Q1 (20 July 2021 Shift 1)

Let $[x]$ denote the greatest integer $\leq x$, where $\mathrm{x} \in \mathbf{R}$. If the domain of the real valued function $\mathrm{f}(\mathrm{x})=\sqrt{\frac{[\mathrm{x}]\mid-2}{\mid[\mathrm{x}]\mid-3}}$ is $(-\infty, a) \cup[b, c) \cup[4, \infty), a<b<c$, then the value of $\mathrm{a}+\mathrm{b}+\mathrm{c}$ is:

(1) 8

(2) 1

(3) -2

(4) -3

Q2 (20 July 2021 Shift 2)

Let $\mathrm{f}: \mathbf{R}-\left\{\frac{\alpha}{6}\right\} \rightarrow \mathbf{R}$ be defined by $\mathrm{f}(\mathrm{x})=\frac{5 \mathrm{x}+3}{6 \mathrm{x}-\alpha}$.

Then the value of α for which $(f o f)(x)=x$, for all $\mathrm{x} \in \mathbf{R}-\left\{\frac{\alpha}{6}\right\}$, is

(1) No such α exists

(2) 5

(3) 8

(4) 6

Q3 (20 July 2021 Shift 2)

The number of solutions of the equation

$\log _{(x+1)}\left(2 x^{2}+7 x+5\right)+\log _{(2 x+5)}(x+1)^{2}-4=0$

$\mathrm{x}>0$, is

Q4 (22 July 2021 Shift 1)

Let $[x]$ denote the greatest integer less than or equal to x. Then, the values of $\mathrm{x} \in \mathbf{R}$ satisfying the equation $\left[\mathrm{e}^{\mathrm{x}}\right]^{2}+\left[\mathrm{e}^{\mathrm{x}}+1\right]-3=0$ lie in the interval :

(1) $\left[0, \frac{1}{\mathrm{e}}\right)$

(2) $[\log_c 2, \log_c 3)$

(3) $[1, \mathrm{e})$

(4) $[0, \log_c 2)$

Q5 (22 July 2021 Shift 1)

If the domain of the function $f(x)=\frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha+\beta$ is equal to :

(1) $\frac{3}{2}$

(2) 2

(3) $\frac{1}{2}$

(4) 1

Q6 (22 July 2021 Shift 1)

Let $\mathrm{A}=\{0,1,2,3,4,5,6,7\}$. Then the number

of bijective functions $f: \mathrm{A} \rightarrow \mathrm{A}$ such that $f(1)+f(2)=3-f(3)$ is equal to

Q7 (25 July 2021 Shift 1)

Let $\mathrm{g}: \mathbf{N} \rightarrow \mathbf{N}$ be defined as

$g(3n+1)=3n+2$

$g(3n+2)=3n+3$

$g(3n+3)=3n+1$, for all $n \geq 0$

Then which of the following statements is true ?

(1) There exists an onto function $\mathrm{f}: \mathbf{N} \rightarrow \mathbf{N}$ such

that $\mathrm{fog}=\mathrm{f}$

(2) There exists a one-one function $\mathrm{f}: \mathbf{N} \rightarrow \mathbf{N}$ such

that $\mathrm{fog}=f$

(3) gogog $= \mathrm{g}$

(4) There exists a function $\mathrm{f}: \mathbf{N} \rightarrow \mathbf{N}$ such that gof $= \mathrm{f}$

Q8 (25 July 2021 Shift 2)

The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is:

(1) 2

(2) 3

(3) 1

(4) 4

Q9 (25 July 2021 Shift 2)

Consider function $f: \mathrm{A} \rightarrow \mathrm{B}$ and

$\mathrm{g}: \mathrm{B} \rightarrow \mathrm{C}(\mathrm{A}, \mathrm{B}, \mathrm{C} \subseteq \mathbf{R})$ such that $(\mathrm{gof})^{-1}$ exists then:

(1) f and g both are one-one

(2) f and g both are onto

(3) f is one-one and g is onto

(4) f is onto and g is one-one

Q10 (27 July 2021 Shift 1)

