AE4423
Airline Planning
& Optimization

Assignment 1 - Group 10

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Introduction

An airline in Sweden has the intention to start operations serving 15 locations in a regional network. For this it provides a data set containing airport location and population data for the 15 respective airports. One of these airports, Malmo (ESMS) serves as the hub from and to which passengers will transfer on to other flights in the network. The aim is to develop a network and fleet plan for direct flights using kerosene aircraft as well as an additional network and fleet plan for routes using both kerosene and electric aircraft.

Demand data has been provided for 10 airports, 5 of which match with 5 of the locations that the airline intends to serve. These are ESGG, ESPA, ESMS, ESSA and ESGJ. The assignment is divided in two parts. The first part extrapolates the demand data from 5 airports to the remaining 10 airports in the data set. Next, this extrapolated data of the demand is used in a leg based model in order to generate the optimal network for the airline. The aim here is to maximize profit, in other words maximize the yield and minimize the operating costs. A fleet of 3 types of aircraft can be leased in order to start operations. The aim is to find the optimal amount of aircraft, the optimal network and the flight frequency per pair of airports.

For problem 2, a similar method is used with the difference that the fleet of aircraft can be expanded with two types of electric aircraft as the airline aspires to meet environmental goals. Instead, a route based network is generated in this section in which routes start and end at the hub, with a maximum of two intermediate stops along the way. The latter is to accommodate for low demand markets within the network. Similar to problem 1, the aim is find the optimal amount of aircraft that need to be leased, the optimal network and flight frequency per route. The solutions for both problem 1 and 2 will be generated for a typical week of flight operations in which 7 days a week, 10 hours of operations per day is possible in order to preserve sufficient time for maintenance and no operations in periods with no demand.

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Problem 1A

2.1. Context & calibration

The given data set includes population data of 15 Swedish airports. The aim is to determine the demand between these 15 airports in 2030 when the airline will start their operations. Demand data from 2020 is known for 10 airports out of which 5 match with the earlier mentioned 15 airports. To make a forecast for the demand, given the population and the distance between two cities, the gravity model, as seen in Equation 2.5, is applied. Using both the population data as well as the demand data of the 5 matching airports, the coefficients k, $b1$ and $b2$ can be calibrated. How this is done is explained in subsection 2.1.2.

$$D_{ij} = k\frac{(pop_i pop_j)^{b1}}{(f * d_{ij})^{b2}} \tag{2.1}$$

The parameters and coefficients in the gravity model are explained in section 2.1.1 and 2.1.2 respectively.

2.1.1. Parameters

Parameter	Description
D_{ij}	Demand between airports i and j
pop_i, pop_j	Population in region of airports i and j
f	Fuel cost, [USD/gallon]
d_{ij}	Distance between airports i and j
pg	Annual population growth

Table 2.1

The population data pop_i, pop_j, fuel cost f, and annual population growth pg are provided in a data set. The distance d_{ij} will be calculated using known coordinates of each airport.

2.1.2. Coefficients

Table 2.2

Coefficients	Description
k	Scaling factor
$b1$	Coefficient
$b2$	Coefficient representing fuel-cost-distance impedance factor

The distance d_{ij} between airports is determined by using the longitude (λ) and latitude (φ) coordinates provided in the data set through the Haversine formula specified as:

$$\triangle\sigma_{ij} = 2arcsin\sqrt{sin^2\frac{\varphi_i - \varphi_j}{2} + cos\varphi_i cos\varphi_j sin^2\frac{\lambda_i - \lambda_j}{2}} \tag{2.2}$$

where $\triangle\sigma_{ij}$ is the length of an arc between airports i and j. φ_i, φ_j, λ_i, and λ_j are the variables for the coordinates of each airport respectively. By multiplying the arc with the radius of the earth (6371 km), the distance d_{ij} can be given by:

$$d_{ij} = \triangle\sigma_{ij} \times R_E \tag{2.3}$$

Now both the distance as the population is known, the calibration of the model can start. This is done by first linearizing it using a logarithm:

$$log\left(D_{ij}\right) = log\left(k\frac{(pop_i pop_j)^{b1}}{(f * d_{ij})^{b2}}\right) \tag{2.4}$$

This can be rewritten to:

$$log\left(D_{ij}\right) = log(k) + b1 \cdot log\left(pop_i pop_j\right) - b2 \cdot log\left(f \cdot d_{ij}\right) \tag{2.5}$$

with the linearized gravity function, an 'ordinary least squares' (OLS) action is applied using the Python function `"scipy.optimize.curve_fit"` to gather the values for k, $b1$ and $b2$ that result in the best fit for the model. The result is displayed in Table 2.3.

