Q1 (20 July 2021 Shift 1)

The coefficient of x^{256} in the expansion of $(1x)^{101}\left(x^2+x+1\right)^{100}$ is:

(1) ${}^{100}C_{16}$

(2) ${}^{100}C_{15}$

(3) $-{}^{100}C_{16}$

(4) $-{}^{100}C_{15}$

Q2 (20 July 2021 Shift 1)

The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is ___

Q3 (20 July 2021 Shift 2)

For the natural numbers m, n, if $(1-y)^m(1+y)^n=1+a_1y+a_2y^2+\ldots+a_{m+n}y^{m+n}$

and $a_1=a_2=10$, then the value of $(m+n)$ is equal to:

(1) 88

(2) 64

(3) 100

(4) 80

Q4 (22 July 2021 Shift 1)

Let $f:\mathbf{R}\rightarrow\mathbf{R}$ be a function defined as

$$f(x)=\begin{cases}3\left(1-\frac{|x|}{2}\right) & \text{if} \quad |x|\leq 2\\ 0 & \text{if} \quad |x|>2\end{cases}$$

Let $g:\mathbf{R}\rightarrow\mathbf{R}$ be given by $g(x)=f(x+2)-f(x-2)$

If n and m denote the number of points in $\mathbf{R}$ where

g is not continuous and not differentiable,

respectively, then $n+m$ is equal to ____.

Q5 (22 July 2021 Shift 1)

If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180 , then r is equal to ____.

Q6 (25 July 2021 Shift 1)

If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \ldots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$, then the value of γ is :

(1) $\frac{a^2+b}{3a^3}$

(2) $\frac{a+b}{3a^2}$

(3) $\frac{b^2}{3a^3}$

(4) $\frac{a+b^2}{3a^3}$

Q7 (25 July 2021 Shift 1)

The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is

Q8 (25 July 2021 Shift 1)

The term independent of ' x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equalto

Q9 (25 July 2021 Shift 2)

The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3} + 3^{1/4}\right)^{12}$ is:

(1) 89

(2) 27

(3) 35

(4) 43

Q10 (25 July 2021 Shift 2)

If the greatest value of the term independent of ' x ' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of ' a ' is equal to:

(1) -1

(2) 1

(3) -2

(4) 2

Q11 (25 July 2021 Shift 2)

The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ___

(1) 3

(2) 4

(3) 2

(4) 1

Q12 (25 July 2021 Shift 2)

Let $\mathrm{n} \in \mathbf{N}$ and $[\mathrm{x}]$ denote the greatest integer less than or equal to x. If the sum of $(\mathrm{n}+1)$ terms $^{\mathrm{n}}\mathrm{C}_0, 3\cdot{}^{\mathrm{n}}\mathrm{C}_1, 5\cdot{}^{\mathrm{n}}\mathrm{C}_2, 7\cdot{}^{\mathrm{n}}\mathrm{C}_3, \ldots\ldots$ is equal to $2^{100}\cdot 101$ then $2\left[\frac{\mathrm{n}-1}{2}\right]$ is equal to ___

Q13 (25 July 2021 Shift 2)

If the co-efficient of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to ___

Q14 (27 July 2021 Shift 1)

If the coefficients of x^7 in $\left(\mathrm{x}^2+\frac{1}{\mathrm{bx}}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b

is equal to:

(1) 2

(2) -1

(3) 1

(4) -2

Q15 (27 July 2021 Shift 2)

A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3 \sqrt{25^{n-1}+7}}+3^{\left(-\frac{1}{8}\right) \log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right) \log_{33}\left(5^{x-1}+1\right)}$ is equal to 180, is :

(1) 0

(2) -1

(3) 2

(4) 1

Answer Key

Q1 (2)	**Q2** (21)	**Q3** (4)	**Q4** (4)
Q5 (8)	**Q6** (3)	**Q7** (1)	**Q8** (210)
Q9 (4)	**Q10** (4)	**Q11** (1)	**Q12** (98)
Q13 (55)	**Q14** (3)	**Q15** (4)	

Q1

$(1-x)^{160} \cdot \left(x^2+x+1\right)^{10} \cdot (1-x)$

$= \left((1-x)\left(x^2+x+1\right)\right)^{100}(1-x)$

$= \left(1^3-x^3\right)^{100}(1-x)$

$= \underbrace{\left(1-x^3\right)^{100}}_{\text{No term of } x^{256}} - \underbrace{x\left(1-x^3\right)^{100}}_{\text{We findcofficicntof } x^{255}}$

Required coefficient $(-1) \times \left(-{}^{100}\mathrm{C}_{ss}\right)$

$= {}^{100}\mathrm{C}_{85} = {}^{10}\mathrm{C}_{15}$

Q2

$\left(4^{1/4}+5^{1/6}\right)^{120}$

$\mathrm{T}_{\mathrm{r}+1} = {}^{120}\mathrm{C}_{\mathrm{r}}\left(2^{1/2}\right)^{120-\mathrm{r}}(5)^{\mathrm{r}/6}$

for rational terms $\mathrm{r} = 6\lambda \quad 0 \leq \mathrm{r} \leq 120$ so total no of forms are 21.

