MATLAB Tutorial 12

ENME 303 Computational Methods for Engineers

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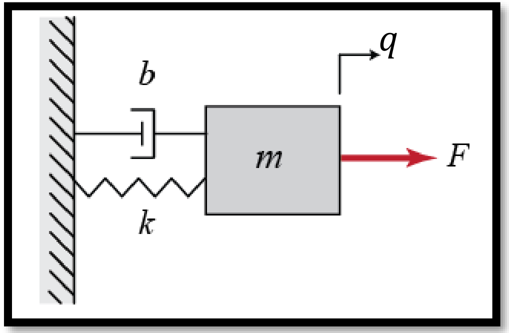
UMBC

Mass Spring Damper

- Consider the mass spring damper system:



$$\boxed{m\ddot{q} + b\dot{q} + kq = F}$$

- Rewrite the given second order ODE as a system of first order ODEs.

Mass Spring Damper

$$\boxed{m\ddot{q} + b\dot{q} + kq = F}$$

- Let:

$$\begin{cases} x_1 \triangleq q \\ x_2 \triangleq \dot{q} \end{cases} \rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{F - bx_2 - kx_1}{m} \end{cases}$$

- x_i are called **states** of the system and x is called the **state vector**:

$$x \triangleq \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \dot{x} \triangleq \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{F - bx_2 - kx_1}{m} \end{bmatrix}$$

- Write $\dot{x}$ vector in the linear form:

$$\dot{x} = Ax + Bu$$

Mass Spring Damper

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- Write $\dot{x}$ vector in the linear form:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\dfrac{k}{m} & -\dfrac{b}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} F$$

- This is **State Space** representation of a **Linear System**.

Mass Spring Damper

- Consider the unforced mass spring damper system:

$$\dot{x} = Ax$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} x$$

$$\begin{cases} m = 10 \\ k = 5 \\ b = 3 \end{cases}$$

- Can you tell how the solutions (states) will evolve by looking at the eigenvalues of A?
- Use the MATLAB function ode45 to solve this linear system.
- The solution of $\dot{x} = Ax$ is $x(t) = e^{At}x(0)$. Compare this solution with the solution you get from the previous part.

Mass Spring Damper

- Consider the following system:

$$\dot{x} = Ax + Bu$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F$$

$$\begin{cases} m = 10 \\ k = 5 \\ b = 3 \\ F = 1 \end{cases}$$

- Use the MATLAB function ode45 to solve this linear system.
- Plot the solutions (states) of the systems over time.
- Change k, b, m to investigate the effect of those parameters on your solution.
- Plot the **phase-plot**, that is plotting states with respect to each other ($x_2(t)\ vs\ x_1(t)$)

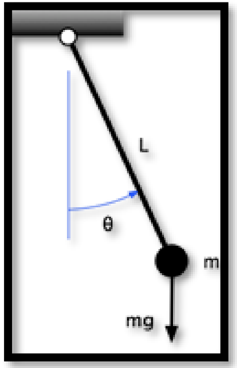
How About Non-Linear Systems?!

We can't write non-linear system as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

Simple Pendulum

- Consider the simple pendulum system:



$$\boxed{l\ddot{\theta} + g\sin(\theta) = 0}$$

$$\begin{cases} g = 10 \\ l = 10 \end{cases}$$

- Use the MATLAB function [ode45] to solve this non-linear system.
- Plot the solutions (states) of the systems over time.

Thanks!