Table 2.3: Coefficients of the gravity model

Coefficient	Value
k	0.0135
b1	0.4563
b2	0.2837

2.2. Population forecasting

The data provided is from 2020 whilst the aim is to forecast the population data for 2030. Given an annual growth factor pg of 1.1 %, a forecast of the population data can be made by a simple equation $pop \times (1+gr)^b$ in where gr is the coefficient for the annual growth and pop is the size of the population in the base year, in this case 2020. The amount of years b is 10. Thus the population of airports i and j respectively can be computed for 2030 by adding an exponential component for both the populations:

$$log\left(D_{ij}\right) = log\left(k\frac{(pop_i * 1.01^{*10} pop_j * 1.01^{10})^{b1}}{(f * d_{ij})^{b2}}\right) \tag{2.6}$$

Table 2.4 shows the population forecast for the 15 relevant airports in the network in 2030 using Equation 2.6.

Table 2.4: Population forecast of 2030 for the 15 airports in Sweden

Airport ICAO code	Airport	Population (2020)	Population (2030)
Malmö	ESMS	344166	380173
Luleå	ESPA	48728	53826
Stockholm	ESSA	975551	1077615
Gothenburg	ESGG	579281	639887
Hagfors	ESOH	22344	24682
Halmstad	ESMT	120265	132847
Hemavan	ESUT	1346	1487
Jönköping	ESGJ	171592	189544
Kalmar	ESMQ	105884	116962
Kerlskoga	ESKK	71862	79380
Karlstad	ESOK	64491	71238
Kiruna	ESNQ	19737	21802
Hagshult	ESMV	60070	66355
Hallviken	ESNA	5433	6001
Hedlanda	ESNC	1734	1915

2.3. Demand forecasting

Using the calibrated gravity model and the estimated values, the demand per pair of airports can be generated for the year 2030 as can be seen in Table 2.5. Note that the table is symmetrical.

Table 2.5: Demand per pair of airports in 2030

	ESMS	ESPA	ESSA	ESGG	ESOH	ESMT	ESUT	ESGJ	ESMQ	ESKK	ESOK	ESNQ	ESMV	ESNA	ESNC
ESMS	0	83	413	405	75	235	16	231	192	133	127	53	153	33	21
ESPA	83	0	157	111	27	53	10	65	51	46	43	35	40	17	9
ESSA	413	157	0	570	148	269	30	351	273	268	236	96	211	64	41
ESGG	405	111	570	0	113	313	22	374	232	203	198	70	225	45	29
ESOH	75	27	148	113	0	50	6	66	47	59	62	17	39	12	8
ESMT	235	53	269	313	50	0	11	169	120	90	86	33	114	21	13
ESUT	16	10	30	22	6	11	0	13	10	9	9	6	8	4	2
ESGJ	231	65	351	374	66	169	13	0	149	125	116	41	163	26	17
ESMQ	192	51	273	232	47	120	10	149	0	85	78	32	97	20	13
ESKK	133	46	268	203	59	90	9	125	85	0	105	29	72	19	13
ESOK	127	43	236	198	62	86	9	116	78	105	0	27	67	18	12
ESNQ	53	35	96	70	17	33	6	41	32	29	27	0	25	11	6
ESMV	153	40	211	225	39	114	8	163	97	72	67	25	0	16	10
ESNA	33	17	64	45	12	21	4	26	20	19	18	11	16	0	5
ESNC	21	9	41	29	8	13	2	17	13	13	12	6	10	5	0

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Problem 1B

3.1. Context

With the demand forecast now generated, the weekly flight frequency plan for the airline can be computed. The aim is to lease a fleet consisting of a set of 3 types of aircraft. Appendix A displays the parameters per aircraft type.

By using the network and fleet mathematical model and computing this in Python using GUROBI, the optimal network can be determined with the objective that profits are maximized. In other words it means maximizing the difference between yield and costs for the network. The network and fleet model from the lectures is adjusted to our goals. An important difference with the model from the lectures is that there is no budget cap, meaning the correlated constraint does not uphold in this model. By leasing, the aircraft investment costs are now a weekly occurrence which contrasts what the lecture slides displayed with its one time purchasing costs. Costs in general are build up differently which will be further explained in subsection 3.2.4.

3.2. Mathematical model

The relevant indices, sets, parameters and decision variables for the mathematical model are given in the following sections. This is followed by a thorough explanation of the objective function and the constraints active on the model.

