1 a) $$y(t) = \frac{1}{\omega}\int_0^t g(t-\tau)\sinh(\omega\tau)\,d\tau$$

$$y''(t) - \omega^2 y(t) = g(t) \quad , \quad y(0) = y'(0) = 0$$

$$\overset{\mathcal{L}}{\Rightarrow} s^2 Y \cancel{- s y(0)} \cancel{- y'(0)} - \omega^2 Y = \mathcal{L}(g(t))$$

$$(s^2 - \omega^2) Y = \mathcal{L}(g(t))$$

$$y = \mathcal{L}^{-1}(Y) = \mathcal{L}^{-1}\left(\mathcal{L}(g(t)) \cdot \frac{1}{s^2 - \omega^2}\right)$$

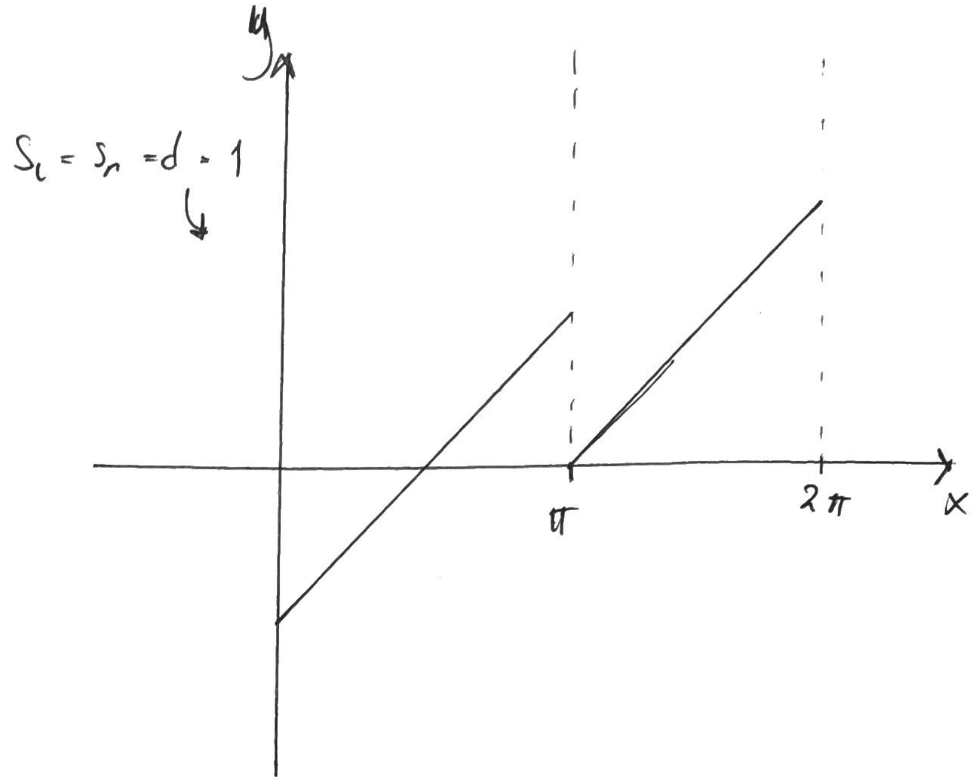
$$\Rightarrow y(t) = \mathcal{L}^{-1}\left(\mathcal{L}(g(t)) \cdot \frac{\omega}{s^2 - \omega^2} \cdot \frac{1}{\omega}\right) = \frac{1}{\omega} \cdot g(t) * \sinh(\omega t)$$

$$\Rightarrow y(t) = \int_0^t g(t-\tau)\sinh(\omega\tau)\,d\tau \quad \square$$

$\boxed{2}$ a)

$$f(t) = \begin{cases} 1-(\pi-t)s_l, & 0<t<\pi \\ 1-d+(t-\pi)s_r, & \pi<t<2\pi \end{cases}$$

2



b)

