Optimal Strategic Quantizer Design via Dynamic Programming

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Abstract

This paper is concerned with the quantization setting where the encoder and the decoder have misaligned objectives. We first motivate the problem via a toy example which demonstrates the intricacies of the strategic quantization problem, specifically shows that iterative optimization of the decoder and the encoder mappings may not converge to a local optimum. As a remedy, we propose a dynamic programming based optimal optimization method, inspired by the early works in the quantization theory. We then extend our approach to variable-rate (entropy-coded) quantization. We finally present numerical results obtained via the proposed algorithm.

Introduction

Consider the following quantization problem: An encoder observes a realization of source $X \in \mathcal{X}$ generated from a probability distribution μ and maps it into a message in a discrete set $Z \in \mathcal{Z}$, via a quantizer $Q : \mathcal{X} \to \mathcal{Z}$ subject to a cardinality constraint $|\mathcal{Z}| \leq M$. The decoder generates an action (reconstruction) $Y \in \mathcal{Y}$ based on the message Z it receives. The objectives of the encoder and the decoder are to minimize $D_E \triangleq \mathbb{E}\{\eta_e(X,Y)\}$ and $D_D \triangleq \mathbb{E}\{\eta_d(X,Y)\}$ respectively, where distortion functions are misaligned, i.e., $\eta_e \neq \eta_d$. The encoder designs Q *ex-ante* i.e., before seeing the realization of X, based only on the statistics and the objectives. The distortion functions η_e and η_d, the shared prior μ, and the quantizer Q are common knowledge. Then, what is Q at the equilibrium?

This problem, which we call *strategic quantization*, is the main subject of this paper. This game setting, without any quantization constraints, i.e., if M is arbitrarily large, is known as the Bayesian Persuasion [1].

Two most relevant studies to this work are by Dughmi and Xu [2] and by Aybas and Turkel [3]. In [2] authors address the algorithmic (complexity) aspect of the problem and show that it is NP-hard in general. In [3], authors present an exhaustive search based algorithm along with several complementary theoretical results.

Here, we consider the strategic quantization problem described above, via the lens of quantization theory. We first demonstrate, via a toy example, that the standard method of optimization by iteratively enforcing necessary conditions of optimality, known as the Lloyd-Max-I method[4], may not yield a locally optimal strategic quantizer. Inspired by the early work in quantization theory [5, 6], we then develop dynamic programming based algorithms that yield the globally optimal solutions of this problem.

We note, in passing, that the quantizers arise as equilibrium strategies in a related but distinctly different information transmission game: the cheap talk [7]. In this setting, the encoder determines its mapping after it sees the realization, i.e., the objective of the encoder is to minimize a functional of the form $\eta_e(X,Y)$ rather than $\mathbb{E}\{\eta_e(X,Y)\}$. This difference stems from the lack of the commitment assumption in the cheap talk setting.

Preliminaries

Notation

In this paper, random variables are denoted by capital letters, their sample values are denoted by the respective lower case letters, and their alphabets are denoted by the respective calligraphic letters. This alphabet may be finite, countably infinite, or a continuum, like an interval $[a,b] \subset \mathbb{R}$. The expectation operator is denoted by $\mathbb{E}\{\cdot\}$. The uniform distribution over an interval $[a,b]$ and the scalar Gaussian with mean a, variance b are denoted by $U[a,b]$ and $\mathbb{N}(a,b)$.