3.2.1. Indices and sets

Table 3.1: Indices and sets used in the mathematical models

Indices	Description	Sets
N	Set of airports	$N \in [1...15]$
K	Types of aircraft	$K \in [1,2,3]$

3.2.2. Decision variables

The decision variables are the variable the the model will be optimized for. They can be any positive integer value.

Table 3.2: The Decision variable for the network and fleet based model

Decision variables	Description
w_{ij}	Flow of passengers from airport i to airport j with transfer at hub
x_{ij}	Direct flow of passengers from airport i to airport j
z_{ij}^k	Number of flights from airport i to airport j with aircraft type k
AC^k	Number of aircraft type k

3.2.3. Parameters

Table 3.3: Parameters used in the network and fleet based model

Parameters	Description	Value/Unit
D_{ij}	Travel demand between airport i and j	-
g_h	Binary variable: 0 if airport h is hub, 1 if airport h is not a hub	-
d_{ij}	Distance between airports i and j	Km
s^k	Number of seats per aircraft type k	-
$Y_{EUR_{ij}}$	Yield between origin i and destination j	€/RPK
LF	Average Load Factor	-
TAT^k	Average Turn Around Time (TAT) per aircraft type k	mins
BT^k	Average utilisation by aircraft type k	hours/week
C_L^k	Weekly cost of leasing per aircraft type k	€/week
C_X^k	Fixed operating cost per aircraft type k	€
$C_{T_{ij}}^k$	Time-based costs between i and j per aircraft type k	€
$C_{F_{ij}}^k$	Fuel costs between i and j per aircraft type k	USD
$C_{E_{ij}}^k$	Energy costs between i and j per (electric) aircraft type k	€
C_{ij}^k	Total operational costs between i and j per aircraft type k	€
c_T^k	Time cost parameter for aircraft type k	€/hour
c_F^k	Fuel cost parameter for aircraft type k	€
V^k	Airspeed per aircraft type k	Km/h
f	Fuel costs	1.42 USD/Gallon
CT^k	Additional charging time per (electric) aircraft type k	mins
e	energy price	0.07 USD/kWh
G^k	Energy per fully recharged aircraft k	kWh
R^k	Range of aircraft	Km

3.2.4. Objective function

The objective function to be optimized is stated in Equation 3.1. As can be seen, it consists of three main parts. The first part represents the total yield, the second part is the sum of all the operating costs. Lastly a sum of all the weekly least cost are subtracted to calculate the profit. Note the implementation of the 10% discount for passengers that transfer at hub.

$$\text{Max Profit} = \sum_{i\in N}\sum_{j\in N}\left[Yield \times d_{ij}\left(x_{ij} + 0.9 \times w_{ij}\right) - \sum_{k\in K}\left(C_{ij}^k \times z_{ij}^k\right)\right] - \sum_{k\in K}\left(C_L^k \times AC^k\right) \tag{3.1}$$

The yield $Y_{EUR_{ij}}$ between origin i and destination j, expressed in € per Revenue-Passenger-Kilometer (RPK) is given by:

$$Y_{EUR_{ij}} = 5.9 \cdot d_{ij}^{-0.76} + 0.043 \tag{3.2}$$

Important to note is that contrary to the lectures, the yield in this case is a variable depending on the distance of a flight leg. Larger distances result in lower yield rates.

For passengers connecting at the hub, the yield is 10% lower. Next, the cost c of operating a certain aircraft k is taken into account. It consists of four components. Three of which are given by the following equations:

$$C^k_{T_{ij}} = c^k_T \frac{d_{ij}}{V^k} \tag{3.3}$$

$$C^k_{F_{ij}} = \frac{c^k_F \cdot f}{1.5} d_{ij} \tag{3.4}$$

$$C^k_{E_{ij}} = e \cdot G^k \frac{d_{ij}}{R^k} \tag{3.5}$$

Together with the fixed operating costs C^k_X the total operating cost per aircraft type k is given by:

$$C^k_{ij} = \left(0.7 + 0.3 \times g_i \times g_j\right) \times \left(C^k_X + C^k_{T_{ij}} + C^k_{F_{ij}}\right) + C^k_{E_{ij}} \tag{3.6}$$

Note that if either airport i or airport j is the hub, that a 30% discount is applied to all cost except for the energy costs.

A difference between calculating the cost this way, and the how it is done in the models from the lectures, is that the cost components already account for the distance d_{ij} for each flight leg contrary to the CASK (Cost Available Seat Kilometre) used in the lectures. The CASK would simply be multiplied by the distance in the objective function. However, due to the inclusion of operating costs C^k_X that are independent of the distance, it won't work that way.

