Q1 (20 July 2021 Shift 1)

Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is:

(1) $\frac{1}{66}$

(2) $\frac{1}{11}$

(3) $\frac{1}{9}$

(4) $\frac{2}{11}$

Q2 (20 July 2021 Shift 1)

The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$, is:

(1) $\frac{7}{36}$

(2) $\frac{2}{9}$

(3) $\frac{1}{6}$

(4) $\frac{1}{4}$

Q3 (20 July 2021 Shift 2)

Let A, B and C be three events such that the probability that exactly one of A and B occurs is $(1-\mathrm{k})$, the probability that exactly one of B and C occurs is $(1-2\mathrm{k})$, the probability that exactly one of C and A occurs is $(1-\mathrm{k})$ and the probability of all A, B and C occur simultaneously is k^2, where $0<\mathrm{k}<1$. Then the probability that at least one of A, B and C occur is :

(1) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

(2) greater than $\frac{1}{2}$

(3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(4) exactly equal to $\frac{1}{2}$

Q4 (22 July 2021 Shift 1)

Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

(1) $\frac{45}{162}$
(2) $\frac{23}{81}$
(3) $\frac{22}{81}$
(4) $\frac{43}{162}$

Q5 (25 July 2021 Shift 1)

Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x-3| < 1\}$

(2) $\{x \in \mathbf{R} : |x-2| \leq 1\}$

(3) $\{x \in \mathbf{R} : |x-1| < 1\}$

(4) $\{x \in \mathbf{R} : |x-5| \leq 1\}$

Q6 (25 July 2021 Shift 2)

Let x be a random variable such that the probability function of a distribution is given by

$P(X=0)=\frac{1}{2}, P(X=j)=\frac{1}{3^j}(j=1,2,3,\ldots,\infty)$ Then the mean of the distribution and P(X is positive and even) respectively are:

(1) $\frac{3}{8}$ and $\frac{1}{8}$
(2) $\frac{3}{4}$ and $\frac{1}{8}$
(3) $\frac{3}{4}$ and $\frac{1}{9}$
(4) $\frac{3}{4}$ and $\frac{1}{16}$

Q7 (25 July 2021 Shift 2)

A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is ___

Q8 (27 July 2021 Shift 1)

The probability that a randomly selected 2 -digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of $3\}$ is equal to

(1) $\frac{1}{6}$

(2) $\frac{2}{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{3}$

Q9 (27 July 2021 Shift 2)

A student appeared in an examination consisting of 8 true-false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least 'n' correct answers is less than $\frac{1}{2}$, is

(1) 5

(2) 6

(3) 3

(4) 4

Answer Key

Q1 (2)	**Q2** (2)	**Q3** (2)	**Q4** (4)
Q5 (1)	**Q6** (2)	**Q7** (4)	**Q8** (3)
Q9 (1)			

Q1

AAEIIMNNOTX

--------M---------

Total words with M at fourth Place $= \frac{10!}{2!2!2!}$

Total words $= \frac{11!}{2!2!2!}$

Required probability $= \frac{10!}{11!} = \frac{1}{11}$

Q2

$D < 0$

$\Rightarrow 4(a+4)^2 - 4(-5a+64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$\Rightarrow (a+16)(a-3) < 0$

$\Rightarrow \mathrm{a} \in (-16, 3)$

$\therefore$ Possible a : $\{-5, -4, \ldots\ldots, 3\}$

$\therefore$ Required probability $= \frac{8}{36}$

$= \frac{2}{9}$

Q3

$\mathrm{P}(\overline{\mathrm{A}} \cap \mathrm{B}) + \mathrm{P}(\mathrm{A} \cap \overline{\mathrm{B}}) = 1 - \mathrm{k}$

$\mathrm{P}(\overline{\mathrm{A}} \cap \mathrm{C}) + \mathrm{P}(\mathrm{A} \cap \overline{\mathrm{C}}) = 1 - 2\mathrm{k}$

$\mathrm{P}(\overline{\mathrm{B}} \cap \mathrm{C}) + \mathrm{P}(\mathrm{B} \cap \overline{\mathrm{C}}) = 1 - \mathrm{k}$

$\mathrm{P}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = \mathrm{k}^2$

$\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - 2\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 1 - \mathrm{k}$

$\mathrm{P}(\mathrm{B}) + \mathrm{P}(\mathrm{C}) - 2\mathrm{P}(\mathrm{B} \cap \mathrm{C}) = 1 - \mathrm{k}$

