Tutorial 3

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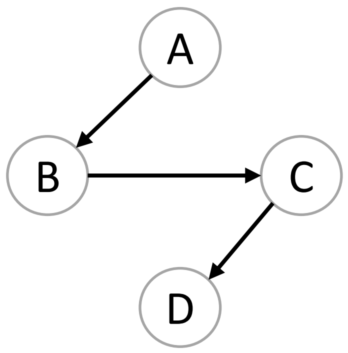
COMPSCI: WEEK 14.1

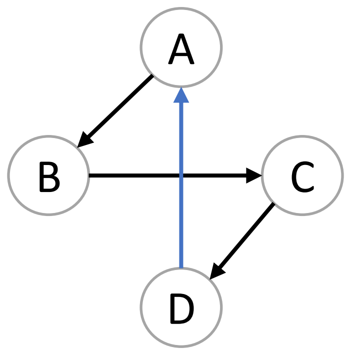
OUTLINE

- Question 1: Graph Connectivity
- Question 2: Graph Connectivity
- Question 3: Graph Connectivity
- Question 4: Graph Coloring
- Question 5: Shortest Path
- Question 6: Shortest Path
- Question 7: Shortest Path
- Question 8: Shortest Path
- Question 9: Minimum Spanning Tree
- Question 10: Minimum Spanning Tree

Question 1: Graph Connectivity

- Let G1 be a digraph with 4 strongly connected components. Obtain a graph G2 from G1 by adding a single directed arc to G1.
- What is the smallest number of strongly connected components G2 might have? Describe an example that would result in this minimum case.





No pairs of vertices are mutually reachable in G1

➔ It would have 4 strongly connected components

➔ Every vertex is an SCC by itself

We join everything in a cycle

➔ We end up with 1 component (minimum case)

Question 2 (A): Graph Connectivity

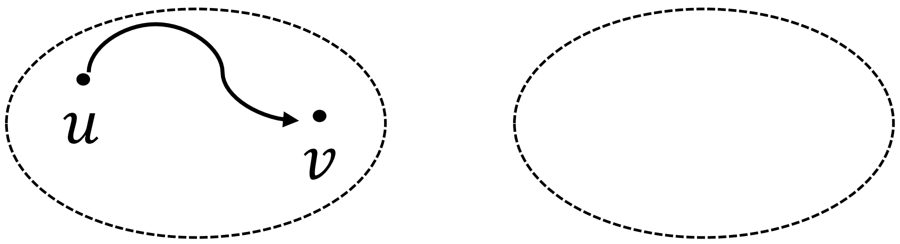
- If there exists $u, v \in G$ such that there is a path from u to v and from v to u, the digraph is strongly connected.

- False
- Definition: A diagraph is strongly connected if and only if for **every pair of nodes** $u, v \in G$, there is a path from u to v and from v to u

Question 2 (B): Graph Connectivity

- If there exists $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph is weakly connected.

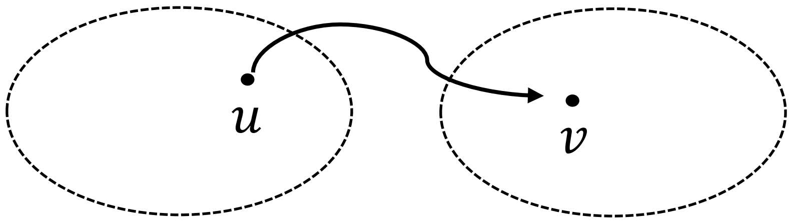
- False
- Consider if its underlying graph has two components, and u, v are from one of the two.



Question 2 (C): Graph Connectivity

- If there exists $u, v \in G$ such that there is a path from u to v but not from v to u, the digraph contains no strong components.

- False.
- Consider a digraph with two strong components, and only an arc (u,v) links a node u in one component to another node v in the other component.