Let the domain of the function

$\mathrm{f}(\mathrm{x}) = \log_4\left(\log_5\left(\log_3\left(18\mathrm{x} - \mathrm{x}^2 - 77\right)\right)\right)$ be (a, b)

Then the value of the integral

$\int_a^b \frac{\sin^3 x}{\left(\sin^3 x + \sin^3(a+b-x)\right)} dx$ is equal to

Q11 (27 July 2021 Shift 1)

Let $\mathrm{S} = \{1, 2, 3, 4, 5, 6, 7\}$. Then the number of

possible functions $\mathrm{f}: \mathrm{S} \rightarrow \mathrm{S}$ such that $\mathrm{f}(\mathrm{m} \cdot \mathrm{n}) = \mathrm{f}(\mathrm{m}) \cdot \mathrm{f}(\mathrm{n})$

for every $m, n \in S$ and $m \cdot n \in S$ is equal to

Q12 (27 July 2021 Shift 2)

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as

$f(x+y)+f(x-y)=2 f(x) f(y), f\left(\frac{1}{2}\right)=-1$. Then

the value of $\sum_{k=1}^{20} \frac{1}{\sin(k) \sin(k+f(k))}$ is equal to:

(1) $\operatorname{cosec}^2(21) \cos(20) \cos(2)$

(2) $\sec^2(1) \sec(21) \cos(20)$

(3) $\operatorname{cosec}^2(1) \operatorname{cosec}(21) \sin(20)$

(4) $\sec^2(21) \sin(20) \sin(2)$

Q13 (27 July 2021 Shift 2)

Let $\alpha=\max_{x \in R}\left\{8^{2 \sin 3x} \cdot 4^{4 \cos 3x}\right\}$ and

$\beta=\min_{x \in R}\left\{8^{2 \sin 3x} \cdot 4^{4 \cos 3x}\right\}$. If $8x^2+bx+c=0$ is a

quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$,

then the value of $c-b$ is equal to :

(1) 42

(2) 47

(3) 43

(4) 50

Answer Key

Q1 (3)	**Q2** (2)	**Q3** (1)	**Q4** (4)
Q5 (1)	**Q6** (720)	**Q7** (1)	**Q8** (1)
Q9 (3)	**Q10** (1)	**Q11** (490)	**Q12** (3)
Q13 (1)			

Q1

For domain,

$\frac{|[x]|-2}{|[x]|-3} \geq 0$

Case I : When $|[\mathrm{x}]|-2 \geq 0$

and $|[x]|-3>0$

$\therefore \mathrm{x} \in(-\infty,-3) \cup[4, \infty)$

Case II : When $|[x]|-2 \leq 0$

and $|[x]|-3<0$

$\therefore \mathrm{x} \in[-2,3)$

So, from (1) and (2) we get

Domain of function

$=(-\infty,-3) \cup[-2,3) \cup[4, \infty)$

$\therefore(a+b+c)=-3+(-2)+3=-2(a<b<c)$

$\Rightarrow$ Option (3) is correct.

Q2

$f(x)=\frac{5 x+3}{6 x-\alpha}=y$ ....(1)

$5 x+3=6 x y-\alpha y$

$x(6 y-5)=\alpha y+3$

$\mathrm{x}=\frac{\alpha \mathrm{y}+3}{6 \mathrm{y}-5}$

$\mathrm{f}^{-1}(\mathrm{x})=\frac{\alpha \mathrm{x}+3}{6 \mathrm{x}-5}$ ....(2)

$\mathrm{fo}\ \mathrm{f}(\mathrm{x})=\mathrm{x}$

$\mathrm{f}(\mathrm{x})=\mathrm{f}^{-1}(\mathrm{x})$

From eq. (i) & (ii) Clearly $(\alpha=5)$

Q3

$\log _{(x+1)}\left(2 x^{2}+7 x+5\right)+\log _{(2 x+5)}(x+1)^{2}-4=0$

$\log _{(x+1)}(2 x+5)(x+1)+2 \log _{(2 x+5)}(x+1)=4$

$\log_{(x+1)}(2x+5)+1+2\log_{(2x+5)}(x+1)=4$

Put $\log_{(x+1)}(2x+5)=t$

$t+\frac{2}{t}=3 \Rightarrow t^2-3t+2=0$

$t=1,2$

$\log_{(x+1)}(2x+5)=1 \& \quad \log_{(x+1)}(2x+5)=2$

$x+1=2x+3$

$\& \quad 2x+5=(x+1)^2$

$x=-4$ (rejected) $\quad x^2=4 \Rightarrow x=2,-2$ (rejected)

