**Q1 (20 July 2021 Shift 1)**

Let P be a plane passing through the points $(1,0,1),(1,-2,1)$ and $(0,1,-2)$. Let a vector $\overrightarrow{\mathrm{a}}=\alpha\hat{\mathrm{i}}+\beta\hat{\mathrm{j}}+\gamma\hat{\mathrm{k}}$ be such that $\overrightarrow{\mathrm{a}}$ is parallel to the plane P, perpendicular to $(\hat{\mathrm{i}}+2\hat{\mathrm{j}}+3\hat{\mathrm{k}})$ and $\overrightarrow{\mathrm{a}}\cdot(\hat{\mathrm{i}}+\hat{\mathrm{j}}+2\hat{\mathrm{k}})=2$, then $(\alpha-\beta+\gamma)^2$ equals ___

**Q2 (20 July 2021 Shift 1)**

If the shortest distance between the lines $\overrightarrow{\mathrm{r}}_1=\alpha\hat{\mathrm{i}}+2\hat{\mathrm{j}}+2\hat{\mathrm{k}}+\lambda(\hat{\mathrm{i}}-2\hat{\mathrm{j}}+2\hat{\mathrm{k}}),\lambda\in\mathbf{R},\alpha>0$ and $\overrightarrow{\mathrm{r}_2}=-4\hat{\mathrm{i}}-\hat{\mathrm{k}}+\mu(3\hat{\mathrm{i}}-2\hat{\mathrm{j}}-2\hat{\mathrm{k}}),\mu\in\mathbf{R}$ is 9 , then

$\alpha$ is equal to ___

**Q3 (20 July 2021 Shift 2)**

The lines $x=ay-1=z-2$ and $x=3y-2=bz-2,(ab\neq 0)$ are coplanar, if :

(1) $\mathrm{b}=1,\mathrm{a}\in\mathrm{R}-\{0\}$

(2) $\mathrm{a}=1,\ \mathrm{b}\in\mathrm{R}-\{0\}$

(3) $\mathrm{a}=2,\ \mathrm{b}=2$

(4) $\mathrm{a}=2,\ \mathrm{b}=3$

**Q4 (20 July 2021 Shift 2)**

Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let $Q$ be the mirror image of the point $(2,3,-1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is on the plane P ?

(1) $(-1,1,2)$

(2) $(1,1,1)$

(3) $(1,1,2)$

(4) $(1,2,2)$

**Q5 (22 July 2021 Shift 1)**

Let L be the line of intersection of planes

$\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If

$P(\alpha,\beta,\gamma)$ is the foot of perpendicular on L from the

point $(1,2,0)$, then the value of $35(\alpha+\beta+\gamma)$ is

equal to :

(1) 101

(2) 119

(3) 143

(4) 134

**Q6 (22 July 2021 Shift 1)**

If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and

$4(x-2)=2(y-\lambda)=(z-3), \lambda\in\mathbf{R}$ is $\frac{1}{\sqrt{38}}$, then

the integral value of $\lambda$ is equal to :

(1) 3

(2) 2

(3) 5

(4) -1

**Q7 (25 July 2021 Shift 1)**

Let the foot of perpendicular from a point $P(1,2,-1)$ to the straight line $L:\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ be N.

Let a line be drawn from P parallel to the plane $x+y+2z=0$ which meets L at point Q. If $\alpha$ is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to

(1) $\frac{1}{\sqrt{5}}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{1}{2\sqrt{3}}$

**Q8 (25 July 2021 Shift 2)**

If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co-planar, then the value of k is ___

**Q9 (27 July 2021 Shift 1)**

Let the plane passing through the point $(-1,0,-2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$ be $ax+by+cz+8=0$. Then the value of $a+b+c$ is equal to:

(1) 3

(2) 8

(3) 5

(4) 4

**Q10 (27 July 2021 Shift 1)**

Let a plane P pass through the point $(3,7,-7)$ and contain the line, $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$. If distance of the plane P from the origin is d, then $d^2$ is equal to

**Q11 (27 July 2021 Shift 2)**

For real numbers $\alpha$ and $\beta\neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$, lies on the plane $x+2y-z=8$, then $\alpha-\beta$ is equal to :

