Q1 (20 July 2021 Shift 1)

Let y = y(x) be the solution of the differential equation $x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx$ $-1 \leq x \leq 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves $x = 0, x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is:

(1) $\frac{1}{8}(\pi - 1)$

(2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{4}(\pi - 2)$

(4) $\frac{1}{6}(\pi - 1)$

Q2 (20 July 2021 Shift 1)

Let $\mathrm{y} = \mathrm{y}(\mathrm{x})$ be the solution of the differential equation $\mathrm{e}^{\mathrm{x}}\sqrt{1 - \mathrm{y}^2}\mathrm{dx} + \left(\frac{\mathrm{y}}{\mathrm{x}}\right)\mathrm{dy} = 0, \mathrm{y}(1) = -1$.

Then the value of $(\mathrm{y}(3))^2$ is equal to:

(1) $1 - 4\mathrm{e}^3$

(2) $1 - 4\mathrm{e}^6$

(3) $1 + 4\mathrm{e}^3$

(4) $1 + 4\mathrm{e}^6$

Q3 (20 July 2021 Shift 2)

Let $y = y(x)$ satisfies the equation $\frac{dy}{dx} - |A| = 0$,

for all $x > 0$, where $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$. If $\mathrm{y}(\pi) = \pi + 2$, then the value of $\mathrm{y}\left(\frac{\pi}{2}\right)$ is :

(1) $\frac{\pi}{2} + \frac{4}{\pi}$

(2) $\frac{\pi}{2} - \frac{1}{\pi}$

(3) $\frac{3\pi}{2} - \frac{1}{\pi}$

(4) $\frac{\pi}{2} - \frac{4}{\pi}$

Q4 (20 July 2021 Shift 2)

Let a curve $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be given by the solution of the differential equation

$\cos\left(\frac{1}{2}\cos^{-1}(\mathrm{e}^{-\mathrm{x}})\right)\mathrm{dx}=\sqrt{\mathrm{e}^{2\mathrm{x}}-1}\mathrm{dy}$

If it intersects y -axis at $\mathrm{y}=-1$, and the intersection point of the curve with x -axis is $(\alpha, 0)$, then e^{α} is equal to

Q5 (22 July 2021 Shift 1)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential equation $\operatorname{cosec}^2 x dy+2dx=(1+y\cos 2x)\operatorname{cosec}^2 x dx$, with $\mathrm{y}\left(\frac{\pi}{4}\right)=0$. Then, the value of $(\mathrm{y}(0)+1)^2$ is equal to:

(1) $\mathrm{e}^{1/2}$

(2) $\mathrm{e}^{-1/2}$

(3) e^{-1}

(4) e

Q6 (22 July 2021 Shift 1)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx=(x+2)dy$, $y(1)=1$. If the domain of $y=y(x)$ is an open interval (α, β), then $l\alpha+\beta|$ is equal to ____.

Q7 (25 July 2021 Shift 1)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential equation $\frac{dy}{dx}=1+xe^{y-x}, -\sqrt{2}<x<\sqrt{2}, y(0)=0$ then, the minimum value of $\mathrm{y}(\mathrm{x}), \mathrm{x}\in(-\sqrt{2}, \sqrt{2})$ is equal to :

(1) $(2-\sqrt{3})-\log_e 2$

(2) $(2+\sqrt{3})+\log_e 2$

(3) $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$

(4) $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$

Q8 (25 July 2021 Shift 1)

Let $y=y(x)$ be solution of the following differential equation

$e^y \frac{dy}{dx}-2e^y \sin x+\sin x \cos^2 x=0, y\left(\frac{\pi}{2}\right)=0$

If $y(0)=\log_c\left(\alpha+\beta e^{-2}\right)$, then $4(\alpha+\beta)$ is equal to

Q9 (25 July 2021 Shift 2)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential equation $x dy=\left(y+x^3 \cos x\right) dx$ with $y(\pi)=0$, then $\mathrm{y}\left(\frac{\pi}{2}\right)$ is equal to:

