University of Louisville Department of Physics

## **LCR Circuits**

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"Before I came here I was confused about this subject. Having listened to your lecture I am still confused.

But on a higher level"

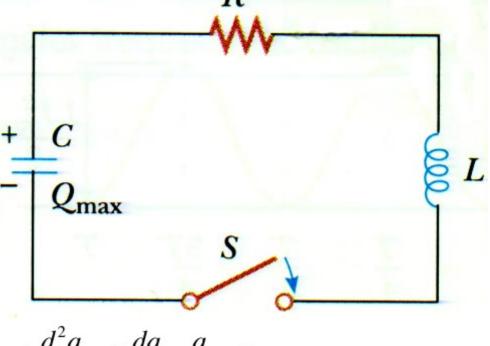
Enrico Fermi



- Oscillations in an LC circuit will continue indefinitely (similar to mechanical oscillations without damping). The stored energy in the electric and magnetic fields oscillates in the same way as the KE and PE in mechanical systems. The oscillations will gradually be attenuated by introduction of a resistance to the circuit. In the analogous mechanical system oscillations are damped by introduction of a frictional force. The resistance causes the system to gradually dissipate (thermal) energy i<sup>2</sup>R.
- Applying the loop theorem to the LCR circuit at right (where the capacitor is initially fully charged as the switch is closed) we obtain

$$\frac{q}{C} - L\frac{di}{dt} - iR = 0$$

which using i = - dq/dt becomes



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

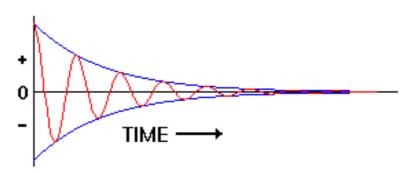
• The general solution of this differential equation has the form

$$q(t) = q_m e^{-Rt/2L} \cos(\omega' t + \phi)$$

where 
$$\omega' = \omega \left(1 - \frac{R^2C}{4L}\right)^{\frac{1}{2}}$$
. Note that when R is small  $\omega' = \omega = 1/\sqrt{(LC)}$  - the natural

frequency of the circuit without the resistance R.

- For small R, the charge on the capacitor (and the current in the circuit) decreases exponentially as shown at right.
- Don't forget an LCR circuit behaves in exactly the same way as a damped mechanical oscillator. See for example this page.





Famous Physicists at a party: Pauli came late, but was mostly excluded from things, so he split.



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