Q1 (22 July 2021 Shift 1)

Let a line $\mathrm{L}: 2\mathrm{x}+\mathrm{y}=\mathrm{k}, \mathrm{k}>0$ be a tangent to the hyperbola $x^2-y^2=3$. If L is also a tangent to the parabola $\mathrm{y}^2=\alpha \mathrm{x}$, then α is equal to:

(1) 12

(2) -12

(3) 24

(4) -24

Q2 (25 July 2021 Shift 1)

The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2-9y^2+32x+36y-164=0$, and its foci is :

(1) $16x^2-9y^2+32x+36y-36=0$

(2) $9\mathrm{x}^2-16\mathrm{y}^2+36\mathrm{x}+32\mathrm{y}-144=0$

(3) $16x^2-9y^2+32x+36y-144=0$

(4) $9\mathrm{x}^2-16\mathrm{y}^2+36\mathrm{x}+32\mathrm{y}-36=0$

Answer Key

Q1 (4) **Q2** (1)

Q1

Tangent to hyperbola of

Slope $\mathrm{m}=-2$ (given)

$y=-2x \pm \sqrt{3(3)}$

$\left(\mathrm{y}=\mathrm{mx} \pm \sqrt{\mathrm{x}^2\ \mathrm{m}^2-\mathrm{b}^2}\right)$

$\Rightarrow y+2x=\pm 3 \Rightarrow 2x+y=3 \quad (k>0)$

For parabola $\mathrm{y}^2=\alpha \mathrm{x}$

$\mathrm{y}=\mathrm{mx}+\frac{\alpha}{4\ \mathrm{m}}$

$\Rightarrow \mathrm{y}=-2\mathrm{x}+\frac{\alpha}{-8}$

$\Rightarrow \frac{\alpha}{-8}=3$

$\Rightarrow =-24.$

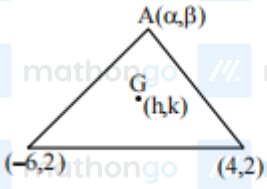
Q2

Given hyperbola is $16(x+1)^2-9(y-2)^2=164+16-36=144$

$\Rightarrow \frac{(x+1)^2}{9}-\frac{(y-2)^2}{16}=1$

Eccentricity, $\mathrm{e}=\sqrt{1+\frac{16}{9}}=\frac{5}{3}$

$\Rightarrow$ foci are $(4,2)$ and $(-6,2)$



Let the centroid be (h, k)

\& $\mathrm{A}(\alpha, \beta)$ be point on hyperbola

So $\mathrm{h}=\frac{\alpha-6+4}{3}, \mathrm{k}=\frac{\beta+2+2}{3}$

$\Rightarrow \alpha=3\ \mathrm{h}+2, \beta=3\mathrm{k}-4$

(α, β) lies on hyperbola so $16(3\ \mathrm{h}+2+1)^2-9(3\mathrm{k}-4-2)^2=144$

$\Rightarrow 144(\ \mathrm{h}+1)^2-81(\mathrm{k}-2)^2=144$

$\Rightarrow 16\left(h^2+2h+1\right)-9\left(k^2-4k+4\right)=16$

$\Rightarrow 16x^2-9y^2+32x+36y-36=0$