Q1 (20 July 2021 Shift 1)

Let a function $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\sin x-e^{x} & \text{if } x \leq 0 \\ a+[-x] & \text{if } 0<x<1 \\ 2x-b & \text{if } x \geq 1\end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on R, then $(a+b)$ is equal to:

(1) 4

(2) 3

(3) 2

(4) 5

Q2 (20 July 2021 Shift 2)

Let a function g : $[0,4] \rightarrow \mathbf{R}$ be defined as

$$g(x)=\begin{cases}\max_{0 \leq t \leq x}\left\{t^{3}-6t^{2}+9t-3\right\}, & 0 \leq x \leq 3 \\ 4-x & , 3<x \leq 4\end{cases}$$

then the number of points in the interval $(0,4)$

where g(x) is NOT differentiable, is

Q3 (22 July 2021 Shift 1)

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\frac{x^{3}}{(1-\cos 2x)^{2}} \log_{e}\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^{2}}\right) & , \quad x \neq 0 \\ \alpha & , \quad x=0\end{cases}.$$

If f is continuous at $x=0$, then α is equal to:

(1) 1

(2) 3

(3) 0

(4) 2

Q4 (25 July 2021 Shift 1)

Let f : $\mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x)=\begin{cases}\frac{\lambda|x^2-5x+6|}{\mu(5x-x^2-6)}, & x<2\\ \frac{\operatorname{tn}(x-2)}{e^{x-[x]}} & ,x>2\\ \mu & ,x=2\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at $x=2$, then $\lambda+\mu$ is equal to

(1) e(-e + 1)

(2) $e(e-2)$

(3)

(4) 2e − 1

Q5 (25 July 2021 Shift 2)

Consider the function $\mathrm{f}(\mathrm{x})=\frac{\mathrm{P}(\mathrm{x})}{\sin(\mathrm{x}-2)}, \quad \mathrm{x}\neq 2$

$=7 \quad , \quad x=2$

Where $\mathrm{P}(\mathrm{x})$ is a polynomial such that $\mathrm{P}''(\mathrm{x})$ is always a constant and $\mathrm{P}(3)=9$. If $\mathrm{f}(\mathrm{x})$ is continuous at $\mathrm{x}=2$, then $\mathrm{P}(5)$ is equal to ___

Q6 (27 July 2021 Shift 1)

Let f : $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \rightarrow \mathbf{R}$ be defined as

$$f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}} & , \quad -\frac{\pi}{4}<x<0\\ b & , \quad x=0\\ e^{\cot 4x/\cot 2x} & , \quad 0<x<\frac{\pi}{4}\end{cases}$$

If f is continuous at $\mathrm{x}=0$, then the value of $6\mathrm{a}+\mathrm{b}^2$ is equal to:

(1) $1-\mathrm{e}$

(2) e-1

(3) $1+\mathrm{e}$

(4) e

Q7 (27 July 2021 Shift 1)

Let f : $[0,3] \rightarrow \mathbf{R}$ be defined by

$\mathrm{f(x)} = \min\{\mathrm{x} - [\mathrm{x}], 1 + [\mathrm{x}] - \mathrm{x}\}$

where [x] is the greatest integer less than or equal to x. Let P denote the set containing all $x \in [0,3]$ where f is discontinuous, and Q denote the set containing all $\mathrm{x} \in (0,3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to

Q8 (27 July 2021 Shift 2)

Let $f : [0,\infty) \rightarrow [0,3]$ be a function defined by $f(x) = \begin{cases} \max\{\sin t : 0 \leq t \leq x\}, 0 \leq x \leq \pi \\ 2 + \cos x, & x > \pi \end{cases}$

Then which of the following is true ?

(1) f is continuous everywhere but not differentiable exactly at one point in $(0,\infty)$

(2) f is differentiable everywhere in $(0,\infty)$

(3) f is not continuous exactly at two points in $(0,\infty)$

(4) f is continuous everywhere but not differentiable exactly at two points in $(0,\infty)$

Answer Key

Q1 (2)	**Q2** (1)	**Q3** (1)	**Q4** (1)
Q5 (39)	**Q6** (3)	**Q7** (5)	**Q8** (2)

Q1

Continuous at $\mathrm{x}=0$

$\mathrm{f}\left(0^{+}\right)=\mathrm{f}\left(0^{-}\right) \Rightarrow \mathrm{a}-1=0-\mathrm{e}^{0}$

$\Rightarrow \mathrm{a}=0$

Continuous at $\mathrm{x}=1$

$\mathrm{f}\left(1^{+}\right)=\mathrm{f}\left(1^{-}\right)$

$\Rightarrow 2(1)-\mathrm{b}=\mathrm{a}+(-1)$

$\Rightarrow \mathrm{b}=2-\mathrm{a}+1 \Rightarrow \mathrm{b}=3$

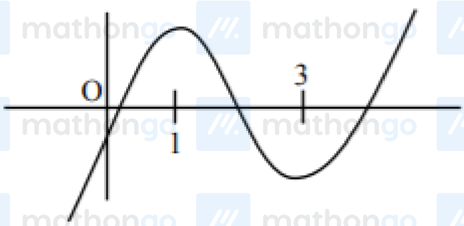
$\therefore \mathrm{a}+\mathrm{b}=3$

Q2

$f(x)=x^{3}-6 x^{2}+9 x-3$

$f^{\prime}(x)=3 x^{2}-12 x+9=3(x-1)(x-3)$

$f(1)=1 f(3)=-3$



$$g(x)=\left[\begin{array}{cc} f(x) & 0 \leq x \leq 1 \\ 0 & 1 \leq x \leq 3 \\ -1 & 3<x \leq 4 \end{array}\right.$$

