Q1 (20 July 2021 Shift 2)

If sum of the first 21 terms of the series

$\log_{9^{1/2}} x+\log_{9^{1/3}} x+\log_{9^{1/4}} x+\ldots \ldots$, where $x>0$ is

504 , then x is equal to

(1) 243

(2) 9

(3) 7

(4) 81

Q2 (20 July 2021 Shift 2)

For $\mathrm{k} \in \mathrm{N}$, let

$\frac{1}{\alpha(\alpha+1)(\alpha+2) \ldots \ldots . .(\alpha+20)}=\sum_{\mathrm{K}=0}^{20} \frac{\mathrm{A}_{\mathrm{k}}}{\alpha+\mathrm{k}}$,

where $\alpha>0$. Then the value of $100\left(\frac{\mathrm{A}_{14}+\mathrm{A}_{15}}{\mathrm{A}_{13}}\right)^{2}$ is

equal to

Q3 (20 July 2021 Shift 2)

Let $\left\{a_{n}\right\}_{n=1}^{\infty}$ be a sequence such that $a_{1}=1, a_{2}=1$

and $\mathrm{a}_{\mathrm{n}+2}=2 \mathrm{a}_{\mathrm{n}+1}+\mathrm{a}_{\mathrm{n}}$ for all $\mathrm{n} \geq 1$. Then the value

of $47 \sum_{n=1}^{\infty} \frac{a_{n}}{2^{3 n}}$ is equal to

Q4 (22 July 2021 Shift 1)

Let S_{n} denote the sum of first n -terms of an arithmetic progression. If $S_{10}=530, S_{5}=140$, then $\mathrm{S}_{20}-\mathrm{S}_{6}$ is equal to :

(1) 1862

(2) 1842

(3) 1852

(4) 1872

Q5 (22 July 2021 Shift 1)

The sum of all the elements in the set $\{\mathrm{n} \in \{1, 2, \ldots, , 100\}$ à¥¤ H.C.F. of n and 2040 is $1\}$ is equal to ____.

Q6 (25 July 2021 Shift 1)

Let S_n be the sum of the first n terms of an arithmetic progression. If $\mathrm{S}_{3\mathrm{n}} = 3\ \mathrm{S}_{2\mathrm{n}}$, then the value of $\frac{S_{4n}}{S_{2\mathrm{n}}}$ is :

(1) 6

(2) 4

(3) 2

(4) 8

Q7 (25 July 2021 Shift 1)

If the value of

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots\ldots \text{ upto } \infty\right)^{\log_{(0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots \text{ upto } \infty\right)}$$

is l, then l^2 is equal to

Q8 (25 July 2021 Shift 2)

If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^{\mathrm{n}}\mathrm{n}}{2}\right]$ is equal to:

(1) 0

(2) 4

(3) -2

(4) 2

Q9 (27 July 2021 Shift 1)

If $\log_3 2, \log_3(2^x-5), \log_3\left(2^x-\frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

Q10 (27 July 2021 Shift 2)

If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to :

(1) 4

(2) 3

(3) 0

(4) 1

Answer Key

Q1 (4)	**Q2** (9)	**Q3** (7)	**Q4** (1)
Q5 (1251)	**Q6** (1)	**Q7** (3)	**Q8** (2)
Q9 (3)	**Q10** (3)		

Q1

$\mathrm{s} = 2\log_9 \mathrm{x} + 3\log_9 \mathrm{x}^{+} \ldots\ldots + 22\log_9 \mathrm{x}$

$\mathrm{s} = \log_9 \times (2 + 3 + \ldots\ldots + 22)$

$\mathrm{s} = \log_9 \mathrm{X} \left\{\frac{21}{2}(2 + 22)\right\}$

Given $252\log_9 \mathrm{x} = 504$

$\Rightarrow \log_9 \mathrm{x} = 2 \Rightarrow \mathrm{x} = 81$

Q2

$\frac{1}{\alpha(\alpha+1)\ldots(\alpha+20)} = \sum_{\mathrm{k}=0}^{20} \frac{\mathrm{A_k}}{\alpha+\mathrm{k}}$

$\mathrm{A}_{14} = \frac{1}{(-14)(-13)\ldots(-1)(1)\ldots(6)} = \frac{1}{14!\cdot 6!}$

$\mathrm{A}_{15} = \frac{1}{(-15)(-14)\ldots(-1)(1)\ldots(5)} = \frac{1}{15!\cdot 5!}$

$\mathrm{A}_{13} = \frac{1}{(-13)\ldots(-1)(1)\ldots(7)} = \frac{-1}{13!\cdot 7!}$

$\frac{\mathrm{A}_{14}}{\mathrm{A}_{13}} = \frac{1}{14!\cdot 6!} \times -13 \times 7! = \frac{-7}{14} = -\frac{1}{2}$

$\frac{\mathrm{A}_{15}}{\mathrm{A}_{13}} = -\frac{1}{15!\times 5!} \times -13! \times 7! = \frac{42}{15\times 14} = \frac{1}{5}$

$100\left(\frac{\mathrm{A}_{14}}{\mathrm{A}_{13}} + \frac{\mathrm{A}_{15}}{\mathrm{A}_{13}}\right)^2 = 100\left(-\frac{1}{2} + \frac{1}{5}\right)^2 = 9$

