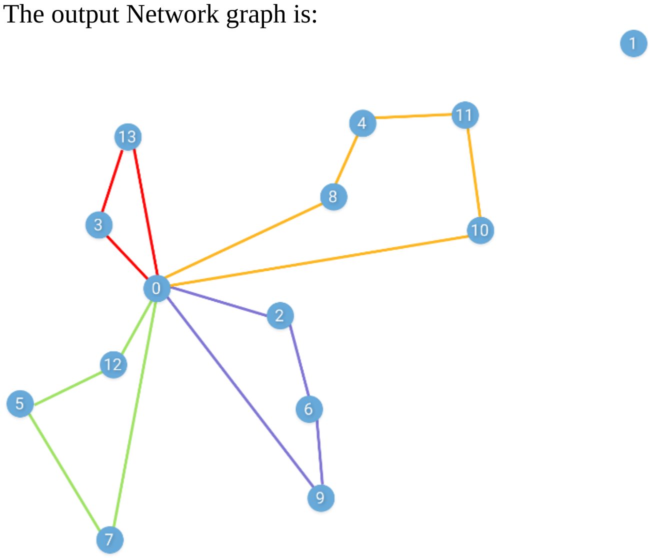

Figure 13: Network graph

OUTPUT ANALYSIS

For the analysis, we have compared TSP and VRP to get an idea about the feasibility and distance covered, it given below in figure 13.

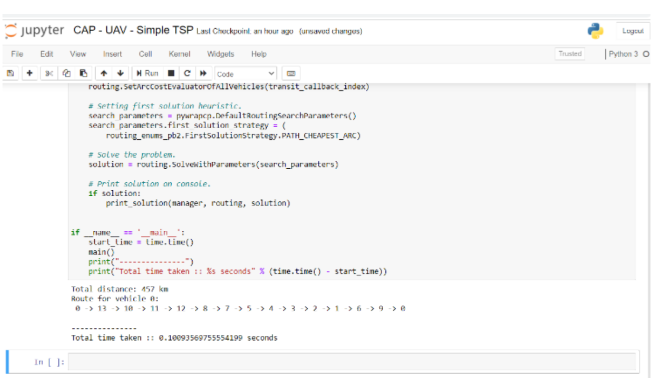

Figure 13: TRP and VRP Comparison

Since TSP has 1 vehicle, the maximum distance is covered by it which is practically not possible for a drone due to it's limited capacity and power. Whereas VRP is distributing into routes based on the constraints given to it thus this is an effective way of delivering goods.

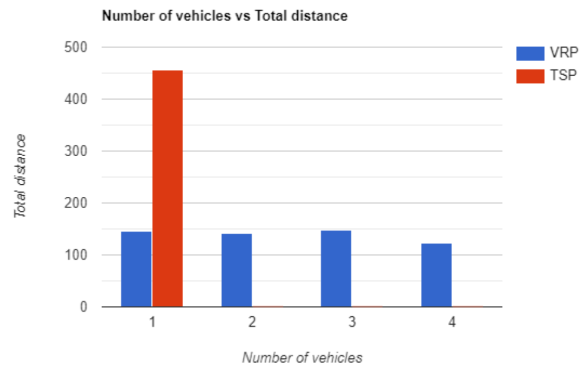

Figure 14: Output Analysis

Another analysis is made with different number of nodes and different number of drones in order to understand how it is impacting the total distance and other constraints.
First experiment is carried out with 14 nodes as show in figure 15.

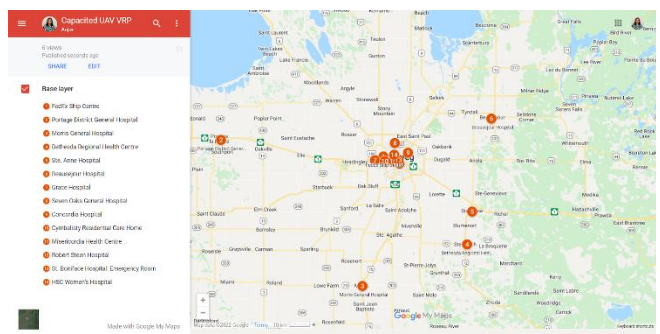

Figure 15: Output Analysis with nodes=14

Second experiment is carried out with 10 nodes as show in figure 16.