g(x) is continuous

$$g^{\prime}(x)=\left[\begin{array}{cc} 3(x-1)(x-3) & 0 \leq x \leq 1 \\ 0 & 1 \leq x \leq 3 \\ -1 & 3<x \leq 4 \end{array}\right.$$

g(x) is non-differentiable at $\mathrm{x}=3$

Q3

For continuity $\lim_{x\to 0} \frac{x^3}{4\sin^4 x}\left(\ln\left(1+12xe^{-2x}\right)-2\ell n\left(1-xe^{-x}\right)\right)$

$=\alpha$

$\lim_{x\to 0}\frac{1}{4x}\left[2xe^{-2x}+2xe^{-x}\right]=\alpha$

$=\frac{1}{4}(4)=\alpha=1$

Q4

$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} e^{\frac{\sin(x-2)}{x-2}}=e^1$

$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}\frac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)}=-\frac{\lambda}{\mu}$

For continuity $\mu=\mathrm{e}=-\frac{\lambda}{\mu}\Rightarrow\mu=\mathrm{e},\lambda=-\mathrm{e}^2$

$\lambda+\mu=\mathrm{e}(-\mathrm{e}+1)$

Q5

$$f(x)=\begin{cases}\frac{\mathrm{P(x)}}{\sin(\mathrm{x}-2)}, & \mathrm{x}\neq 2\\ 7 & \mathrm{x}=2\end{cases}$$

$\mathrm{P}''(\mathrm{x})=$ const. $\Rightarrow \mathrm{P(x)}$ is a 2 degree polynomial

$f(\mathrm{x})$ is cont. at $\mathrm{x}=2$

$f\left(2^+\right)=f\left(2^-\right)$

$\lim_{x\to 2^+}\frac{\mathrm{P(x)}}{\sin(\mathrm{x}-2)}=7$

$\lim_{x\to 2^+}\frac{(x-2)(ax+b)}{\sin(x-2)}=7\Rightarrow 2a+b=7$

$\mathrm{P(x)}=(\mathrm{x}-2)(\mathrm{ax}+\mathrm{b})$

$\mathrm{P}(3)=(3-2)(3\mathrm{a}+\mathrm{b})=9\Rightarrow 3\mathrm{a}+\mathrm{b}=9$

$a=2,b=3$

$\mathrm{P}(5)=(5-2)(2.5+3)=3.13=39$

Q6

$\lim_{x\to 0} f(x)=b$

$\lim_{x\to 0^+} xe^{\frac{\cot 4x}{\cot 2x}}=e^{\frac{1}{2}}=b$

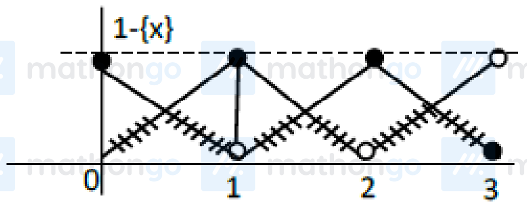
$\lim_{x\to 0^-}(1+|\sin x|)^{\frac{3a}{\sin x|}}=e^{3a}=e^{\frac{1}{2}}$

$\lim_{x\to 0^-}(1+|\sin x|)\sin x] = e^{3a} = e^{\frac{1}{2}}$

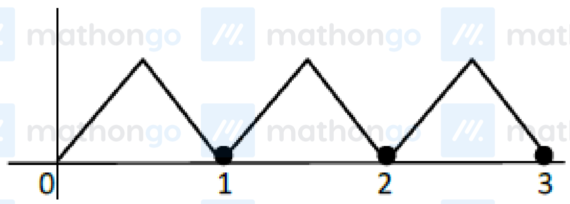
$a = \frac{1}{6} \Rightarrow 6a = 1$

$\left(6a+b^2\right) = (1+e)$

Q7



$1-\{x\} = 1-x;\ 0 \le x < 1$

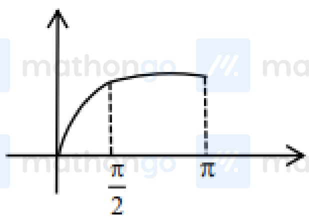


Non differentiable at

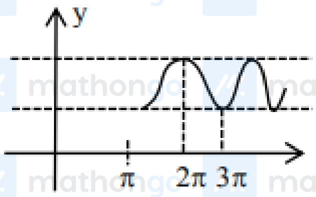
$x = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$

Q8

Graph of $\max\{\sin t : 0 \le t \le x\}$ in $x \in [0, \pi]$

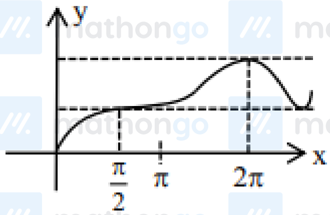


& graph of cos for $x \in [\pi, \infty)$



So graph of

$$f(x)=\begin{cases}\max\{\sin t : 0\le t\le x, & 0\le x\le \pi\\ 2+\cos x & x>h\end{cases}$$



f(x) is differentiable everywhere in $(0,\infty)$