(1) 5

(2) 9

(3) 3

(4) 7

**Q12 (27 July 2021 Shift 2)**

The distance of the point $\mathrm{P}(3,4,4)$ from the point of intersection of the line joining the points. $\mathrm{Q}(3,-4,-5)$ and $\mathrm{R}(2,-3,1)$ and the plane $2\mathrm{x}+\mathrm{y}+\mathrm{z}=7$, is equal to

# Answer Key

**Q1** (81) **Q2** (6) **Q3** (1) **Q4** (4)

**Q5** (2) **Q6** (1) **Q7** (3) **Q8** (1)

**Q9** (4) **Q10** (3) **Q11** (4) **Q12** (7)

**Q1**

Equation of plane :

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$$

$\Rightarrow 3x - z - 2 = 0$

$\overrightarrow{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \|$ to $3x - z - 2 = 0$

$\Rightarrow 3\alpha - 8 = 0$ ....(1)

$\overrightarrow{a} \perp i + 2\hat{j} + 3\hat{k}$

$\Rightarrow \alpha + 2\beta + 38 = 0$ ...(2)

$\vec{a}.(\hat{i} + \hat{j} + 2\hat{k}) = 0$

$\Rightarrow \alpha + \beta + 28 = 2$ ...(3)

on solving $1, 2 \& 3$

$\alpha = 1, \quad \beta = -5, 8 = 3$

So $(\alpha - \beta + 8) = 81$

**Q2**

If $\overrightarrow{r} = \overrightarrow{a} + \lambda\overrightarrow{b}$ and $\overrightarrow{r} = \overrightarrow{c} + \lambda\overrightarrow{d}$

then shortest distance between two lines is $L = \frac{(\overrightarrow{a} - \overrightarrow{c}) \cdot (\overrightarrow{b} \times \overrightarrow{d})}{|b \times d|}$

$\therefore \overrightarrow{a} - \overrightarrow{c} = ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k})$

$\frac{\overrightarrow{b} \times \overrightarrow{d}}{|b \times d|} = \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3}$

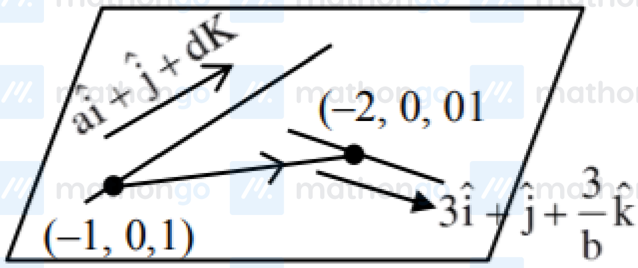
$\therefore ((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}) \cdot \frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = 9$

or $\alpha = 6$

**Q3**

$\frac{x+1}{a} = y = \frac{z-1}{a}$

$\frac{x+2}{3} = y = \frac{z}{3/b}$




lines are Co-planar

$$\begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix} = 0 \Rightarrow -\left(\frac{3}{b}-a\right)-1(a-3)=0$$

$a-\frac{3}{b}-a+3=0$

$b=1, a \in R-\{0\}$

**Q4**

Plane p is $\perp^r$ to line

$\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$

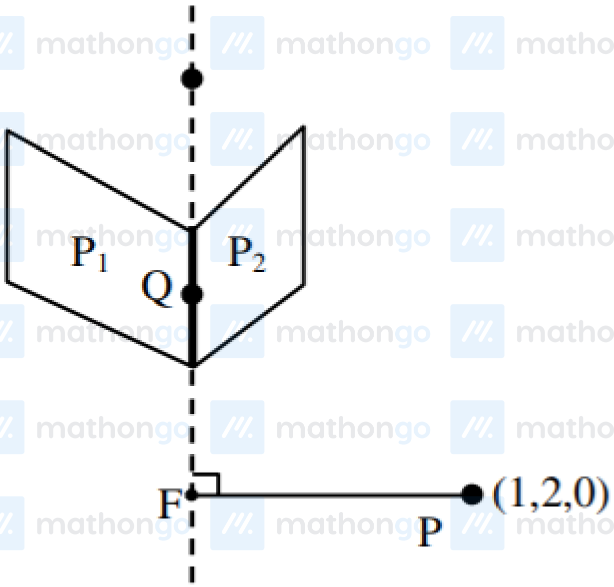
& passes through pt. $(2,3)$ equation of plane $p$ $2(x-2)+1(y-3)+1(z+1)=0$

$2x+y+z-6=0$

pt $(1,2,2)$ satisfies above equation

**Q5**

$$P_1: x-y+2z=2$$
$$P_2=2x+y-3=2$$




Let line of Intersection of planes $P_1$ and $P_2$ cuts xy plane in point Q.

