MATLAB Tutorial 10

ENME 303 Computational Methods for Engineers

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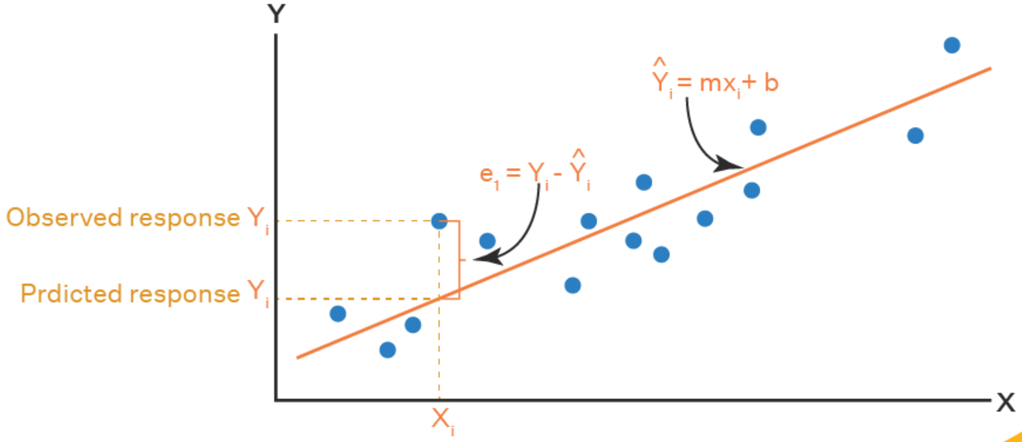
Linear Regression

- Prediction Error:
  - $e_i \triangleq y_i - \hat{y}_i = y_i - mx_i - c$
- Prediction Cost:
  - $J = \sum_{i=1}^{n} e_i^2$

Goal: Find m and c such that J is minimized



Least Squares Solution

- Let write $y_i = \varphi_i \theta$
- Where $\varphi_i = [x_i \quad 1]$ and $\theta = \begin{bmatrix} m \\ c \end{bmatrix}$
- So $\hat{y} = \Phi\Theta$
- $E \triangleq y - \hat{y} = y - \Phi\Theta$
- $J = E^T E = \Theta^T \Phi^T \Phi \Theta - 2\Theta^T \Phi^T y + y^T y$
- Minimization: $\frac{\partial J}{\partial \Theta} = 0 \rightarrow \Theta = (\Phi^T \Phi)^{-1} \Phi^T y$
- Compare with:
  - $x = (A^T A)^{-1} A^T b$

Practice

- We want to fit a linear model to the measured data. In particular, we will develop a model that best describes the measurements of x and y.

$$y = mx + c$$

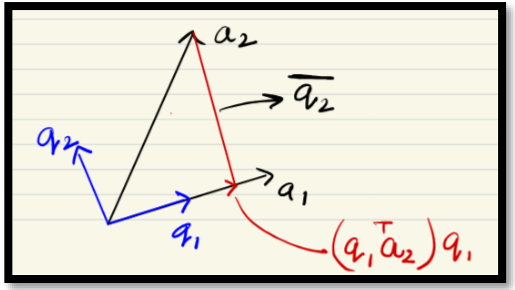
- Data Generation:
  - Consider the line $y = x + 3$
  - Add some noise to it using randn
- Use least square solution to fit a line to the generated data.

Orthogonalization

- Orthonormal vectors:
  - $q_i^T q_j = \begin{cases} 0 & if\ i \neq j \\ 1 & if\ i = j \end{cases}$
- Orthogonalization:
  - Given A, find Q such that $\mathcal{R}(A) = \mathcal{R}(Q)$ and $Q^T Q = I$
- Orthogonalization Process:

$$A = [a_1 \; \cdots \; a_n]$$
$$q_1 \triangleq \hat{a}_1 = a_1 / \|a\|$$
$$\bar{q}_2 \triangleq a_2 - (q_1^T a_2) q_1$$
$$q_2 \triangleq \bar{q}_2 / \|q_2\|$$

$$\bar{q}_3 \triangleq a_3 - (q_1^T a_3) q_1 - (q_2^T a_3) q_2$$
$$q_3 = \frac{\bar{q}_3}{\|q_3\|}.$$



QR Factorization

$$\underset{n\times m}{A} = [a_1 \; a_2 \; \cdots \; a_m]$$

$$= \underset{n\times m}{[q_1 \; q_2 \; \cdots \; q_m]} \cdot \underset{m\times m}{\begin{bmatrix} q_1^T a_1 & q_1^T a_2 & q_1^T a_3 & q_1^T a_4 & \cdots \\ & q_2^T a_2 & q_2^T a_3 & q_2^T a_4 & \cdots \\ & & q_3^T a_3 & q_3^T a_4 & \cdots \\ & & & q_4^T a_4 & \cdots \end{bmatrix}}$$

$$= Q \; R$$

Upper triangular Matrix.

Solving $Ax = b$

$$\underset{n\times m}{A}\ \underset{m\times 1}{x} = \underset{n\times 1}{b}$$

$$Q\,R\,x = b$$

$$\Rightarrow Q^T Q\,R\,x = Q^T b$$

$$\Rightarrow R x = Q^T b$$

$$\downarrow$$

Solve by back substitution!!

Practice

- Find orthonormal matrix Q from the columns of A.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & 3 & 6 \end{bmatrix}$$

- Find R and write A in QR form.
- Check $Q^T Q = I$
- $Q^{-1} = Q^T$

Thanks!