Strategic Quantization Problem

The encoder and the decoder have distortion functions $\eta_e(x,y)$ and $\eta_d(x,y)$ respectively that depend on the source realization $x \in \mathcal{X}$ and action taken by the decoder $y \in \mathcal{Y}$, where $\mathcal{X}$ is a compact metric space and $\mathcal{Y}$ is compact. The set of Borel probabilities over $\mathcal{X}$, a compact metric space in weak* topology, is denoted by $\Delta(\mathcal{X})$. The agents share a prior belief about X, $\mu_0 \in \Delta(\mathcal{X})$ which is common knowledge. A strategic (fixed-rate) quantizer is a measurable mapping $Q : \mathcal{X} \to \mathcal{Z}$ where $\mathcal{Z}$ denotes the compact metric space of messages that satisfies and $|\mathcal{Z}| \leq M$ for a given quantization resolution $M \in \mathbb{Z}^+$. Any quantizer induces a distribution τ over the messages given μ_0. A variable rate constraint is in the form of $-\int \log \tau d(\tau) \leq H_0$ instead of the fixed rate constraint $|\mathcal{Z}| \leq M$.

The timing of the game is as follows. First, the encoder designs a quantizer Q, based on the common knowledge, and announces to the decoder. Then, nature randomly selects a realization x from $\mathcal{X}$ according to the common prior μ_0. The encoder generates a message $z \in \mathcal{Z}$ through the announced quantizer and transmits to the decoder noiselessly. The decoder, upon observing $z \in \mathcal{Z}$, takes an action $r \in \mathcal{R}$. The solution concept sought after here is the encoder-preferred perfect Bayesian equilibrium.

A Toy Example

Consider $X \sim U[-1,1]$, $M=3$, with $\eta_e(x,y) = (x^3-y)^2$ and $\eta_d(x,y) = (x-y)^2$. The boundaries are parametrized as $[-1,r_1), [r_1,r_2)$, and $[r_2,1]$ for some $r_1, r_2 \in [-1,1]$ that satisfy $r_2 \geq r_1$. Then, the decoder reconstructions (actions) are:

$$y_1 = \frac{\int_{-1}^{r_1} \frac{1}{2} a da}{\int_{-1}^{r_1} \frac{1}{2} da} = \frac{1+r_1}{2},\ y_2 = \frac{\int_{r_1}^{r_2} \frac{1}{2} a da}{\int_{r_1}^{r_2} \frac{1}{2} da} = \frac{r_2+r_1}{2},\ y_3 = \frac{\int_{r_2}^{1} \frac{1}{2} a da}{\int_{r_2}^{1} \frac{1}{2} da} = \frac{r_2+1}{2}. \quad (1)$$

The cost function is then

$$J(r_1, r_2) = \int_{-1}^{r_1} (u^3 - \frac{1 + r_1}{2})^2 du + \int_{r_1}^{r_2} (u^3 - \frac{r_2 + r_1}{2})^2 du + \int_{r_2}^{1} (u^3 - \frac{1 + r_2}{2})^2 du \quad (2)$$

Applying the KKT optimality conditions $\frac{\partial J}{\partial r_1} = \frac{\partial J}{\partial r_2} = 0$ yields, after straightforward algebra, that the only non-degenerate solution is $r_1 = -0.7403$ and $r_2 = 0.7403$. We note that iteratively enforcing optimality conditions for the encoder and the decoder (as in Lloyd-Max algorithm) results in $r_1 \uparrow 0$ and $r_2 \downarrow 0$ since each iteration pushes the boundaries towards origin. Hence, straightforward enforcement of optimality conditions does not yield a locally optimal solution, since any perturbation of $r_1 = r_2 = 0$ would be preferred by the encoder to $r_1 = r_2 = 0$.

Dynamic Programming Based Algorithms

Although we have formulated the problem in its most general form above, we next focus on discrete sources to develop algorithms. We note however that while the algorithms presented here are designed for discrete sources, they can be readily generalized to continuous settings.

Let X be a discrete source taking values from $\mathcal{X} = \{x_1, ..., x_K\}, x_1 < x_2 < ... < x_K$, with a probability mass function $p_X(x_k) = p_k, 1 \leq k \leq K$. For ease of notation, we define $\mathcal{X}_t = \{x_1, \ldots, x_K, x_{K+1}\}$, where $x_{K+1} = x_K + 1$. The set $\mathcal{X}$ is divided into regions by the encoder as $[r_{m-1}, r_m), 1 \leq m \leq M$ with $r_m \in \mathcal{X}_t, 0 \leq m \leq M, r_0 < r_1 < ... < r_M$ called the decision levels. The quantizer output, called the representative levels for each region of the source, $y \in \{y_m, 1 \leq m \leq M\}$, satisfies

$$Q(x) = y_m, \forall x \in [r_{m-1}, r_m), m = 1 \ldots M. \quad (3)$$

Let $g(\cdot)$ be the indexing function, i.e., $g(x_k) = k, \ \forall x_k \in \mathcal{X}_t$.

