Q1 (20 July 2021 Shift 1)

If the value of $\lim_{x \to 0}(2-\cos x\sqrt{\cos 2x})\left(\frac{x+2}{x^2}\right)$ is equal to e^n, then a is equal to ___

Q2 (20 July 2021 Shift 2)

If $f : \mathbf{R} \to \mathbf{R}$ is given by $f(\mathrm{x}) = \mathrm{x} + 1$, then the value of
$\lim_{n \to \infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \ldots\ldots + f\left(\frac{5(n-1)}{n}\right)\right]$
is:

(1) $\frac{3}{2}$

(2) $\frac{5}{2}$

(3) $\frac{1}{2}$

(4) $\frac{7}{2}$

Q3 (20 July 2021 Shift 2)

If $\lim_{x \to 0} \frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10, \alpha, \beta, \gamma \in \mathbf{R}$, then the value of $\alpha + \beta + \gamma$ is

Q4 (27 July 2021 Shift 1)

Let $\mathrm{f} : \mathbf{R} \to \mathbf{R}$ be a function such that $\mathrm{f}(2) = 4$ and $\mathrm{f}'(2) = 1$. Then, the value of $\lim_{\mathrm{x} \to 2} \frac{\mathrm{x}^2 \mathrm{f}(2) - 4\mathrm{f}(\mathrm{x})}{\mathrm{x} - 2}$ is equal to :

(1) 4

(2) 8

(3) 16

(4) 12

Q5 (27 July 2021 Shift 2)

The value of $\lim_{x \to 0}\left(\frac{x}{\sqrt[8]{1-\sin x} - \sqrt[8]{1+\sin x}}\right)$ is equal to:

(1) 0

(2) 4

(3) -4

(4) -1

Answer Key

Q1 (3) **Q2** (4) **Q3** (3) **Q4** (4)

Q5 (3)

Q1

$\lim_{x\to 0}(2-\cos x\sqrt{\cos x})^{\frac{x+2}{x^2}}$

form: 1^{∞}

$=\mathrm{e}^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now $\operatorname{limt}_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$=$

$\operatorname{limt}_{x\to 0}\frac{\sin x\sqrt{\cos 2x}-\cos x\times\frac{1}{2\sqrt{\cos 2x}}\times(-2\sin 2x)}{2x}$

(by L ' Hospital Rule)

$\operatorname{limt}_{x\to 0}\frac{\sin x\cos 2x+\sin 2x\cdot\cos x}{2x}$

$=\frac{1}{2}+1=\frac{3}{2}$

So, $\mathrm{e}^{\lim_{x\to 0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$=\mathrm{e}^{\frac{3}{2}\times 2}=\mathrm{e}^3$

$\Rightarrow a=3$

Q2

$\mathrm{I}=\sum_{\mathrm{r}=0}^{\mathrm{n}-1}\mathrm{f}\left(\frac{5\mathrm{r}}{\mathrm{n}}\right)\frac{1}{\mathrm{n}}$

$\mathrm{I}=\int_0^1\mathrm{f}(5\mathrm{x})\mathrm{dx}$

$\mathrm{I}=\int_0^1(5\mathrm{x}+1)\mathrm{dx}$

$\mathrm{I}=\left[\frac{5\mathrm{x}^2}{2}+\mathrm{x}\right]_0^1$

$\mathrm{I}=\frac{5}{2}+1=\frac{7}{2}$

Q3

$\lim_{x\to 0}\frac{\alpha x\left(1+x+\frac{x^2}{2}\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}\right)+\gamma x^2(1-x)}{x^3}$

$\lim_{x\to 0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)}{x^3}=10$

For limit to exist

$\alpha-\beta=0,\alpha+\frac{\beta}{2}+\gamma=0$

$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10$

$\beta=\alpha, \gamma=-3 \frac{\alpha}{2}$

Put in (i)

$\frac{\alpha}{2}-\frac{\alpha}{3}+\frac{3 \alpha}{2}=10$

$\frac{\alpha}{6}+\frac{3 \alpha}{2}=10 \Rightarrow \frac{\alpha+9 \alpha}{6}=10$

$\Rightarrow \alpha=6$

$\alpha=6, \beta=6, \gamma=-9$

$\alpha+\beta+\gamma=3$

Q4

Apply L'Hopital Rule $\lim _{\mathrm{x} \rightarrow 2}\left(\frac{2 \mathrm{x} f(2)-4 f^{\prime}(\mathrm{x})}{1}\right)$

$=\frac{4(4)-4}{1}=12$

Q5

$\lim _{x \rightarrow 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$=\lim _{x \rightarrow 0}\left(\frac{\mathrm{x}}{\sqrt[8]{1-\sin \mathrm{x}}-\sqrt[8]{1+\sin \mathrm{x}}}\right)$

$=\lim _{x \rightarrow 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$

$\left(\frac{(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})}{\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x}}\right)$

$\left(\frac{(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})}{\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x}}\right)$

$\left(\frac{(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})}{\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x}}\right)$

$=\lim _{x \rightarrow 0}\left(\frac{x}{1-\sin x-(1+\sin x)}\right)$

$(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})$

$(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})$

$=\lim _{x \rightarrow 0} \frac{x}{(-2 \sin x)}(\sqrt[8]{1-\sin x}+\sqrt[8]{1+\sin x})$

$(\sqrt[4]{1-\sin x}+\sqrt[4]{1+\sin x})(\sqrt[2]{1-\sin x}+\sqrt[2]{1+\sin x})$

$=\lim _{x \rightarrow 0}\left(-\frac{1}{2}\right)(2)(2)(2)\left\{\because \lim _{x \rightarrow 0} \frac{\sin x}{x}=1\right\}=-4$