$\Rightarrow$ z -coordinate of point Q is zero

$$\Rightarrow \left.\begin{array}{r} x-y=2 \\ \text{and } 2x+y=2 \end{array}\right\} \Rightarrow x=\frac{4}{3}, y=\frac{-2}{3}$$

$\Rightarrow Q\left(\frac{4}{3}, \frac{-2}{3}, 0\right)$

Vector parallel to the line of intersection $\vec{a}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}=-\hat{i}+5\hat{j}+3\hat{k}$

Equation of Line of intersection

$\frac{x-\frac{4}{3}}{-1}=\frac{y+\frac{2}{3}}{5}=\frac{z-0}{3}=\lambda(\text{ say })$

Let coordinates of foot of perpendicular be $F\left(-\lambda+\frac{4}{3}, 5\lambda-\frac{2}{3}, 3\lambda\right)$

$\overrightarrow{PF}=\left(-\lambda+\frac{1}{3}\right)\hat{i}+\left(5\lambda-\frac{8}{3}\right)\hat{j}+(3\lambda)\hat{k}$

$\overrightarrow{PF}\cdot\vec{a}=0$

$\Rightarrow \lambda-\frac{1}{3}+25\lambda\frac{-40}{3}+9\lambda=0$

$\Rightarrow 35\lambda=\frac{41}{3} \Rightarrow \lambda=\frac{41}{105}$

Now, $\alpha=-\lambda\frac{4}{3}, \beta=5\lambda-\frac{2}{3}, \gamma-3\lambda$

$\Rightarrow \alpha+\beta+\gamma=7\lambda+\frac{2}{3}$

$= 7\left(\frac{41}{105}\right) + \frac{2}{3}$

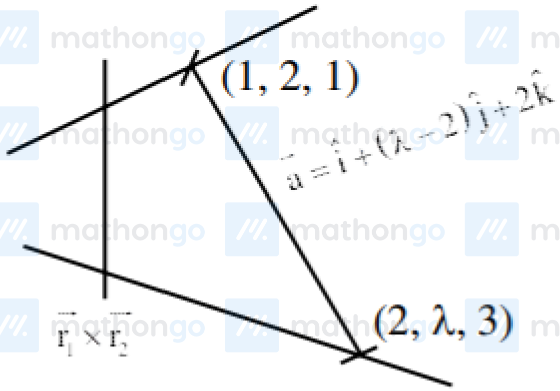
$= \frac{51}{15}$

$\Rightarrow 35(\alpha + \beta + \gamma) = \frac{51}{15} \times 35 = 119$

**Q6**

$L_1 : \frac{(x-1)}{2} = \frac{(y-2)}{1} = \frac{(z-1)}{3} \quad \vec{r}_1 = 2\hat{i} + \hat{j} + 3\hat{k}$

$L_2 : \frac{(x-2)}{1} = \frac{y-\lambda}{2} = \frac{z-3}{4} \quad \vec{r}_2 = \hat{i} + 2\hat{j} + 4\hat{k}$




Shortest distance = Projection of $\vec{a}$ on $r_1 \times r_2$

$= \frac{\left|\vec{a} \cdot \left(\vec{r}_1 \times \vec{r}_2\right)\right|}{\left|\vec{r}_1 \times \vec{r}_2\right|}$

$\left|\vec{a} \cdot \left(\vec{r}_1 \times \vec{r}_2\right)\right| = \begin{vmatrix} 1 & \lambda - 2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14 - 5\lambda|$

$|\vec{r}_1 \times \vec{r}_2| = \sqrt{38}$

$\therefore \frac{1}{\sqrt{38}} = \frac{|14-5\lambda|}{\sqrt{38}}$

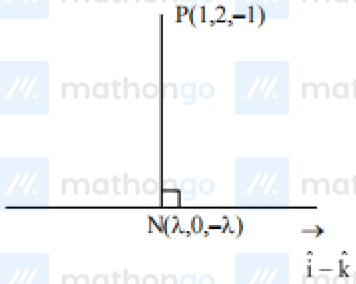
$\Rightarrow |14 - 5\lambda| = 1$

$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \frac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$.

