1. Let G_1 be a digraph with 4 strongly connected components. Obtain a graph G_2 from G_1 by adding a single directed arc to G_1. What is the smallest number of strongly connected components G_2 might have? Describe an example that would result in this minimum case.

2. Determine the following statements are True/False for a digraph.

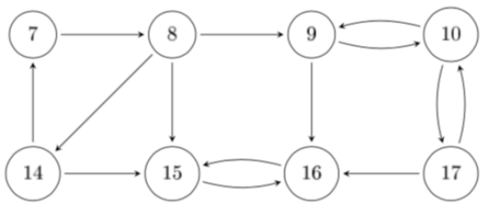
   (A) If there exist $u, v \in G$ such that there is a path from u to v and from v to u, the digraph is strongly connected. TRUE/FALSE?

   (B) If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph is weakly connected. TRUE/FALSE?

   (C) If there exist $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph contains no strong components. TRUE/FALSE?

   (D) If there exist $u, v \in G$ such that there is no path from u to v or there is no path from v to u, the digraph is not strongly connected. TRUE/FALSE?

3. Consider the following directed graph G. What is the strongly connected components of the digraph G if each such component is given by its set of nodes? Please show your working to justify your answer. [**8 marks**]



4. Consider the graph with vertices 0, 1, 2, 3, 4 and edges

   $0:\ 1,2,3$
   $1:\ 2,3,4$
   $2:\ 3,4$
   $3:$
   $4:$

   Which of the following statement is true?

   (A) The graph has a 2- and a 4-coloring but no 3-coloring.

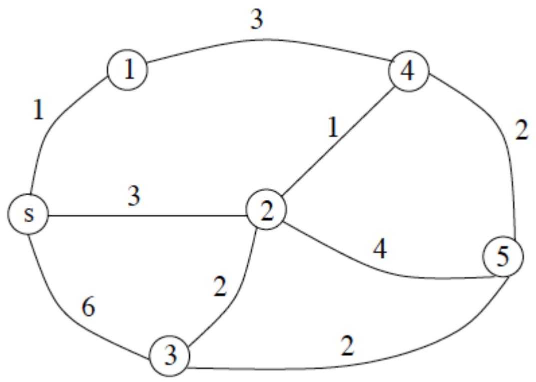
   (B) The graph has a 4-coloring but no 2- or 3-coloring.

   (C) The graph has a 2- and a 3-coloring, but no 4-coloring

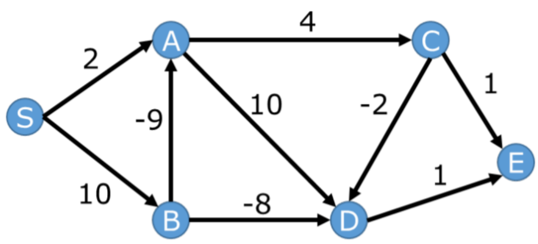
   (D) The graph has a 3- and 4-coloring but no 2-coloring.

   (E) The graph has neither a 2-coloring, nor a 3- or a 4-coloring.

5. Consider the single-source shortest path problem with source s in the weighted digraph G shown above. Find the minimum weight paths from the vertex s to all the other vertices in the graph below using Dijkstra's algorithm. Show the value of the distance vector after each step.



6. Given the graph G shown, we find the shortest paths from node S using the Bellman-Ford algorithm. How many iterations does it take before the algorithm converges to the solution?



7. Let G be a weighted graph with positive edge weights. Let $\{u, v\}$ be an edge in the graph. It is known that the shortest path from a source vertex s to u has weight 100 and the shortest path from s to v has weight 95. Which of the following statement is always TRUE?

   (A) weight($\{u, v\}$)> 5

   (A) weight($\{u, v\}$)$= \min(95, 100)$

   (A) weight($\{u, v\}$)< 5

   (A) weight($\{u, v\}$)≥ 5

   (A) weight($\{u, v\}$)$= \max(95, 100)$

8. Consider the digraph G with nodes 0, 1, 2 and arcs (0, 1) with weight 5, (0, 2) with weight 2, (1, 2) with weight -2. If we are solving the SSSP problem for node 0 then which of the following is true?

   (A) Dijkstra gives wrong answer; Bellman-Ford gives right answer.

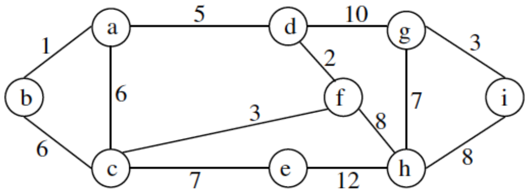
   (B) Dijkstra and Bellman-Ford both give right answer.

   (C) Dijkstra and Bellman-Ford both give wrong answer.

   (D) There is no solution for this digraph.

   (E) Dijkstra gives right answer; Bellman-Ford gives wrong answer.

9. There are 8 cities and there are roads in between some of the cities. Somehow all the roads are damaged simultaneously. We have to repair the roads to connect the cities again. There is a fixed cost to repair a particular road. Design an algorithm to find out the minimum cost to connect all the cities by repairing roads. The city map and their re-paring costs are illustrated as follows. Print out the minimum cost to connect all the cities by your program.



10. An undirected weighted graph G has n vertices. The cost matrix of G is given by an $n \times n$ square matrix whose diagonal entries are all equal to 0 and whose non-diagonal entries are all equal to 2. Which of the following statements is TRUE?

(A) G has a minimum spanning tree of weight 0.

(B) G has multiple minimum spanning trees, each of weight 2(n-1).

(C) G has no minimum spanning tree.

(D) G has a unique minimum spanning tree of weight 2(n-1).

(E) G has multiple minimum spanning trees, each of different weights