3.2.5. Constraints and assumptions

The first constraint in the model is that the sum of the direct passenger flow x_{ij} and the passenger flow that travels via the hub w_{ij} cannot exceed the demand q_{ij} in order to minimize spoilage:

$$x_{ij} + w_{ij} \leq q_{ij} \quad \forall i, j \in N \tag{3.7}$$

The second constraint is the condition that the amount of passengers going through a hub airport can only be present in case a hub airport exists. This is allocated by binary variables for g_h as explained further in Table 3.3:

$$w_{ij} \leq q_{ij} \times g_i \times g_j \quad \forall i, j \in N \tag{3.8}$$

Next is the preservation of flow in each flight leg in the network. This means the sum of the direct passengers x_{ij} and the passengers flying through the hub w_{ij} should not exceed the total seat capacity on that leg. Important to note that for the latter, a load factor (LF) of 0.8 is used to account for a cabin capacity of 80%:

$$x_{ij} + \sum_{m \in N} w_{im} \times \left(1 - g_j\right) + \sum_{m \in N} w_{mj} \times \left(1 - g_i\right) \leq \sum_{k \in K} z^k_{ij} \times s^k \times LF \quad \forall i, j \in N \tag{3.9}$$

Additionally, a constraint is set to preserve that the sum of aircraft that arrives at an airport also leaves that airport. The number of flights z per aircraft type k is used to account for this:

$$\sum_{j \in N} z^k_{ij} = \sum_{j \in N} z^k_{ji} \quad \forall i \in N, \forall k \in K \tag{3.10}$$

The total operation time of all aircraft should not exceed the maximum utilisation rate of the fleet. Important here is to account for a 50% higher TAT for flights that go to the hub. The average utilisation per aircraft is 70 hours a week, given 10 hours of operations per day:

$$\sum_{i \in N} \sum_{j \in N} \left(\frac{d_{ij}}{sp^k} + TAT(1 + \left(0.5 \times \left(1 - g_j\right)\right) \right) \times z^k_{ij} \leq BT^k \times AC^k \quad \forall k \in K \tag{3.11}$$

The last constraint is set to make sure that aircraft's do not fly on flight legs that are longer than their maximum fly range.:

$$z_{ij}^k \leq a_{ij}^k \quad \rightarrow a_{ij}^k = \begin{cases} 10000 \text{ if } d_{ij} \leq \text{ Range }^k \\ 0 \quad \text{otherwise} \end{cases} \quad \forall i,j \in N, \forall k \in K \tag{3.12}$$

3.3. Implementation

Implementing this model is done with the use of Python. First the data is generated and stored in dictionaries and data frames for easy access. For this a Generate_data() function was written that reads the given excel file. All the given parameters are also stored by this function. After this, the model structure can be initialized and decision variables can be created. To do so data frames formatted to store information for each flight leg are created for x, w and z. These data frames are filled with zeros, some of which will be replaced with actual decision variables, in the form of GUROBI variables that can only be positive, integer values. Replacing these decision variables is done in a way that prevents the model from growing to big. For instance, there be will no decision variable created for z_{ij}^k if R^k is smaller than d_{ij}. This effectively also takes care of constraint Equation 3.12. This will also be the case if the runway of either airport i or airport j is to short for aircraft k. A similar approach is used to take care of constraint Equation 3.8: no decision variable will be created for w_{ij} if either airport i or airport j is the hub. After all the decision variables are created and stored in the correct dictionary or data frame, the next step of implementing the constraints can start. This is a just a process of iterating though all the relevant decision variables and implementing the mathematical constraint as stated in subsection 3.2.5 into the GUROBI model. After which the objective function can be created similarly. GUROBI can than optimize the objective function for all the decision variables given the constraints. The results are shown in the next section.

3.4. Results

By running the model, the optimal network that results in the highest profit for the airline can be determined. The optimal profit was determined to be 39359.8 €. Figure 3.1 shows the optimal network for the airline. Blue represents flights flown by aircraft type 1 and red represents the flights flown by aircraft type 2.

The frequency per leg in the network is displayed in Table 3.5. The frequency is the number of flights from airport i to airport j per aircraft type k per week, displayed by the decision variable z_{ij}^k. Aircraft type 3 had a frequency of 0 for every airport route, thus it is not included in the table. Airports ESNA and ESNC do not show any flight frequencies due to the fact their runway lengths can not facilitate landing and takeoff of the 3 aircraft types available. No flights are facilitated to or from ESUT. This is not surprising given the demand to and from this airport is low, as can be seen in Table 2.5. Is is important to notice that there were no transferring passengers w_{ij} in the network. An explanation could be the low economic viability to transport passengers via two flight legs from their origin to their destination as in practice a passenger only pays for one flight. This however should be further investigated to better understand the dynamics of the optimisation model.

