

Q1 (22 July 2021 Shift 1)

The number of solutions of  $\sin^7 x + \cos^7 x = 1$ ,  $x \in [0, 4\pi]$  is equal to

(1) 11

(2) 7

(3) 5

(4) 9

Q2 (25 July 2021 Shift 1)

The sum of all values of  $x$  in  $[0, 2\pi]$ , for which

$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$ , is equal to :

(1)  $8\pi$ (2)  $11\pi$ (3)  $12\pi$ (4)  $9\pi$

Answer Key

Q1 (3)

Q2 (4)

Q1

$$\sin^7 x \leq \sin^2 x \leq 1 \dots (1)$$

$$\text{and } \cos^7 x \leq \cos^2 x \leq 1 \dots (2)$$

$$\text{also } \sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \text{equality must hold for (1) \& (2)} \Rightarrow \sin^7 x = \sin^2 x \& \cos^7 x = \cos^2 x$$

$$\Rightarrow \sin x = 0 \& \cos x = 1$$

or

$$\cos x = 0 \& \sin x = 1$$

$$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$$

$$\Rightarrow 5 \text{ solutions}$$

Q2

$$(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$$

$$\Rightarrow 2 \sin \frac{5x}{2} \left\{ \cos \frac{3x}{2} + \cos \frac{x}{2} \right\} = 0$$

$$\Rightarrow 2 \sin \frac{5x}{2} \left\{ 2 \cos x \cos \frac{x}{2} \right\} = 0$$

$$2 \sin \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$$

$$\cos \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{So sum} = 6\pi + \pi + 2\pi = 9\pi$$