Q1 (20 July 2021 Shift 1)

Let $A=\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}$, $a \in \mathbf{R}$ be written as $P+Q$ where P is a symmetric matrix and Q is skew symmetric matrix. If $\det(Q)=9$, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 36

(2) 24

(3) 45

(4) 18

Q2 (20 July 2021 Shift 1)

Let $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$ and $B=7A^{20}-20A^{7}+2I$,

where I is an identity matrix of order 3×3. If $B=\left[b_{ij}\right]$, then b_{13} is equal to ___

Q3 (20 July 2021 Shift 2)

Let $A=\left\{a_{ij}\right\}$ be a 3×3 matrix, where

$$a_{ij}=\begin{cases}(-1)^{j-i} & \text{if } i<j\\ 2 & \text{if } i=j\\ (-1)^{i+j} & \text{if } i>j\end{cases}$$

then $\det\left(3\operatorname{Adj}\left(2A^{-1}\right)\right)$ is equal to

Q4 (22 July 2021 Shift 1)

Let $A=[a_i]$ be a real matrix of order 3×3, such that $a_{i1}+a_2+a_3=1$, for $i=1,2,3$. Then, the sum of all the entries of the matrix A^3 is equal to:

(1) 2

(2) 1

(3) 3

(4) 9

Q5 (22 July 2021 Shift 1)

Let $A=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$. Then the number of 3×3

matrices B with entries from the set $\{1,2,3,4,5\}$

and satisfying $\mathrm{AB}=\mathrm{BA}$ is

Q6 (25 July 2021 Shift 1)

Let

$\mathrm{S}=\left\{\mathrm{n}\in\mathrm{N}\begin{pmatrix}0 & \mathrm{i}\\ 1 & 0\end{pmatrix}^{\mathrm{n}}\begin{pmatrix}\mathrm{a} & \mathrm{b}\\ \mathrm{c} & \mathrm{d}\end{pmatrix}=\begin{pmatrix}\mathrm{a} & \mathrm{b}\\ \mathrm{c} & \mathrm{d}\end{pmatrix}\forall\mathrm{a},\mathrm{b},\mathrm{c},\mathrm{d}\in\mathbf{R}\right\}$

where $i=\sqrt{-1}$. Then the number of 2 -digit numbers in the set S is

Q7 (25 July 2021 Shift 2)

If $\mathrm{P}=\begin{bmatrix}1 & 0\\ 1/2 & 1\end{bmatrix}$, then P^{s0} is:

(1) $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$

(2) $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & 25\\ 0 & 1\end{bmatrix}$

(4) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$

Q8 (27 July 2021 Shift 1)

Let $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$. If $\mathrm{A}^{-1}=\alpha\mathrm{I}+\beta\mathrm{A},\alpha,\beta\in\mathbf{R},\mathrm{I}$ is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to :

(1) 5

(2) $\frac{8}{3}$

(3) 2

(4) 4

Q9 (27 July 2021 Shift 2)

Let A and B be two 3×3 real matrices such that $\left(A^2 - B^2\right)$ is invertible matrix. If $A^5 = B^5$ and $A^3 B^2 = A^2 B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :

(1) 2

(2) 4

(3) 1

(4) 0

Q10 (27 July 2021 Shift 2)

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \ldots . + A^{20}$ then the sum of all the elements of the matrix M is equal to

Answer Key

Q1 (1) **Q2** (910) **Q3** (108) **Q4** (3)

Q5 (3125) **Q6** (11) **Q7** (1) **Q8** (4)

Q9 (4) **Q10** (2020)

Q1

$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in R$

and $P = \frac{A+A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$

and $Q = \frac{A-A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

As, $\det(Q) = 9$

$\Rightarrow (a-3)^2 = 36$

$\Rightarrow a = 3 \pm 6$

$\therefore \quad a = 9, -3$

$\therefore \quad \det.(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$

$= 0 - \frac{(a-3)^2}{4} = 0$, for $a = -3$

$= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12)$, for $a = 9$

$\therefore$ Modulus of the sum of all possible values of

det. $(P) = |-36| + |0| = 36$ Ans.