(1) $\frac{\pi^2}{4}+\frac{\pi}{2}$

(2) $\frac{\pi^2}{2}+\frac{\pi}{4}$

(3) $\frac{\pi^2}{2}-\frac{\pi}{4}$

(4) $\frac{\pi^2}{4}-\frac{\pi}{2}$

Q10 (25 July 2021 Shift 2)

Let a curve $y=f(x)$ pass through the point $\left(2, \left(\log_{\mathrm{e}} 2\right)^2\right)$ and have slope $\frac{2\mathrm{y}}{\mathrm{x}\log_{\mathrm{e}} \mathrm{x}}$ for all positive real value of x. Then the value of $\mathrm{f}(\mathrm{e})$ is equal to ___

Q11 (27 July 2021 Shift 1)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be solution of the differential equation

$\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with $y(0)=0$

If $\mathrm{y}\left(-\frac{2}{3}\log_{\mathrm{e}} 2\right)=\alpha \log_{\mathrm{e}} 2$, then the value of α is equal to:

(1) $-\frac{1}{4}$

(2) $\frac{1}{4}$

(3) 2

(4) $-\frac{1}{2}$

Q12 (27 July 2021 Shift 1)

If $\mathrm{y}=\mathrm{y}(\mathrm{x}), \quad \mathrm{y} \in\left[0, \frac{\pi}{2}\right)$ is the solution of the

differential equation

$\sec y \frac{d y}{d x}-\sin (x+y)-\sin (x-y)=0$, with $y(0)=0$

then $5 \mathrm{y}^{\prime}\left(\frac{\pi}{2}\right)$ is equal to

Q13 (27 July 2021 Shift 2)

Let $y=y(x)$ be the solution of the differential equation $\left(x-x^3\right) d y=\left(y+y x^2-3 x^4\right) d x, x>2$

If $\mathrm{y}(3)=3$, then $\mathrm{y}(4)$ is equal to:

(1) 4

(2) 12

(3) 8

(4) 16

Q14 (27 July 2021 Shift 2)

Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential

equation $\mathrm{dy}=\mathrm{e}^{(x+y} d x ; \alpha \in \mathbf{N}$. If $y\left(\log _e 2\right)=\log _e 2$

and $\mathrm{y}(0)=\log _e\left(\frac{1}{2}\right)$, then the value of α is equal

to

Answer Key

Q1 (1)	**Q2** (2)	**Q3** (1)	**Q4** (2)
Q5 (3)	**Q6** (4)	**Q7** (4)	**Q8** (4)
Q9 (1)	**Q10** (1)	**Q11** (1)	**Q12** (2)
Q13 (2)	**Q14** (2)		

Q1

We have

$\frac{dy}{dx} = \frac{x\left(\frac{y}{x}\cdot\tan\frac{y}{x}-1\right)}{x\tan\frac{y}{x}}$

$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{y}}{\mathrm{x}} - \cot\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

Put $\frac{y}{x} = v$

$\Rightarrow \mathrm{y} = \mathrm{vn}$

$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = \mathrm{v} + \frac{\mathrm{ndv}}{\mathrm{dx}}$

Now, we get

$v + n\frac{dv}{dx} = v - \cot(v)$

$\Rightarrow \int(\tan)\mathrm{dv} = -\int\frac{\mathrm{dx}}{\mathrm{x}}$

$\therefore \ell\mathrm{n}\left|\sec\left(\frac{\mathrm{y}}{\mathrm{x}}\right)\right| = -\ell\mathrm{n}|\mathrm{x}| + \mathrm{c}$

$\mathrm{As}\left(\frac{1}{2}\right) = \left(\frac{\mathrm{y}}{\mathrm{x}}\right) \Rightarrow \mathrm{C} = 0$

$\therefore \sec\left(\frac{\mathrm{y}}{\mathrm{x}}\right) = \frac{1}{\mathrm{x}}$

$\Rightarrow \cos\left(\frac{y}{x}\right) = x$

$\therefore y = x\cos^{-1}(x)$

So, required bounded area

$= \int_0^{1/\sqrt{2}} \mathrm{x}\left(\cos^{-1}\mathrm{x}\right)\mathrm{dx} = \left(\frac{\pi-1}{8}\right)$

(II) (I)

(I.B.P.)