The optimal decision levels (chosen by the encoder) and the optimal representation levels (chosen by the decoder) are denoted by r_m^* and y_m^*, respectively. We set $r_0^* = x_1$, and $r_M^* = x_{K+1}$. The combinations of intervals in $[x_1, x_{K+1})$ are given by $\mathcal{S} = \{[\alpha, \beta) : \alpha, \beta \in \mathcal{X}_t, x_1 \leq \alpha < \beta \leq x_{K+1}\}$. The optimal encoder and decoder distortions are given by D_e^* and D_d^*, respectively.

Fixed Rate

The encoder chooses $\{r_m, 0 \leq m \leq M\}$ that minimize $\mathbb{E}\{\eta_e(X, Y)\}$,

$$D_e(r_0, r_1, \ldots, r_M) = \mathbb{E}\{\eta_e(X, Y)\} = \sum_{m=1}^{M} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_e(x_k, y_m) p_k, \quad (4)$$

where y_m are chosen by the decoder to minimize $D_d = \mathbb{E}\{\eta_d(X, Y)\}$,

$$y_m = \underset{\substack{t \in \mathcal{X} \\ r_{m-1} \leq t < r_m}}{\arg\min} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_d(x_k, t) p_k. \quad (5)$$

For each $[\alpha, \beta) \in \mathcal{S}$, the 1-level distortion $D_1(\alpha, \beta)$ is defined as the distortion to the encoder due to quantizing the interval $[\alpha, \beta)$ with one representation level:

$$D_1(\alpha, \beta) \triangleq D_1(x_{g(\alpha)}, x_{g(\beta)}) = \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_e(x_k, y)p_k, \tag{6}$$

where y is given by

$$y = \underset{\substack{t \in \mathcal{X} \\ \alpha \leq t < \beta}}{\arg\min} \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_d(x_k, t)p_k. \tag{7}$$

The m-level distortion for an interval $[\alpha, \beta) \in \mathcal{S}$ due to quantizing the interval with m representation levels, is defined as

$$D_m(\alpha, \beta) \triangleq \min_{\substack{r_1, r_2, \ldots, r_{m-1} \in \mathcal{X} \\ \alpha = r_0 < r_1 < \ldots < r_{m-1} < r_m = \beta}} \sum_{i=1}^{m} \sum_{k=g(r_{i-1})}^{g(r_i)-1} \eta_e(x_k, y_i)p_k, \tag{8}$$

where y_m is given by (7) as

$$y_m = \underset{\substack{t \in \mathcal{X} \\ r_{m-1} \leq t < r_m}}{\arg\min} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_d(x_k, t)p_k. \tag{9}$$

D_m can be written in terms of D_1 as

$$D_m(\alpha, \beta) = \min_{\substack{r_1, r_2, \ldots, r_{m-1} \in \mathcal{X} \\ \alpha = r_0 < r_1 < \ldots < r_{m-1} < r_m = \beta}} \sum_{i=1}^{m} D_1(r_{i-1}, r_i). \tag{10}$$

Substituting $m = M$ in (10) to get the M-level distortion,

$$D_M(\alpha, \beta) = \min_{\substack{r_1, r_2, \ldots, r_{M-1} \in \mathcal{X} \\ \alpha = r_0 < r_1 < \ldots < r_{M-1} < r_M = \beta}} \sum_{i=1}^{M} D_1(r_{i-1}, r_i). \tag{11}$$

