

Mathematical Induction

JEE Main 2021 (July) Chapter-wise Questions

Questions with Answer Keys

MathonGo

Q1 (22 July 2021 Shift 1)

The number of elements in the set $\{n \in \{1, 2, 3, \dots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ____.

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Questions with Answer Keys

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Answer Key

Q1 (96)

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Hints and Solutions

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Q1 mathongo mathongo mathongo mathongo mathongo mathongo mathongo
 $11^n > 10^n + 9^n$ mathongo mathongo mathongo mathongo mathongo mathongo
 $\Rightarrow 11^n - 9^n > 10^n$ mathongo mathongo mathongo mathongo mathongo mathongo
 $\Rightarrow (10+1)^n - (10-1)^n > 10^n$ mathongo mathongo mathongo mathongo mathongo
 $\Rightarrow \{^nC_1 \cdot 10^{n-1} + ^nC_3 10^{n-3} + ^nC_5 10^{n-5} + \dots\} > 10^n$ mathongo mathongo mathongo
 $\Rightarrow 2n \cdot 10^{n-1} + 2 \{^nC_3 10^{n-3} + ^nC_5 10^{n-5} + \dots\} > 10^n$ mathongo mathongo mathongo
.....(1)

For $n = 5$ mathongo mathongo mathongo mathongo mathongo mathongo
 $10^5 + 2 \{^5C_3 10^2 + ^5C_5\} > 10^5$ (True) mathongo mathongo mathongo mathongo mathongo
For $n = 6, 7, 8, \dots, 100$

$2n10^{n-1} > 10^n$ mathongo mathongo mathongo mathongo mathongo mathongo
 $\Rightarrow 2n10^{n-1} + 2 \{^nC_3 10^{n-3} + ^nC_5 10^{n-5} + \dots\} > 10^n$ mathongo mathongo mathongo
 $\Rightarrow 11^n - 9^n > 10^n$ For $n = 5, 6, 7, \dots, 100$

For $n = 4$, Inequality (1) is not satisfied \Rightarrow Inequality does not hold good for $N = 1, 2, 3, 4$ mathongo

So, required number of elements = 96

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