$\therefore$ option (1) is correct.

Q2

$\mathrm{e^x}\sqrt{1-\mathrm{y}^2}\mathrm{dx}+\frac{\mathrm{y}}{\mathrm{x}}\mathrm{dy}=0$

$\Rightarrow \mathrm{e^x}\sqrt{1-\mathrm{y}^2}\mathrm{dx}+\frac{-\mathrm{y}}{\mathrm{x}}\mathrm{dy}$

$\Rightarrow \int \frac{-\mathrm{y}}{\sqrt{1-\mathrm{y}^2}}\mathrm{dy}=\int_{\mathrm{II}}^{\mathrm{e^x}}{}^{\mathrm{x}}\mathrm{dx}$

$\Rightarrow \sqrt{1-\mathrm{y}^2}=\mathrm{e^x}(\mathrm{x}-1)+\mathrm{c}$

$\text{Given : At } \mathrm{x}=1, \mathrm{y}=-1$

$\Rightarrow 0=0+\mathrm{c} \Rightarrow \mathrm{c}=0$

$\therefore \sqrt{1-\mathrm{y}^2}=\mathrm{e^x}(\mathrm{x}-1)$

$\text{At } \mathrm{x}=3 \quad 1-\mathrm{y}^2=\left(\mathrm{e}^3 2\right)^2 \Rightarrow \mathrm{y}^2=1-4\mathrm{e}^6$

Q3

$|\mathrm{A}|=-\frac{\mathrm{y}}{\mathrm{x}}+2\sin\mathrm{x}+2$

$\frac{\mathrm{dy}}{\mathrm{dx}}=|\mathrm{A}|$

$\frac{\mathrm{dy}}{\mathrm{dx}}=-\frac{\mathrm{y}}{\mathrm{x}}+2\sin\mathrm{x}+2$

$\frac{dy}{dx}+\frac{y}{x}=2\sin x+2$

$\mathrm{I.F.}=\mathrm{e}^{\int\frac{1}{\mathrm{x}}\mathrm{dx}}=\mathrm{x}$

$\Rightarrow \mathrm{yx}=\int \mathrm{x}(2\sin\mathrm{x}+2)\mathrm{dx}$

$xy=x^2-2x\cos x+2\sin x+c$

Now $x=\pi, y=\pi+2$

Use in (i) $\mathrm{c}=0$

Now (i) be comes $xy=x^2-2x\cos x+2\sin x$

put $\mathrm{x}=\pi/2$

$\frac{\pi}{2}\mathrm{y}=\left(\frac{\pi}{2}\right)^2-2\cdot\frac{\pi}{2}\cos\frac{\pi}{2}+2\sin\frac{\pi}{2}$

$\frac{\pi}{2}\mathrm{y}=\frac{\pi^2}{4}+2$

Q4

$\cos\left(\frac{1}{2}\cos^{-1}(\mathrm{e}^{-x})\right)\mathrm{dx}=\sqrt{\mathrm{e}^{2\mathrm{x}}-1}\mathrm{dy}$

Put $\cos^{-1}(\mathrm{e}^{-x})\theta, \theta\in[0,\pi]$

$\cos\theta=\mathrm{e}^{-x} \Rightarrow 2\cos^2\frac{\theta}{2}-1=\mathrm{e}^{-x}$

$\cos\frac{\theta}{2}=\sqrt{\frac{\mathrm{e}^{-\mathrm{x}}+1}{2}}=\sqrt{\frac{\mathrm{e^x}+1}{2\mathrm{c^x}}}$