The M levels of quantization in the interval $[r_0, r_M)$ can be solved in terms of $M-1$ levels of quantization in the interval $[r_0, r_{M-1})$ and 1 level of quantization in the interval $[r_{M-1}, r_M)$. For this, D_M is written in terms of the $(M-1)^{th}$-level distortion of the interval $[r_0, r_{M-1})$, and 1-level distortion of the interval $[r_{M-1}, r_M)$ as

$$D_M(r_0, r_M) = \min_{\substack{r_{M-1} \in \mathcal{X} \\ r_0 < r_{M-1} < r_M}} \left(D_{M-1}(r_0, r_{M-1}) + D_1(r_{M-1}, r_M)\right). \tag{12}$$

A similar equation can be written for any $m, 1 \leq m \leq M$, for intervals starting with r_0, that is, intervals of the form $[r_0, \alpha), r_0 < \alpha \leq r_M$,

$$D_m(r_0, \alpha) = \min_{\substack{t \in \mathcal{X} \\ r_0 < t < \alpha}} \left(D_{m-1}(r_0, t) + D_1(t, \alpha)\right). \tag{13}$$

The α value from (13) gives the optimal $(m-1)^{th}$ decision level for m-level quantization of the interval $[r_0, \alpha)$, $r_{m-1}(r_0, \alpha)$,

$$r_{m-1}(r_0, \alpha) = \underset{\substack{t \in \mathcal{X} \\ r_0 < t < \alpha}}{\arg\min} [D_{m-1}(r_0, t) + D_1(t, \alpha)]. \tag{14}$$

The algorithm requires a forward pass and a backward pass. During the forward pass, each interval in $\mathcal{S}$ with the first element as r_0, $[r_0, \alpha), \alpha \in \mathcal{X}, r_0 < \alpha \leq r_M$ is quantized into $m = 1, \ldots, M$ regions, and the $(m-1)^{th}$ decision level is given by (14), for each m.

During the backward pass, r^*_{M-1} is found by applying (14) on the interval $[r^*_0, r^*_M)$:

$$r^*_{M-1} = r_{M-1}(r^*_0, r^*_M). \tag{15}$$

This is done iteratively for $[r^*_0, r^*_m), m = M, \ldots, 2$ to get the decision levels $\{r^*_m\}$ as

$$r^*_{m-1} = r_{m-1}(r^*_0, r^*_m). \tag{16}$$

The optimal representative levels, $\{y^*_m\}$ are found using (9),

$$y^*_m = \underset{\substack{t \in \mathcal{X} \\ r_{m-1} \leq t < r_m)}}{\arg\min} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_d(x_k, y_m) p_k \tag{17}$$

The algorithm (presented in Algorithm 1) takes as input the source alphabet $\mathcal{X}$, source distribution $p_X(\cdot)$, the number of levels of quantization M, the encoder distortion function $\eta_e(\cdot, \cdot)$, and the decoder distortion function $\eta_d(\cdot, \cdot)$. The outputs of the algorithm are the decision levels chosen by the encoder $\{r^*_m, 0 \leq m \leq M\}$, and the representative levels chosen by the decoder $\{y^*_m, 1 \leq m \leq M\}$.

We define some notation used in the algorithm here. Let $int([\alpha, \beta), \gamma)$ be defined as the distortion to the encoder in quantizing the interval $[\alpha, \beta)$ to the point γ, and $dec(\alpha, \beta)$ be the representative level that gives the minimum distortion to the decoder in quantizing the interval $[\alpha, \beta)$:

$$int([\alpha, \beta), \gamma) \triangleq \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_e(x_k, \gamma) p_k, \quad dec(\alpha, \beta) \triangleq \underset{\substack{t \in \mathcal{X} \\ \alpha \leq t < \beta}}{\arg\min} \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_d(x_k, t) p_k. \tag{18}$$