$\boxed{4}$ b) $a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt$

$$a_0 = \frac{1}{2\pi}\left\{\int_0^{\pi} 1-\pi s_L + s_L t \, dt + \int_{\pi}^{2\pi} 1-d+s_R t - \pi s_R \, dt\right\}$$

$$= \frac{1}{2\pi}\left\{\left[t - \pi s_L t + \frac{1}{2}s_L t^2\right]_0^{\pi} + \left[t - dt - \pi s_R t + \frac{1}{2}s_R t^2\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\cancel{\pi} - \pi^2 s_L + \frac{1}{2}s_L\pi^2 \cancel{+2\pi} \cancel{-d2\pi} \cancel{-2\pi^2 s_R} \cancel{+2s_R\pi^2}\right.$$

$$\left.\cancel{-(\pi - d\pi} - \pi^2 s_R + \frac{1}{2}\pi^2 s_R)\right\}$$

$$= \frac{1}{2\pi}\left(-\frac{1}{2}s_L\pi^2 + 2\pi - d\pi + \frac{1}{2}\pi^2 s_R\right)$$

$$= 1 - \frac{d}{2} + \frac{(s_R - s_L)\pi}{4} = a_0$$

2 b) Forts. (1)

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos nt \, dt$$

$$\Rightarrow \frac{1}{\pi}\left\{ \int_0^{\pi} (1 - S_L\pi + S_L t)\cos nt \, dt + \int_{\pi}^{2\pi} (1 - d + S_R t)\cos nt \, dt \right\}$$

$$= \frac{1}{\pi}\left\{ \left[\cancel{\frac{1 - S_L\pi}{n}\sin(nt)}\right]_0^{\pi} + S_L\left[\cancel{\frac{t\sin nt}{n}} + \frac{\cos nt}{n^2}\right]_0^{\pi} \right.$$

$$\left. + \left[\cancel{\frac{1 - d - \pi S_R}{n}\sin nt}\right]_{\pi}^{2\pi} + S_R\left[\cancel{\frac{t\sin nt}{n}} + \frac{\cos nt}{n^2}\right]_{\pi}^{2\pi} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{S_L\cos n\pi}{n^2} - \frac{S_L}{n^2} + \frac{S_R}{n^2} - \frac{S_R\cos n\pi}{n^2} \right\}$$

$$= \frac{1}{\pi}\left\{ \frac{S_L - S_R}{n^2}(-1)^n + \frac{S_R - S_L}{n^2} \right\} = \frac{1}{\pi}\left\{ \frac{S_R - S_L}{n^2} - \frac{S_R - S_L}{n^2}(-1)^n \right\}$$

$$= \frac{1}{\pi}\left(\frac{S_R - S_L}{n^2}\right)\left(1 - (-1)^n\right) \qquad a_n = \frac{S_R - S_L}{\pi n^2}\left(1 - (-1)^n\right)$$

$$a_1 = \frac{2(S_R - S_L)}{\pi\, 4} \qquad a_2 = 0 \qquad a_3 = \frac{2(S_R - S_L)}{\pi\, 9} \qquad a_4 = 0$$

a_n er bare ikke-null når n er odde altså blir

$$a_{n\,\text{odde}} = a_{2m-1} = \frac{2(S_R - S_L)}{\pi(2m-1)^2}$$

$\boxed{2}$ b) Forts. (2)

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin nt\,dt = \frac{1}{\pi}\left(\int_0^{\pi}(1 - S_L\pi + S_L t)\sin nt\,dt\right.$$

$$\left. + \int_\pi^{2\pi}(1-d-S_R t - S_R\pi)\sin nt\,dt\right)$$

$$\pi b_n = \left[-\frac{1-S_L\pi}{n}\cos nt\right]_0^{\pi} + S_L\left[-\frac{t\cos nt}{n} + \frac{\sin nt}{n^2}\right]_0^{\pi}$$

$$+\left[-\frac{1-d-\pi S_R}{n}\cos nt\right]_\pi^{2\pi} - S_R\left[-\frac{t\cos nt}{n} + \frac{\sin nt}{n^2}\right]_\pi^{2\pi}$$

$$= -\left(\frac{1-S_L\pi}{n}\cos n\pi - \frac{1-S_L\pi}{n}\right) - S_L\left(\frac{\pi\cos n\pi}{n}\right)$$

$$-\left(\frac{1-d-\pi S_R}{n} - \frac{1-d-\pi S_R}{n}\cos n\pi\right) - S_R\left(\frac{2\pi}{n} - \frac{\pi\cos n\pi}{n}\right)$$

