CS 583 – Assignment 1

Theoretical assignments

1. a.the number of independent parameters for P(X2, X3, ..., Xn, Y2, Y3, ..., Ym) with binary variables= $\mathbf{2^{(n-1)+(m-1)}}$ **-1**

b. P(X2, X3, ..., Xn, Y2, Y3, ..., Ym) with 3 variables = $\mathbf{3^{(n-1)+(m-1)}}$ **- 1**

c. For i possibles values- **(i^n * i^m) - (n + m - 1)**

d. P(Y2, Y3, …, Ym | X2, X3, …, Xn) for binary variables = $\mathbf{2^{(n-1)}(2^{(m-1)}-1)}$

e.P(Y2, Y3, …, Ym | X2, X3, …, Xn) for 3 possible values= $\mathbf{3^{(n-1)}(3^{(m-1)}-1)}$

f. (2 * 3 * ….*n) (2 *3 * …* m -1) that is (Summation of i from i = 2 to n) * (Summation of i from i = 2 to (m -1))

2. a. P(A, B, C, D, E, G, H, J) =

P(A) * P(B) * P(C | A, B) * P(D | C) * P(E) * P(G | D, E, J) * P(H | G) * P(J | C)

2 b. **Root nodes:** A and B, so 2 * n parameters.

- **C:** 2 parents (A, B), so 2 * n * (n - 1) = 2 * n^2 - 2 * n parameters.
- **D:** 1 parent (C), so 1 * n * (n - 1) = n^2 - n parameters.
- **E:** No parents, so n parameters.
- **G:** 3 parents (D, E, J), so 3 * n * (n - 1)^2 = 3 * n^3 - 9 * n^2 + 6 * n parameters.
- **H:** 1 parent (G), so n^2 - n parameters.
- **J:** 1 parent (C), so n^2 - n parameters.

Total parameters: 2 * n + 2 * n^2 - 2 * n + n^2 - n + n + 3 * n^3 - 9 * n^2 + 6 * n + n^2 - n + n^2 - n = 3 n^3 - 7 n^2 + 3 n + 2

Redundancies:

- C: subtract 2 (from A, B).
- D: subtract 1 (from C).
- G: subtract 3 (from D, E, J).

Total Independent Parameters:

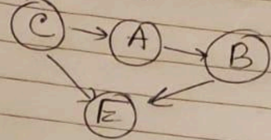
Total parameters - Redundancies = (3 * n^3 - 7 * n^2 + 3 * n + 2) - (2 + 1 + 3) = $3n^3 - 7n^2$ -8

2. c. Are the following independence statements true or false?
i. $A \perp B$ - True
ii. $A \perp B \mid C$ - False
iii. $A \perp B \mid J$ - False
iv. $A \perp B \mid G$ - False
v. $A \perp B \mid E$ - True
vi. $A \perp B \mid H$ - False
vii. $A \perp H$ - False
viii. $A \perp H \mid J$ - False
ix. $A \perp H \mid D, J$ - False
x. $D \perp J$ - True
xi. $B \perp E$ - True
xii. $B \perp E \mid J$ - False
xiii. $B \perp E \mid J, H$ - False

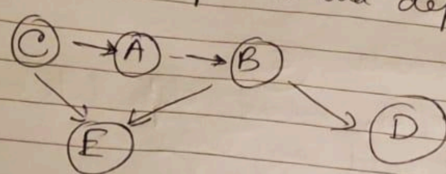
3. a. C, A, B, E, D, G
C: No parents, since it is the first in the order and there is no prior information.
A: No parents, as nothing comes before A to condition on.
B: Must have A as a parent because there's a direct arrow from A to B in the DAG.
E: Should have B and C as parents, due to arrows from both to E.
D: Should have B as a parent, due to the arrow from B to D.
G: Should have E as a parent, due to the arrow from E to G.

The minimal I-Map would then be: $C \rightarrow E \leftarrow B \rightarrow D$ and $B \rightarrow E \rightarrow G$, with $A \rightarrow B$.

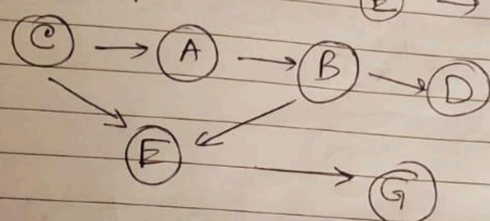
E: B & C are parents of E in P-map, so we add edges →



D: D has B as parent and dependent on B, so we add,

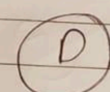


G: since E is the only parent of G, need
G is dependent on E →

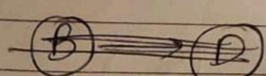


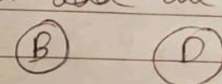
b. D, B, A, ~~A~~ E, C, G

D : root node



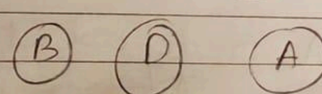
B : B is ~~…~~ not dependent on D as it has a direct edge.
we don't add an edge





