Q1 (20 July 2021 Shift 2)

Consider a triangle having vertices $A(-2,3), B(1,9)$ and $C(3,8)$. If a line L passing through the circum-centre of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is

Q2 (27 July 2021 Shift 1)

Let P and Q be two distinct points on a circle which has center at $C(2,3)$ and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set $\{P, Q\}$ is equal to

(1) $\{(4,0),(0,6)\}$

(2) $\{(2+2\sqrt{2}, 3-\sqrt{5}),(2-2\sqrt{2}, 3+\sqrt{5})\}$

(3) $\{(2+2\sqrt{2}, 3+\sqrt{5}),(2-2\sqrt{2}, 3-\sqrt{5})\}$

(4) $\{(-1,5),(5,1)\}$

Q3 (27 July 2021 Shift 2)

The point $P(a, b)$ undergoes the following three transformations successively:

(a) reflection about the line $y = x$.

(b) translation through 2 units along the positive direction of x -axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a + b$ is equal to :

(1) 13

(2) 9

(3) 5

(4) 7

Q4 (27 July 2021 Shift 2)

Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point:

(1) $(1, 2)$

(2) $(2, 2)$

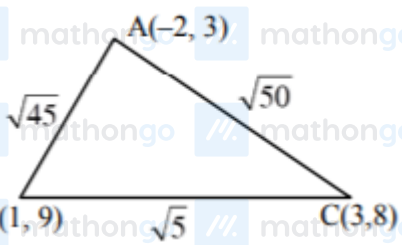
(3) $(2, 1)$

(4) $(1, 3)$

Answer Key

Q1 (9) **Q2** (4) **Q3** (2) **Q4** (2)

Q1



$(\sqrt{50})^2 = (\sqrt{45})^2 + (\sqrt{5})^2$

$\angle B = 90^\circ$

Circum-center $= \left(\frac{1}{2}, \frac{11}{2}\right)$

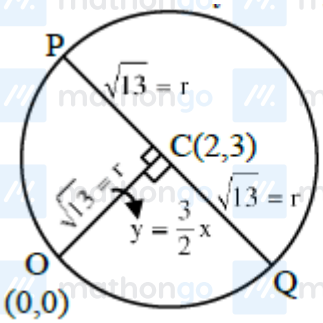
Mid point of BC $= \left(2, \frac{17}{2}\right)$

Line : $\left(y - \frac{11}{2}\right) = 2\left(x - \frac{1}{2}\right) \Rightarrow y = 2x + \frac{9}{2}$

Passing though $\left(0, \frac{\alpha}{2}\right)$

$\frac{\alpha}{2} = \frac{9}{2} \Rightarrow \alpha = 9$

Q2



$\tan\theta = -\frac{2}{3}$

Using symmetric from of line P, Q : $(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta)$

$\left(2 \pm \sqrt{13} \cdot \left(-\frac{3}{\sqrt{13}}\right), 3 \pm \sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right)$

$(-1, 5) \& (5, 1)$

Q3

Image of $A(a, b)$ along $y = x$ is $B(b, a)$. Translating

it 2 units it becomes $C(b + 2, a)$.

Now, applying rotation theorem

$-\frac{1}{2} + \frac{7}{\sqrt{2}}i = ((b + 2) + ai)\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\frac{-1}{\sqrt{2}} + \frac{7}{\sqrt{2}}i = \left(\frac{b+2}{\sqrt{2}} - \frac{a}{\sqrt{2}}\right) + i\left(\frac{b+2}{\sqrt{2}} + \frac{a}{\sqrt{2}}\right)$

$\Rightarrow b - a + 2 = -1$

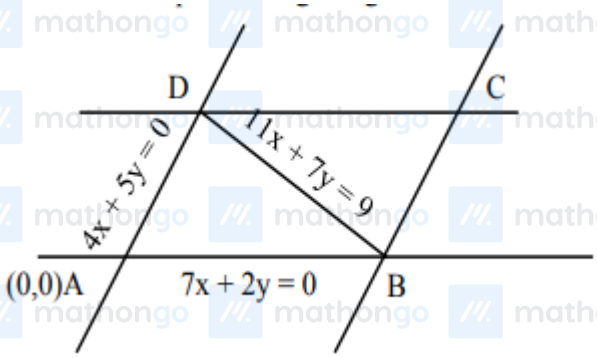
and $b + 2 + a = 7$

$\Rightarrow a = 4; b = 1$

$\Rightarrow 2a + b = 9$

Q4

Both the lines pass through origin.



point D is equal of intersection of $4x + 5y = 0 \& 11x + 7y = 9$

So, coordinates of point $D = \left(\frac{5}{3}, -\frac{4}{3}\right)$

Also, point B is point of intersection of $7x + 2y = 0 \& 11x + 7y = 9$

So, coordinates of point $B = \left(-\frac{2}{3}, \frac{7}{3}\right)$

diagonals of parallelogram intersect at middle

let middle point of $B, D \Rightarrow \left(\frac{\frac{5}{3}-\frac{2}{3}}{2}, \frac{\frac{-4}{3}+\frac{7}{3}}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$

equation of diagonal AC

$\Rightarrow (y - 0) = \frac{\frac{1}{\alpha}-0}{\frac{1}{\alpha}-0}(\pi - 0)$

$y = x$

diagonal AC passes through $(2, 2)$.