$$= -\left(\frac{(-1)^n}{n}(1-S_L\pi) - \frac{1}{n}(1-S_L\pi)\right) - S_L\pi\frac{(-1)^n}{n}$$

$$-\left(\frac{1}{n}(1-d-\pi S_R) - \frac{(-1)^n}{n}(1-d-\pi S_R)\right) - S_R\left(\frac{2\pi}{n} - \frac{\pi(-1)^n}{n}\right)$$

$$= \frac{1}{n}(1-S_L\pi) - \frac{(-1)^n}{n}(1-S_L\pi) - S_L\frac{\pi(-1)^n}{n} + \frac{(-1)^n}{n}(1-d-\pi S_R)$$

$$-\frac{1}{n}(1-d-\pi S_R) + \frac{S_R\pi(-1)^n}{n} - \frac{S_R 2\pi}{n}$$

$$\Rightarrow b_n = \frac{1-S_L\pi}{n\pi}\left(1-(-1)^n\right) - S_L\frac{(-1)^n}{n} + \frac{1+d-S_R\pi}{\pi n}\left((-1)^n-1\right)$$

$$+\frac{S_R}{n}\left((-1)^n-2\right)$$

☒ b) forts. (3)

Når n en partall så får man:

$$b_{n\ like} = b_{2m} = -\frac{S_L}{2m} - \frac{S_R}{2m} = -\frac{S_L + S_R}{2m}$$

for b når n odde får man:

$$b_{n\ odde} = b_{2m-1} - \frac{1 - S_L\pi}{(2m-1)\pi}\cdot 2 + \frac{S_L}{2m-1} + \frac{1-d-S_R\pi}{\pi(2m-1)}(-2)$$

$$+ \frac{S_R}{2m-1}(-3)$$

$$= \frac{2}{(2m-1)\pi} - \frac{2S_L}{2m-1} + \frac{SL}{2m-1} - \frac{2}{\pi(2m-1)} + \frac{2d}{\pi(2m-1)}$$

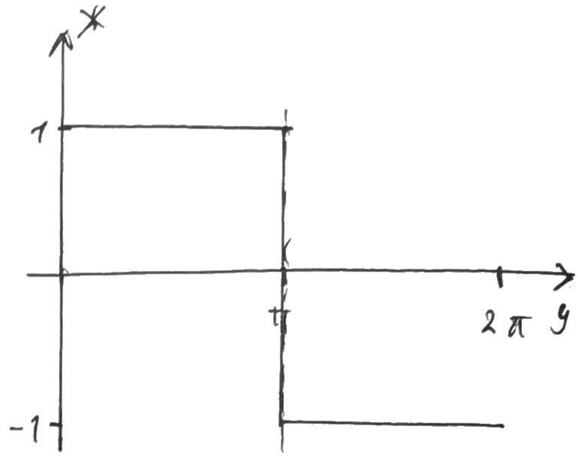
$$+ \frac{2SR}{2m-1} - \frac{3S_R}{2m-1}$$

$$= -\frac{SL}{2m-1} + \frac{2d}{\pi(2m-1)} - \frac{S_R}{2m-1} = \frac{2d}{\pi(2m-1)} - \frac{S_R - S_L}{2m-1}$$

$$= \frac{1}{2m-1}\left(\frac{2d}{\pi} - (S_R + S_L)\right).$$

2 c) i) $S_L = S_R = 0 \quad d = 2$

$$f(t) = \begin{cases} 1, & 0 < t < \pi \\ -1, & \pi < t < 2\pi \end{cases}$$



$$a_0 = 1 - \frac{2}{2} + \frac{(0-0)\pi}{4} = 0$$

$$a_{2n} = 0$$

$$a_{2m-1} = \frac{2(0-0)}{\pi(2m-1)^2} = 0$$

$$b_{2m} = -\frac{0+0}{2m} = 0$$

$$b_{2m-1} = \frac{1}{2m-1}\left(\frac{2\cdot 2}{\pi} - (0+0)\right) = \frac{4}{(2m-1)\pi}$$

