Q1 (20 July 2021 Shift 1)

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(\mathrm{z}) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here $\arg(\mathrm{z})$ denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$

(2) $-\frac{3\pi}{4}$

(3) $-\frac{\pi}{4}$

(4) $\frac{3\pi}{4}$

Q2 (20 July 2021 Shift 2)

If the real part of the complex number

$(1-\cos\theta+2\,\mathrm{i}\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the

value of the integral $\int_0^\theta \sin x\,dx$ is equal to:

(1) 1

(2) 2

(3) -1

(4) 0

Q3 (22 July 2021 Shift 1)

Let n denote the number of solutions of the

equation $\mathrm{z}^2 + 3\overline{\mathrm{z}} = 0$, where z is a complex

number. Then the value of $\sum_{\mathrm{k}=0}^{\infty} \frac{1}{\mathrm{n}^{\mathrm{k}}}$ is equal to

(1) 1

(2) $\frac{4}{3}$

(3) $\frac{3}{2}$

(4) 2

Q4 (25 July 2021 Shift 2)

The equation of a circle is

$\mathrm{Re}(z^2) + 2(\mathrm{Im}(z))^2 + 2\,\mathrm{Re}(z) = 0$, where $z = x + iy$

A line which passes through the center of the given circle and the vertex of the parabola,

$x^2 - 6x - y + 13 = 0$, has y -intercept equal to ___

Q5 (27 July 2021 Shift 1)

Let C be the set of all complex numbers. Let $S_1 = \left\{z \in C \middle| |z - 3 - 2i|^2 = 8\right\}$

$S_2 = \{z \in C \mid \mathrm{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \| z - \bar{z} | \geq 8\}$

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is

equal to

(1) 1

(2) 0

(3) 2

(4) Infinite

Q6 (27 July 2021 Shift 2)

Let $\mathbb{C}$ be the set of all complex numbers. Let

$S_1 = \{z \in \mathbb{C} : |z - 2| \leq 1\}$ and

$S_2 = \{z \in \mathbb{C} : z(1 + i) + \bar{z}(1 - i) \geq 4\}$.

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for

$z \in S_1 \cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$

(2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$

(4) $\frac{5+2\sqrt{2}}{4}$

Q7 (27 July 2021 Shift 2)

If the real part of the complex number $z=\frac{3+2i\cos\theta}{1-3i\cos\theta}, \theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to

Answer Key

Q1 (3) **Q2** (1) **Q3** (2) **Q4** (1)

Q5 (1) **Q6** (4) **Q7** (1)

Q1

As $|z\omega| = 1$

$\Rightarrow$ If $|z| = r$, then $|\omega| = \frac{1}{r}$

Let $\arg(\mathrm{z}) = \theta$

$\therefore \arg(\omega) = \left(\theta - \frac{3\pi}{2}\right)$

So, $\mathrm{z} = \mathrm{re}^{\mathrm{i}\theta}$

$\Rightarrow \overline{\mathrm{z}} = \mathrm{re}^{\mathrm{i}(-\theta)}$

$\omega = \frac{1}{\mathrm{r}}\mathrm{e}^{1\left(\theta - \frac{3\pi}{2}\right)}$

Now, consider

$\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i}\right)$

$= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i)$

$\therefore$ prin $\arg\left(\frac{1-2\overline{z}\omega}{1+3\overline{z}\omega}\right)$

$=$ prin $\arg\left(\frac{1-2\overline{z}\omega}{1+3\overline{z}\omega}\right)$

$= \left(-\frac{1}{2}(1+i)\right)$

$= -\left(\pi - \frac{\pi}{4}\right) = \frac{-3\pi}{4}$

So, option (2) is correct.

Q2

$z = \frac{1}{1-\cos\theta+2i\sin\theta}$

$= \frac{2\sin^2\frac{\theta}{2}-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$