So, $x=2$

No. of solution $=1$

Q4

$[e^x]^2+[e^x+1]-3=0$

$\Rightarrow [e^x]^2+[e^x]+1-3=0$

Let $[e^x]=t$

$\Rightarrow t^2+t-2=0$

$\Rightarrow t=-2,1$

$[e^x]=-2 \quad$ (Not possible)

or $[e^x]=1 \quad \therefore 1 \leq e^x<2$

$\Rightarrow \quad \ln(1) \leq x<\ln(2)$

$\Rightarrow \quad 0 \leq x<\ln(2)$

$\Rightarrow \quad x \in[0, \ln 2)$

Q5

$O \leq x^2-x+1 \leq 1$

$\Rightarrow x^2-x \leq 0$

$\Rightarrow x \in[0,1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right) \leq \frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\leq 1$

$\Rightarrow 0<2\mathrm{x}-1\leq 2$

$1<2\mathrm{x}\leq 3$

$\frac{1}{2}<x\leq\frac{3}{2}$

Taking intersection $x\in\left(\frac{1}{2},1\right]$

$\Rightarrow\alpha=\frac{1}{2},\beta=1$

$\Rightarrow\alpha=\frac{1}{2},\beta=1$

Q6

$f(1)+f(2)=3-f(3)$

$\Rightarrow \mathrm{f}(1)+\mathrm{f}(2)=3+\mathrm{f}(3)=3$

The only possibility is: $0+1+2=3$

$\Rightarrow$ Elements $1,2,3$ in the domain can be mapped

with $0,1,2$ only.

So number of bijective functions.

$=\underline{3}\times\lfloor 5=720$

Q7

$g:\mathrm{N}\rightarrow\mathrm{N}\quad g(3\mathrm{n}+1)=3\mathrm{n}+2$

$g(3n+2)=3n+3$

$g(3n+3)=3n+1$

$$g(x)=\begin{bmatrix} x+1 & x=3k+1 \\ x+1 & x=3k+2 \\ x-2 & x=3k+3 \end{bmatrix}$$

$$g(g(x))=\begin{bmatrix} x+2 & x=3k+1 \\ x-1 & x=3k+2 \\ x-1 & x=3k+3 \end{bmatrix}$$

$$g(g(g(x)))=\begin{bmatrix} x & x=3k+1 \\ x & x=3k+2 \\ x & x=3k+3 \end{bmatrix}$$

If $f:\mathrm{N}\rightarrow\mathrm{N}$, f is a one-one function such that

$f(\mathrm{g}(\mathrm{x}))=f(\mathrm{x}) \Rightarrow \mathrm{g}(\mathrm{x})=\mathrm{x}$, which is not the case

If f $f: \mathrm{N} \rightarrow \mathrm{N} f$ is an onto function

such that $f(\mathrm{g}(\mathrm{x}))=f(\mathrm{x})$,

one possibility is $f(x)=\begin{cases} n & x=3n+1 \\ n & x=3n+2 \quad n \in N_0 \\ n & x=3n+3 \end{cases}$

Here $f(\mathrm{x})$ is onto, also $f(\mathrm{g}(\mathrm{x}))=f(\mathrm{x}) \forall \mathrm{x} \in \mathrm{N}$

