Q1 (20 July 2021 Shift 1)

If in a triangle ABC, $AB = 5$ units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\triangle ABC$ is 5 units, then the area (in sq. units) of $\triangle ABC$ is:

(1) $10 + 6\sqrt{2}$

(2) $8 + 2\sqrt{2}$

(3) $6 + 8\sqrt{3}$

(4) $4 + 2\sqrt{3}$

Q2 (20 July 2021 Shift 2)

Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to:

(1) $\frac{\sqrt{5}+1}{4}$

(2) $\frac{\sqrt{5}-1}{2}$

(3) $\frac{\sqrt{2}-1}{2}$
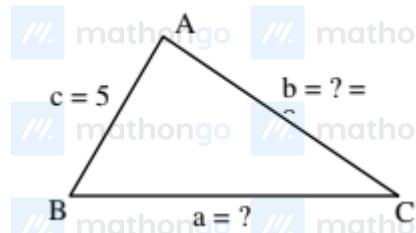
(4) $\frac{\sqrt{5}-1}{4}$

Answer Key

Q1 (3)   **Q2** (2)

Q1

As, $\cos B = \frac{3}{5}$

$\Rightarrow B = 53°$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c$

$\Rightarrow C = 30°$

Now, $\frac{b}{\sin B} = 2R$

$\Rightarrow b = 2(5)\left(\frac{4}{5}\right) = 8$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$

$\Rightarrow a^2 - 6a - 3\,g = 0$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\Rightarrow 3 + 4\sqrt{3}$　(Reject $a = 3 - 4\sqrt{3}$)

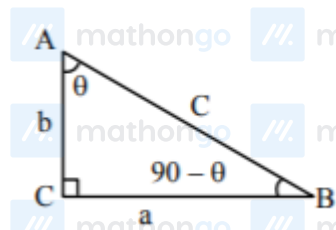Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct.

Q2

$< \text{A} = \theta$

$< \text{B} = 90 - \theta$

$a = \text{smallest side}$

$c^2 = a^2 + b^2$

$\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2}$

$\frac{b^2 c^2}{a^2} = b^2 + c^2$

Use $a = 2R \sin \text{A} = 2R \sin \theta$

$b = 2R \sin \text{B} = 2R \sin(90 - \theta) = 2R \cos \theta$

$c = 2R \sin \text{C} = 2 \sin 90° = 2R$

$\frac{4R^2 \cos^2 \theta}{4R^2 \sin^2 \theta} = 4R^2 \cos^2 \theta + 4R^2$

$\cos^2 \theta = \sin^2 \theta \cos^2 \theta + \sin^2 \theta$

$1 - \sin^2 \theta = \sin^2 \theta \left(1 - \sin^2 \theta\right) + \sin^2 \theta$

$\sin^2 \theta = \frac{3 - \sqrt{5}}{2}$

$\Rightarrow \sin \theta = \frac{\sqrt{5} - 1}{2}$