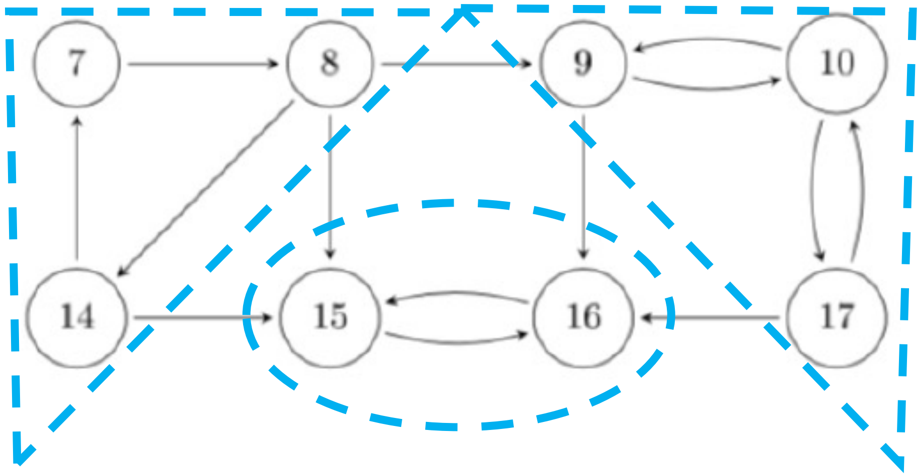
Question 2 (D): Graph Connectivity

- If there exist $u, v \in G$ such that there is no path from u to v or there is no path from v to u, the digraph is not strongly connected.

- True.
- It is the negation of the definition of strongly connected digraph.

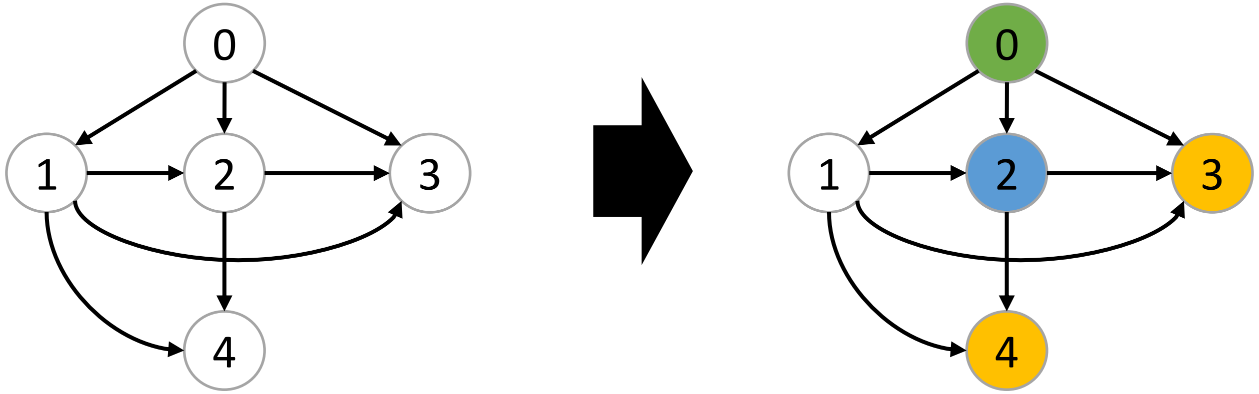
Question 3: Graph Connectivity

- What are the strongly connected components of the digraph G if each such component is given by its set of nodes? Please show your working to justify your answer.



Question 4: Graph Coloring

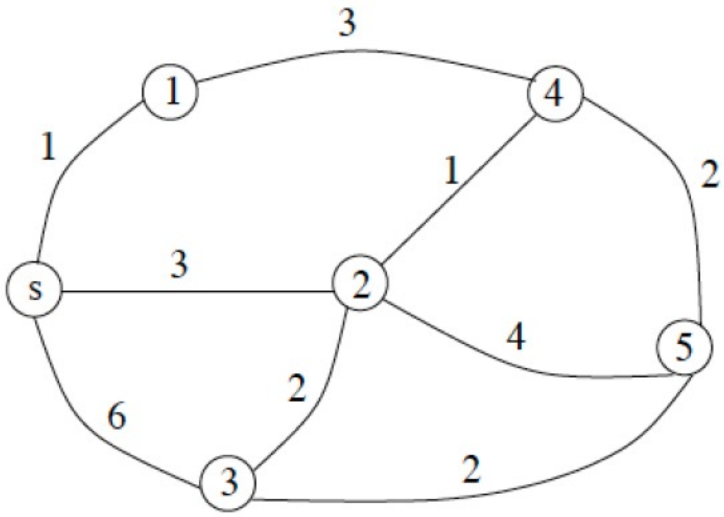
- Consider the graph with vertices 0, 1, 2, 3, 4 and edges, which of the following statement is true?
  A. The graph has neither a 2-coloring, nor a 3- or a 4-coloring.
  B. The graph has a 2- and a 4-coloring but no 3-coloring.
  C. The graph has a 4-coloring but no 2- or 3-coloring.
  D. The graph has a 2- and a 3-coloring, but no 4-coloringe The graph has a 3- and 4-coloring but no 2-coloring.