Let $dp_t(\alpha, m)$ be the distortion to the encoder in dividing the interval $[r_0, \alpha)$ to $[r_0, t)$ and $[t, \alpha)$, $r_0 < t < \alpha$, where $[r_0, t)$ is to be quantized to $m-1$ levels and $[t, \alpha)$ is to be quantized to 1 level:

$$dp_t(\alpha, m) \triangleq D_{m-1}(r_0, t) + D_1(t, \alpha). \tag{19}$$

Algorithm 1 Fixed Rate Strategic Quantization

Input: $\mathcal{X}, p_X(\cdot), M, \eta_e(\cdot,\cdot), \eta_d(\cdot,\cdot)$

Output: $\{r^*_m, 0 \le m \le M\}$, $\{y^*_m, 1 \le m \le M\}$, D^*_e, D^*_d

Initialization: $r^*_0 = x_1$, $r^*_M = x_{K+1}$

$\mathcal{S} \leftarrow \{[\alpha,\beta) : \alpha,\beta \in \mathcal{X}, x_1 \le \alpha < \beta \le x_{K+1}\}$

for $[\alpha,\beta) \in \mathcal{S}$ **do**

  $y \leftarrow dec(\alpha,\beta)$

  $D_1(\alpha,\beta) \leftarrow int([\alpha,\beta], y)$

end for

for $m \leftarrow 2, 3, ..., M$ **do**

  for $\alpha \in \mathcal{X} \backslash \{x_1\}$ **do**

    $r_{M-1}(r^*_0,\alpha) \leftarrow \underset{\substack{t\in\mathcal{X} \\ r^*_0<t<\alpha}}{\arg\min}(dp_t(\alpha,m))$

    $D_m(r^*_0,\alpha) \leftarrow dp_{r_{M-1}(r^*_0,\alpha)}(\alpha,m)$

  end for

end for

for $m \leftarrow M, M-1, ..., 2$ **do**

  $r^*_{m-1} = r_{m-1}(r^*_0, r^*_m)$

end for

for $m \leftarrow 1, 2, ..., M$ **do**

  $y^*_m \leftarrow dec(r^*_{m-1}, r^*_m)$

end for

$D^*_e \leftarrow \sum_{m=1}^{M} \sum_{k=g(r^*_{m-1})}^{g(r^*_m)-1} \eta_e(x_k, y^*_m) p_k$

$D^*_d \leftarrow \sum_{m=1}^{M} \sum_{k=g(r^*_{m-1})}^{g(r^*_m)-1} \eta_d(x_k, y^*_m) p_k$

Variable Rate

Let $p(\alpha,\beta)$ be the probability that x lies in the interval $[\alpha,\beta)$,

$$p(\alpha,\beta) = \sum_{k=g(\alpha)}^{g(\beta)-1} p_k. \tag{20}$$

Let $prob_m, 1 \le m \le M$ be the probability of the quantizer mapping an input x to y_m with decision levels $\{r_m\}$,

$$prob_m = p(r_{m-1}, r_m). \tag{21}$$

Let $H(r_0, r_1, ..., r_m)$ be the entropy of quantizing to $\{r_i, 0 \le i \le m\}$ decision levels and $H(\alpha,\beta)$ be the entropy of quantizing the interval $[\alpha,\beta)$ to a single level, given by

$$H(r_0, r_1, ..., r_m) = \sum_{i=1}^{m} -prob_i \log_2(prob_i), \quad H(\alpha,\beta) = -p(\alpha,\beta)\log_2(p(\alpha,\beta)). \tag{22}$$