Q3

$(1-y)^m(1+y)^n$

Coefficient of y $(a_1) = 1.{}^nC_1 + {}^mC_1(-1)$

$= \mathrm{n}-\mathrm{m} = 10 \quad \ldots\ldots(1)$

Coefficient of y^2 $(\mathrm{a}_2) = 1.{}^{\mathrm{n}}\mathrm{C}_2 - {}^{\mathrm{m}}\mathrm{C}_1 \cdot {}^{\mathrm{n}}\mathrm{C}_1.+1.{}^{\mathrm{m}}\mathrm{C}_2 = 10$

$= \frac{\mathrm{n}(\mathrm{n}-1)}{2} - \mathrm{m}\cdot\mathrm{n} + \frac{\mathrm{m}(\mathrm{m}-1)}{2} = 10$

$\mathrm{m}^2+\mathrm{n}^2-2\mathrm{mn}-(\mathrm{n}+\mathrm{m}) = 20$

$(\mathrm{n}-\mathrm{m})^2-(\mathrm{n}+\mathrm{m}) = 20$

$\mathrm{n}+\mathrm{m} = 80$ ....(2)

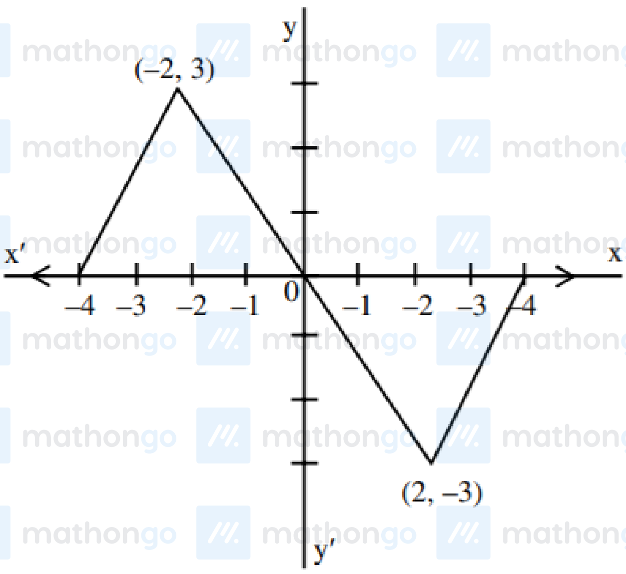
By equation (1) & (2) $\mathrm{m} = 35, \mathrm{n} = 45$

Q4

$$f(x-2)\begin{cases}\frac{3x}{2} & -4 \le x \le -2\\ -\frac{3x}{2} & -2 < x \le 0\\ 0 & x \in (-\infty,-4)\cup(0,+\infty)\end{cases}$$

$$f(x+2)\begin{cases}\frac{3x}{2}+6 & 0 \le x \le 2\\ -\frac{3x}{2}+6 & 2 \le x \le 4\\ 0 & x \in (-\infty,0)\cup(4,+\infty)\end{cases}$$

$$g(x)=f(x+2)-f(x-2)\begin{cases}\frac{3x}{2}+6 & -4 \le x \le -2\\ -\frac{3x}{2} & -2 < x < 2\\ \frac{3x}{2}-6 & 2 \le x \le 4\\ 0 & x \in (-\infty,-4)\cup(4,+\infty)\end{cases}$$



