Q1 (20 July 2021 Shift 1)

Let 'a' be a real number such that the function

$\mathrm{f(x)} = \mathrm{ax}^2 + 6\mathrm{x} - 15, \mathrm{x} \in \mathbf{R}$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15, x \in \mathbf{R}$ has a:

(1) local maximum at $\mathrm{x} = -\frac{3}{4}$

(2) local minimum at $\mathrm{x} = -\frac{3}{4}$

(3) local maximum at $x = \frac{3}{4}$

(4) local minimum at $\mathrm{x} = \frac{3}{4}$

Q2 (20 July 2021 Shift 2)

The sum of all the local minimum values of the

twice differentiable function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$f(\mathrm{x}) = \mathrm{x}^3 - 3\mathrm{x}^2 - \frac{3f''(2)}{2}\mathrm{x} + f''(1)$ is :

(1) -22

(2) 5

(3) -27

(4) 0

Q3 (22 July 2021 Shift 1)

Let f : $\mathbf{R} \rightarrow \mathbf{R}$ be defined as

$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x & , \quad x > 0 \\ 3xe^x & , x \leq 0 \end{cases}$. Then f is

increasing function in the interval

(1) $\left(-\frac{1}{2}, 2\right)$

(2) $(0, 2)$

(3) $\left(-1, \frac{3}{2}\right)$

(4) $(-3, -1)$

Q4 (25 July 2021 Shift 1)

Let $\mathrm{f(x)} = 3\sin^4 \mathrm{x} + 10\sin^3 \mathrm{x} + 6\sin^2 \mathrm{x} - 3$,

$x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is:

(1) increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(2) decreasing in $\left(0, \frac{\pi}{2}\right)$

(3) increasing in $\left(-\frac{\pi}{6}, 0\right)$

(4) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Q5 (25 July 2021 Shift 1)

The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

(1) 2

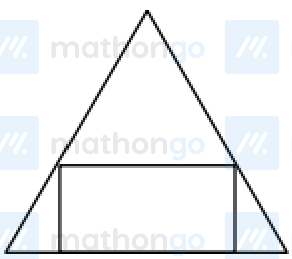
(2) 4

(3) 6

(4) 1

Q6 (25 July 2021 Shift 2)

If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ___



Q7 (27 July 2021 Shift 2)

Let $f : (a, b) \rightarrow \mathbf{R}$ be twice differentiable function such that $f(\mathrm{x}) = \int_{\mathrm{a}}^{\mathrm{x}} \mathrm{g(t)dt}$ for a differentiable function $\mathrm{g(x)}$. If $f(\mathrm{x}) = 0$ has exactly five distinct roots in (a, b), then $\mathrm{g(x)g'(x)} = 0$ has at least :

(1) twelve roots in (a, b)

(2) five roots in (a, b)

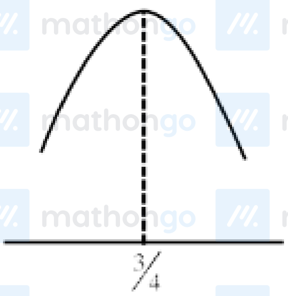
(3) seven roots in (a, b)

(4) three roots in (a, b)

Answer Key

Q1 (1)	**Q2** (3)	**Q3** (3)	**Q4** (4)
Q5 (1)	**Q6** (3)	**Q7** (3)	

Q1



$\frac{-B}{2A}=\frac{3}{4}$

$\Rightarrow \frac{-(6)}{2a}=\frac{3}{4}$

$\Rightarrow a=\frac{-6\times 4}{6} \Rightarrow a=-4$

$\therefore g(x)=4x^2-6x+15$

Local max. at $\mathrm{x}=\frac{-\mathrm{B}}{2\mathrm{~A}}=-\frac{(-6)}{2(-4)}$

$=\frac{-3}{4}$

Q2

$\mathrm{f}(\mathrm{x})=\mathrm{x}^3-3\mathrm{x}^2-\frac{3}{2}\mathrm{f}'(2)\mathrm{x}+\mathrm{f}''(1) \ldots .(\mathrm{i})$

$\mathrm{f}(\mathrm{x})=3\mathrm{x}^2-6\mathrm{x}-\frac{3}{2}\mathrm{f}'' \ldots .(ii)$

$\mathrm{f}''(\mathrm{x})=6\mathrm{x}-6 \ldots (\mathrm{iii})$

Now is 3^{rd} equation $f''(2)=12-6=6$

$\mathrm{f}''(11=0)$

Use (ii)

$f'(x)=3x^2-6x-\frac{3}{2}f''(2)$

$f(x)=3x^2-6x-\frac{3}{2}\times 6$

$f(x)=3x^2-6x-9$

$\mathrm{f}(\mathrm{x})=0$

$3x^2-6x-9=0$

$\Rightarrow \mathrm{x}=-1\&3$

Use (iii)

$f''(x) = 6x - 6$

$\mathrm{f}''(-1) = -12 < 0$ maxima

$\mathrm{f}''(3) = 12 > 0$ minima.

