$\boxed{1}$ a) $\dfrac{1}{s^2 \cancel{\cdot} (s^2+1)} = \dfrac{1}{s^2} \cdot \dfrac{1}{s^2+1}$

$= \mathcal{L}(t) \cdot \mathcal{L}(\sin t) = t * \sin t$

$\int_0^t \underbrace{(t-\tau)}_{u} \underbrace{\sin \tau}_{v'} \, d\tau =$

$u = t - \tau \qquad v = -\cos \tau$

$u' = -1 \qquad v' - \sin \tau$

$(t-\tau)(-\cos\tau)\Big|_0^t - \int_0^t -1 \cdot (-\cos\tau)\, d\tau$

$- \sin \tau \Big|_0^t$

$\cancel{(t-t)(-\cos t)} - \sin t$

$-\left((t-0)\underbrace{(-\cos(0))}_{=-1} - \cancel{\sin(0)}\right)$

$= \underline{\underline{-\sin t + t}} \; \cancel{- t - \sin t}$

$\boxed{1}$ b) $\mathcal{L}^{-1}\left(\frac{s}{s^2+2s+1}\right) \Rightarrow \frac{s}{(s+1)^2} = \cancel{\frac{s}{s+1} \cdot \frac{1}{s+1}}$

$= \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2} = \frac{1}{s+1} - \frac{1}{(s+1)^2}$

$= \mathcal{L}(e^{-t}) - \mathcal{L}(te^{-t})$

$= \underline{\underline{(1-t)e^{-t}}}$

c) $\frac{2s}{(s^2+1)^2} = 2\frac{s}{s^2+1} \cdot \frac{1}{s^2+1}$

$= 2 \cdot \mathcal{L}(\cos(t)) \cdot \mathcal{L}(\sin(t))$

$= 2\int_0^t \cos(\tau) \cdot \sin(t-\tau)\, d\tau$

$= 2\int_0^t \cos^2\tau \sin t - \cos t \sin\tau\cos\tau\, d\tau$

$\Leftrightarrow$

$\Rightarrow \mathcal{L}^{-1}\left(\frac{2s}{(s^2+1)^2}\right) = \underline{\underline{t \cdot \sin t}}$

$\boxed{1}$ d)

$$(s-3)^{-5} = \frac{1}{(s-3)^5}$$

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}} \Rightarrow n+1=5$$

$$\underline{n = 4}$$

$\cancel{n!}$ $4! = \underline{24}$

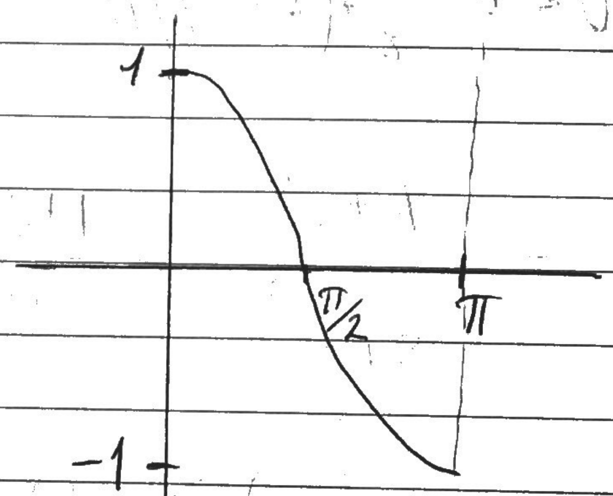
$$\mathcal{L}(t^4) = \frac{24}{s^5}$$

s-shift $\Rightarrow \mathcal{L}^{-1}\left(\frac{1}{(s-3)^5}\right)$

$$\Rightarrow \underline{\underline{e^{3t}\, t^4 \cdot \frac{1}{24}}}$$

$\boxed{2}$ a) $f(t) = (u(t) - u(t-\pi))\cos t$

$$= u(t)\cos t - u(t-\pi)\cos t$$

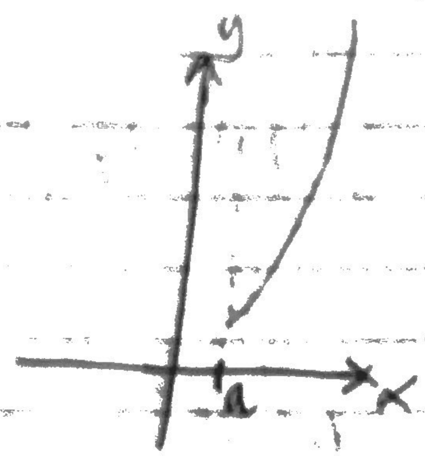


**2** a) $f(t) = u(t)\cos t - u(t-\pi)\cos t$

$$\mathcal{L}(f)(s) = \int_0^\infty e^{-st}\cos t\,dt - \int_\pi^\infty e^{-st}\cos t\,dt$$

$$= \frac{s}{s^2+1} - \frac{se^{-\pi s}}{s^2+1} = \underline{\underline{\frac{s(1-e^{-\pi s})}{s^2+1}}}$$