Regarding the fleet, the following aircraft count was selected to be leased every week (Table 3.4):

Table 3.4: Fleet configuration by aircraft type for a week for the network and fleet model

Aircraft type	*Count*
AC 1	4
AC 2	1
AC 3	0

The network described above results in the following performances:

- Net profit: €39360
- Total flow: 15120 passengers
- Total capacity: 15128 seats
- Spillage: 4273 passengers

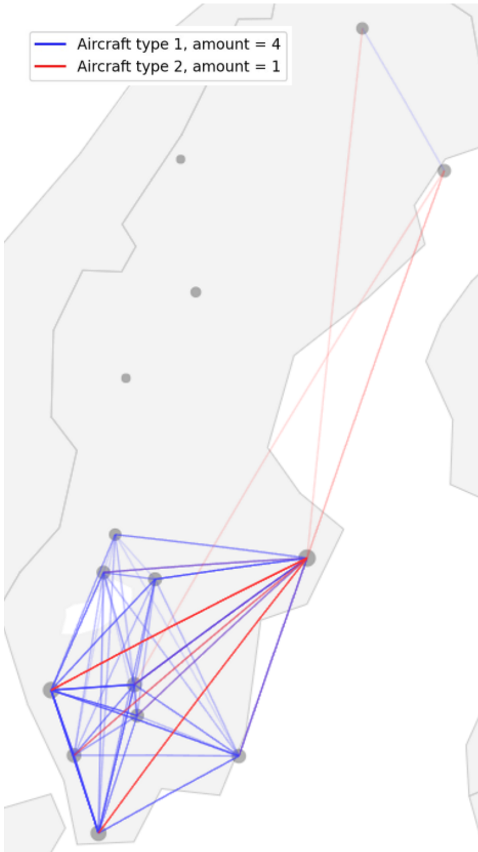

Figure 3.1: Optimal network for the airline using the network and fleet based model. Blue and red flights are done by aircraft types 1 and 2 respectively. The opacity of the line indicates the frequency for each leg, and the grey dots are the airports, which size is scaled by the log of their population size.

- Spoilage: 8 passengers

Table 3.5: The flight frequency per aircraft type between each airport pair, a dashed line indicates that no flights will be provided for that leg. The color of the number indicates the aircraft type and matches with Figure 3.1.

	ESMS		ESPA		ESSA		ESGG		ESOH		ESMT		ESUT		ESGJ		ESMQ		ESKK		ESOK		ESNQ		ESMV		ESNA		ESNC	
AC type	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
ESMS	-	-	-	-	-	7	11	-	2	-	6	-	-	-	6	-	5	-	3	-	3	-	-	-	4	-	-	-	-	-
ESPA	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-
ESSA	-	7	-	2	-	-	-	10	4	-	1	4	-	-	8	1	6	1	7	-	5	1	-	1	4	1	-	-	-	-
ESGG	11	-	-	-	-	10	-	-	3	-	8	-	-	-	10	-	6	-	5	-	5	-	-	-	6	-	-	-	-	-
ESOH	2	-	-	-	4	-	3	-	-	-	1	-	-	-	1	-	1	-	1	-	1	-	-	-	1	-	-	-	-	-
ESMT	6	-	-	-	1	4	8	-	1	-	-	-	-	-	4	-	3	-	2	-	2	-	-	-	3	-	-	-	-	-
ESUT	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
ESGJ	6	-	-	1	8	1	10	-	1	-	4	-	-	-	-	-	4	-	3	-	3	-	-	-	4	-	-	-	-	-
ESMQ	5	-	-	-	6	1	6	-	1	-	3	-	-	-	4	-	-	-	2	-	2	-	-	-	2	-	-	-	-	-
ESKK	3	-	-	-	7	-	5	-	1	-	2	-	-	-	3	-	2	-	-	-	3	-	-	-	2	-	-	-	-	-
ESOK	3	-	-	-	5	1	5	-	1	-	2	-	-	-	3	-	2	-	3	-	-	-	-	-	1	-	-	-	-	-
ESNQ	-	-	1	-	-	1	-	-	-	-	-	-	-	-	-	-	-	-	—	-	-	-	-	-	-	-	-	-	-	-
ESMV	4	-	-	-	4	1	6	-	1	-	3	-	-	-	4	-	2	-	2	-	1	-	-	-	-	-	-	-	-	-
ESNA	-	-	-	-	-	-	-	-	-	-	-	-	—	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
ESNC	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

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Problem 2

4.1. Context

Building up from problem 1, the airline now requires to expand its fleet with two types of electrical aircraft to meet the environmental goals. For this a charging station for the electric aircraft is required to be installed at the hub airport of Malmo (ESMS). With the aim to make the most use of the range capacity of the aircraft and to maximise revenue, the airline considers using two stop triangular routes in which a flight departs from the hub airport, sequentially serves two other airports and then returns back to the hub to refuel/recharge the batteries (no refueling/recharging is possible on-route).