$= \frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$

$\operatorname{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$

$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$

$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$

$3\cos^2\frac{\theta}{2}=\frac{3}{2}$

$\cos^2\frac{\theta}{2}=\frac{1}{2}$

$\frac{\theta}{2}=n\pi\pm\frac{\pi}{4}$

$\theta=2n\pi\pm\frac{\pi}{2}$

$\theta=2\mathrm{n}\pi\pm\frac{\pi}{2}$

$\theta\in(0,\pi)$

$\left.\theta=\frac{\pi}{2}\right]$

$\int_0^{\frac{\pi}{2}}\sin\theta d\theta-[-\cos\theta]_0^{\frac{\pi}{2}}$

$=-(0-1)$

$=1$

Q3

$\mathrm{z}^2+3\overline{\mathrm{z}}=0$

Put $\mathrm{z}=\mathrm{x}+\mathrm{iy}$

$\Rightarrow x^2-y^2+2ixy+3(x-iy)=0$

$\Rightarrow\left(x^2-y^2+3x\right)+i(2xy-3y)=0+i0$

$\therefore x^2-y^2+3x=0\ldots\ldots(1)$

$2xy-3y=0\ldots\ldots(2)$

$x=\frac{3}{2},y=0$

Put $\mathrm{x}=\frac{3}{2}$ in equation (1)

$\frac{9}{4}-y^2+\frac{9}{2}=0$

$y^2=\frac{27}{4}\Rightarrow y=\pm\frac{3\sqrt{3}}{2}$

$\therefore(x,y)=\left(\frac{3}{2},\frac{3\sqrt{3}}{2}\right),\left(\frac{3}{2},\frac{-3\sqrt{3}}{2}\right)$

Put $y=0\Rightarrow x^2-0+3x=0$

$x=0,-3$

$\therefore(x,y)=(0,0),(-3,0)$

$\therefore$ No of solutions $= n = 4$

$$\sum_{K=0}^{\infty}\left(\frac{1}{n^k}\right) = \sum_{K=0}^{\infty}\left(\frac{1}{4^k}\right)$$
$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots$$
$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

Q4

Equation of circle is $\left(x^2 - y^2\right) + 2y^2 + 2x = 0$ $\mathrm{x}^2 + \mathrm{y}^2 + 2\mathrm{x} = 0$

Centre : $(-1, 0)$ Parabola : $\mathrm{x}^2 - 6\mathrm{x} - \mathrm{y} + 13 = 0$

$(x-3)^2 = y - 4$

Vertex : $(3, 4)$ Equation of line $\equiv \mathrm{y} - 0 = \frac{4-0}{3+1}(\mathrm{x}+1)$

$y = x + 1$

y -intercept $= 1$

Q5

$S_1 : |z - 3 - 2i|^2 = 8$

$|z - 3 - 2i| = 2\sqrt{2}$

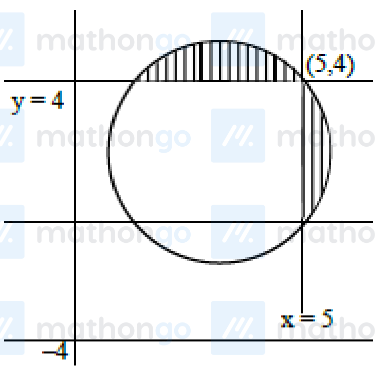
$(x-3)^2 + (y-2)^2 = (2\sqrt{2})^2$

$S_2 : x \geq 5$

$S_3 : |z - \bar{z}| \geq 8$

$|2iy| \geq 8$

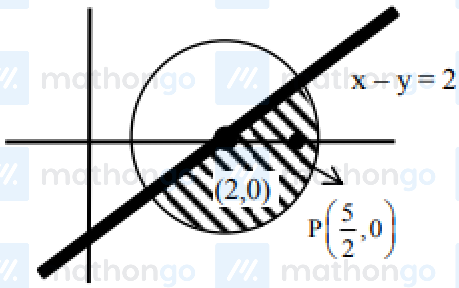
$2|y| \geq 8 \quad \therefore y \geq 4, y \leq -4$



$n(S_1 \cap S_2 \cap S_3) = 1$

Q6

$|t-2| \leq 1$ Put $t = x + iy$



$(x-2)^2 + y^2 \leq 1$

Also, $t(1+i) + \bar{t}(1-i) \geq 4$

Gives $x - y \geq 2$

Let point on circle be $A(2+\cos\theta, \sin\theta)$

$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$= \cos^2\theta - \cos\theta + \frac{1}{4} + \sin^2\theta$

$= \frac{5}{4} - \cos\theta$

For $(AP)^2$ maximum $\theta = -\frac{3\pi}{4}$

$(AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}}$

Q7

$\operatorname{Re}(z) = \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, $\sin^2 3\theta + \cos^2\theta = 1$