$\sqrt{\frac{e^x+1}{2c^x}}dx = \sqrt{e^{2x}-1}dy$

$\frac{1}{\sqrt{2}}\int\frac{\mathrm{dx}}{\sqrt{\mathrm{e^x}}\sqrt{\mathrm{e}^x-1}} = \int \mathrm{dy}$

Put $\mathrm{e^x} = \mathrm{t}, \frac{\mathrm{dt}}{\mathrm{dx}} = \mathrm{e^x}$

$\frac{1}{\sqrt{2}}\int\frac{\mathrm{dt}}{\mathrm{e}^x\sqrt{\mathrm{e}^x}\sqrt{\mathrm{e}^x-1}} = \int \mathrm{d\ y}$

$\int\frac{\mathrm{dt}}{\mathrm{t}\sqrt{t^2-\mathrm{t}}} = \sqrt{2}\mathrm{y}$

Put $\mathrm{t} = \frac{1}{\mathrm{z}}, \frac{\mathrm{dt}}{\mathrm{dz}} = -\frac{1}{\mathrm{z}^2}$

$\int\frac{-\frac{\mathrm{d}z}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}-\frac{1}{z}}} = \sqrt{2y}$

$-\int\frac{\mathrm{d}z}{\sqrt{1-z}} = \sqrt{2}y$

$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2(1-\mathrm{e}^{-x})^{1/2} = \sqrt{2y} + c\xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-\mathrm{e}^{-x})^{1/2} = \sqrt{2}(\mathrm{y}+1)$, passes through $(\alpha, 0)$

$2(1-\mathrm{e}^{-a})^{1/2} = \sqrt{2}$

$\sqrt{1-\mathrm{e}^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-\mathrm{e}^{-\alpha} = \frac{1}{2}$

$\mathrm{e}^{-a} = \frac{1}{2} \Rightarrow \mathrm{e}^{a} = 2$

Q5

$\frac{\mathrm{dy}}{\mathrm{dx}} + 2\sin^2\mathrm{x} = 1 + \mathrm{y}\cos 2\mathrm{x}$

$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} + (-\cos 2\mathrm{x})\mathrm{y} = \cos 2\mathrm{x}$

$\mathrm{I.F.} = \mathrm{e}^{\int -\cos 2\mathrm{xdx}} = \mathrm{e}^{-\frac{\sin 2\mathrm{x}}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right) = \int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx + c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right) = -e^{-\frac{\sin 2x}{2}} + c$

Given

$\mathrm{y}\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 0 = -\mathrm{e}^{-1/2} + \mathrm{c} \Rightarrow \mathrm{c} = \mathrm{e}^{-1/2}$

$\Rightarrow y\left(\mathrm{e}^{-\frac{\sin 2\mathrm{x}}{2}}\right) = -\mathrm{e}^{-\frac{\sin 2\mathrm{x}}{2}} + \mathrm{e}^{-1/2}$

at $\mathrm{x}=0$

$\mathrm{y}=-1+\mathrm{e}^{-1/2}$

$\Rightarrow \mathrm{y}(0)=-1+\mathrm{e}^{-1/2} \Rightarrow (\mathrm{y}(0)+1)^{2}=\mathrm{e}^{-1}$

Q6

$\mathrm{y}+1=\mathrm{Y} \Rightarrow \mathrm{dy}=\mathrm{dY}$

$\mathrm{x}+2=\mathrm{X} \Rightarrow \mathrm{dx}=\mathrm{dX}$

$\Rightarrow \left(\mathrm{Xe}^{\frac{\mathrm{Y}}{\mathrm{X}}}+\mathrm{Y}\right) \mathrm{dX}=\mathrm{XdY}$

$\Rightarrow \mathrm{XdY}-\mathrm{YdX}=\mathrm{Xe}^{\mathrm{Y}/\mathrm{x}} \mathrm{dX}$