$\Rightarrow$ Option (1) is correct

Q2

Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = I + C$

where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$C^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$

$$C^3=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}=C^4=C^5=\ldots\ldots$$

$B=7\,A^{20}-20\,A^7+2I$

$=7(I+C)^{20}-20(I+C)^7+2I$

$=7\left(I+20C+{}^{20}C_2C^2\right)-20\left(I+7C+{}^{7}C_2C^2\right)+2I$

So

$b_{13}=7\times{}^{20}C_2-20\times{}^{7}C_2=910$

Q3

$$A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$$

$|A|=4$

$\left|3\operatorname{adj}\left(2A^{-1}\right)\right|=\left|3.2^2\operatorname{adj}\left(A^{-1}\right)\right|$

$=12^3\left|\operatorname{adj}\left(A^{-1}\right)\right|=12^3\left|A^{-1}\right|^2=\frac{12^3}{|A|^2}=\frac{12^3}{16}=108$

Q4

$$A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$$

Let $x=\begin{bmatrix}1\\1\\1\end{bmatrix}$

$$AX=\begin{bmatrix}a_{11}+a_{12}+a_{13}\\a_{21}+a_{22}+a_{23}\\a_{31}+a_{32}+a_{33}\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\Rightarrow AX=X$

Replace X by AX $A^2X=AX=X$

Replace X by AX $A^3X=AX=X$

Let $A^3=\begin{bmatrix}x_1&x_2&x_3\\y_1&y_2&y_3\\z_1&z_2&z_3\end{bmatrix}$

$$A^3\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

Sum of all the element $= 3$

Q5

Let matrix $B=\begin{bmatrix}a & b & c\\ d & e & f\\ g & n & i\end{bmatrix}$

$\because \quad AB = BA$

$$\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}=\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}d&e&f\\a&b&c\\g&h&i\end{bmatrix}=\begin{bmatrix}b&a&c\\e&d&f\\h&g&i\end{bmatrix}$$

$\Rightarrow d=b, e=a, f=c, g=h$

$\therefore$ Matrix $B=\begin{bmatrix}a&b&c\\b&a&c\\g&g&i\end{bmatrix}$

No. of ways of selecting a, b, c, g, i

$= 5\times5\times5\times5\times5$

$= 5^5 = 3125$

$\therefore$ No. of Matrices B $= 3125$

Q6

Let $X=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ $\& A=\begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^n$

$\Rightarrow AX = IX$

$\Rightarrow A = I$

$\Rightarrow \begin{pmatrix}0 & i\\ 1 & 0\end{pmatrix}^n = I$

$\Rightarrow A^8=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$\Rightarrow$ n is multiple of 8

So number of 2 digit numbers in the set $\mathrm{S} = 11(16, 24, 32, \ldots\ldots..96)$

Q7

$$\mathrm{P} = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$\mathrm{P}^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\mathrm{P}^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$\mathrm{P}^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$\vdots$

$$\therefore \mathrm{P}^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

Q8

$$\mathrm{A} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, |\mathrm{A}| = 6$$

$$\mathrm{A}^{-1} = \frac{\operatorname{adj} \mathrm{A}}{|\mathrm{A}|} = \frac{1}{6} \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left.\begin{array}{l} \alpha + \beta = \frac{2}{3} \\ \beta = -\frac{1}{6} \end{array}\right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$$4(\alpha - \beta) = 4(1) = 4$$

Q9

$$\mathrm{C} = \mathrm{A}^2 - \mathrm{B}^2; |\mathrm{C}| \neq 0$$

$$\mathrm{A}^5 = \mathrm{B}^5 \text{ and } \mathrm{A}^3\,\mathrm{B}^2 = \mathrm{A}^2\,\mathrm{B}^2$$

Now, $A^5 - A^3B^2 = B^5 - A^2B^3$

$$\Rightarrow \mathrm{A}^3\left(\mathrm{A}^2 - \mathrm{B}^2\right) + \mathrm{B}^3\left(\mathrm{A}^2 - \mathrm{B}^2\right) = 0$$

$$\Rightarrow \left(\mathrm{A}^3 + \mathrm{B}^3\right)\left(\mathrm{A}^2 - \mathrm{B}^2\right) = 0$$

Post multiplying inverse of $A^2 - B^2$:

$A^3 + B^3 = 0$

Q10

$$A^n = \begin{bmatrix} 1 & n & \frac{n^2+n}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

So, required sum

$= 20 \times 3 + 2 \times \left(\frac{20 \times 21}{2}\right) + \sum_{r=1}^{20} \left(\frac{r^2+r}{2}\right)$

$= 60 + 420 + 105 + 35 \times 41 = 2020$