Algorithm 2 Variable Rate Strategic Quantization

Input: $\mathcal{X}, p_X(\cdot), \lambda, \eta_e(\cdot,\cdot), \eta_d(\cdot,\cdot)$
Output: M^*,$\{r_m^*, 0 \le m \le M^*\}$, $\{y_m^*, 1 \le m \le M^*\}$, $H(r_0^*, ..., r_M^*)$, D_e^*, D_d^*
Initialization: $r_0^* = x_1$, $r_M^* = x_{K+1}$, th, $M = 2$
$\mathcal{S} \leftarrow \{[\alpha, \beta) : \alpha, \beta \in \mathcal{X}, x_1 \le \alpha < \beta \le x_{K+1}\}$
for $[\alpha, \beta) \in \mathcal{S}$ **do**
  $y \leftarrow dec(\alpha, \beta)$
  $D_1(\alpha, \beta) \leftarrow int([\alpha, \beta), y) + \lambda H(\alpha, \beta)$
end for
repeat
  for $m \leftarrow 2, 3, ..., M$ **do**
    for $\alpha \in \mathcal{X} \backslash \{x_1\}$ **do**
      $r_{m-1}(r_0^*, \alpha) \leftarrow \underset{\substack{t \in \mathcal{X} \\ r_0^* < t < \alpha}}{\arg\min}(dp_t(\alpha, m))$
      $D_m(r_0^*, \alpha) \leftarrow dp_{r_{m-1}(r_0^*, \alpha)}(\alpha, m)$
    end for
  end for
  for $m \leftarrow M, M-1, ..., 2$ **do**
    $r_{m-1}^* = r_{m-1}(r_0^*, r_m^*)$
  end for
  for $m \leftarrow 1, 2, ..., M$ **do**
    $y_m^* \leftarrow dec(r_{m-1}^*, r_m^*)$
  end for
  $H(r_0^*, r_1^*, ..., r_M^*) \leftarrow \sum_{m=1}^{M} H(r_{m-1}^*, r_m^*)$
  $D(\lambda, M) \leftarrow D_e(r_0^*, r_1^*, \ldots, r_M^*) + \lambda H(r_0^*, r_1^*, ..., r_M^*)$
  $M = M + 1$
until convergence in $D(\lambda, M)$: $\frac{D(\lambda,M) - D(\lambda,M-1)}{D(\lambda,M-1)} < th$
$M^* \leftarrow M$
$D_e^* \leftarrow \sum_{m=1}^{M^*} \sum_{k=g(r_{m-1}^*)}^{g(r_m^*)-1} \eta_e(x_k, y_m^*) p_k$
$D_d^* \leftarrow \sum_{m=1}^{M^*} \sum_{k=g(r_{m-1}^*)}^{g(r_m^*)-1} \eta_d(x_k, y_m^*) p_k$

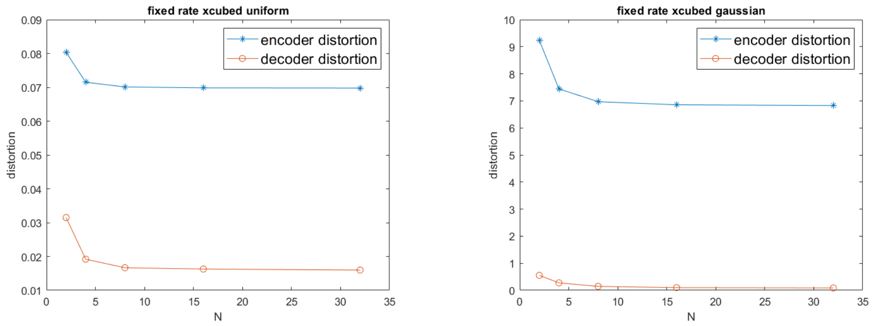
Let λ be the Lagrange parameter. For a given M, the encoder minimizes the following over $\{r_m, 0 \le m \le M\}$

$$D(\lambda, M) = \mathbb{E}\{\eta_e(X, Y)\} + \lambda H(r_0, ..., r_M) = \sum_{m=1}^{M} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_e(x_k, y_m) p_k + \lambda H(r_0, ..., r_M),$$

where y_m are chosen by the decoder:

$$y_m = \underset{\substack{t \in \mathcal{X} \\ r_{m-1} \le t < r_m}}{\arg\min} \sum_{k=g(r_{m-1})}^{g(r_m)-1} \eta_d(x_k, t) p_k. \tag{23}$$

