Q1 (20 July 2021 Shift 1)

Let $\mathrm{A}=\left[\mathrm{a}_{\mathrm{y}}\right]$ be a 3×3 matrix, where $a_{ij}=\begin{cases}1 & , \text{ if } i=j \\ -x & , \text{ if } |i-j|=1 \\ 2x+1 & , \text{ otherwise}\end{cases}$

Let a function $\mathrm{f}: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $\mathrm{f}(\mathrm{x})=\operatorname{det}(\mathrm{A})$.

Then the sum of maximum and minimum values of f on $\mathbf{R}$ is equal to:

(1) $-\frac{20}{27}$

(2) $\frac{88}{27}$

(3) $\frac{20}{27}$

(4) $-\frac{88}{27}$

Q2 (20 July 2021 Shift 1)

Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\left|\begin{array}{lll}x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c\end{array}\right|=2$$

then value of λ^2 is equal to ___

Q3 (20 July 2021 Shift 2)

The value of $\mathrm{k} \in \mathbf{R}$, for which the following system

of linear equations

$3x-y+4z=3$

$x+2y-3z=-2$

$6x+5y+kz=-3$

has infinitely many solutions, is :

(1) 3

(2) -5

(3) 5

(4) -3

Q4 (22 July 2021 Shift 1)

The values of λ and μ such that the system of equations $x+y+z=6, 3x+5y+5z=26$

$\mathrm{x}+2\mathrm{y}+\lambda\mathrm{z}=\mu$ has no solution, are :

(1) $\lambda=3, \mu=5$

(2) $\lambda=3, \mu\neq 10$

(3) $\lambda\neq 2, \mu=10$

(4) $\lambda=2, \mu\neq 10$

Q5 (25 July 2021 Shift 1)

The values of a and b, for which the system of

equations $2x+3y+6z=8$

$x+2y+az=5$

$3x+5y+9z=b$

has no solution, are:

(1) $a=3, b\neq 13$

(2) $a\neq 3, b\neq 13$

(3) $a\neq 3, b=3$

(4) $a=3, b=13$

Q6 (25 July 2021 Shift 1)

Let $\mathrm{M}=\left\{\mathrm{A}=\begin{pmatrix}\mathrm{a} & \mathrm{b}\\ \mathrm{c} & \mathrm{d}\end{pmatrix}: \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\in\{\pm 3, \pm 2, \pm 1, 0\}\right\}$

Define $\mathrm{f}: \mathrm{M}\rightarrow \mathbf{Z}$, as $\mathrm{f}(\mathrm{A})=\det(\mathrm{A})$, for all $\mathrm{A}\in\mathrm{M}$

where $\mathbf{Z}$ is set of all integers. Then the number of $\mathrm{A}\in\mathrm{M}$ such that $\mathrm{f}(\mathrm{A})=15$ is equal to

Q7 (25 July 2021 Shift 2)

The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq \mathrm{x} \leq \frac{\pi}{4}$ is:

(1) 4

(2) 1

(3) 2

(4) 3

Q8 (27 July 2021 Shift 1)

For real numbers α and β, consider the following system of linear equations :

$\mathrm{x}+\mathrm{y}-\mathrm{z}=2, \mathrm{x}+2\mathrm{y}+\alpha\mathrm{z}=1, 2\mathrm{x}-\mathrm{y}+\mathrm{z}=\beta$

If the system has infinite solutions, then $\alpha+\beta$ is equal to

Q9 (27 July 2021 Shift 1)

Let $f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$

Then the maximum value of $\mathrm{f}(\mathrm{x})$ is equal to

Answer Key

Q1 (4)	**Q2** (1)	**Q3** (2)	**Q4** (4)
Q5 (1)	**Q6** (16)	**Q7** (2)	**Q8** (5)
Q9 (6)			

