Q1 (20 July 2021 Shift 2)

Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4, 1)$ and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$

If $\frac{r_1}{r_2} = a + b\sqrt{2}$, then $a + b$ is equal to :

(1) 3

(2) 11

(3) 5

(4) 7

Q2 (22 July 2021 Shift 1)

Let the circle $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$

(2) $100 < C < 165$

(3) $81 < C < 156$

(4) $100 < C < 156$

Q3 (27 July 2021 Shift 1)

Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to:

(1) 2

(2) $(3\sqrt{2} + 2)$

(3) 4

(4) $3(\sqrt{2} - 1)$

Q4 (27 July 2021 Shift 1)

Let $A=\left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 2x^2+2y^2-2x-2y=1\right\}$

$B=\left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid 4x^2+4y^2-16y+7=0\right\}$ and

$C=\left\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x^2+y^2-4x-2y+5 \leq r^2\right\}$

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$

(2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$

(4) $1+\sqrt{5}$

Q5 (27 July 2021 Shift 2)

Consider a circle C which touches the y -axis at

$(0,6)$ and cuts off an intercept $6\sqrt{5}$ on the x -axis.

Then the radius of the circle C is equal to :

(1) $\sqrt{53}$

(2) 9

(3) 8

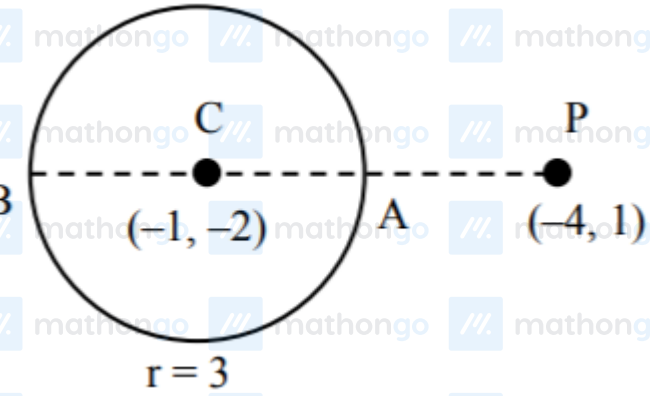
(4) $\sqrt{82}$

Answer Key

Q1 (3) **Q2** (4) **Q3** (3) **Q4** (3)

Q5 (2)

Q1



Centre of smallest circle is A

Centre of largest circle is B $r_2 = |CP - CA| = 3\sqrt{2} - 3$

$r_3 = CP + CB = 3\sqrt{2} + 3$

$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = \frac{(3\sqrt{2}+3)^2}{9} = (\sqrt{2}+1)^2 = 3 + 2\sqrt{2}$

$a = 3, b = 2$

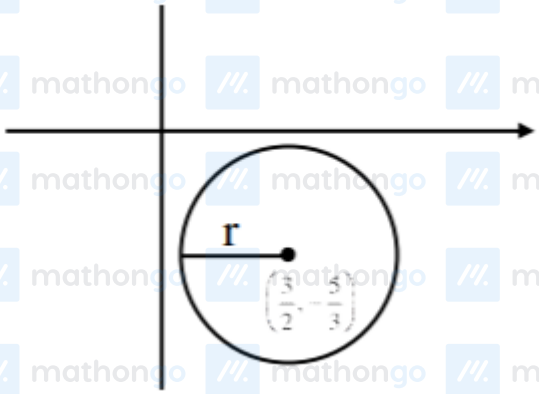
Q2

$S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$\Rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$

Centre $\equiv (-g, -f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$

radius $= r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$



Now, $\Rightarrow r < \frac{3}{2}$

$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$

$\Rightarrow C > 100$

$\ldots\ldots(1)$

Now point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$

$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$

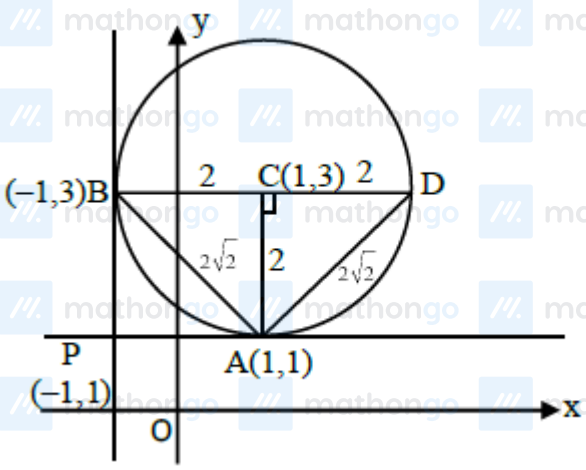
$$\boxed{C < 156}$$

From $(1)\&(2)$

$$\boxed{100 < C < 156}$$

Ans.

Q3



$\Delta ABD = \frac{1}{2} \times 2 \times 4$

$= 4$

Q4

$S_1 : x^2 + y^2 - x - y - \frac{1}{2} = 0; C_1 \left(\frac{1}{2}, \frac{1}{2}\right)$

$r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}} = 1$

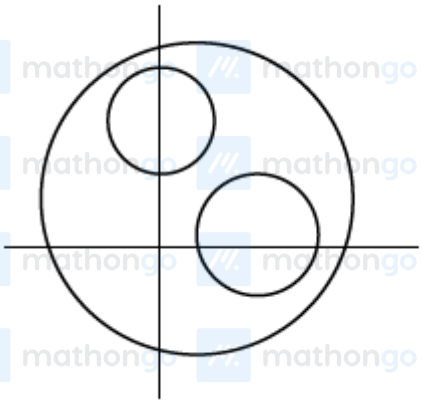
$S_2 : x^2 + y^2 - 4y + \frac{7}{4} = 0; C_2 : (0, 2)$

$r_2 = \sqrt{4 - \frac{7}{4}} = \frac{3}{2}$

$S_3 : x^2 + y^2 - 4x - 2y + 5 - r^2 = 0$

$C_3 : (2, 1)$

$r_3 = \sqrt{4 + 1 - 5 + r^2} = |r|$



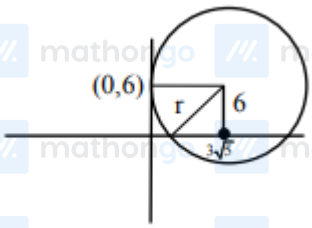
$C_1C_3 = \sqrt{\frac{5}{2}}$

$\sqrt{\frac{5}{2}} \leq |r - 1| \Rightarrow \left.\begin{array}{l} r \leq 1 + \sqrt{\frac{5}{2}} \\ r \geq \frac{3}{2} + \sqrt{5} \end{array}\right\}$

$C_2C_3 = \sqrt{5} \leq \left|r - \frac{3}{2}\right|$

$\left.\begin{array}{l} r - \frac{3}{2} \geq \sqrt{5} \\ r - \frac{3}{2} \leq -\sqrt{5} \end{array}\right\}$

Q5



$$r = \sqrt{6^2 + (3\sqrt{5})^2}$$

$$= \sqrt{36 + 45} = 9$$