4.2. Mathematical model

Similar to problem 1B, all the indices, sets, parameters and decision variables are displayed in the following sections followed by the objective function and the constraints on the model.

4.2.1. Indices and sets

In the table below the sets and indices used in the route based model are shown. Note that $R1$ is the set of routes that start at the hub, fly to another airport and return to the hub. $R2$ is the set of routes that consists of triangular routes.

Table 4.1: Indices and sets for the route based model

Indices	Description	Sets
N	Set of airports	$n \in [1, ..., 15]$
K	Types of aircraft	$k \in [1, ..., 5]$
R	Set of routes	$r, n \in [R1, R2]$

4.2.2. Decision variables

The same decision variables are used in this model as in the model above, however, they are expanded to be per routes. w_{ij}^{rn} for example represents the flow of passengers that transfer at the hub from route r to route n and have the departure airport i and final destination of airport j. The direct flow x_{ij}^{r} is flow that is direct in the sense that the passengers stay on the route. The Decision variables for this model are show in Table 4.2:

Table 4.2: Decision variables for the route based model

Decision variables	Description
w_{ij}^{rn}	Flow of passengers from airport i to airport j per route r per route n
x_{ij}^{r}	Direct flow of passengers from airport i to airport j per route r
z_{r}^{k}	Number of flights per route r with aircraft type k
AC^{k}	Number of aircraft type k

4.2.3. Parameters

The parameters for this model are the same as for problem 1B, and are displayed in Table 3.3. The exception are the following parameters, listed in Table 4.3

Table 4.3: Additional parameters to table Table 4.2 used in the route based model

Parameters	*Description*	*Value/Unit*
S_{i}^{r}	Subsequent nodes	-
P_{i}^{r}	Precedent nodes	-
δ_{ij}^{r}	Auxiliary parameters between i, j and h per route r (= 1 when ij is part of route r)	-

4.2.4. Objective function

The objective again is the difference between yield and costs that is to be maximized. Contrary to problem 1B, the network now uses route based flights serving up to 3 stops. This is modeled by using indices r and n. Note again the 10% discount for passengers that transfer at the hub. Due to economy of scale, a 30% discount on the total cost (excluding energy cost) can also be applied on all routes leaving and arriving at the hub.

$$\text{Max Profit} = \sum_{r\in R}\sum_{i\in N}\sum_{j\in N}\left[Yield \times d_{ij}\left(x_{ij}^{r} + 0.9\times\sum_{n\in R} w_{ij}^{rn}\right) - \sum_{k\in K}\left(C_{ij}^{k}\times\delta_{ij}^{r}\times z_{r}^{k}\right)\right] - \sum_{k\in K}\left(C_{L}^{k}\times AC^{k}\right) \tag{4.1}$$

4.2.5. Constraints and assumptions

Total demand constraint: Similar to problem 1A, the total flow of direct passengers on a route r and the sum of passengers that transfer through the hub on route r and n should not be larger than the total demand:

$$\sum_{r\in R}\left(x_{ij}^{r} + \sum_{n\in R} w_{ij}^{rn}\right) \leq q_{ij} \quad \forall i,j\in N \tag{4.2}$$

Direct demand constraint: The flow of direct passengers on route r should not be higher than the product of demand and the condition that the connection between i and j lies on that route r:

$$x_{ij}^{r} \leq q_{ij}\times\delta_{ij}^{r} \quad \forall r\in R, \quad i,j\in N \tag{4.3}$$

Indirect demand constraint: The flow of the passengers that transfer at the hub airport h should not exceed demand for that connection. And should be zero if the connection from an airport i to the hub h is not part of route r and the connection from a hub h to an airport j is not part of route n:

$$w_{ij}^{rn} \leq q_{ij}\times\delta_{ih}^{r}\times\delta_{hj}^{n} \quad \forall r,n\in R, \quad i,j\in N \tag{4.4}$$