$\Rightarrow \mathrm{d}\left(\frac{\mathrm{Y}}{\mathrm{X}}\right) \mathrm{e}^{-\frac{\mathrm{Y}}{\mathrm{X}}}=\frac{\mathrm{dX}}{\mathrm{X}}$

$-\mathrm{e}^{-\mathrm{Y}/\mathrm{x}}=\ell|\mathrm{X}|+\mathrm{c}$

$(3,2) \rightarrow -\mathrm{e}^{-2/3}=\ell|3|+\mathrm{c}$

$-\mathrm{e}^{-\frac{t}{\mathrm{x}}}=\ln|\mathrm{X}|-\mathrm{e}^{-\frac{2}{3}}-\ell\mathrm{n}3$

$\mathrm{e}^{-\frac{\mathrm{Y}}{\mathrm{x}}}=\mathrm{e}^{2/3}+\ln 3-\ln|\mathrm{X}|>0$

$\ln|\mathrm{X}|<\mathrm{e}^{2/3}+\ell\mathrm{n}3 \rightarrow \lambda$

$\mathrm{lx}+2\ |<\mathrm{e}^{\lambda}$

$-\mathrm{e}^{\lambda}<\mathrm{x}+2<\mathrm{e}^{\lambda}$

$-\mathrm{e}^{\lambda}-2<\mathrm{x}<\mathrm{e}^{\lambda}-2$

α β

$\alpha+\beta=-4 \Rightarrow| \alpha+\beta\mathrm{I}=4$

Q7

$\frac{dy-dx}{e^{y-x}}=xdx$

$\Rightarrow \frac{dy-dx}{e^{y-x}}=xdx$

$\Rightarrow -\mathrm{e}^{x-y}=\frac{x^{2}}{2}+c$

At $x=0, y=0 \Rightarrow c=-1$

$\Rightarrow \mathrm{e}^{x-y}=\frac{2-x^{2}}{2}$

$\Rightarrow y=x-\ln\left(\frac{2-x^{2}}{2}\right)$

$\Rightarrow \frac{dy}{dx}=1+\frac{2x}{2-x^{2}}=\frac{2+2x-x^{2}}{2-x^{2}}$

$\sqrt{1-\sqrt{1}+\sqrt{2}+\sqrt{2}}$

So minimum value occurs at $\mathrm{x}=1-\sqrt{3}\; y(1-\sqrt{3})=(1-\sqrt{3})-\ell n\left(\frac{2-(4-2\sqrt{3})}{2}\right)$

$=(1-\sqrt{3})-\ln(\sqrt{3}-1)$

Q8

Let $\mathrm{e}^{y}=\mathrm{t}$

$\Rightarrow \frac{\mathrm{dt}}{\mathrm{dx}}-(2\sin \mathrm{x})\mathrm{t}=-\sin \mathrm{x}\cos^{2}\mathrm{x}$

I.F. $=\mathrm{e}^{2\mathrm{wax}}$

$\Rightarrow \mathrm{t}\cdot \mathrm{e}^{2\cos x}=\int \mathrm{e}^{2\cos x}\cdot\left(-\sin x\cos^{2}x\right)\mathrm{dx}$

at $x=\frac{\pi}{2}, y=0 \Rightarrow C=\frac{3}{4}$

$\Rightarrow \mathrm{e}^{y}=\frac{1}{2}\cos^{2}x-\frac{1}{2}\cos x+\frac{1}{4}+\frac{3}{4}\cdot e^{-20\omega x}$

$\Rightarrow y=\log\left[\frac{\cos^{2}x}{2}-\frac{\cos x}{2}+\frac{1}{4}+\frac{3}{4}e^{-2\cos x}\right]$

Put $\mathrm{x}=0$

$\Rightarrow y=\log\left[\frac{1}{4}+\frac{3}{4}\mathrm{e}^{-2}\right] \Rightarrow \alpha=\frac{1}{4}, \beta=\frac{3}{4}$

