

Questions with Answer Keys

MathonGo

Q1 (22 July 2021 Shift 1)

The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is _____.

Q2 (25 July 2021 Shift 1)

The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

(1) $\frac{8}{3}$

(2) $\frac{17}{3}$

(3) $\frac{13}{3}$

(4) $\frac{7}{3}$

Q3 (27 July 2021 Shift 1)

If the area of the bounded region

$$R = \left\{ (x, y) : \max \{0, \log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2 \right\}$$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of

$(\alpha + \beta - 2\gamma)^2$ is equal to :

(1) 8

(2) 2

(3) 4

(4) 1













































































































































Q4 (27 July 2021 Shift 2)

The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to :-

(1) $\frac{16}{3}$

(2) $\frac{2}{3}$

(3) $\frac{9}{2}$

 (4) $\frac{4}{3}$	 mathongo	 mathongo	 mathongo	 mathongo	 mathongo	 mathongo
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Answer Key

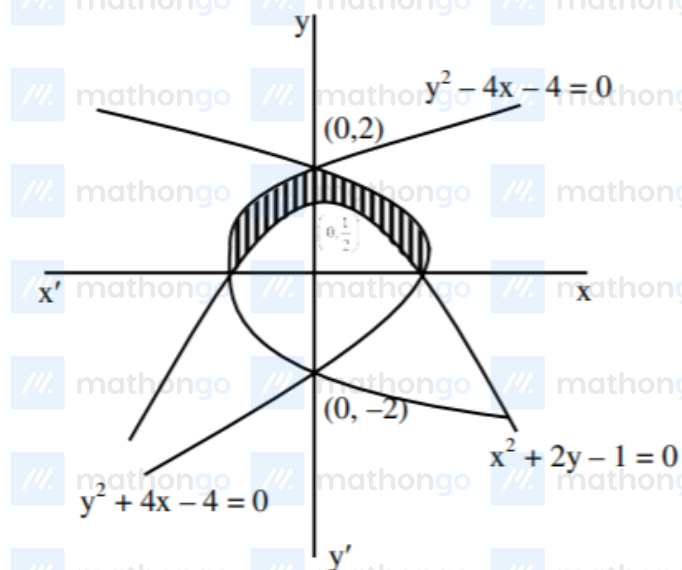
Q1 (2)

Q2 (4)

Q3 (2)

Q4 (3)

Q1

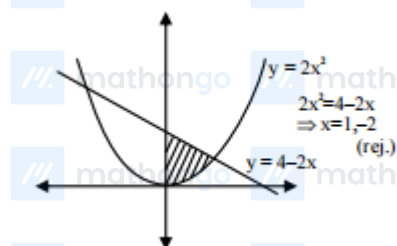


Required Area (shaded)

$$= 2 \left[\int_0^2 \left(\frac{4-y^2}{4} \right) dy - \int_0^1 \left(\frac{1-x^2}{2} \right) dx \right]$$

$$= 2 \left[\frac{4}{3} - \frac{1}{3} \right] = (2)$$

Q2



$$\text{Required area} = \int_0^1 (4 - 2x - 2x^2) dx = 4x - x^2 - \frac{2x^3}{3} \Big|_0^1$$

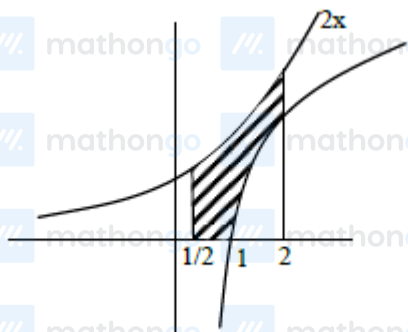
$$= 4 - 1 - \frac{2}{3} = \frac{7}{3}$$

Q3

Hints and Solutions

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$$R = \left\{ (x, y) \cdot \max(0, \log_e x) \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2 \right\}$$



$$\int_{1/2}^2 2^x dx - \int_{1/2}^2 \ln x dx$$

$$\Rightarrow \left[\frac{2^x}{\ln 2} \right]_{1/2}^2 - [x \ln x - x]_{1/2}^2$$

$$\Rightarrow \frac{(2^2) - 2^{1/2}}{\log_0 2} - (2 \ln 2 - 1)$$

$$\Rightarrow \frac{(2^2 - \sqrt{2})}{\log_2 2} - 2 \ln 2 + 1$$

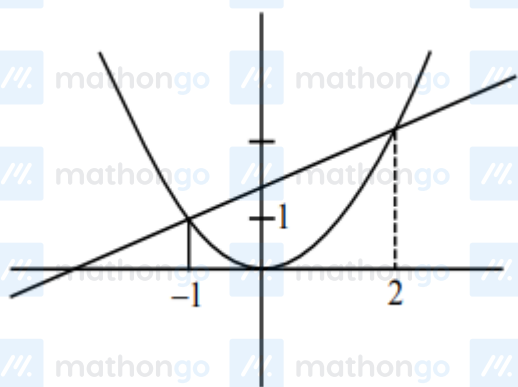
$$\therefore \alpha = 2^2 - \sqrt{2}, \beta = -2, \gamma = 1$$

$$\Rightarrow (\alpha + \beta + 2\gamma)^2$$

$$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2$$

$$\Rightarrow (\sqrt{2})^2 = 2$$

Q4



$$y - x = 2, x^2 = y$$

$$\text{Now, } x^2 = 2 + x$$

Hints and Solutions

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$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\text{Area} = \int_{-1}^2 (2+x-x^2)$$

$$= \left| 2x + \frac{x^2}{2} - \frac{x^3}{3} \right|_{-1}^2$$

$$= \left(4 + 2 - \frac{8}{3} \right) - \left(-2 + \frac{1}{2} + \frac{1}{3} \right)$$

$$= 6 - 3 + 2 - \frac{1}{2} = \frac{9}{2}$$