Q1 (20 July 2021 Shift 1)

Let T be the tangent to the ellipse $\mathrm{E}: \mathrm{x}^2+4 \mathrm{y}^2=5$ at the point $\mathrm{P}(1,1)$. If the area of the region bounded by the tangent T, ellipse E, lines $\mathrm{x}=1$ and $\mathrm{x}=\sqrt{5}$ is $\alpha \sqrt{5}+\beta+\gamma \cos ^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to ____

Q2 (22 July 2021 Shift 1)

Let $\mathrm{E}_1: \frac{\mathrm{x}^2}{\mathrm{a}^2}+\frac{\mathrm{y}^2}{\mathrm{b}^2}=1, \mathrm{a}>\mathrm{b}$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is :

(1) $\frac{-1+\sqrt{5}}{2}$
(2) $\frac{-1+\sqrt{8}}{2}$
(3) $\frac{-1+\sqrt{3}}{2}$
(4) $\frac{-1+\sqrt{6}}{2}$

Q3 (25 July 2021 Shift 1)

Let an ellipse $E: \frac{\mathrm{x}^2}{\mathrm{a}^2}+\frac{\mathrm{y}^2}{\mathrm{b}^2}=1, \mathrm{a}^2>\mathrm{b}^2$, passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus $\mathrm{F}(\alpha, 0), \alpha>0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(1) $\frac{8}{3}$
(2) $\frac{4}{3}$
(3) $\frac{16}{3}$
(4) 3

Q4 (25 July 2021 Shift 2)

If a tangent to the ellipse $x^2+4y^2=4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point:

(1) $(\sqrt{3}, 0)$

(2) $(\sqrt{2}, 0)$

(3) $(1,1)$

(4) $(-1,1)$

Q5 (27 July 2021 Shift 1)

A ray of light through $(2,1)$ is reflected at a point P on the y -axis and then passes through the point $(5,3)$. If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be:

(1) $11x+7y+8=0$ or $11x+7y-15=0$

(2) $11x-7y-8=0$ or $11x+7y+15=0$

(3) $2x-7y+29=0$ or $2x-7y-7=0$

(4) $2x-7y-39=0$ or $2x-7y-7=0$

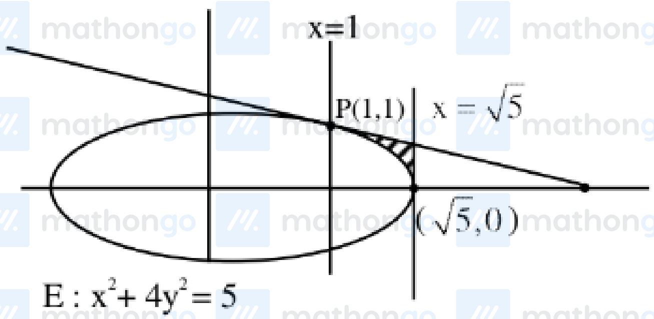
Q6 (27 July 2021 Shift 2)

Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at $(3,-4)$, one focus at $(4,-4)$ and one vertex at $(5,-4)$. If $mx-y=4,\ m>0$ is a tangent to the ellipse E, then the value of $5\ m^2$ is equal to

Answer Key

Q1 (1)	**Q2** (1)	**Q3** (3)	**Q4** (1)
Q5 (3)	**Q6** (3)		

Q1



Tangent at P : $x + 4y = 5$

Required Area $= \int_1^{\sqrt{5}} \left(\frac{5-x}{4} - \frac{\sqrt{5-x^2}}{2}\right) dx$

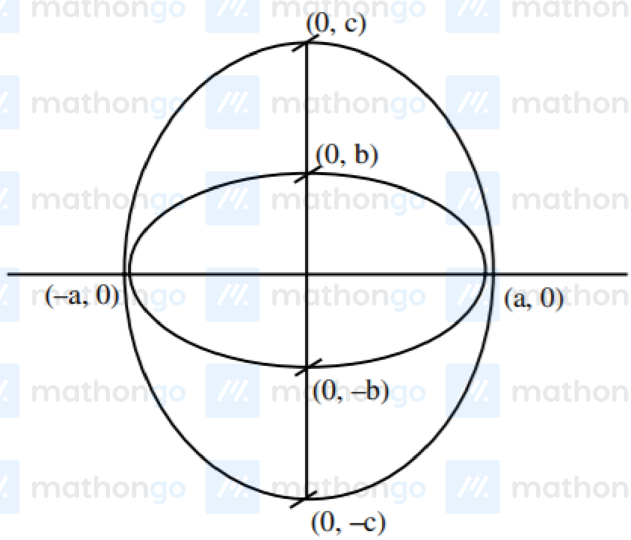
$= \left[\frac{5x}{4} - \frac{x^2}{8} - \frac{x}{4}\sqrt{5-x^2} - \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$

$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

It we assume $\alpha, \beta, \gamma, \in$ Q (Not given in question) then $\alpha = \frac{5}{4}, \beta = -\frac{5}{4} \& \gamma = -\frac{5}{4}$

$|\alpha + \beta + \gamma| = 1.25$

Q2



$e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{a^2}{c^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{a^2}{c^2}$