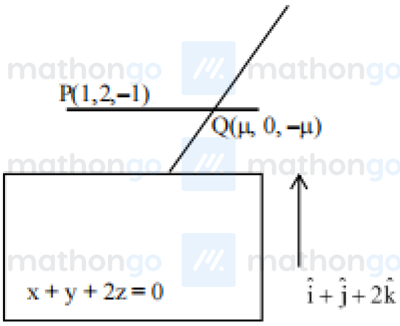
**Q7**




$\overrightarrow{PN} \cdot (\hat{i} - \hat{k}) = 0$

$\Rightarrow N(1, 0, -1)$

Now,




$\overrightarrow{PQ} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 0$

$\Rightarrow \mu = -1$

$\Rightarrow Q(-1, 0, 1)$

$\overrightarrow{PN} = 2\hat{j}$ and $\overrightarrow{PQ} = 2\hat{i} + 2\hat{j} - 2\hat{k}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

**Q8**

$$\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$(k+1)[2-6] - 4[1-9] + 6[2-6] = 0$

$k = 1$

**Q9**

Normal of req. plane $(2\hat{i}+\hat{j}-\hat{k})\times(\hat{i}-\hat{j}-\hat{k})$

$=-2\hat{i}+\hat{j}-3\hat{k}$

Equation of plane $-2(x+1)+1(y-0)-3(z+2)=0$

$-2x+y-3z-8=0$

$2x-y+3z+8=0$

$a+b+c=4$

**Q10**

$\overrightarrow{\mathrm{BA}}=(\hat{\mathrm{i}}+4\hat{\mathrm{j}}-5\hat{\mathrm{k}})$

$\overrightarrow{\mathrm{BA}}=(\hat{\mathrm{i}}+4\hat{\mathrm{j}}-5\hat{\mathrm{k}})$

$\overrightarrow{\mathrm{BA}}\times\vec{\ell}=\overrightarrow{\mathrm{n}}=\begin{vmatrix}\hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}}\\ -3 & 2 & 1\\ 1 & 4 & -5\end{vmatrix}$

$\hat{\mathrm{a}}+\mathrm{b}\hat{\mathrm{j}}+\mathrm{c}\hat{\mathrm{k}}=-14\hat{\mathrm{i}}-\hat{\mathrm{j}}(\mathrm{m})+\hat{\mathrm{k}}(-14)$

$\mathrm{a}=1,\ \mathrm{b}=1,\mathrm{c}=1$

Plane is $(\mathrm{x}-2)+(\mathrm{y}-3)+(\mathrm{z}+\mathrm{z})=0$

$\mathrm{x}+\mathrm{y}+\mathrm{z}-3=0$

$\mathrm{d}=\sqrt{3}\Rightarrow\mathrm{d}^2=3$

**Q11**

First line is $(\phi+\alpha,2\phi+1,3\phi+1)$

and second line is $(q\beta+4,3q+6,3q+7)$

For intersection $\phi+\alpha=\mathrm{q}\beta+4 \quad \ldots$ (i)

$2\phi+1=3\mathrm{q}+6$

...(i)

$3\phi+1=3\mathrm{q}+7$

for (ii) & (iii) $\phi=1,\mathrm{q}=-1$

So, from (i) $\alpha+\beta=3$

Now, point of intersection is $(\alpha+1,3,4)$

It lies on the plane.

Hence, $\alpha = 5 \& \beta = -2$

**Q12**

$\overrightarrow{\mathrm{QR}} : -\frac{x-3}{1} = \frac{y+4}{-1} = \frac{z+5}{-6} = r$

$\Rightarrow (\mathrm{x}, \mathrm{y}, \mathrm{z}) \equiv (\mathrm{r}+3, -\mathrm{r}-4, -6\mathrm{r}-5)$

Now, satisfying it in the given plane.

We get $\mathrm{r} = -2$.

so, required point of intersection is $\mathrm{T}(1, -2, 7)$.

Hence, $\mathrm{PT} = 7$.