Use (i)

$\mathrm{f(x)} = \mathrm{x}^3 - 3\mathrm{x}^2 - \frac{3}{2}\mathrm{f}''(2)\mathrm{x} + \mathrm{f}''(1)$

$f(x) = x^3 - 3x^2 - \frac{3}{2} \times 6 \times x + 0$

$f(x) = x^3 - 3x^2 - 9x$

$\mathrm{f}(3) = 27 - 27 - 9 \times 3 = -27$

Q3

For $x > 0 f'(x) = -4x^2 + 4x + 3$

$\mathrm{f(x)}$ is increasing in $\left(-\frac{1}{2}, \frac{3}{2}\right)$

For $x \leq 0 f'(x) = 3e^x(1 + x)$

$\mathrm{f}'(\mathrm{x}) > 0 \forall \mathrm{x} \in (-1, 0)$

$\Rightarrow \mathrm{f(x)}$ is increasing in $(-1, 0)$

So, in complete domain $\mathrm{f(x)}$ is increasing in $\left(-1, \frac{3}{2}\right)$

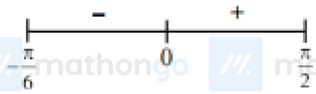
Q4

$f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3, x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

$f'(\mathrm{x}) = 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x$

$= 6\sin x\cos x\left(2\sin^2 x + 5\sin x + 2\right)$

$=6\sin x\,\mathrm{cosx}(2\sin x + 1)(\sin + 2)$



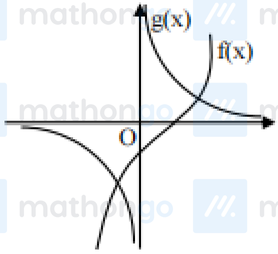
Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Q5

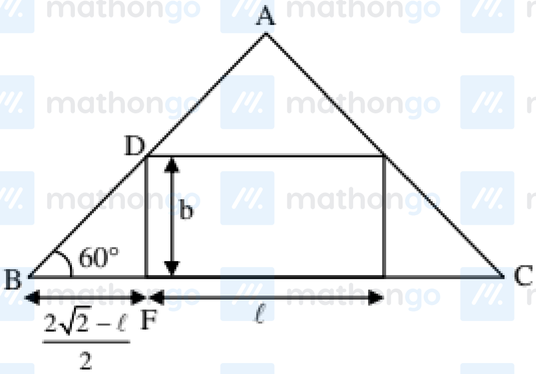
$e^{6x}-e^{4x}-2e^{3x}-12e^{2x}+e^{x}+1=0$

$\Rightarrow \left(e^{3x}-1\right)^{2}-e^{x}\left(e^{3x}-1\right)=12e^{2x}$

$\left(e^{3x}-1\right)^{2}\left(e^{x}-e^{-x}-e^{-2x}\right)=12$



Q6



In $\triangle DBF$

$\tan 60^{\circ}=\frac{2b}{2\sqrt{2}-\ell} \Rightarrow b=\frac{\sqrt{3}(2\sqrt{2}-\ell)}{2}$

$A=$ Area of rectangle $=\ell \times b$

$A=\ell \times \frac{\sqrt{3}}{2}(2\sqrt{2}-\ell)$

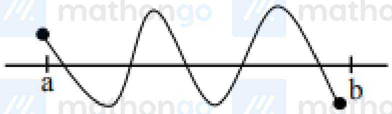
$\frac{dA}{d\ell}=\frac{\sqrt{3}}{2}(2\sqrt{2}-\ell)-\frac{\ell.\sqrt{3}}{2}=0$

$\ell=\sqrt{2}$

$A=\ell \times b=\sqrt{2} \times \frac{\sqrt{3}}{2}(\sqrt{2})=\sqrt{3}$

$\Rightarrow A^{2}=3$

Q7



$\mathrm{f}(\mathrm{x})=\int_{\mathrm{a}}^{\mathrm{x}} \mathrm{g}(\mathrm{t}) \mathrm{dt}$

$\mathrm{f}(\mathrm{x}) \rightarrow 5$

$\mathrm{f}^{\prime}(\mathrm{x}) \rightarrow 4$

$\mathrm{g}(\mathrm{x}) \rightarrow 4$

$\mathrm{g}^{\prime}(\mathrm{x}) \rightarrow 3$