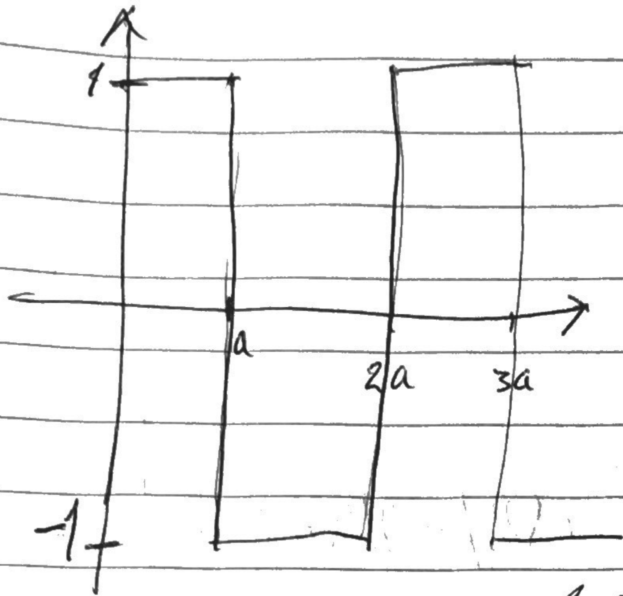
b) $f(t) = u(t-a)t^2$, $a>0$ konstant.




$$\mathcal{L}(f)(s) = \mathcal{L}(u(t-a)t^2) = e^{-as}\mathcal{L}((t+a)^2)$$

$$= e^{-as}\mathcal{L}(t^2+2at+a^2) = \underline{\underline{e^{-as}\left(\frac{2}{s^3}+\frac{2a}{s^2}+\frac{a^2}{s}\right)}}$$

$\boxed{2}$ e) $f(t) = u(t) + 2\sum_{i=1}^{\infty}(-1)^i u(t-ia)$, $a>0$




$$\mathcal{L}(f)(s) = \sum_{i=1}^{\infty}(-1)^i \int_{ai}^{a(i+1)} e^{-st}\,dt =$$

$$\Rightarrow \sum_{i=0}^{\infty} \frac{1-e^{-sa}}{s}\left(-e^{-sa}\right)^i = \frac{1-e^{-sa}}{s}\cdot\frac{1}{1-\left(-e^{-sa}\right)}$$

$$\Rightarrow \frac{1-e^{-sa}}{s\left(1+e^{-sa}\right)} = \underline{\underline{\frac{1}{s}\tanh\left(\frac{sa}{2}\right)}}$$

$\boxed{3}$ $f$ er definert for $t \geq 0$ og er av eksponensiell orden $e^{kt}$. $f'$ er stykkevis kont. og har et endelig antal endelige diskontinuiteter ~~for~~ for $t = t_1, t_2, t_3, \dots$

Vil vise:

$$\mathcal{L}(f')(s) = s\mathcal{L}(f)(s) - f(0) - \sum_{j=1}^{p} e^{-t_j s}\left(f(t_j+) - f(t_j-)\right)$$

$$\mathcal{L}(f')(s) = \int_0^\infty e^{-st} f'(t)\,dt \qquad s > k$$

$$= \int_0^{t_1} e^{-st} f'(t)\,dt + \sum_{j=1}^{n-1} \int_{t_j}^{t_{j+1}} e^{-st} f'(t)\,dt + \int_{t_p}^{\infty} e^{-st} f'(t)\,dt$$

$$= s\int_0^{t_1} e^{-st} f(t)\,dt + e^{-st_1} f(t_1-) - e^{-s\cdot 0} f(0) \dots$$

$$= s\int_0^\infty e^{-st} f(t)\,dt - f(0) + \sum_{j=1}^{p} e^{-st_j}\left(f(t_i+) + f(t_1-)\right) \qquad \square$$

$\boxed{4}$ $\mathcal{L}^{-1}(F)(t) = f(t)$

a) $k \neq 0 \Rightarrow \underline{\mathcal{L}^{-1}(F \circ \mathcal{T}_k)(t) = \frac{1}{k} f(\frac{1}{k})}$

$\underline{\mathcal{T}_k(t) := kt \,, \quad F \circ \mathcal{T}_k(t) = F(\mathcal{T}_k(t)) = F(kt)}$

$$\mathcal{L}\left(\frac{1}{k} f\left(\frac{t}{k}\right)\right)(s) = \left(F \circ \mathcal{T}_k(t)\right)(s) = \int_0^\infty \frac{1}{k} f\left(\frac{t}{k}\right) e^{-st} dt$$

$$= \int_0^\infty e^{-skx} f(x)\, dx = F(ks) = \left(F \circ \mathcal{T}_k(t)\right)(s) \quad \square$$

b) $\underline{\mathcal{L}^{-1}\left(\frac{F}{h_2}\right)(t) = \int_0^t \int_0^u f(v)\, dv\, du} \qquad h_i := t^i$

$\underline{\text{Påstår at: } \mathcal{L}\left(\int_0^t f(x)\, dx\right)(s) = \frac{\mathcal{L}(t)(s)}{s}}$

$$\mathcal{L}\left(\int_0^t g(x)\, dx\right)(s) = \frac{\mathcal{L}(g)(s)}{s}$$

$$\mathcal{L}\left(\int_0^t \int_0^x f(u)\, du\, dx\right)(s) = \frac{\mathcal{L}\left(\int_0^x f(u)\, du\right)(s)}{s}$$

$$= \frac{\mathcal{L}(f)(s)}{s^2} \quad \square$$