Q8

$|x|^2-|x|-12=0$

$(|x|+3)(|x|-4)=0$

$|x|=4 \Rightarrow x=\pm 2$

Q9

$\therefore (\text{ gof })^{-1}$ exist $\Rightarrow$ gof is bijective

$\Rightarrow' f'$ must be one-one and ' g ' must be ONTO

Q10

For domain

$\log_5\left(\log_3\left(18x-x^2-77\right)\right)>0$

$\log_3\left(18x-x^2-77\right)>1$

$18x-x^2-77>3$

$x^2-18x+80<0$

$\mathrm{x} \in(8,10)$

$\Rightarrow \mathrm{a}=8$ and $\mathrm{b}=10$

$I=\int_a^b \frac{\sin^3 x}{\sin^3 x+\sin^3(a+b-x)} dx$

$I=\int_a^b \frac{\sin^3(a+b-x)}{\sin^3 x+\sin^3(a+b-x)}$

$2\mathrm{I}=(\mathrm{b}-\mathrm{a}) \Rightarrow \mathrm{I}=\frac{\mathrm{b}-\mathrm{a}}{2}(\because \mathrm{a}=8$ and $\mathrm{b}=10)$

$\mathrm{I}=\frac{10-8}{2}=1$

Q11

$\mathrm{F(mn)=f(m)\cdot f(n)}$

Put $\mathrm{m=1f(n)=f(1)\cdot f(n)\Rightarrow f(1)=1}$

Put $\mathrm{m=n=2}$

$$\mathrm{f(4)=f(2).f(2)}\begin{cases}\mathrm{f(2)=1\Rightarrow f(4)=1}\\ \text{ or}\\ \mathrm{f(2)=2\Rightarrow f(4)=4}\end{cases}$$

Put $\mathrm{m=2,n=3}$

$$f(6)=f(2)\cdot f(3)\begin{cases}\text{ when } f(2)=1\\ f(3)=1\text{ to }7\\ f(2)=2\\ f(3)=1\text{ or }2\text{ or }3\end{cases}$$

$\mathrm{f(5),f(7)}$ can take any value

Total $|\mathrm{x}|\times 7\times 1\times 7\times 1\times 7$

$|\mathrm{x}|\times 3\times 1\times 7\times 1\times 7$

$=490$

Q12

$f(x)=\cos\lambda x$

$\because \mathrm{f}\left(\frac{1}{2}\right)=-1$

So, $-1=\cos\frac{\lambda}{2}$

$\Rightarrow\lambda=2\pi$

Thus $\mathrm{f(x)=\cos 2\pi x}$

Now k is natural number

Thus $\mathrm{f(k)=1}$

$\sum_{k=1}^{20}\frac{1}{\sin k\sin(k+1)}=\frac{1}{\sin 1}\sum_{k=1}^{20}\left[\frac{\sin((k+1)-k)}{\sin k\cdot\sin(k+1)}\right]$

$=\frac{1}{\sin 1}\sum_{k=1}^{20}(\cot\mathrm{k}-\cot(\mathrm{k}+1))$

$=\frac{\cot 1-\cot 21}{\sin 1}=\operatorname{cosec}^2 1\operatorname{cosec}(21)\cdot\sin 20$

Q13

$\alpha=\max\left\{8^{2\sin 3x}\cdot 4^{4\cos 3x}\right\}$

$=\max\left\{2^{6\sin 3x}\cdot 2^{\operatorname{sos} 3x}\right\}$

$=\max\left\{2^{6\sin 3x+\operatorname{sos} 3x}\right\}$

and $\beta=\min\left\{8^{2\sin 3\mathrm{x}}\cdot 4^{4\cos 3\mathrm{x}}\right\}=\min\left\{2^{6\sin 3x+8\cos 3\mathrm{x}}\right\}$

Now range of $6\sin 3x+8\cos 3x$

$=\left[-\sqrt{6^2+8^2},+\sqrt{6^2+8^2}\right]=[-10,10]$

$\alpha=2^{10}\&\beta=2^{-10}$

So, $\alpha^{1/5}=2^2=4$

$\Rightarrow\beta^{1/5}=2^{-2}=1/4$

quadratic $8\mathrm{x}^2+\mathrm{bx}+\mathrm{c}=0,\mathrm{c}-\mathrm{b}=$

$8\times$[(product of roots]+(sum of roots)

$=8\times\left[4\times\frac{1}{4}+4+\frac{1}{4}\right]=8\times\left[\frac{21}{4}\right]=42$