$\Rightarrow c^2 = \frac{a^4}{b^2} \Rightarrow c = \frac{a^2}{b}$

Also $b = ce$

$\Rightarrow c = \frac{b}{e}$

$\frac{b}{e} = \frac{a^2}{b}$

$\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1+\sqrt{5}}{2}$

Q3

$\frac{3}{2a^2} + \frac{1}{b^2} = 1$ and $1 - \frac{b^2}{a^2} = \frac{1}{3}$

$\Rightarrow a^2 = 3b^2 = 3$

$\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1 \ldots$(i)

Its focus is $(1, 0)$

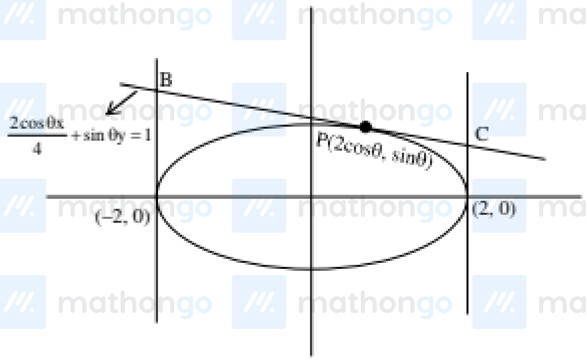
Now, eqn of circle is

$(x-1)^2 + y^2 = \frac{4}{3}$ .....(ii)

Solving (i) and (ii) we get $y = \pm\frac{2}{\sqrt{3}}, x = 1$

$\Rightarrow PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$

Q4



$\frac{x^2}{4}+\frac{y^2}{1}=1$

Equation of tangent is $(\cos\theta)x+2\sin\theta y=2$

$\mathrm{B}\left(-2,\frac{1+\cos\theta}{\sin\theta}\right),\quad \mathrm{C}\left(2,\frac{1-\cos\theta}{\sin\theta}\right)$

$\mathrm{B}\left(-2,\cot\frac{\theta}{2}\right)\quad \mathrm{C}\left(2,\tan\frac{\theta}{2}\right)$

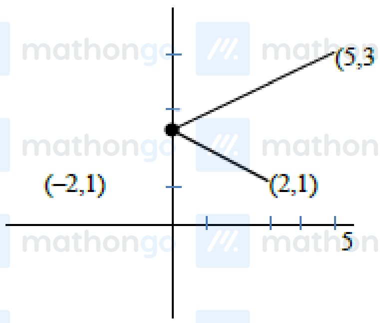
Equation of circle is

$(x+2)(x-2)+\left(y-\cot\frac{\theta}{2}\right)\left(y-\tan\frac{\theta}{2}\right)=0$

$\mathrm{x}^2-4+\mathrm{y}^2-\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\mathrm{y}+1=0$

so, $(\sqrt{3},0)$ satisfying option (1)

Q5



Equation of reflected Ray $y-1=\frac{2}{7}(x+2)$

$7y-7=2x+4$

$2x-7y+11=0$

Let the equation of other directrix is

$2x-7y+\lambda$

Distance of directrix from Focub $\frac{a}{e}-ae=\frac{8}{\sqrt{53}}$

$3a-\frac{a}{3}=\frac{8}{\sqrt{53}}$ or $a=\frac{3}{\sqrt{53}}$

Distance from other focus $\frac{\mathrm{a}}{\mathrm{e}}+\mathrm{ae}$

$3a+\frac{a}{3}=\frac{10a}{3}=\frac{10}{3}\times\frac{3}{\sqrt{53}}=\frac{10}{\sqrt{53}}$

Distance between two directrix $=\frac{2\mathrm{a}}{\mathrm{e}}$

$=2\times3\times\frac{3}{\sqrt{53}}=\frac{18}{\sqrt{53}}$

$\left|\frac{\lambda-11}{\sqrt{53}}\right|=\frac{18}{\sqrt{53}}$

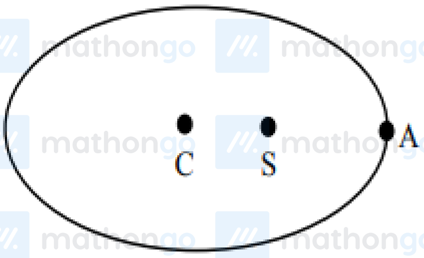
$\lambda-11=18$ or -18

$\lambda=29$ or -7

$2x-7y-7=0$ or $2x-7y+29=0$

Q6

Given $\mathrm{C}(3,-4), \mathrm{S}(4,-4)$



and $\mathrm{A}(5,-4)$

Hence, $\mathrm{a}=2 \& \mathrm{ae}=1$

$\Rightarrow \mathrm{e}=\frac{1}{2}$

$\Rightarrow \mathrm{b}^2=3$

So, $E: \frac{(x-3)^2}{4}+\frac{(y+4)^2}{3}=1$

Intersecting with given tangent.

$\frac{x^2-6x+9}{4}+\frac{m^2x^2}{3}=1$

Now, $\mathrm{D}=0$ (as it is tangent)

So, $5\ \mathrm{m}^2=3$