Q1

$$A=\begin{bmatrix}1 & -x & 2x+1\\ -x & 1 & -x\\ 2x+1 & -x & 1\end{bmatrix}$$

$$|A|=4x^3-4x^2-4x=f(x)$$

$$f'(x)=4\left(3x^2-2x-1\right)=0$$

$$\Rightarrow x=1; x=\frac{-1}{3}$$

$$\therefore \underbrace{f(1)=-4}_{\text{min}}; \underbrace{f\left(-\frac{1}{3}\right)=\frac{20}{27}}_{\text{max.}}$$

$$\text{Sum}=-4+\frac{20}{27}=-\frac{88}{27}$$

Q2

$$\begin{vmatrix}x+a-c & x+b & x+a\\ x-1 & x+c & x+b\\ x-b+d & x+d & x+c\end{vmatrix}=2$$

$$C_2\rightarrow C_2-C_3$$

$$\Rightarrow\begin{vmatrix}x-2\lambda & \lambda & x+a\\ x-1 & \lambda & x+b\\ x+2\lambda & \lambda & x+c\end{vmatrix}=2$$

$$R_2\rightarrow R_2-R_1,\quad R_3\rightarrow R_3-R_1$$

$$\Rightarrow\lambda\begin{vmatrix}x-2\lambda & 1 & x+a\\ 2\lambda-1 & 0 & \lambda\\ 4\lambda & 0 & 2\lambda\end{vmatrix}=2$$

$$\Rightarrow 1\left(4\lambda^2-4\lambda^2+2\lambda\right)=2$$

$$\Rightarrow\lambda^2=1$$

Q3

$$\begin{vmatrix}3 & -1 & 4\\ 1 & 2 & -3\\ 6 & 5 & \mathrm{K}\end{vmatrix}=0$$

$$\Rightarrow 3(2\,\mathrm{K}+15)+\mathrm{K}+18-28=0$$

$$\Rightarrow 7\,\mathrm{K}+35=0\Rightarrow\mathrm{K}=-5$$

Q4

$x+y+z=6$ ...(i)

$3x+5y+5z=26 \quad \ldots$(ii)

$\mathrm{x}+2\mathrm{y}+\lambda\mathrm{z}=\mu$...(iii)

$5\times$(i) $-$ (ii) $\Rightarrow 2\mathrm{x}=4\Rightarrow \mathrm{x}=2$

$\therefore$ from (i) and (ii)

$\mathrm{y}+\mathrm{z}=4$....(iv)

$2\mathrm{y}+\lambda\mathrm{z}=\mu-2$....(v)

(v) $-2\times$ (iv) $\Rightarrow (\lambda-2)\mathrm{z}=\mu-10$

$\Rightarrow \mathrm{z}=\frac{\mu-10}{\lambda-2}$ & $\mathrm{y}=4-\frac{\mu-10}{\lambda-2}$

$\therefore$ For no solution $\lambda=2$ and $\mu\neq 10$.

Q5

$$D=\begin{vmatrix}2&3&6\\1&2&a\\3&5&9\end{vmatrix}=3-a$$

$$D=\begin{vmatrix}2&3&8\\1&2&5\\3&5&b\end{vmatrix}=b-13$$

If $a=3, b\neq 13$, no solution.

Q6

$|\mathrm{A}|=\mathrm{ad}-\mathrm{bc}=15$

where $a,b,c,d\in\{\pm3,\pm2,\pm1,0\}$

Case I $\mathrm{ad}=9$ & $\mathrm{bc}=-6$

For ad possible pairs are $(3,3),(-3,-3)$ For be possible pairs are $(3,-2),(-3,2),(-2,3),(2,-3)$ So total matrix $=2\times4=8$ Case II $\mathrm{ad}=6$ & $\mathrm{bc}=-9$

Similarly total matrix $=2\times4=8$

$\Rightarrow$ Total such matrices are $=16$

Q7

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, \frac{-\pi}{4} \le x \le \frac{\pi}{4}$$

Apply : $R_1 \rightarrow R_1 - R_2 \& R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\therefore x = \frac{\pi}{4}$

Q8

For infinite solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2 + \alpha) = 0$

$\Rightarrow \alpha = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$1(1 + 2\beta) - 2(1 + 4) - (\beta - 2) = 0$

$\beta - 7 = 0$

$\beta = 7$

$\therefore \alpha + \beta = 5$ Ans.

Q9

$$\left|\begin{array}{ccc} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{array}\right| \left(\begin{array}{l} R_1 \rightarrow R_1 - R_2 \\ \& R_2 \rightarrow R_2 - R_3 \end{array}\right)$$

$$-2\left(\cos^2 x\right)+2\left(2+2\cos 2x+\sin^2 x\right)$$

$$4+4\cos 2x-2\left(\cos^2 x-\sin^2 x\right)$$

$$f(x)=4+\underbrace{2\cos 2x}_{\max -1}$$

$$f(x)_{\max}=4+2=6$$