

Q1 (22 July 2021 Shift 1)

The number of elements in the set $\{n \in \{1, 2, 3, \dots, 100\} : \forall \alpha$

$(11)^n > (10)^n + (9)^n\}$ is _____.

Answer Key

Q1 (96)

Q1

$$11^n > 10^n + 9^n$$

$$\Rightarrow 11^n - 9^n > 10^n$$

$$\Rightarrow (10 + 1)^n - (10 - 1)^n > 10^n$$

$$\Rightarrow \{ {}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \dots \} > 10^n$$

$$\Rightarrow 2n \cdot 10^{n-1} + 2 \{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \dots \} > 10^n$$

$$\dots \dots (1)$$

For $n = 5$

$$10^5 + 2 \{ {}^5C_3 10^2 + {}^5C_5 \} > 10^5 \text{ (True)}$$

For $n = 6, 7, 8, \dots, 100$

$$2n10^{n-1} > 10^n$$

$$\Rightarrow 2n10^{n-1} + 2 \{ {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \dots \} > 10^n$$

$$\Rightarrow 11^n - 9^n > 10^n \text{ For } n = 5, 6, 7, \dots, 100$$

For $n = 4$, Inequality (1) is not satisfied \Rightarrow Inequality does not hold good for $N = 1, 2, 3, 4$

So, required number of elements = 96