

$$= \int_0^1 y^3 dy + \int_0^1 (y^2 + y) dy$$

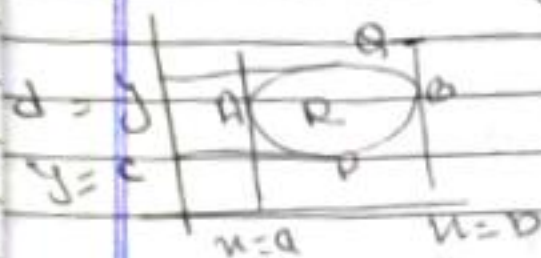
$$= \left[\frac{y^4}{4} + \frac{y^3}{3} + \frac{y^2}{2} \right]_0^1 = \left[\frac{1}{4} + \frac{1}{3} + \frac{1}{2} \right] = \frac{3}{4}$$

(*) Green's theorem's proof and statement

Statement: If $f_1(x, y)$ and $f_2(x, y)$, $\frac{\partial f_1}{\partial x}$ and $\frac{\partial f_2}{\partial y}$ be continuous functions defined in a region R of the xy -plane bounded by a simple closed curve C ,

$$\text{then } \oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx dy$$

$$= \int_C (f_1 dx + f_2 dy)$$



Proof:

consider such a region R of the xy -plane bounded by any simple closed curve (path) C which is cut by a plane line parallel to the y -coordinate axis so that the curve C is broken into two curves.

Let the region R be bounded by the line $y=c$, $y=d$, $x=g_1(y)$, $x=g_2(y)$ the curves A, B, P, Q .

Case - I: $P=Q$ (0)

$$= \int_0^1 y^2 \cdot 2y dy + (y^2 + y) dy$$

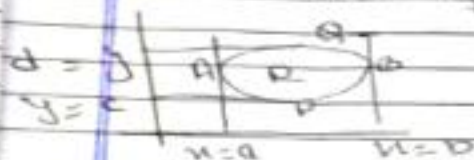
$$= \left[\frac{2y^5}{5} + \frac{y^3}{3} + \frac{y^2}{2} \right]_0^1 = \left[\frac{2}{5} + \frac{1}{3} + \frac{1}{2} \right] = \frac{37}{30}$$

⑧ Green's theorem & proof and statement

Statement 8 If $f_1(x, y)$ and $f_2(x, y)$, f_1 and f_2 be continuous functions defined in a region R of the xy plane bounded by a closed curve C ,

$$\text{then } \oint_C \mathbf{r} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx dy$$

$$= \int_C (f_1 dx + f_2 dy)$$



Proof 8

consider such a region R of the xy -plane bounded by any simple closed curve (path) C which is cut by a plane line parallel to the x -ordinate axis & that the curve C is broken into two curves.

Let the region R be bounded by the line $y=c$ & $y=d$ & the curves A, B, P, Q .

Case - I & $P = Q$ (non)

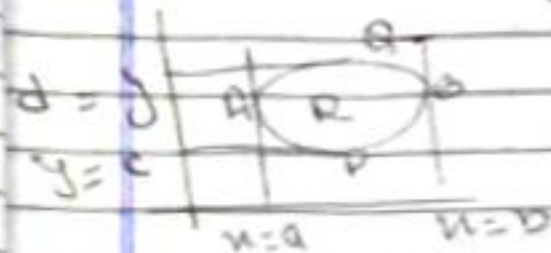
$$= \int_0^1 y^2 \cdot 2y dy + (y^2 + y) dy$$

$$= \left[\frac{2y^5}{5} + \frac{y^3}{3} + \frac{y^2}{2} \right]_0^1 = \left[\frac{2}{5} + \frac{1}{3} + \frac{1}{2} \right] = \frac{17}{30}$$

(*) Green's theorem & proof and statement

Statement 8 If $f_1(x, y)$ and $f_2(x, y)$ i.e. and f_2 be continuous functions defined in a region R of the xy plane bounded by a closed curve C ,

$$\text{then } \oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(f_2 \frac{\partial f_1}{\partial x} - f_1 \frac{\partial f_2}{\partial y} \right) dx dy$$



$$= \int_C (f_1 dx + f_2 dy)$$

Proof 3

consider such a region R of the xy -plane bounded by any simple closed curve (path) C which is cut by a plane line parallel to the co-ordinate axes $P-Q$ that the curve C is broken into two curves.

Let the region R be bounded by the line $y=g(x)$ and the curves A, B, P, Q .

Case - I 8 $P=Q$ (0)

$$= \int_0^1 y^2 \cdot 2y dy + (y^2 + y) dy$$

$$= \left[\frac{2y^5}{5} + \frac{y^3}{3} + \frac{y^2}{2} \right]_0^1 = \left[\frac{2}{5} + \frac{1}{3} + \frac{1}{2} \right] = \frac{37}{30}$$

(*) Green's theorem 3 proof and statement

Statement 3 If $f_1(x, y)$ and $f_2(x, y)$ are continuous functions defined in a region R of the xy -plane bounded by a closed curve C , then

$$\oint_C f_1 dx + f_2 dy = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \right) dx dy$$

$$= \int_C (f_1 dx + f_2 dy)$$

Proof 3

consider such a region R of the xy -plane bounded by any simple closed curve (path) C which is cut by a plane line parallel to the co-ordinate axes so that the curve C is broken into two curves.

Let the region R be bounded by the line $y = c$ and the curves A, B, P, Q .

Case - I 3 $P = Q$ (closed curve)

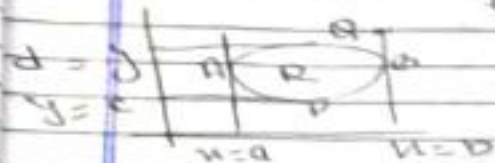
$$= \int_0^1 y^2 dy + (y^2 + 1) dy$$

$$= \left[\frac{y^3}{3} + \frac{y^2}{2} + y \right]_0^1 = \left[\frac{1}{3} + \frac{1}{2} + 1 \right] = \frac{11}{6}$$

(5) Green's Theorem & proof and statement

Statement If $f_1(x, y)$ and $f_2(x, y)$ i.e. P and Q be continuous functions defined in a region R of the xy -plane bounded by a closed curve C ,

$$\text{then } \oint_C P dx + Q dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$



$$= \int_C (f_1 dx + f_2 dy)$$

Proof 3

consider with a region R of the xy -plane bounded by any simple closed curve (path) C which is cut by a plane line parallel to the x -coordinate axis & that the curve C is broken into two curves.

Let the region R be bounded by the line $y=c$ & the curves A, B, C, D .

Case - I & $P = Q = 0$



