15 Smooth Trajectory Generator

Let $a_{\max} > 0$ and $v_{\max} > 0$. Let $x_{\rm r}$ denote the distance to the next waypoint. Suppose that QC accelerate at the maximum acceleration, cruises at the maximum velocity, and then decelerates at maximum acceleration to reach the desired waypoint. Let T_1 be the time to reach the maximum velocity. Then,

$$T_1 = \frac{v_{\max}}{a_{\max}}. \tag{184}$$

and

$$x(T_1) = \frac{1}{2}a_{\max}T_1^2 = \frac{1}{2}\frac{v_{\max}^2}{a_{\max}} \tag{185}$$

If $|x_{\rm r}| < 2x(T_1)$, then the QC velocity does not reach $v_{\max}$. In this case, assume that the QC accelerates to the maximum velocity and decelerates to zero velocity in the same time. Let $\bar{T}_1$ denote the time instant at which maximum velocity $\bar{v}_{\max}$ is reached. Then,

$$\bar{T}_1 = \frac{\bar{v}_{\max}}{a_{\max}}, \tag{186}$$

and

$$x(\bar{T}_1) = \frac{1}{2}\frac{\bar{v}_{\max}^2}{a_{\max}}. \tag{187}$$

Note that $x_{\rm r} = 2x(\bar{T}_1)$, and thus

$$\bar{v}_{\max} = \sqrt{x_{\rm r}a_{\max}}, \tag{188}$$

and

$$\bar{T}_1 = \sqrt{\frac{x_{\rm r}}{a_{\max}}}. \tag{189}$$

For $t \in (0, \bar{T}_1)$,

$$x(t) = \begin{cases} \frac{1}{2}a_{\max}t^2, & t \in [0, \bar{T}_1), \\ \frac{1}{2}a_{\max}\bar{T}_1^2 + \bar{v}_{\max}(t - \bar{T}_1) - \frac{1}{2}a_{\max}(t - \bar{T}_1)^2, & t \in [\bar{T}_1, 2\bar{T}_1), \end{cases} \tag{190}$$

Next, consider the case where $|x_{\rm r}| \geq 2x(T_1)$, then the QC velocity reaches $v_{\max}$. In this case, assume that the QC accelerates to the maximum velocity during first T_1 seconds, cruises at the maximum for $T_2 - T_1$ seconds and decelerates to zero velocity in T_1 seconds. Then,

$$T_1 = \frac{v_{\max}}{a_{\max}}. \tag{191}$$

and

$$x(T_1) = \frac{1}{2}a_{\max}T_1^2 = \frac{1}{2}\frac{v_{\max}^2}{a_{\max}}, \tag{192}$$

$$x(T_2) = x(T_1) + v_{\max}(T_2 - T_1), \tag{193}$$

$$x(T_3) = x(T_2) + \frac{1}{2}a_{\max}T_1^2 = x(T_1) + v_{\max}(T_2 - T_1) + \frac{1}{2}a_{\max}T_1^2 = \frac{v_{\max}^2}{a_{\max}} + v_{\max}(T_2 - T_1). \tag{194}$$

Note that $x_{\rm r} = x(T_3)$, and thus

$$T_2 = T_1 + \left(\frac{x_r}{v_{\max}} - \frac{v_{\max}}{a_{\max}}\right). \tag{195}$$

$$x(t) = \begin{cases} \frac{1}{2}a_{\max}t^2, & t \in [0, T_1), \\ \frac{1}{2}a_{\max}T_1^2 + v_{\max}(t - T_1), & t \in [T_1, T_2), \\ \frac{1}{2}a_{\max}T_1^2 + v_{\max}(T_2 - T_1) + v_{\max}(t - T_2) - \frac{1}{2}a_{\max}(t - T_2)^2, & t \in [T_2, T_3). \end{cases} \tag{196}$$