Q9

$xdy=\left(y+x^{3}\cos x\right)dx$

$xdy=ydx+x^{3}\cos xdx$

$\frac{xdy-ydx}{x^{2}}=\frac{x^{3}\cos xdx}{x^{2}}$

$\frac{d}{dx}\left(\frac{y}{x}\right)=\int x\cos xdx$

$\Rightarrow \frac{y}{x}=x\sin x-\int 1\cdot \sin xdx$

$\frac{y}{x}=x\sin x+\cos x+C$

$\Rightarrow 0=-1+C \Rightarrow C=1, x=\pi, y=0$

so $\frac{y}{x}=x\sin x+\cos x+1$

$y=x^{2}\sin x+x\cos x+x \quad x=\frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right)=\frac{\pi^{2}}{4}+\frac{\pi}{2}$

Q10

$y' = \frac{2y}{x\ell nx}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n \mid \ell nx| + C$

put $x = 2, y = (\ell n2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

Q11

$\frac{dy}{dx} = e^{3x} \cdot e^{4y} \Rightarrow \int e^{-4y} dy = \int e^{3x} dx$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x}-7}{-3}$

$e^{4y} = \frac{3}{7-4e^{3x}} \Rightarrow 4y = \ln\left(\frac{3}{7-4e^{3x}}\right)$

$4y = \ell n\left(\frac{3}{6}\right)$ when $x = -\frac{2}{3}\ell n 2$

$y = \frac{1}{4}\ln\left(\frac{1}{2}\right) = -\frac{1}{4}\ln 2$

Q12

$\sec y \frac{dy}{dx} = 2\sin x \cos y$

$\sec^2 y dy = 2\sin x dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$\sec^2 y \frac{dy}{dx} = 2\sin x$

$5\frac{dy}{dx} = 2$

Q13

$\left(x-x^{3}\right) d y=\left(y+y x^{2}-3 x^{4}\right) d x$

$\Rightarrow x d y-y d x=\left(y x^{2}-3 x^{4}\right) d x+x^{3} d y$

$\Rightarrow \frac{x d y-y d x}{x^{2}}=(y d x+x d y)-3 x^{2} d x$

$\Rightarrow \mathrm{d}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)=\mathrm{d}(\mathrm{xy})-\mathrm{d}\left(\mathrm{x}^{3}\right)$

Integrate $\Rightarrow \frac{\mathrm{y}}{\mathrm{x}}=\mathrm{xy}-\mathrm{x}^{3}+\mathrm{c}$

given $f(3)=3 \Rightarrow \frac{3}{3}=3 \times 3-3^{3}+\mathrm{c}$

$\Rightarrow \mathrm{c}=19$

$\therefore \frac{y}{x}=x y-x^{3}+19$

at $x=4, \frac{y}{4}=4 y-64+19$

$15 \mathrm{y}=4 \times 45$

$\Rightarrow \mathrm{y}=12$

Q14

$\int \mathrm{e}^{-y} \mathrm{dy}=\int \mathrm{e}^{\mathrm{ax}} \mathrm{dx}$

$\Rightarrow \mathrm{e}^{-y}=\frac{\mathrm{e}^{\mathrm{ax}}}{\alpha}+\mathrm{c}$

$\operatorname{Put}(\mathrm{x}, \mathrm{y})=(\ell \mathrm{n} 2, \ell \mathrm{n} 2)$

$\frac{-1}{2}=\frac{2^{\alpha}}{\alpha}+\mathrm{C}$

Put $(x, y) \equiv(0,-\ell n 2)$ in (i)

$-2=\frac{1}{\alpha}+\mathrm{C}$

(ii) - (iii)

$\frac{2^{\alpha}-1}{\alpha}=\frac{3}{2}$

$\Rightarrow \alpha=2$ (as $\alpha \in \mathbb{N}$)