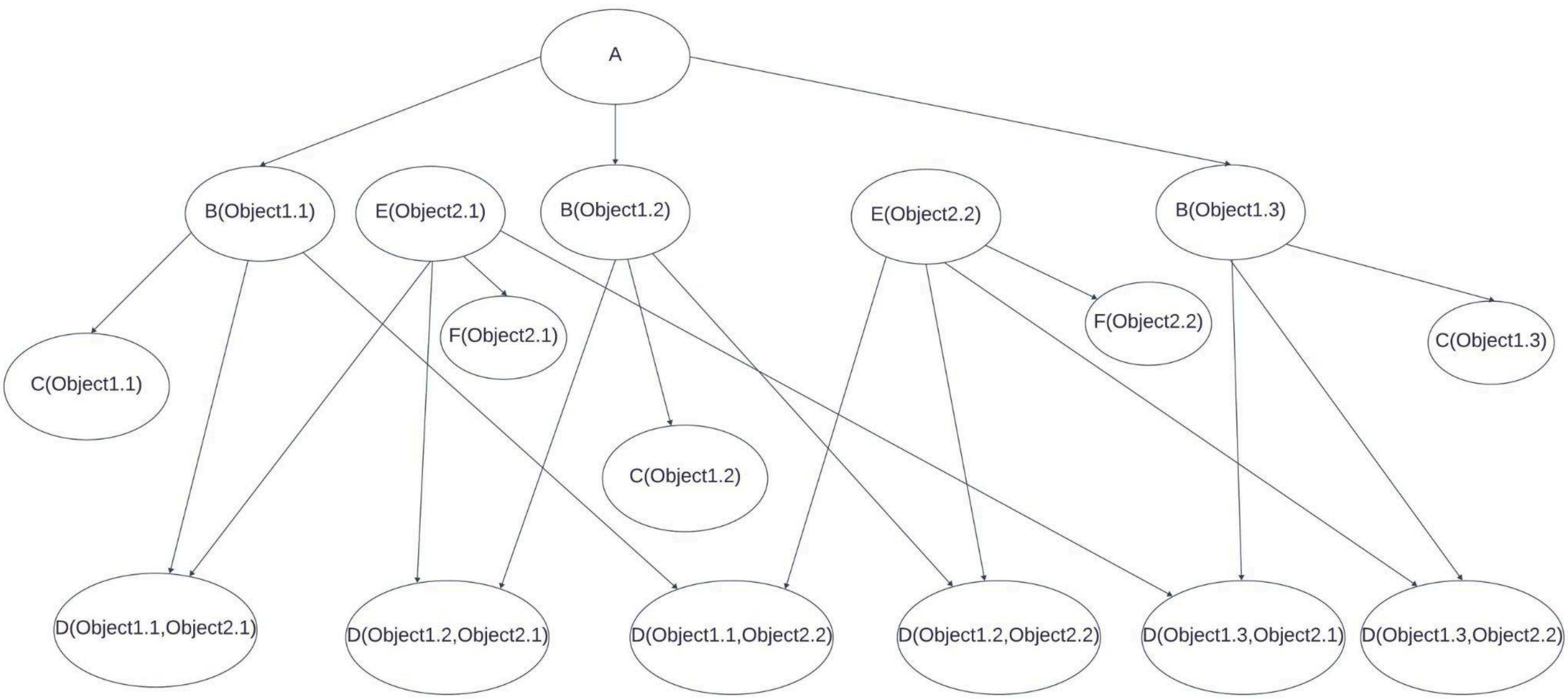
Problem 1-

Probability distribution -

$$P(A) \prod_{i=1}^{n} P(B_i/A)\, P(C_i/B_i) \prod_{j=1}^{m} P(E_j) P(F_j/E_j)\, P(D_{ij}/B_i, E_j)$$

here π is product of probability



Problem 2.

(a) P(C) = ∑P(A). ∑P(B|A). P(C|B)

Case 1

P(C=T)= P(A=T).[P(B=T | A=T).P(C=T | B=T)

+ $P(B=F|A=T).P(C=T|B=F)$]