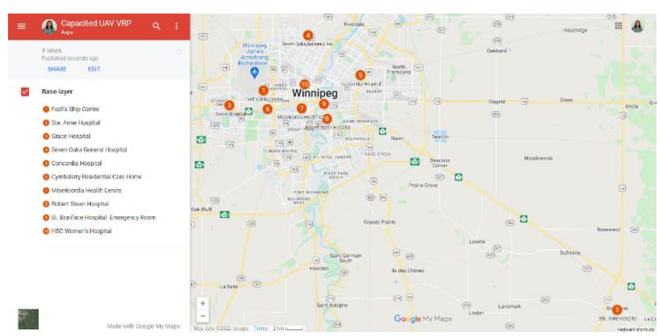

Figure 16: Output Analysis with nodes=10

First experiment is carried out with 7 nodes as show in figure 17.

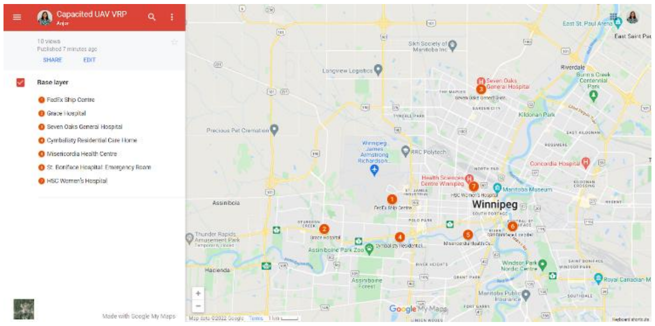

Figure 17: Output Analysis with nodes=17

So, the overall comparison of VRP with different nodes and different number of drones is given in figure 18. It is showing number of nodes and total distance covered by each done whereas #2 represents two number of drones, #3 represents 3 number of drone and #4 represents four number of drone respectively. Hence it can be shown that if we have a less number of drones, then less routes are covering and missing a lot of nodes. And with the increase in drones, the distance covered is getting increased as well and covering the maximum nodes as well.

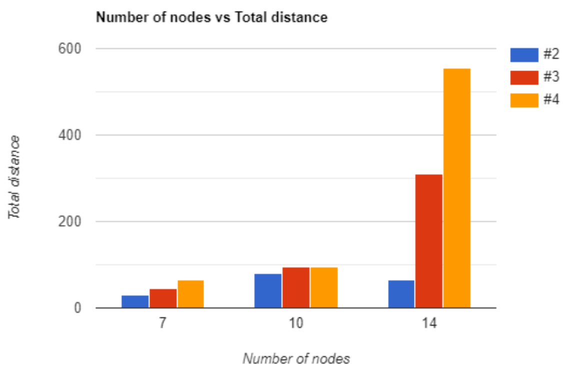

Figure 18: Overall Output Analysis

CONCLUSION AND FUTURE WORK

VRP is a critical issue in transportation and a key component of supply chain management. Numerous earlier research have studied and solved difficulties with the basic VRP and its derivatives. However, in VRP problems, elements like vehicle speed, vehicle type selection, vehicle load, and alternate path selection are frequently disregarded, despite the fact that these factors frequently present in real-world situations. This paper demonstrates the basic working of savings algorithm and compare the results of TSP and VRP. There is a lot which can be done in VRP like additional constraints can be applied. Common challenges such as the time window problem, penalties, problems involving several depots, and the truck and tail routing difficulty, for example, might be added to improve the algorithm's applicability to reality. In addition, future research will focus on a multi-objective problem that considers carbon footprint minimization, routing time, and routing distance.

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