Q1 (27 July 2021 Shift 2)

Let $\mathbf{N}$ be the set of natural numbers and a relation

R on $\mathbf{N}$ be defined by $R = \left\{(x, y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\right\}$

Then the relation R is :

(1) symmetric but neither reflexive nor transitive

(2) reflexive but neither symmetric nor transitive

(3) reflexive and symmetric, but not transitive

(4) an equivalence relation

Q2 (27 July 2021 Shift 2)

Let $A = \left\{n \in \mathbf{N} \mid n^2 \leq n + 10,000\right\}, B = \{3k + 1 \mid k \in \mathbf{N}\}$

and $C = \{2k \mid k \in N\}$, then the sum of all the elements

of the set $A \cap (B - C)$ is equal to

Answer Key

Q1 (2) **Q2** (832)

Q1

$x^3 - 3x^2y - xy^2 + 3y^3 = 0$

$\Rightarrow x\left(x^2 - y^2\right) - 3y\left(x^2 - y^2\right) = 0$

$\Rightarrow (x - 3y)(x - y)(x + y) = 0$

Now, $x = y \quad \forall (x, y) \in N \times N$ so reflexive

But not symmetric \& transitive See, $(3, 1)$ satisfies but $(1, 3)$ does not. Also $(3, 1)$& $(1, -1)$ satisfies but $(3, -1)$ does not

Q2

$\mathrm{B} - \mathrm{C} \equiv \{7, 13, 19, \ldots 97, \ldots\}$

Now, $\mathrm{n}^2 - \mathrm{n} \leq 100 \times 100$

$\Rightarrow \mathrm{n}(\mathrm{n} - 1) \leq 100 \times 100$

$\Rightarrow \mathrm{A} = \{1, 2, \ldots, 100\}$

So, $A \cap (B - C) = \{7, 13, 19, \ldots, 97\}$

Hence, sum $= \frac{16}{2}(7 + 97) = 832$