$\mathrm{P}(\mathrm{C}) + \mathrm{P}(\mathrm{A}) - 2\mathrm{P}(\mathrm{A} \cap \mathrm{C}) = 1 - 2\mathrm{k}$

$(1) + (2) + (3)$

$\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) + \mathrm{P}(\mathrm{C}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}) - \mathrm{P}(\mathrm{B} \cap \mathrm{C})$

$-P(C \cap A) = \frac{-4k+3}{2}$

So

$P(A \cup B \cup C) = \frac{-4k+3}{2} + k^2$

$P(A \cup B \cup C) = \frac{2k^2-4k+3}{2}$

$= \frac{2(k-1)^2+1}{2}$

$P(A \cup B \cup C) > \frac{1}{2}$

Q4

$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad |A| = ad - bc$

Total case $= 6^4$

For non-singular matrix $|A| \neq 0 \Rightarrow ad - bc \neq 0$

$\Rightarrow ad \neq bc$

And a, b, c, d are all different numbers in the set $\{1, 2, 3, 4, 5, 6\}$

Now for $ad = bc$

(i) $6 \times 1 = 2 \times 3$

$$\left.\begin{array}{l} \Rightarrow a = 6,\ b = 2, c = 3,\ d = 1 \\ \text{or } a = 1,\ b = 2, c = 3,\ d = 6 \\ \qquad \vdots \end{array}\right\} \text{8 such cases}$$

(ii) $6 \times 2 = 3 \times 4$

$$\left.\begin{array}{l} \Rightarrow a = 6,\ b = 3, c = 4,\ d = 2 \\ \text{or } a = 2,\ b = 3, c = 4,\ d = 6 \\ \qquad \vdots \end{array}\right\}$$

favourable cases

$= {}^6C_4 \mid 4 - 16$

required probability

$= \frac{{}^6C_4|4-16}{6^4} = \frac{43}{162}$

Q5

required probability $= \frac{{}^9C_3 \cdots {}^6}{4^9}$
$= \frac{{}^9C_3}{27} \cdot \left(\frac{3}{4}\right)^9$
$= \frac{28}{9} \cdot \left(\frac{3}{4}\right)^9 \Rightarrow k = \frac{28}{9}$
Which satisfies $|x - 3| < 1$

Q6

mean $= \sum x_1 p_i = \sum_{r=0}^{\infty} r \cdot \frac{1}{3^r} = \frac{3}{4}$

$p(x \text{ is even }) = \frac{1}{3^2} + \frac{1}{3^4} + \ldots \infty$
$= \frac{\frac{1}{9}}{1-\frac{1}{9}} = \frac{1/9}{8/9} = \frac{1}{8}$

Q7

$P(\text{Head}) = \frac{1}{2}$

$1 - P(\text{ All tail }) \geq 0.9$

$1 - \left(\frac{1}{2}\right)^n \geq 0.9$
$\Rightarrow \left(\frac{1}{2}\right)^n \leq \frac{1}{10}$
$\Rightarrow n_{\min} = 4$

Q8

Total number of cases $= {}^{90}C_1 = 90$ Now, $2^n - 2 = (3-1)^n - 2$

${}^nC_0 3^n - {}^nC_1 \cdot 3^{n-1} + \ldots + (-1)^{n-1} \cdot {}^nC_{n-1} 3 + (-1)^n \cdot {}^nC_n - 2$

$3\left(3^{n-1} - n3^{n-2} + \ldots + (-1)^{n-1} \cdot n\right) + (-1)^n - 2$
$(2^n - 2)$ is multiply of 3 only when n is odd Req. Probability $= \frac{45}{90} = \frac{1}{2}$

Q9

$P(E) < \frac{1}{2}$

$\Rightarrow \sum_{r=n}^{8} {}^8C_r \left(\frac{1}{2}\right)^{8-T} \left(\frac{1}{2}\right)^r < \frac{1}{2}$
$\Rightarrow \sum_{r=n}^{8} {}^8C_r \left(\frac{1}{2}\right)^8 < \frac{1}{2}$
$\Rightarrow {}^8C_n + {}^8C_{n+1} + \ldots + {}^8C_8 < 128$

$\Rightarrow 256-\left({}^8C_0+{}^8C_1+\ldots.+{}^8C_{n-1}\right)<128$

$\Rightarrow {}^8C_0+{}^8C_1+\ldots.+{}^8C_{n-1}>128$

$\Rightarrow n-1 \geq 4$

$\Rightarrow n \geq 5$