$\mathrm{n}=0$

$\mathrm{m}=4 \Rightarrow (\mathrm{n}+\mathrm{m}=4)$

Q5

$\left(2x^r+\frac{1}{x^2}\right)^{10}$

General term $= {}^{10}C_R\left(2x^2\right)^{10-R}x^{-2R}$

$\Rightarrow 2^{10-R\,10}C_R = 180\ldots\ldots\ldots(1)$

$\&(10-R)r-2R=0$

$r=\frac{2R}{10-R}$

$r=\frac{2(R-10)}{10-R}+\frac{20}{10-R}$

$\Rightarrow \mathrm{r}=-2+\frac{20}{10-\mathrm{R}} \ldots \ldots(2)$

$\mathrm{R}=8$ or 5 reject equation (1) not satisfied

At $\mathrm{R}=8$

$2^{10-\mathrm{R}^{10}} \mathrm{C}_{\mathrm{R}}=180 \Rightarrow \mathrm{r}=8$

Q6

$(a-b)^{-1}+(a-2b)^{-1}+\ldots . .+(a-nb)^{-1}$
$=\frac{1}{\mathrm{a}} \sum_{\mathrm{t}=1}^{\mathrm{n}}\left(1-\frac{\mathrm{rb}}{\mathrm{a}}\right)^{-1}$
$=\frac{1}{\mathrm{a}} \sum_{\mathrm{i}=1}^{\mathrm{n}}\left\{\left(1+\frac{\mathrm{rb}}{\mathrm{a}}+\frac{\mathrm{r}^{2} \mathrm{~b}^{2}}{\mathrm{a}^{2}}\right)+(\text { terms to be neglected })\right\}$
$=\frac{1}{a}\left[n+\frac{n(n+1)}{2} \cdot \frac{b}{a}+\frac{n(n+1)(2n+1)}{6} \cdot \frac{b^{2}}{a^{2}}\right]$
$=\frac{1}{a}\left[n^{3}\left(\frac{b^{2}}{3a^{2}}\right)+\ldots .\right]$
So $\gamma=\frac{b^{2}}{3a^{3}}$

Q7

Coeff. of middle term in $(1+\mathrm{x})^{20}={ }^{20}\mathrm{C}_{10}$

\& Sum of Coeff. of two middle terms in $(1+x)^{19}={ }^{19}C_{9}+{ }^{19}C_{10}$
So required ratio $=\frac{{ }^{20}\mathrm{C}_{10}}{{ }^{9}\mathrm{C}_{9}+{ }^{19}\mathrm{C}_{10}}=\frac{{ }^{2}\mathrm{C}_{10}}{{ }^{2}\mathrm{C}_{10}}=1$

Q8

$\left(\left(\mathrm{x}^{1/3}+1\right)-\left(\frac{\mathrm{x}^{1/2}+1}{\mathrm{x}^{1/2}}\right)\right)^{10}$
$=\left(x^{1/3}-\frac{1}{x^{1/2}}\right)^{10}$

Now General Term
$\mathrm{T}_{\mathrm{r}+1}={ }^{10}\mathrm{C}_{\mathrm{r}}\left(\mathrm{x}^{1/3}\right)^{10-\mathrm{r}} \cdot\left(-\frac{1}{\mathrm{x}^{1/2}}\right)^{\mathrm{r}}$
For independent term $\frac{10-\mathrm{r}}{3}-\frac{\mathrm{r}}{2}=0 \Rightarrow \mathrm{r}=4$

$\Rightarrow \mathrm{T}_{5}={ }^{10}\mathrm{C}_{4}=210$

Q9

$T_{r+1} = {}^{12}C_r\left(2^{1/3}\right)^r \cdot \left(3^{1/4}\right)^{12-r}$

T_{t+1} will be rational number when $r = 0, 3, 6, 9, 12$

$\& r = 0, 4, 8, 12$

$\Rightarrow r = 0, 12$

$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$

$= 24 + 16 = 43$

Q10

$T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$

$r = 0, 1, 2, \ldots, 10$

T_{i+1} will be independent of x

when $10 - 2r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5(x\sin\alpha)^5 \times \left(\frac{a\cos\alpha}{x}\right)^5$

$= {}^{10}C_5 \times a^5 \times \frac{1}{2^5}(\sin 2\alpha)^5$

will be greatest when $\sin 2\alpha = 1$

$\Rightarrow {}^{10}C_5\frac{a^5}{2^5} = {}^{10}C_5 \Rightarrow a = 2$

Q11

Let $P = \left(1 + \frac{1}{10^{100}}\right)^{10^{|x}},$

Let $x = 10^{100}$

$\Rightarrow P = \left(1 + \frac{1}{x}\right)^x$

$\Rightarrow P = 1 + (x)\left(\frac{1}{x}\right) + \frac{(x)(x-1)}{\lfloor 2} \cdot \frac{1}{x^2}$