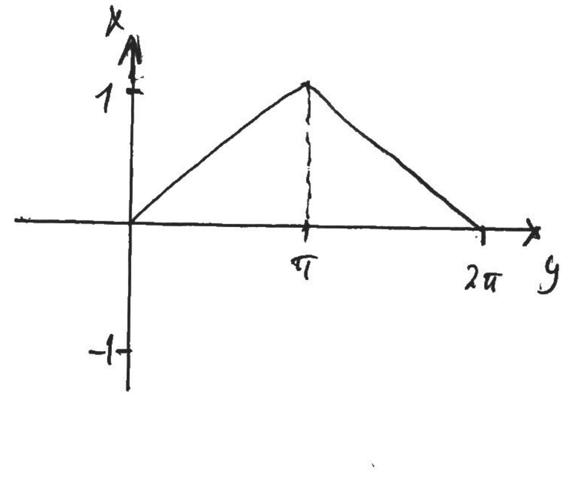
$$f(t) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

$$\sim \frac{4}{\pi}\sum_{m=1}^{\infty} \frac{\sin(2m-1)t}{2m-1}$$

ii) $S_L = -S_R = 1/\pi \quad d = 0$

$$f(t) = \begin{cases} t/\pi & 0 < t < \pi \\ 2 - t/\pi & \pi < t < 2\pi \end{cases}$$

$$a_0 = 1 - \frac{0}{2} + \frac{(-1/\pi - 1/\pi)\pi}{4} = 1/2$$



$$a_{2m} = 0$$

$$a_{2m-1} = \frac{2(-1/\pi - 1/\pi)}{\pi(2m-1)^2} = -\frac{4}{(2m-1)^2\pi^2}$$

$$b_{2m} = -\frac{1/\pi - 1/\pi}{2m} = 0$$

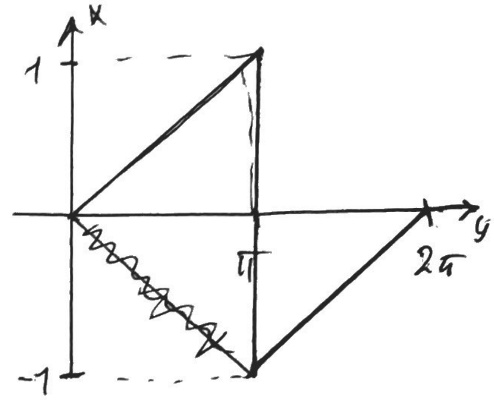
$$b_{2m-1} = \frac{1}{2m-1}\left(\frac{2\cdot 0}{\pi} - \left(-1/\pi - 1/\pi\right)\right) = 0$$

$$f(t) \sim \frac{1}{2} - \frac{4}{\pi^2}\sum_{m=1}^{\infty} \frac{\cos(2m-1)t}{(2m-1)^2}$$

$\boxed{2}$ c) (ii) $s_1 = s_2 = 1/\pi$ $d=2$

8/

$$f(t) = \begin{cases} +t/\pi & 0 < t < \pi \\ -2 + t/\pi & \pi < t < 2\pi \end{cases}$$



$$a_0 = 1 - \frac{2}{2} + \frac{(1/\pi - 1/\pi)\pi}{4} = 0$$

$$a_{2m} = 0$$

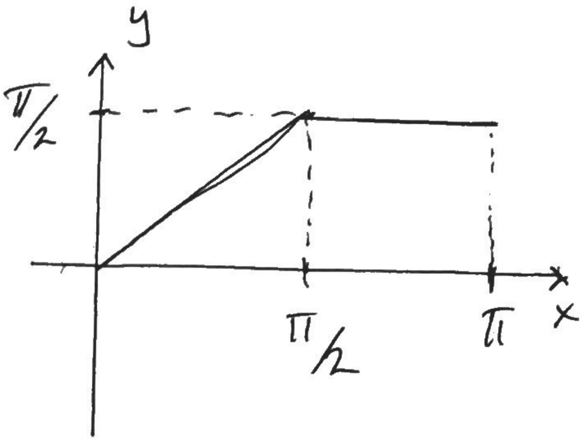
$$a_{2m-1} = \frac{2(1/\pi - 1/\pi)}{\pi(2m-1)^2} = 0$$

