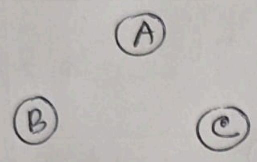
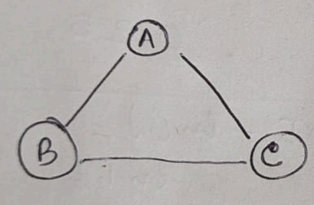
1.

1. a. Markov Network graph -

Create 3 nodes. -

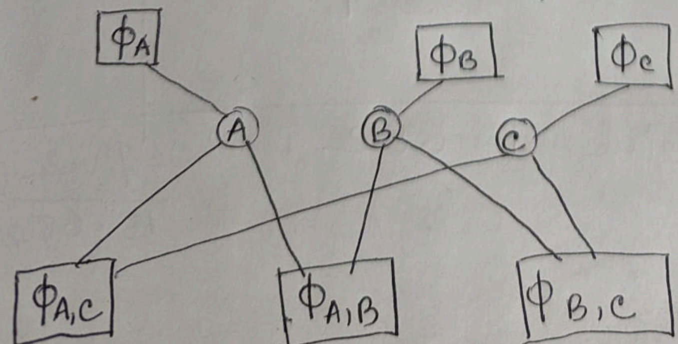


we could connect all 3 nodes with each other because factors $f(A,B)$, $f(B,C)$ & $f(A,C)$ is present which depicts dependency. between A and B, B and C & A and C.



b. <u>Factor graph</u>

- create 3 variable nodes and 6 Factor nodes..
- Connect each variable node to factor node that corresponds, e.g. A will be connected in $f(A)$, $f(A,B)$ & $f(A,C)$



c.

A, B, C	$\phi(A) * \phi(B) * \phi(C) * \phi(A,B) * \phi(A,C) * \phi(B,C)$	$P(A,B,C)$
T T T	$2 * 1 * 1 * 5 * 6 * 1 = 60$	0.0197
T T F	800	0.262
T F T	480	0.157
T F F	64	0.021
F T T	1	0.0003
F T F	480	0.157
F F T	200	0.065
F F F	960	0.315
	3045	

Problem 2.

In Markov network graph, two nodes (variables) are conditionally independent if and only if all paths between the two nodes are blocked by a node in the given set. A path is blocked if it includes a node such that the set of nodes being conditioned on includes that node.

a. $A \perp B$: False because there is a direct connection between A and B through C.

B. $A \perp B \mid C$: True because C separates A from B, blocking the path.

C. $A \perp G \mid D$: False because even though D is given, there is still an indirect path from A to G through C and F that is not blocked by D.

D. $A \perp G \mid D, F$: True because D and F block all paths between A and G.

E. $A \perp H | G$: True because G is the only parent of H and once G is known, H is independent of any other node that is not its descendant.

3.

A	B	C	ø(A) * ø(B) * ø(C) * ø(A, B) * ø(A, C)* ø(B, C)	P (A, B, C)
T	T	T	e^ -(ln(2) + ln(4) – ln(5)) = e^ -ln(1.6) = 0.625	0.033
T	T	F	e^ -(ln(2) - ln(5) + ln(6)) = e^ -ln(2.4) = 0.417	0.022
T	F	T	e^ -(ln(2) - ln(3) + ln(4) + ln(6)) = e^ -ln(16) = 0.062	0.003
T	F	F	e^ -(ln(2) - ln(3)) = e^ -ln(0.66) = 1.515	0.081
F	T	T	e^ -ln(4) = 0.25	0.013
F	T	F	e^ -ln(6) = 0.167	0.008
F	F	T	e^ -(-ln(3) + ln(4) - ln(5) + ln(6)) = e^ -ln(1.6) = 0.625	0.033
F	F	F	e^ -(- ln(3) - ln(5)) = e^ ln(15) = 15	0.803
Total =			18.661	

4. Need to calculate P(Y|X), that is P(Y1, Y2 | X11=T, X21=F, X12=T, X22=T).

P(Y1, Y2 | X11=T, X21=F, X12=T, X22=T) =
1/Z * Φ(x11, y1) * Φ(x21, y1) * Φ(x12, y2) * Φ(x22, y2) * Φ(y1, y2)