The minimum number of colors we need is 4.

Question 5: Shortest Path

- Consider the single-source shortest path problem with source s in the weighted graph G shown below.
- Find the minimum weight paths from the vertex s to all the other vertices in the graph below using Dijkstra's algorithm. Show the value of the distance vector after each step.

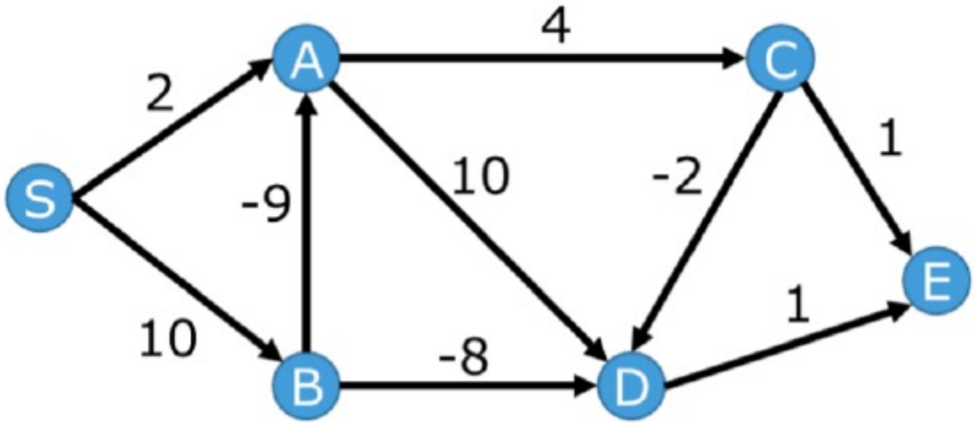


S 1 2 3 4 5

dist = $\{0, 1, 3, 6, \infty, \infty\}$	dist[1] = 1
dist = $\{0, 1, 3, 6, 4, \infty\}$	dist[2] = 3
dist = $\{0, 1, 3, 5, 4, 7\}$	dist[4] = 4
dist = $\{0, 1, 3, 5, 4, 6\}$	dist[3] = 5
dist = $\{0, 1, 3, 5, 4, 6\}$	dist[5] = 6

Question 6: Shortest Path

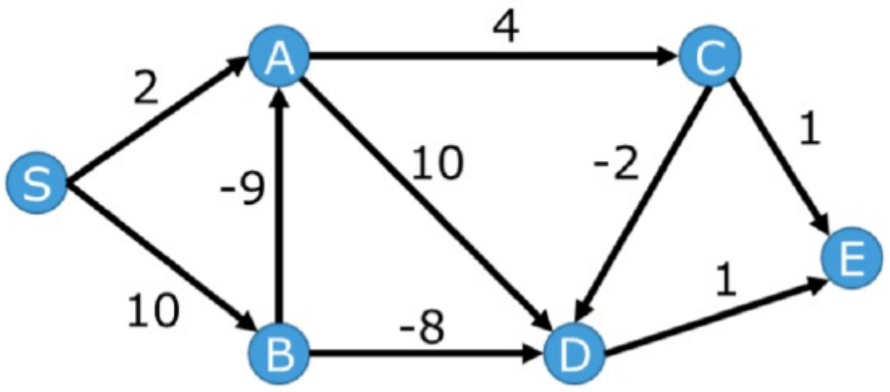
- Consider the single-source shortest path problem with source s in the weighted digraph G shown below.
- Find the shortest paths from node s using the Bellman-Ford algorithm.
- How many iterations does it take before the algorithm converges to the solution?



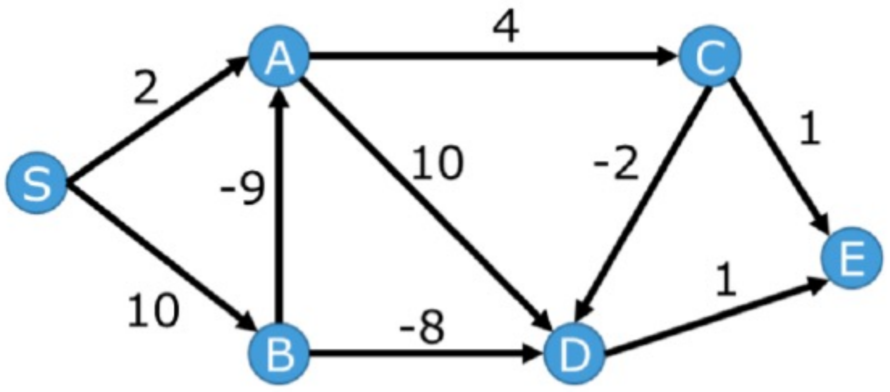
We calculate the distance array dist[x] after each iteration i.
We label S=0, A=1,B=2, C=3, D=4, E=5.