$$b_{2m} = -\frac{1/\pi + 1/\pi}{2m} = -\frac{1}{\pi m}$$

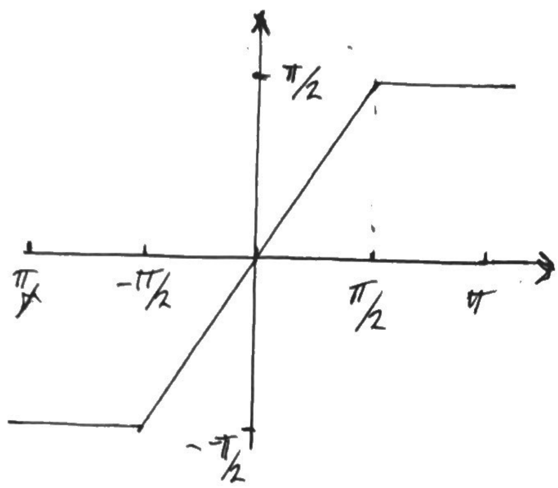
$$b_{2m-1} = \frac{1}{2m-1}\left(\frac{2\cdot 2}{\pi} - \left(1/\pi + 1/\pi\right)\right) = \frac{1}{2m-1}\left(\frac{2}{\pi}\right) = \frac{2}{\pi(2m-1)}$$

$$f(t) \sim \sum_{m=1}^{\infty}\left(-\frac{1}{\pi m}\sin 2mt + \frac{2}{\pi(2m-1)}\sin(2m-1)t\right)$$

[3] $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi/2, & \pi/2 < x < \pi \end{cases}$



[4] a)



b) $f(x) = \begin{cases} -\pi/2, & -\pi < x < -\pi/2 \\ x, & -\pi/2 < x < \pi/2 \\ \pi/2, & \pi/2 < x < \pi \end{cases}$

$$\underline{f_0(x) = \sum_{n=1}^{\infty} b_n \sin(nx)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx) \iff \pi/2\, b_n = \int_0^{\pi/2} x \sin nx\, dx + \int_{\pi/2}^{\pi} \frac{\pi}{2} \sin nx\, dx$$

$$\boxed{\int x \sin kx\, dx = \frac{\sin kx - kx\cos kx}{k^2} + C}$$

$$\Rightarrow \frac{\sin\frac{\pi}{2}n - n\frac{\pi}{2}\cos\frac{\pi}{2}n}{n^2} + \frac{\pi}{2n}\left(-\cos n\pi + \cos\frac{\pi}{2}n\right)$$

$$= \frac{\sin\frac{\pi}{2}n}{n^2} - \frac{\pi}{2n}(-1)^n \longrightarrow$$

4 b) forts.

b_n for n like:

$$\frac{\pi}{2} b_{2m} = \frac{\sin \pi m}{4m^2} - \frac{\pi}{4m}(-1)^{2m} = -\frac{\pi}{4m} \Rightarrow \underline{b_{2m} = \frac{1}{2m}}$$

b_n for n odde:

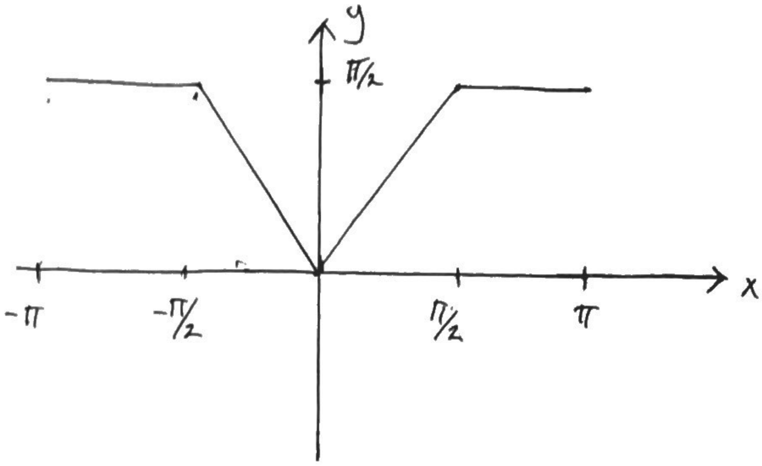
$$\frac{\pi}{2} b_{2m-1} = \frac{\sin \frac{\pi}{2}(2m-1)}{(2m-1)^2} - \frac{\pi}{2}\left(\frac{1}{2m-1}\right)(-1)^{2m-1}$$