When $(Y1, Y2) = AI,AI$ that is $[Y1 = Y2]$

$\Phi1(X11, Y1)= e^{-(1*(-1)+0*1)} = e$

$\Phi1(X12, Y2) = e^{-(1*(-1) + 0*1)} = e$

$\Phi2(X21, Y1) = e^{-(0*1+0(-1))} = 1$

$\Phi2(X22, Y2) = e^{-(1* 1+0(-1))} = 1/e$

$\Phi3(Y1, Y2) = e^{-(1*(-1)+0*1)} = e$

And , $\Phi(x11, y1) * \Phi(x21, y1) * \Phi(x12, y2) * \Phi(x22, y2) * \Phi(y1, y2) = e * e * 1 * 1/e * e =7.38$

Therefore, $P(Y1, Y2 \mid X11=T, X21=F, X12=T, X22=T)$ when $(Y1, Y2) = AI,AI$ $= 1/Z *7.38$.
we will get Z after calculating all 4 combinations of Y1 , Y2.

When $(Y1, Y2) = AI,DB$ that is $[Y1 != Y2]$

$\Phi1(X11, Y1)= e^{-(1*(-1)+0*1)} = e$

$\Phi1(X12, Y2) = e^{-(0*(-1) + 1*1)} = 1/e$

$\Phi2(X21, Y1) = e^{-(0*1+0(-1))} = 1$

$\Phi2(X22, Y2) = e^{-(0* 1+1(-1))} = e$

$\Phi3(Y1, Y2) = e^{-(0*(-1)+1*1)} =1/ e$

And , $\Phi(x11, y1) * \Phi(x21, y1) * \Phi(x12, y2) * \Phi(x22, y2) * \Phi(y1, y2) = e * 1/e * 1 * e * 1/e = 1$

When $(Y1, Y2) =DB, AI$ that is $[Y1 != Y2]$

$\Phi1(X11, Y1)= e^{-(0* (-1)+1*1)} = 1/e$

$\Phi1(X12, Y2) = e^{-(1*(-1) + 0*1)} = e$

$\Phi2(X21, Y1) = e^{-(0*1+0(-1))} = 1$

$\Phi2(X22, Y2) = e^{-(1* 1+0(-1))} = 1/e$

$\Phi3(Y1, Y2) = e^{-(0*(-1)+1*1)} = 1/e$

And , $\Phi(x11, y1) * \Phi(x21, y1) * \Phi(x12, y2) * \Phi(x22, y2) * \Phi(y1, y2) = 1/e * e * 1 * 1/e * 1/e = 1/(e^2) = 0.13$

When (Y1, Y2) = DB,DB that is [Y1 = Y2]

$\Phi1(X11, Y1)= e^{-(0*(-1)+1*1)} = 1/e$

$\Phi1(X12, Y2) = e^{-(0*(-1) + 1*1)} = 1/e$

$\Phi2(X21, Y1) = e^{-(0*1+0(-1))} = 1$

$\Phi2(X22, Y2) = e^{-(0*1+1(-1))} = e$

$\Phi3(Y1, Y2) = e^{-(1*(-1)+0*1)} = e$

And , $\Phi(x11, y1) * \Phi(x21, y1) * \Phi(x12, y2) * \Phi(x22, y2) * \Phi(y1, y2) = 1/e * 1/e * 1 * e * e = 1$

Hence, Z = 7.38 + 1 + 0.13 + 1 = 9.51

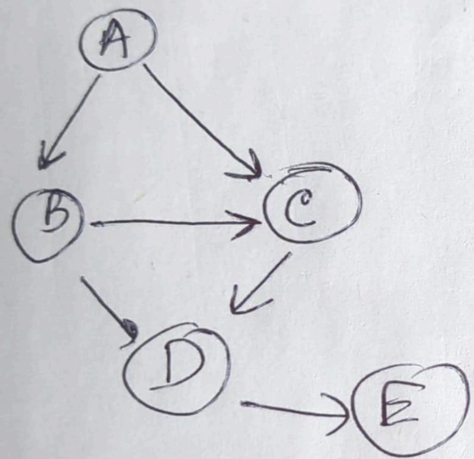
$\phi(AI,AI)=7.38$
$\phi(AI,DB)=1$
$\phi(DB,AI)=0.13$
$\phi(DB,DB)=1$