Flow constraints: For the flow constraints, the route is split in 3 parts. The first part preserves that the sum of the flow of direct and transferring passengers does not exceed the capacity on the route **from the hub node (H)**. The second part preserves that the sum of the flow of direct and transferring passengers that travel between the spokes (i.e. outside the hub) does not exceed the capacity between those spokes. Note that this part is only relevant for routes with more the 2 airports $R2$. The third part ensures that the sum of the flow of direct and indirect passengers does not exceed the capacity on the route **to the hub node(H)**.

$$\sum_{m\in S_H^r} x_{Hm}^r + \sum_{n\in R}\sum_{p\in N}\sum_{m\in S_H^r} w_{pm}^{nr} \le \sum_{k\in K} z_r^k \times s^k \times LF \quad \forall r\in R\left(\text{with } j = S_H^r(1) \ \&\ i = H\right) \tag{4.5}$$

$$\sum_{m\in S_j^r} x_{im}^r + \sum_{m\in P_i^r} x_{mj}^r + \sum_{n\in R}\sum_{p\in N} w_{pj}^{nr} + \sum_{n\in R}\sum_{p\in N} w_{ip}^{rn} \le \sum_{k\in K} z_r^k \times s^k \times LF \quad \forall r\in R2\left(\text{with } i = S_H^r(1) \ \&\ j = S_H^r(2)\text{to the hub node(H)}\right) \tag{4.6}$$

$$\sum_{m\in P_i^r} x_{mH}^r + \sum_{n\in R}\sum_{p\in N}\sum_{m\in P_i^r} w_{mp}^{rn} \le \sum_{k\in K} z_r^k \times s^k \times LF \quad \forall r\in R2\left(\text{with } i = S_H^r(2) \ \&\ \forall r\in R\backslash R2 \text{ with } i = S_H^r(1) \text{ and,for both, j=H}\right) \tag{4.7}$$

Aircraft utilization constraints: This constraint is similar to Equation 3.11 and it makes sure that aircraft's are only utilised a maximum of 70 hours per week, including landing and take off time. Note that the TAT^{rk} is the total turnaround time for all the stops in the route, including a 50% longer TAT to the hub.

$$\sum_{r\in R}\left(\frac{d_r}{sp^k} + TAT^{rk} + \text{Charge}_r^k\right) \times z_r^k \le BT^k \times AC^k, \forall k\in \boldsymbol{K} \tag{4.8}$$

Aircraft allocation constraints: This constraint is similar to Equation 3.12, and it assures that airports don't fly routes that are longer then their range. Since aircraft only refuel or recharge at the hub, the length of the hole route must be within range.

$$z_r^k \le a_r^k \quad \rightarrow a_r^k = \begin{cases} 10000 \text{ if } d_r \le \text{Range}^k \\ 0 \quad \text{otherwise} \end{cases} \quad \forall r\in R, \forall k\in K \tag{4.9}$$

4.3. Implementation

Similarly to the implementation described in section 3.3, model 2 is constructed in 4 steps. Data relevant to the problem is initially generated. This includes all the traffic, the various aircraft models and their respective properties, and the routes available. The next step is to generate the various decisions variables to be adjusted by the model. This model has a lot of decision variables, since x_{ij}^r for instance, does not only have a value for each departure-arrival combination, but for each route-departure-arrival combination. For x_{ij}^{rn} this gets even worse. Note however, that both airport i as airport j must be on route r for x_{ij}^r to be non zero. So although this model has a bigger set of individual variables, by smart implementation, the model can be kept concise. So, decision variable were generated only when it made sense to do so to limit the complexity of the model and improve the optimisation efficiency (this includes aircraft-runway compatibility, route length, aircraft range, and impossible route-airport combinations). This also means that certain constraints were effectively implemented that way, with the direct demand (Equation 4.3), indirect demand (Equation 4.4), and aircraft allocation constraint (Equation 4.8) being enforced at the variable generation. The constraints are then constructed, followed by the objective function, and the model is finally optimised.

4.4. Results

Similar for problem 1B, the optimal network resulting in the highest profit for the airline was determined. Figure 4.2 shows the optimal network for the airline with the inclusion of triangular routes and electric aircraft. The frequency per leg in the network is displayed in Figure 4.1. Again, the frequency is for one standard week of operations.