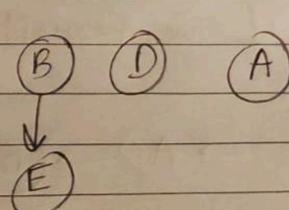
A : $A \perp D \mid B$

$A \perp B$



E : $E \perp D \mid B$

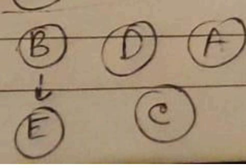
$E \perp A \mid B$

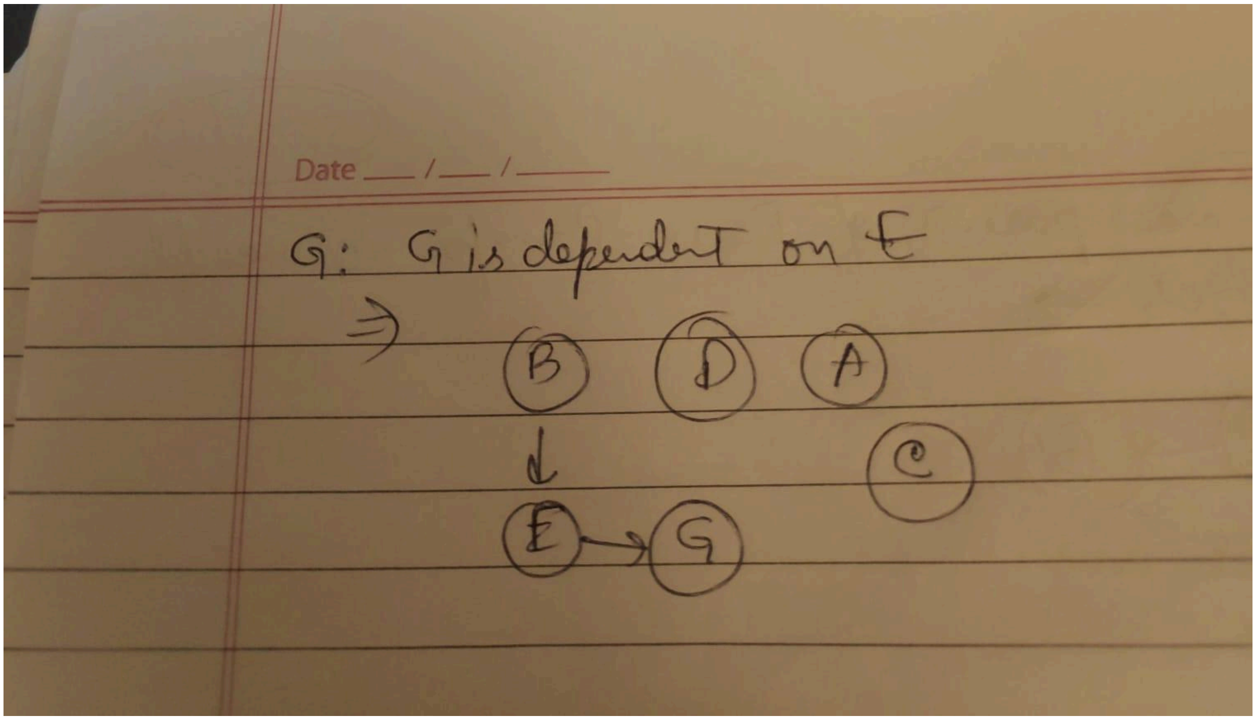


C : $C \perp B \mid E$

$C \perp A \mid B, E$

$C \perp D \perp B$





c. G, E, D, C, B, A

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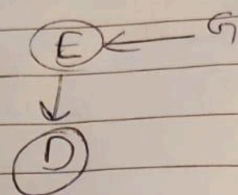
c: G, E, D, C, B, A

G: G in root node

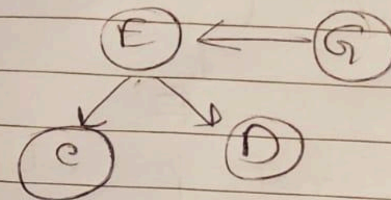
E: $E \leftarrow G$

D: $D \perp G \mid E$

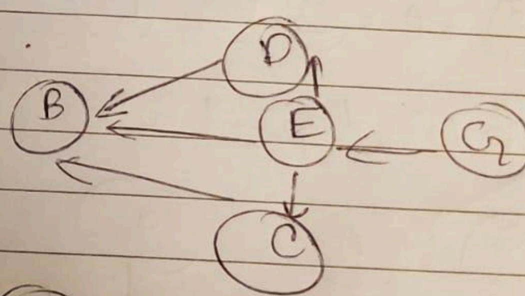


C: $C \perp G \mid E$

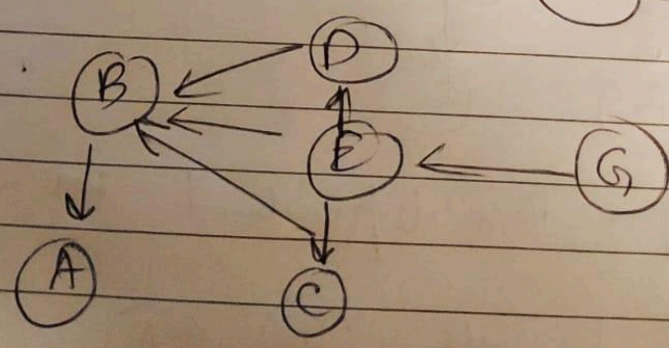
$C \perp D \mid E$



B = $B \perp G \mid E$



A:



d. G, A, C, E, D, B

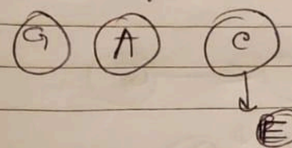
d: G, A, C, E, D, B.

G: root node

A: G A no edge as G is not parent of A

C: G A C as G & A are not parents of C in P-map

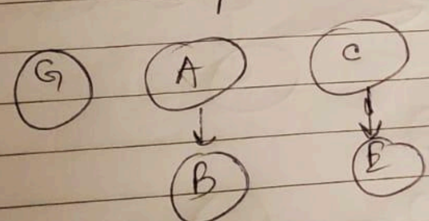
E: E is dependent on C as parent -



D: D has not parents or dependency on

G, A, C → E D

B: B is dependent on A →



Now, revisiting, D is dependent on B,

⇒ minimal I-map