$+ \frac{(x)(x-1)(x-2)}{\lfloor 3} \cdot \frac{1}{x^3} + \ldots$

(upto $10^{100} + 1$ terms) $\Rightarrow P = 1 + 1 + \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 2x^2}\right) + \left(\frac{1}{\lfloor 3} - \ldots\right) + \ldots$ so on

$\Rightarrow P = 2 + ($ Positive value less then $\frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{4} + \ldots)$

Also $e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \ldots$

$\Rightarrow \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \ldots = e - 2$

$\Rightarrow \mathrm{P}=2+($ positive value less then $\mathrm{e}-2) \Rightarrow \mathrm{P} \in(2,3)$

$\Rightarrow$ least integer value of P is 3

Q12

$1 .{ }^{\mathrm{I}} \mathrm{C}_0+3 \cdot{ }^n \mathrm{C}_1+5 \cdot{ }^{\mathrm{n}} \mathrm{C}_2+\ldots+(2 \mathrm{n}+1) \cdot{ }^{\mathrm{n}} \mathrm{C}_{\mathrm{a}}$
$\mathrm{T}_{\mathrm{t}}=(2 \mathrm{r}+1)^{\mathrm{n}} \mathrm{C}_t$

$\mathrm{S}=\Sigma \mathrm{T}_t$

$\mathrm{S}=\Sigma(2 \mathrm{r}+1)^{\mathrm{n}} \mathrm{C}_t=\Sigma 2 \mathrm{r}^{\mathrm{n}} \mathrm{C}_t+\Sigma^{\mathrm{n}} \mathrm{C}_{\mathrm{t}}$
$\mathrm{S}=2\left(\mathrm{n} \cdot 2^{\mathrm{n}-1}\right)+2^{\mathrm{n}}=2^{\mathrm{n}}(\mathrm{n}+1)$

$2^{\mathrm{a}}(\mathrm{n}+1)=2^{100} .101 \Rightarrow \mathrm{n}=100$

$2\left[\frac{\mathrm{n}-1}{2}\right]=2\left[\frac{99}{2}\right]=98$

Q13

${ }^{\mathrm{n}} \mathrm{C}_7 2^{\mathrm{n}-7} \frac{1}{3^7}={ }^{\mathrm{n}} \mathrm{C}_8 2^{\mathrm{n}-8} \frac{1}{3^8}$
$\Rightarrow \mathrm{n}-7=48 \Rightarrow \mathrm{n}=55$

Q14

Coefficient of x^7 in $\left(\mathrm{x}^2+\frac{1}{\mathrm{bx}}\right)^{11}$

${ }^{11} \mathrm{C}_r\left(\mathrm{x}^2\right)^{11-t} \cdot\left(\frac{1}{\mathrm{bx}}\right)^{\mathrm{t}}$

${ }^{11} \mathrm{C}_{\mathrm{r}} \mathrm{x}^{22-3 \mathrm{t}} \cdot \frac{1}{\mathrm{b}^t}$
$22-3 r=7$

$\mathrm{r}=5$

$\therefore{ }^{11} \mathrm{C}_5 \cdot \frac{1}{\mathrm{b}^5} \mathrm{x}^7$
Coefficient of x^{-7} in $\left(\mathrm{x}-\frac{\mathrm{b}}{\mathrm{bx}^2}\right)^{11}$

${ }^{11} \mathrm{C}_r(\mathrm{x})^{11-t} \cdot\left(-\frac{1}{\mathrm{bx}^2}\right)^t$

${ }^{11} \mathrm{C}_r \mathrm{x}^{11-3 t} \cdot \frac{(-1)^t}{\mathrm{b}^t}$
$11-3 r=-7 \therefore r=6$

${ }^{11} \mathrm{C}_6 \cdot \frac{1}{\mathrm{b}^6} \mathrm{x}^{-7}$

${}^{11}\mathrm{C}_5 \cdot \frac{1}{\mathrm{b}^5} = {}^{11}\mathrm{C}_6 \cdot \frac{1}{\mathrm{b}^6}$

Since $b \neq 0 \quad \therefore b = 1$

Q15

${}^{10}\mathrm{C}_8\left(25^{(x-1)}+7\right) \times\left(5^{(x-1)}+1\right)^{-1}=180$

$\Rightarrow \frac{25^{x-1}+7}{5^{(x-1)}+1}=4$

$\Rightarrow \frac{t^2+7}{t+1}=4$

$\Rightarrow t=1,3=5^{x-1}$

$\Rightarrow \mathrm{x}-1=0$ (one of the possible value).

$\Rightarrow \mathrm{x}=1$