Q1 (25 July 2021 Shift 2)

The value of $\cot\frac{\pi}{24}$ is:

(1) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$

(2) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

(3) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$

(4) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

Q2 (27 July 2021 Shift 1)

If $\sin\theta+\cos\theta=\frac{1}{2}$, then

$16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to:

(1) 23

(2) -27

(3) -23

(4) 27

Answer Key

Q1 (2) **Q2** (3)

Q1

$$\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$\theta = \frac{\pi}{24}$$

$$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$$

$$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

Q2

$$\sin\theta + \cos\theta = \frac{1}{2}$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$$

$$\sin 2\theta = -\frac{3}{4}$$

Now:

$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$= 1 - 2\left(-\frac{3}{4}\right)^2$$

$$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$$

$$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$$

$$= \left(3 - 4\sin^2 2\theta\right) \cdot \sin 2\theta$$

$$= \left[3 - 4\left(\frac{9}{16}\right)\right] \cdot \left(-\frac{3}{4}\right)$$

$$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$$

$$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$$

$$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$$