We now redefine the 1-level distortion, $D_1(\alpha, \beta)$ for $[\alpha, \beta) \in \mathcal{S}$ that gives the distortion due to quantizing the interval $[\alpha, \beta)$ with one representation level for the given λ



(a)$X \sim U[\frac{1}{2}, \frac{1}{12}]$ (b)$X \sim \mathbb{N}(0, 1)$

Figure 1: Fixed rate quantization of a uniform and a Gaussian source, for $\eta_e(X,Y) = (X^3 - Y)^2$ and $\eta_d(X,Y) = (X-Y)^2$.

parameter as,

$$D_1(\alpha, \beta) = \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_e(x_k, y) p(\alpha, \beta) + \lambda H(\alpha, \beta), \tag{24}$$

where

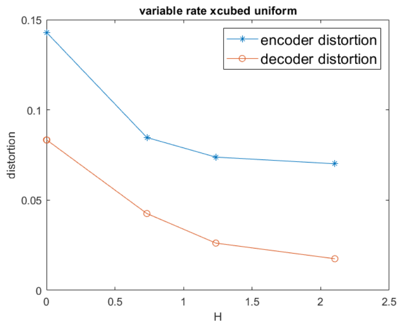
$$y = \arg\min_{\substack{t \in \mathcal{X} \\ \alpha \le t < \beta}} \sum_{k=g(\alpha)}^{g(\beta)-1} \eta_d(x_k, t) p_k. \tag{25}$$

The algorithm proceeds the same as the fixed rate algorithm for the given M. It is iterated over $M = 2, 3\ldots$ until convergence (assuming $D(\lambda, M)$ is monotonic in M) using a threshold parameter th, and the corresponding M is the optimal number of representative levels, M^*, for the given λ.

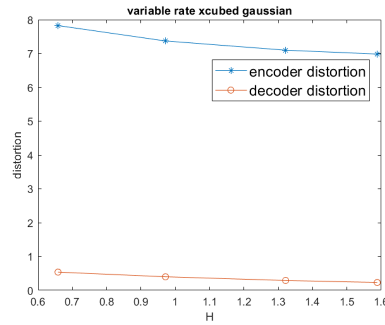
The algorithm (presented in Algorithm 2) takes as input the source alphabet $\mathcal{X}$, source distribution $p_X(\cdot)$, the Lagrange parameter λ, the encoder distortion function $\eta_e(\cdot,\cdot)$, and the decoder distortion function $\eta_d(\cdot,\cdot)$. The outputs of the algorithm are the number of quantization levels M^*, the decision levels chosen by the encoder $\{r_m^*, 0 \le m \le M^*\}$, the representative levels chosen by the decoder $\{y_m^*, 1 \le m \le M^*\}$, the entropy of quantization $H(r_0^*, \ldots, r_{M^*}^*)$, the encoder distortion D_e^*, and the decoder distortion D_d^*.

Numerical Results

We consider Gaussian, $\mathbb{N}(0,1)$ and uniform $U[\frac{1}{2}, \frac{1}{12}]$ sources for $\eta_e(X,Y) = (X^3 - Y)^2$ and $\eta_d(X,Y) = (X-Y)^2$. The encoder and the decoder distortions for fixed and variable rate quantization results are plotted in Figures 1 and 2 respectively. As expected, both distortions monotonically decrease with rate, however, unlike their nonstrategic counterpart, here they stay almost constant as rate increases at high rate region. This is because of the mismatch between the objectives of the encoder and the decoder: even if there was no quantization at all, distortions would not vanish see e.g., [8] for more details.



(a)$X \sim U[\frac{1}{2}, \frac{1}{12}]$



(b)$X \sim \mathbb{N}(0, 1)$

Figure 2: Variable rate quantization of a uniform and a Gaussian source. for $\eta_e(X, Y) = (X^3 - Y)^2$ and $\eta_d(X, Y) = (X - Y)^2$.

Discussion

In this paper, we have developed dynamic programming algorithms for the strategic quantization problem.

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