Initially, we have dist = $[0, \infty, \infty, \infty, \infty, \infty]$

Iteration $i = 0$:
1. Take the arc (S, A) to update dist[A] = 2.
2. Take the arc (S, B) to update dist[B] = 10.
we have dist = $[0,2,10, \infty, \infty, \infty]$



Iteration $i = 1$:
1. Take the arc (A, C) to update dist[3] = 6.
2. Take the arc (A, D) to update dist[4] = 12.
3. Take the arc (B, A) to update dist[1] = 1.
4. Take the arc (B, D) to update dist[4] = 2.
We have dist = $[0,1,10,6,2, \infty]$



Iteration $i = 2$:

1. Take the arc (A, C) to update dist[C] = 5.
2. Take the arc (C, E) to update dist[E] = 7.
3. Take the arc (D, E) to update dist[E] = 3.

We have dist = $[0,1,10,5,2,3]$

All other iterations will give you the same distance array.
So, three iterations will lead to converge.

Question 7: Shortest Path

- Let G be a weighted graph with positive edge weights.
- Let {u,v} be an edge in the graph. It is known that the shortest path from a source vertex **s** to **u** has weight **100** and the shortest path from **s** to **v** has weight **95**.
- Which of the following statement is always TRUE?

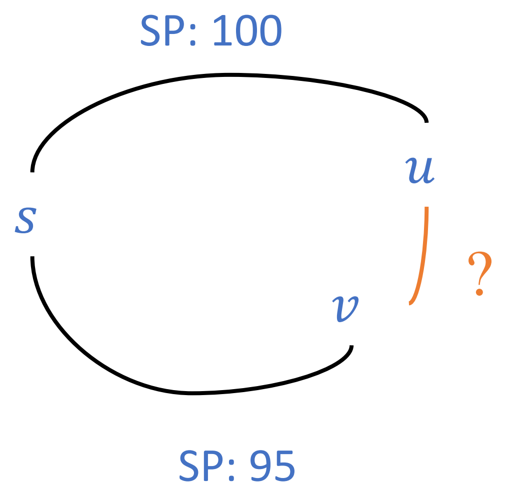
A. $weight(\{u,v\}) > 5$

B. $weight(\{u,v\}) = \min(95, 100)$

C. $weight(\{u,v\}) < 5$

D. $weight(\{u,v\}) \geq 5$

E. $weight(\{u,v\}) = \max(95, 100)$

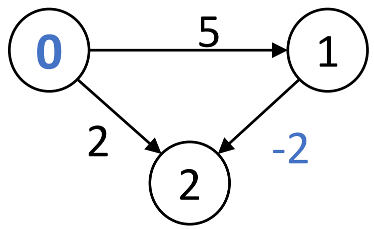


$$dist(s,u) = \min(dist(s,u), dist(s,v) + \underline{dist(v,u)})$$

Question 8: Shortest Path

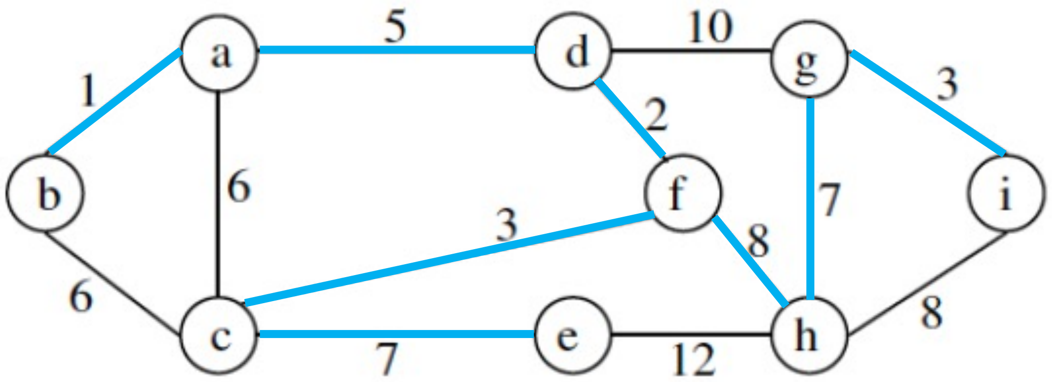
- Consider the digraph G with nodes 0, 1, 2 and arcs (0, 1) with weight 5, (0, 2) with weight 2, (1, 2) with weight -2. If we are solving the SSSP problem for node 0 then which of the following is true?

