

Questions with Answer Keys

MathonGo

Q1 (20 July 2021 Shift 1)

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3z\bar{\omega}}\right)$ is :

(Here $\arg(z)$ denotes the principal argument of complex number z)

- (1) $\frac{\pi}{4}$
- (2) $-\frac{3\pi}{4}$
- (3) $-\frac{\pi}{4}$
- (4) $\frac{3\pi}{4}$

Q2 (20 July 2021 Shift 2)

If the real part of the complex number

$(1 - \cos \theta + 2i \sin \theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the

value of the integral $\int_0^\theta \sin x dx$ is equal to:

- (1) 1
- (2) 2
- (3) -1
- (4) 0

Q3 (22 July 2021 Shift 1)

Let n denote the number of solutions of the

equation $z^2 + 3\bar{z} = 0$, where z is a complex

number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

- (1) 1
- (2) $\frac{4}{3}$
- (3) $\frac{3}{2}$
- (4) 2

Q4 (25 July 2021 Shift 2)

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Questions with Answer Keys

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The equation of a circle is

$\operatorname{Re}(z^2) + 2(\operatorname{Im}(z))^2 + 2 \operatorname{Re}(z) = 0$, where $z = x + iy$

A line which passes through the center of the given circle and the vertex of the parabola,

$x^2 - 6x - y + 13 = 0$, has y -intercept equal to _____

Q5 (27 July 2021 Shift 1)

Let C be the set of all complex numbers. Let $S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$

$S_2 = \{z \in C \mid \operatorname{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is

equal to

(1) 1

(2) 0

(3) 2

(4) Infinite

Q6 (27 July 2021 Shift 2)

Let C be the set of all complex numbers. Let

$S_1 = \{z \in C : |z - 2| \leq 1\}$ and

$S_2 = \{z \in C : z(1+i) + \bar{z}(1-i) \geq 4\}$.

Then, the maximum value of $|z - \frac{5}{2}|^2$ for

$z \in S_1 \cap S_2$ is equal to :

(1) $\frac{3+2\sqrt{2}}{4}$

(2) $\frac{5+2\sqrt{2}}{2}$

(3) $\frac{3+2\sqrt{2}}{2}$

(4) $\frac{5+2\sqrt{2}}{4}$

Complex Number

JEE Main 2021 (July) Chapter-wise Questions

Questions with Answer Keys

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Q7 (27 July 2021 Shift 2)

If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2 \theta$ is equal to

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Complex Number

JEE Main 2021 (July) Chapter-wise Questions

Questions with Answer Keys

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Answer Key

Q1 (3) **Q2 (1)** **Q3 (2)** **Q4 (1)**

Q3 (1) Q4 (4) Q5 (1) Q6 (1) Q7 (1) Q8 (1) Q9 (1) Q10 (1)

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Hints and Solutions

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Q1

As $|z\omega| = 1$ \Rightarrow If $|z| = r$, then $|\omega| = \frac{1}{r}$ Let $\arg(z) = \theta$

$$\therefore \arg(\omega) = \left(\theta - \frac{3\pi}{2} \right)$$

So, $z = re^{i\theta}$

$$\Rightarrow \bar{z} = re^{i(-\theta)}$$

$$\omega = \frac{1}{r} e^{1\left(\theta - \frac{3\pi}{2}\right)}$$

Now, consider

$$\begin{aligned} \frac{1-2\bar{z}\omega}{1+3z\omega} &= \frac{1-2e^{i\left(-\frac{3\pi}{2}\right)}}{1+3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i} \right) \\ &= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i) \end{aligned}$$

$$\therefore \text{prin arg} \left(\frac{1-2\bar{z}\omega}{1+3z\omega} \right)$$

$$= \text{prin arg} \left(\frac{1-2\bar{z}\omega}{1+3z\omega} \right)$$

$$= \left(-\frac{1}{2}(1+i) \right)$$

$$= -\left(\pi - \frac{\pi}{4} \right) = -\frac{3\pi}{4}$$

So, option (2) is correct.

Q2

$$z = \frac{1-\cos\theta+2i\sin\theta}{1-\cos\theta+2i\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2}-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$$

$$= \frac{\sin\frac{\theta}{2}-2i\cos\frac{\theta}{2}}{4\sin\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}$$

$$\operatorname{Re}(z) = \frac{1}{2\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)} = \frac{1}{5}$$

$$\sin\frac{2\theta}{2} + 4\cos^2\frac{\theta}{2} = \frac{5}{2}$$

$$1 - \cos^2\frac{\theta}{2} + 4\cos\frac{\theta}{2} = \frac{5}{2}$$

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Complex Number

JEE Main 2021 (July) Chapter-wise Questions

Hints and Solutions

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 $3 \cos^2 \frac{\theta}{2} = \frac{3}{2}$       

cos² $\frac{\theta}{2}$ = $\frac{1}{2}$  mathongo  mathongo  mathongo  mathongo  mathongo  mathongo  mathongo  mathongo