Table 4.4: Fleet configuration by aircraft type for a week for the route-based network and fleet model

Aircraft type	*Count*
AC 1	0
AC 2	0
AC 3	0
AC 4	0
AC 5	2

The network described above results in the following performances:

- Net profit: €23443
- Total flow: 4444 passengers
- Total capacity: 4492 seats

- Spillage: 14935 passengers
- Spoilage: 36 passengers

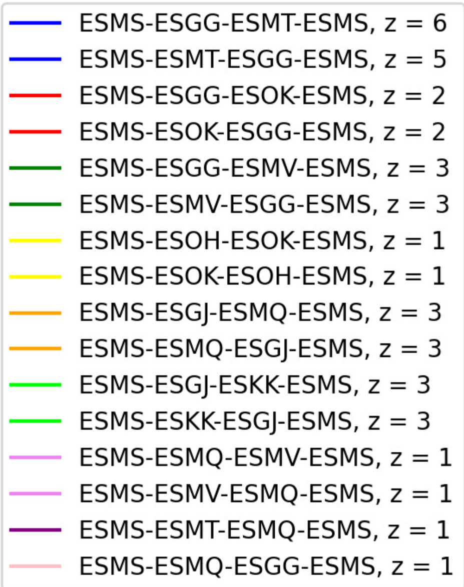

Figure 4.1: All the server routes with their flight frequency, where the colors match with Figure 4.2. If two routes are each others inverse, they are given the same color.

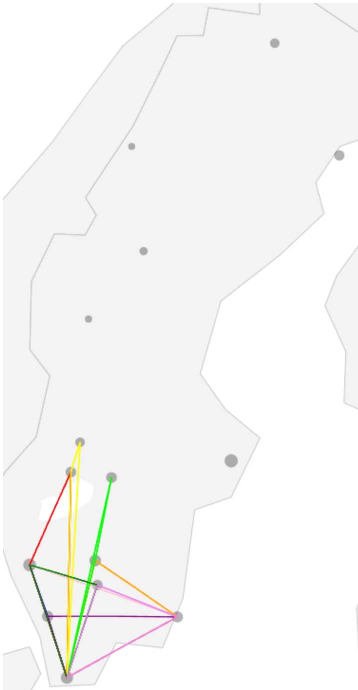

Figure 4.2: Optimal network for the airline using the route-based network and fleet model. Each leg making up a route is colored accordingly. The grey dots are the airports, which size is scaled by the log of their population size.

A total of 16 routes are served. The routes and their frequencies are shown in Figure 4.1 and Figure 4.2. As can be seen in these two figures, all routes start and end at the hub (as required). It is interesting to notice that only short distance triangular routes are served. It makes sense that only short routes are served, since airplanes can only recharge at the hub. This results in the effect that all airports that are remote from the hub, are not served in any route. Therefore all the of demand of the remote airports is missed which could be the reason that the leg based model from chapter 4, where planes can refuel anywhere, is more profitable then this route based model. Next, is also makes sense that serving triangular routes is more profitable because when every route has to start and end at the hub, the only way to allow direct flights between two airports is by triangular routes. Finally, the significant spillage suggests that either the cost of flying a passenger for more legs than they have payed for is never profitable, or an error in the cost/yield estimation could be present in the current model implementation (more in chapter 5)

5
Conclusion

To conclude, despite getting results for both models, a common feature seems to be present in both. Both models upon optimising results in no transfers being done. This can be explained by the fact that both models where built similarly, using the same logic. A systemic error in calculations and methodology is as such likely to be present in both models.

Upon investigating, we have concluded that it is likely to be in the cost/yield determination, as in its current state, the cost of flying a passenger for more legs than they are paying for seems to be immediately resulting in a loss (adjusting the yield/cost to make the overall transport more profitable do result in transfers). All constraints where thoroughly tested, and it was however concluded that the optimisation model implementation itself is likely to be correct.

6 Appendix

Table 6.1: Parameters per aircraft type

Aircraft type k	**1**	**2**	**3**	**4**	**5**
Speed V^k [km/h]	550	820	850	350	480
Seats s	45	70	150	20	48
TAT [mins]	25	35	45	20	25
Charging time CT^k [mins]	-	-	-	20	45
Maximum range R^k [km]	1500	3300	6300	400	1000
Runway required rwr k [m]	1400	1600	1800	750	950
Weekly lease costs C_L^k [€]	15000	34000	80000	12000	22000
Fixed operating costs C_X [€]	300	600	1250	90	120
Time cost parameter c_T^k [€/hr]	750	775	1400	750	750
Fuel cost parameter c_F	1	2	3.75	-	-
Batteries energy G^k [kWh]	-	-	-	2130	8216

6.1. Individual workload

Student names	***Mathematical modeling***	***Programming***	***Reporting***
Hugo Loopik - 4478231	30%	55%	15%
Ruben Ranty - 4378474	30%	20%	50%
Victor Guillet - 4488636	30%	60%	10%