Q3

$a_{n+2} = 2a_{n+1} + a_n$, let $\sum_{n=1}^{\infty} \frac{a_n}{8^n} = P$

Divide by 8^{n} we get

$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$

$\Rightarrow 64\frac{\mathrm{a_{n+2}}}{8^{\mathrm{n+2}}} = \frac{16\mathrm{a_{n+1}}}{8^{\mathrm{n+1}}} + \frac{\mathrm{a_n}}{8^{\mathrm{n}}}$

$64\sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = 16\sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty} \frac{a_n}{8^n}$

$64\left(P - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(P - \frac{a_1}{8}\right) + P$

$\Rightarrow 64\left(P - \frac{1}{8} - \frac{1}{64}\right) = 16\left(P - \frac{1}{8}\right) + P$

$64P - 8 - 1 = 16P - 2 + P$

$47\mathrm{P} = 7$

Q4

$\mathrm{S}_{10}=530 \Rightarrow \frac{10}{2}\{2\mathrm{a}+9\mathrm{~d}\}=530$

$\Rightarrow 2\mathrm{a}+9\mathrm{~d}-106\ldots.(1)$

and $\mathrm{S}_5=140 \Rightarrow \frac{5}{2}\{2\mathrm{a}+4\mathrm{~d}\}=140$

$\Rightarrow 2\mathrm{a}+4\mathrm{~d}=56\ldots..(2)$

$\Rightarrow 5\mathrm{~d}=50 \Rightarrow \frac{\mathrm{d}=10}{\mathrm{d}} \Rightarrow \frac{\mathrm{a}=8}{\mathrm{d}}$

Now, $\mathrm{S}_{20}-\mathrm{S}_6=\frac{20}{2}\{2\mathrm{a}+19\mathrm{~d}\}-\frac{6}{2}\{2\mathrm{a}+5\mathrm{~d}\}$

$=14\mathrm{a}+175\mathrm{~d}$

$=(14\times 8)+(175\times 10)$

$=1862$

Q5

$2040=2^3\times 3\times 5\times 17$

n should not be multiple of $2,3,5$ and 17 . Sum of all $n=(1+3+5\ldots\ldots+99)-(3+9+15+$

$21+\ldots\ldots+99)-(5+25+35+55+65+85+95)$

$-(17)$

$=2500-\frac{17}{2}(3+99)-365-17$

$=2500-867-365-17$

$=1251$

Q6

Let a be first term and d be common diff. of this

A.P.

Given $S_{3\mathrm{~a}}=3S_{2\mathrm{n}}$

$\Rightarrow \frac{3n}{2}[2a+(3n-1)d]=3\frac{2n}{2}[2a+(2n-1)d]$

$\Rightarrow 2\mathrm{a}+(3\mathrm{n}-1)\mathrm{d}=4\mathrm{a}+(4\mathrm{n}-2)\mathrm{d}$

$\Rightarrow 2\mathrm{a}+(\mathrm{n}-1)\mathrm{d}=0$

Now $\frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]} = \frac{2[\underbrace{2a+(n-1)d}_{=0}+3nd]}{[\underbrace{2a+(n-1)d}_{=0}+nd]}$

$= \frac{6nd}{nd} = 6$

Q7

$\ell = (\underbrace{1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3}}_{S} + \ldots$

$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \ldots$

$\frac{S}{3} = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \ldots$

$\frac{-}{2} - \frac{1}{2} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots$

$\frac{2\,S}{3} = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots$

$S = \frac{3}{2}\left(\frac{4/3}{1-1/3}\right) = 3$

Now $\ell = (3)^{\log_{es}\left(\frac{1/3}{1-1/3}\right)}$

$\ell = 3^{\operatorname{lng}_{(10+1)}\left(\frac{1}{2}\right)} = 3^{1/2} = \sqrt{3}$

$\Rightarrow \ell^2 = 3$

Q8

$\sum_{n=8}^{100}\left[\frac{(-1)^n \cdot n}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 + \ldots - 50 + 50 = 4$

Q9

$2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

Let $2^x = t$

$\log_3(t-5)^2 = \log_3 2\left(t - \frac{7}{2}\right)$

$(t-5)^2 = 2t - 7$

$t^2 - 12t + 32 = 0$

$(t-4)(t-8) = 0$

$\Rightarrow 2^x = 4$ or $2^x = 8$

$\mathrm{X}=2$ (Rejected)

Or $\mathrm{x}=3$

Q10

$x=\frac{1}{2}\left(\tan \frac{\pi}{9}+\tan \frac{7 \pi}{18}\right)$
and $2 y=\tan \frac{\pi}{9}+\tan \frac{5 \pi}{18}$
so, $x-2 y=\frac{1}{2}\left(\tan \frac{\pi}{9}+\tan \frac{7 \pi}{18}\right)$
$-\left(\tan \frac{\pi}{9}+\tan \frac{5 \pi}{18}\right)$
$\Rightarrow|x-2 y|=\left|\frac{\cot \frac{\pi}{9}-\tan \frac{\pi}{9}}{2}-\tan \frac{5 \pi}{18}\right|$
$=\left|\cot \frac{2 \pi}{9}-\cot \frac{2 \pi}{9}\right|=0$
$\left(\operatorname{astan} \frac{5 \pi}{18}=\cot \frac{2 \pi}{9} ; \tan \frac{7 \pi}{18}=\cot \frac{\pi}{9}\right)$