Partition function Z=9.51
Normalized probabilities:
$P(AI,AI|X)=1/Z * 7.38 =0.776$
$P(AI,DB|X)= 1/Z * 1 = 0.105$
$P(DB,AI|X)= 1/Z * 0.13 = 0.013$
$P(DB,DB|X)= 1/Z * 1 = 0.105$

The MAP assignment to Y is the one with the highest probability, which is P(AI,AI) with a probability of approximately 0.776.

5.

5. (a) let's enumerate order as
A, B, C, D, E for our graph G



$A \rightarrow B$

$B \rightarrow C$

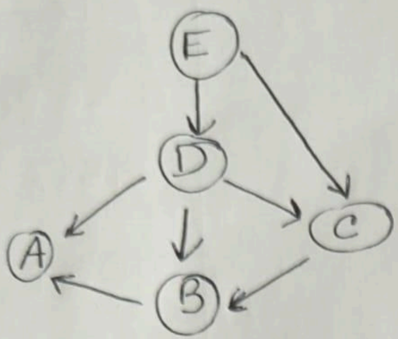
$A \rightarrow C$

$B \rightarrow D$

$C \rightarrow D$

$D \rightarrow E$

5(b) E, D, C, B, A



here, $E \not\perp D, C$

$E \perp A, B \mid D, C$

$D \not\perp E, A, B, C$

$C \not\perp E, D, B$

$C \perp A \mid D, B$

$B \not\perp A, C, D$

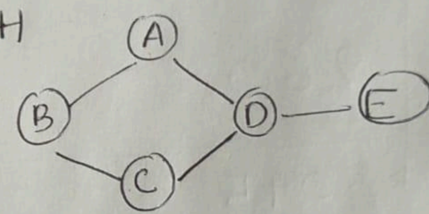
$B \perp E / D, C$

$A \not\perp D, B$

$A \perp E, C \mid D, B$

5 c) Lets compare G_1 with original Markov's graph.

In Original graph H



$A \not\perp B, D$

$A \perp C, E / B, D$

$A \not\perp C, E$

$B \not\perp C, A$

$B \perp D, E / A, C$

$C \not\perp B, D$

$C \perp A, E \mid B, D$

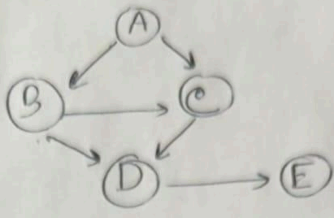
$D \not\perp A, E, C$

$D \perp B \mid A, C$

$E \not\perp D$

$E \perp A, B, C \mid D$

In our graph G_1,



$A \not\perp B, C$

$A \perp D, E \mid B C$

$B \not\perp A, C, D$

$B \perp E \mid D$

$C \not\perp A, B, D$

$C \perp E \mid D$

$D \not\perp B, C, E$

$D \perp A \mid B, C$

$E \not\perp D$

$E \perp A, B, C \mid D$

Hence, except independency of E node, none of other nodes independencies are preserved. It is not a P-Map.

5 d) In our graph G2, missing independencies are $B \perp D \mid A, C$

$C \perp E \mid B, D$

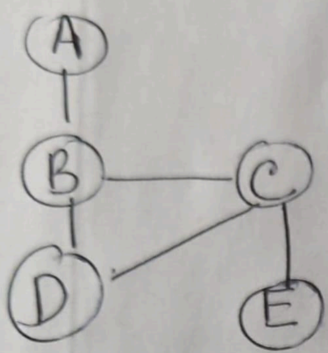
$D \perp B \mid A, C$

$E \perp C \mid D$

It is not a P-Map. for H

6.

(a) Lets use order A, B, C, D, E



(b) Graph H is not a P-Map for G.
It doesn't capture all independencies.
that are in original distribution.
particularly, nodes that are married
(B and D). missing independencies
between A and D that is There in
original graph.