Q1 (20 July 2021 Shift 1)

If α and β are the distinct roots of the equation $\mathrm{x}^2+(3)^{1/4}\mathrm{x}+3^{1/2}=0$, then the value of $\alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)$ is equal to :

(1) 56×3^{25}

(2) 56×3^{24}

(3) 52×3^{24}

(4) 28×3^{25}

Q2 (25 July 2021 Shift 1)

If α,β are roots of the equation $\mathrm{x}^2+5(\sqrt{2})\mathrm{x}+10=0, \alpha>\beta$ and $\mathrm{P_n}=\alpha^{\mathrm{n}}-\beta^{\mathrm{n}}$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to

Q3 (25 July 2021 Shift 2)

If $\mathrm{a}+\mathrm{b}+\mathrm{c}=1, \mathrm{ab}+\mathrm{bc}+\mathrm{ca}=2$ and $\mathrm{abc}=3$, then the value of $a^4+b^4+c^4$ is equal to ___

Q4 (27 July 2021 Shift 1)

Let α,β be two roots of the equation $\mathrm{x}^2+(20)^{1/4}\mathrm{x}+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to

(1) 10

(2) 100

(3) 50

(4) 160

Q5 (27 July 2021 Shift 2)

The number of real roots of the equation

$\mathrm{e}^{4\mathrm{x}}-\mathrm{e}^{3\mathrm{x}}-4\mathrm{e}^{2\mathrm{x}}-\mathrm{e}^{\mathrm{x}}+1=0$ is equal to

Answer Key

Q1 (3) **Q2** (1) **Q3** (13) **Q4** (3)

Q5 (2)

Q1

As, $\left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha$

$\Rightarrow\left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2$ (On squaring)

$\therefore\left(\alpha^4+3\right)=(-)\sqrt{3}\alpha^2$

$\Rightarrow\alpha^8+6\alpha^4+9=3\alpha^4$ (Again squaring)

$\therefore\alpha^8+3\alpha^4+9=0$

$\Rightarrow\alpha^8=-9-3\alpha^4$

(Multiply by α^4)

So, $\alpha^{12}=-9\alpha^4-3\alpha^8$

$\therefore\quad\alpha^{12}=-9\alpha^4-3\left(-9-3\alpha^4\right)$

$\Rightarrow\alpha^{12}=\cancel{-9\alpha^4}+27+\cancel{9\alpha^4}$

Hence, $\boxed{\alpha^{12}=(27)^2}$

$\Rightarrow\left(\alpha^{12}\right)^8=(27)^8$

$\Rightarrow\alpha^{96}=(3)^{24}$

Similarly $\beta^{96}=(3)^{24}$

$\therefore\alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)=(3)^{24}\times52$

$\Rightarrow$ Option (3) is correct.

Q2

$x^2+5\sqrt{2}x+10=0$

$\&\mathrm{p}_{\mathrm{a}}=\alpha^{\mathrm{n}}-\beta^{\mathrm{n}}$ (Given)

Now $\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}=\frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$

$\frac{P_{17}\left(\alpha^{20}-\beta^{20}+5\sqrt{2}\left(\alpha^{19}-\beta^{19}\right)\right)}{P_{18}\left(\alpha^{19}-\beta^{19}+5\sqrt{2}\left(\alpha^{18}-\beta^{18}\right)\right)}$

$\frac{P_{17}\left(\alpha^{19}(\alpha+5\sqrt{2})-\beta^{19}(\beta+5\sqrt{2})\right)}{P_{18}\left(\alpha^{18}(\alpha+5\sqrt{2})-\beta^{18}(\beta+5\sqrt{2})\right)}$

Since $\alpha+5\sqrt{2}=-10/\alpha\ldots\ldots\ldots.(1)$

\& $\beta+5\sqrt{2}=-10/\beta$

Now put there values in above expression $=-\frac{10P_{17}P_{18}}{-10P_{18}P_{17}}=1$

Q3

$a^2+b^2+c^2=(a+b+c)^2-2\Sigma ab=-3$

$(ab+bc+ca)^2=\Sigma(ab)^2+2abc\Sigma a$

$\Rightarrow \Sigma(ab)^2=-2$

$$\begin{aligned} a^4+b^4+c^4 &= \left(a^2+b^2+c^2\right)^2-2\Sigma(ab)^2 \\ &= 9-2(-2)=13 \end{aligned}$$

Q4

$\left(x^2+\sqrt{5}\right)^2=\sqrt{20}x^2$

$x^4=-5 \Rightarrow x^8=25$

$\alpha^8+\beta^8=50$

Q5

$\mathrm{t}^4-\mathrm{t}^3-4\mathrm{t}^2-\mathrm{t}+1=0, \mathrm{e}^x=\mathrm{t}>0$

$\Rightarrow \mathrm{t}^2-\mathrm{t}-4-\frac{1}{\mathrm{t}}+\frac{1}{\mathrm{t}^2}=0$

$\Rightarrow \alpha^2-\alpha-6=0, \alpha=\mathrm{t}+\frac{1}{\mathrm{t}} \geq 2$

$\Rightarrow \alpha=3,-2$ (reject)

$\Rightarrow \mathrm{t}+\frac{1}{\mathrm{t}}=3$

$\Rightarrow$ The number of real roots $=2$