$\frac{\theta}{2} = n\pi \pm \frac{\pi}{4}$

$\theta = 2n\pi \pm \frac{\pi}{2}$

$\theta \in (0, \pi)$

$\theta \in (0, \pi)$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin \theta d\theta = [-\cos \theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -(0 - 1)$$

$z^2 + p\bar{z} = 0$ mathongo mathongo mathongo mathongo mathongo mathongo mathongo

Put $z = x + iy$

$$\Rightarrow x' - y' + 2ixy + 3(x - iy) = 0$$

$$\therefore x^2 - y^2 + 3x = 0 \quad \dots \dots \dots (1)$$

$$2xy - 3y = 0. \dots \dots (2)$$

$x = \frac{3}{2}$, $y = 0$  mathongo  mathongo  mathongo  mathongo  mathongo  mathongo

Put $x = \frac{3}{2}$ in equation (1)

$$\frac{9}{4} - y^2 + \frac{9}{2} = 0$$

$$y^2 = \frac{27}{4} \Rightarrow y = \pm \frac{3\sqrt{3}}{2}$$

∴ $(x, y) \equiv \left(\frac{-1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, -\frac{1}{2}\right)$  mathongo  mathongo  mathongo  mathongo

Put $y = 0 \Rightarrow x^2 - 0 + 3x = 0$

$$\therefore (x, y) \equiv (0, 0), (-3, 0)$$

Hints and Solutions

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∴ No of solutions = $n = 4$

$$\sum_{K=0}^{\infty} \left(\frac{1}{n^k} \right) = \sum_{K=0}^{\infty} \left(\frac{1}{4^k} \right)$$

$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

Q4

Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0 \Rightarrow x^2 + y^2 + 2x = 0$

Centre : $(-1, 0)$ Parabola : $x^2 - 6x - y + 13 = 0$

$$(x - 3)^2 = y - 4$$

Vertex : $(3, 4)$ Equation of line $\equiv y - 0 = \frac{4-0}{3+1}(x + 1)$

$$y = x + 1$$

y -intercept = 1

Q5

$$S_1 : |z - 3 - 2i|^2 = 8$$

$$|z - 3 - 2i| = 2\sqrt{2}$$

$$(x - 3)^2 + (y - 2)^2 = (2\sqrt{2})^2$$

$$S_2 : x \geq 5$$

$$S_3 : |z - \bar{z}| \geq 8$$

$$|2iy| \geq 8$$

$$2|y| \geq 8 \quad \therefore y \geq 4, y \leq -4$$



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Hints and Solutions

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$n(S_1 \cap S_2 \cap S_3) = 1$ mathongo mathongo mathongo mathongo mathongo mathongo

Q6 mathongo mathongo mathongo mathongo mathongo mathongo mathongo

$|t - 2| \leq 1$ Put $t = x + iy$ mathongo mathongo mathongo mathongo mathongo mathongo



($x - 2$)² + $y^2 \leq 1$ mathongo mathongo mathongo mathongo mathongo mathongo

Also, $t(1+i) + \bar{t}(1-i) \geq 4$ mathongo mathongo mathongo mathongo mathongo mathongo

Gives $x - y \geq 2$ mathongo mathongo mathongo mathongo mathongo mathongo

Let point on circle be $A(2 + \cos \theta, \sin \theta)$

$$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right] \quad \text{mathongo mathongo mathongo mathongo mathongo}$$

$$(AP)^2 = \left(2 + \cos \theta - \frac{5}{2}\right)^2 + \sin^2 \theta \quad \text{mathongo mathongo mathongo mathongo}$$

$$= \cos^2 \theta - \cos \theta + \frac{1}{4} + \sin^2 \theta \quad \text{mathongo mathongo mathongo mathongo}$$

$$= \frac{5}{4} - \cos \theta \quad \text{mathongo mathongo mathongo mathongo}$$

For $(AP)^2$ maximum $\theta = -\frac{3\pi}{4}$ mathongo mathongo mathongo mathongo

$$(AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}} \quad \text{mathongo mathongo mathongo mathongo}$$

Q7 mathongo mathongo mathongo mathongo mathongo mathongo

$$\operatorname{Re}(z) = \frac{3-6\cos^2 \theta}{1+9\cos^2 \theta} = 0 \quad \text{mathongo mathongo mathongo mathongo}$$

$$\Rightarrow \theta = \frac{\pi}{4} \quad \text{mathongo mathongo mathongo mathongo}$$

$$\text{Hence, } \sin^2 3\theta + \cos^2 \theta = 1 \quad \text{mathongo mathongo mathongo mathongo}$$

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