A. Dijkstra gives wrong answer; Bellman-Ford gives right answer.
B. Dijkstra and Bellman-Ford both give right answer.
C. Dijkstra and Bellman-Ford both give wrong answer.
D. There is no solution for this digraph.
E. Dijkstra gives right answer; Bellman-Ford gives wrong answer.



Question 9: Minimum Spanning Tree

- Design an algorithm to find out the minimum cost to connect all the cities by repairing roads.



- Sorted Edges: (a,b,1) (d,f,2) (c,f,3) (g,i,3) (a,d,5) (a,c,6) (b,c,6) (c,e,7) (g,h,7) (f,h,8) (h,i,8) (d,g,10) (e,h,12)
- Selected Edges: (a,b,1) (d,f,2) (c,f,3) (g,i,3) (a,d,5) ~~(a,c,6) (b,c,6)~~ (c,e,7) (g,h,7) (f,h,8) ~~(h,i,8) (d,g,10) (e,h,12)~~
- MST = 1+2+3+3+5+7+7+8 = 36

Question 10: Minimum Spanning Tree

- An undirected weighted graph G has n vertices. The cost matrix of G is given by an $n \times n$ square matrix whose diagonal entries are all equal to 0 and whose non-diagonal entries are all equal to 2. Which of the following statements is TRUE?

A. G has a minimum spanning tree of weight 0.

B. G has a unique minimum spanning tree of weight 2(n-1).

C. G has multiple minimum spanning trees, each of different weights

D. G has multiple minimum spanning trees, each of weight 2(n-1).

E. G has no minimum spanning tree.

$$\begin{bmatrix} 0 & 2 & \dots & 2 \\ 2 & 0 & \dots & 2 \\ \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & \dots & 0 \end{bmatrix}$$

Cost of Matrix

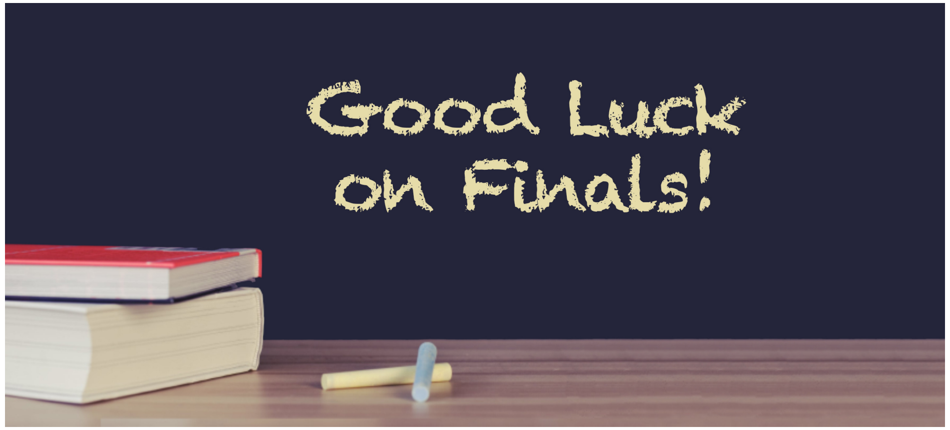
Resources

- Course Website
  - https://ankechiang.github.io/cs220_swu.html
- Lecture Notes
  - https://github.com/ankechiang/ankechiang.github.io/tree/master/cs220_lecture
- Lecture Recordings
  - https://github.com/ankechiang/ankechiang.github.io/tree/master/cs220_recording

Final Exam

Question Types:

1. Multiple Choice Question
2. Short Answer Questions



Topics

1. Complexity [~10]
2. Sorting [~15]
3. Searching [~5]
4. Graph Traversal [~20]
5. Cycles and Girth [~10]
6. Topological Order [~5]
7. Bipartite Graphs (Coloring) [~10]
8. Shortest Path [~15]
9. Minimum Spanning Tree [~10]