$$= \frac{\sin(m\pi - \frac{\pi}{2})}{(2m-1)^2} + \frac{\pi}{2(2m-1)} \qquad \boxed{\sin m\pi - \frac{\pi}{2} = (-1)^{m+1}}$$

$$\Rightarrow b_{2m-1} = \frac{2(-1)^{m+1}}{\pi(2m-1)^2} + \frac{1}{2m-1},$$

$$\underline{\underline{f_o(x) = \sum_{m=1}^{\infty} \left\{ \left[\frac{2(-1)^{m+1}}{\pi(2m-1)^2} + \frac{1}{2m-1}\right] \sin(2m-1)x - \frac{1}{2m} \sin 2mx \right\}}}$$

$\boxed{5}$ a)



$$f(x) = \begin{cases} \pi/2 & , -\pi < x \; -\pi/2 \\ -x & , -\pi/2 < x < 0 \\ x & , 0 < x < \pi/2 \\ \pi/2 & , \pi/2 < x < \pi \end{cases}$$

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b) Fourier cosinusrekke :

$$f(x) = \sum_{n=1}^{\infty} a_n \cos nx + a_0$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} f_e(x)\, dx = \frac{1}{\pi} \int_0^{\pi} f(x)\, dx$$

$$\pi a_0 = \int_0^{\pi/2} x\, dx + \int_{\pi/2}^{\pi} \pi/2 \, dx = 1/2 \left[x^2 \right]_0^{\pi/2} + \frac{\pi}{2} \left[x \right]_{\pi/2}^{\pi}$$

$$\pi a_0 = \frac{\pi^2/4}{2} - 0 + \frac{\pi^2}{2} - \frac{\pi^2/2}{2} = \frac{\pi^2}{8} + \frac{4\pi^2}{8} - \frac{2\pi^2}{8}$$

$$\underline{a_0 = \frac{1}{\pi} \cdot \frac{3\pi^2}{8} = \frac{3\pi}{8}}$$

[5] b) forts.

$$\int x \cos kx \, dx = \frac{kx \sin kx + \cos kx}{k^2} + C$$

- $$\frac{\pi}{2} a_n = \int_0^{\pi/2} x \cos nx + \frac{\pi}{2} \int_{\pi/2}^{\pi} \cos nx \, dx$$

$$\frac{\pi}{2} a_n = \frac{\frac{\pi}{2} n \sin \frac{\pi}{2} n + \cos \frac{\pi}{2} n}{n^2} - \frac{1}{n^2} + \frac{\pi}{2n}\left(\sin n\pi - \sin \frac{\pi}{2} n\right)$$

- $$= \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2} - \frac{1}{n^2} \Rightarrow a_n = \frac{2\cos\left(\frac{n\pi}{2}\right)}{\pi n^2} - \frac{2}{\pi n^2}$$

a_n for n odde:

$$a_{2m-1} = \frac{2\cos\left(\frac{1}{2}\pi(2m-1)\right)}{\pi(2m-1)^2} - \frac{2}{\pi(2m-1)^2} \quad \Big| \quad \cos \tfrac{1}{2}\pi(2m-1) = 0$$

$$a_{2m-1} \Rightarrow \frac{-2}{\pi(2m-1)^2}$$

- a_n for n like:

$$a_{2m} = \frac{2\cos\left(\frac{1}{2}\pi 2m\right)}{4\pi m^2} - \frac{2}{4\pi m^2} \quad \Big| \quad \cos \pi m = (-1)^m$$

$$a_{2m} = \frac{2}{4\pi m^2}\left((-1)^m - 1\right)$$

$$a_{2(2m-1)} = -\frac{1}{4\pi(2m-1)^2}$$

- $$\Rightarrow f_e(x) = \frac{3\pi}{8} + \sum_{m=1}^{\infty} \left\{ -\frac{2}{\pi(2m-1)^2} \cos(2m-1)x - \frac{1}{4\pi(2m-1)^2} \cos 2(2m-1)x \right\}$$