+ $P(A=F)$.[$P(B=T|A=F).P(C=T|B=T)$

+ $P(B=F|A=F).P(C=T|B=F)$

] $=0.4[0.3\cdot0.9+0.7\cdot0.4]+0.6[0.8\cdot0.9+0.2\cdot0.4]$

$=0.4[0.27+0.28]+0.6[0.72+0.08]$

$=0.4\cdot0.55+0.6\cdot0.8$

$=0.22+0.48 \Rightarrow P(C=T) = 0.7$

And $P(C=F) = 1- P(C=T) = 1 - 0.7=0.3$

(b) $P(C|A=T)= \sum P(B|A=T).P(C|B)$

For C=T

$P(C=T|A=T)=P(B=T|A=T)P(C=T|B=T)+P(B=F|A=T)P(C=T|B=F)$

$=0.3\cdot0.9+0.7\cdot0.4 = 0.55$

For C=F: $P(C=F|A=T)=1-0.55=0.45$

(C) $P(C=T|A=T,B=T)= P(C=T|B=T)=0.9$

$P(C=F|A=T,B=T)=1-P(C=T|B=T)=0.1$

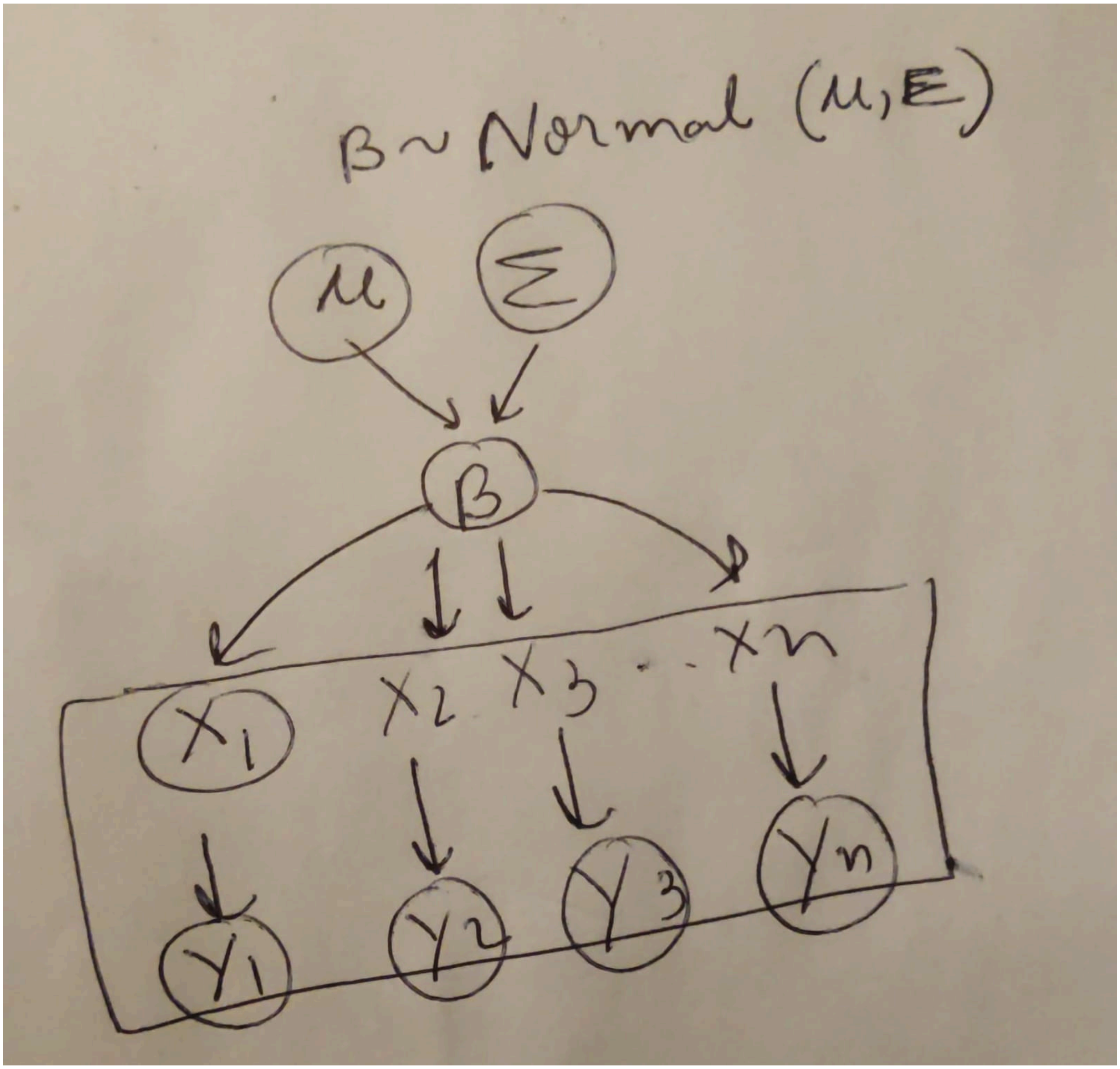
Problem 3

The plate notation for Bayesian Logistic Regression can be represented as follows:

$Y1 \sim \text{Bernoulli}(\text{invLogit}(X1\beta))$ $Y2 \sim \text{Bernoulli}(\text{invLogit}(X2\beta))$ ... $YN \sim \text{Bernoulli}(\text{invLogit}(XN\beta))$

$\beta \sim \text{Normal}(\mu, \Sigma)$.

Each box represents a set of variables or data points.The X boxes represent the input vectors.The Y boxes represent the binary outcomes.The arrows from X to Y indicate that each outcome Y n depends on its corresponding input vector X n through the parameter vector β.The prior distribution of the parameter vector β is represented outside the boxes. In this case, it follows a normal distribution with mean μ and covariance matrix Σ.



Problem 4-

Variable	All Factors	Participates	New Factor After *	New Factor After +
A	P(A),P(B|A),P(D), P(C|B,D),P(E|C)	P(A),P(B|A)	f1(A,B)	t1(B)
B	t1(B),P(D), P(C|B,D),P(E|C)	t1(B),P(C|B,D)	f2(B,C,D)	t2(C,D)
D	P(D),P(E|C),t2(C,D)	P(D),t2(C,D)	f3(C,D)	t3(C)
C	P(E|C),t3(C)	P(E|C),t3(C)	f4(E,C)	t4(E)
Normalize	t4(E)			