Euclidean Distance Algorithms for Portfolio Risk Optimization

Project-I report submitted to

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In

Humanities and Social Sciences

by

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Humanities and Social Sciences

Indian Institute of Technology Kharagpur

Autumn Semester, 2024-25

DECLARATION

I certify that

(a) The work contained in this report has been done by me under the guidance of my supervisor.

(b) The work has not been submitted to any other Institute for any degree or diploma.

(c) I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.

(d) Whenever I have used materials (data, theoretical analysis, figures, and text) from other sources, I have given due credit to them by citing them in the text of the thesis and giving their details in the references. Further, I have taken permission from the copyright owners of the sources, whenever necessary

Date: November 2024 Ankit

DEPARTMENT OF HUMANITIES AND SOCIAL SCIENCES

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

KHARAGPUR - 721302, INDIA

CERTIFICATE

This is to certify that the project report entitled "**Euclidean Distance Algorithms for Portfolio Risk Optimization**" submitted by **Ankit** (Roll No. 20HS20007) to Indian Institute of Technology Kharagpur towards partial fulfillment of requirements for the award of degree of Master of Science in Humanities and Social Sciences is a record of bona fide work carried out by him under my supervision and guidance during Autumn Semester, 2024-25.

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Date: November 2024

Place: Kharagpur

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1 Introduction

In the evolving landscape of finance, portfolio optimization has become crucial for investors aiming to balance risk and return in dynamic markets. Traditional models, such as the mean-variance framework introduced by Markowitz (1952), have laid the groundwork for systematic asset allocation by maximizing returns for a given risk level. However, applying this model effectively in real-world scenarios requires precise estimations of returns and covariances, parameters that are notoriously difficult to predict. Estimation errors can lead to suboptimal portfolio performance, especially in volatile markets (Best & Grauer, 1991; Chopra & Ziemba, 1993). Consequently, there is a need for adaptable portfolio models that can mitigate these limitations and respond dynamically to market changes.

In this study, we explore a hybrid approach to portfolio optimization that integrates traditional strategies with modern machine learning techniques, specifically reinforcement learning (RL). Reinforcement learning frames portfolio allocation as a sequential decision-making problem, where an agent learns to adjust weights over time based on market observations. This approach not only reduces dependency on fixed historical estimates but also adapts continuously to changing data, offering a flexible and data-driven alternative to static models.

This project introduces a custom environment for portfolio optimization, leveraging OpenAI's Gym library. In this environment, an RL agent optimizes portfolio weights by minimizing the Euclidean distance between the portfolio's actual and optimal configurations. The environment incorporates forecasted data for four portfolio strategies: Minimum Variance (Min-Var), Equally Weighted, Hierarchical Risk Parity (HRP), and Hierarchical Equal Risk Contribution (HERC). Each strategy brings unique strengths—such as the risk minimization of Min-Var and the structured diversification of HRP and HERC—that the agent can dynamically balance through weight adjustments.

The RL agent operates in a structured environment (PortfolioOptimizationEnv), where its actions (portfolio weight allocations) are evaluated by how well they reduce the distance from an optimal allocation. The agent observes forecasted distances, learns to balance weights across these strategies, and is rewarded for actions that bring the portfolio closer to its target. This reward structure encourages the agent to develop an adaptive allocation strategy, adjusting weights based on real-time data to optimize risk and return outcomes.

This project combines traditional portfolio methods with modern machine-learning techniques to achieve an advanced, adaptive framework for portfolio construction. By using reinforcement learning to learn optimal weight combinations across various strategies, this approach addresses the limitations of static optimization and provides a model that is both responsive to changing markets and robust against estimation errors. Ultimately, this hybrid model aims to demonstrate the potential for reinforcement learning to enhance portfolio stability and performance, offering a resilient and adaptable solution for investors in complex, real-world market environments.

2 Literature review

The roots of portfolio theory can be traced back to Markowitz's (1952) pioneering mean-variance model, which laid the groundwork for understanding asset allocation by balancing expected returns with risk. According to this model, investors aim to maximize returns for a given risk level, often measured by the Sharpe ratio (Sharpe, 1966). However, in real-world applications, practical limitations arise due to estimation errors in expected returns and covariances, which can compromise the model's effectiveness (Best & Grauer, 1991; Chopra & Ziemba, 1993). These challenges have even impacted the performance of variance-minimizing portfolios, developed to sidestep the need to predict mean returns directly (Jagannathan & Ma, 2003). Such shortcomings underscore the need for more robust optimization approaches to improve out-of-sample performance (De Miguel et al., 2009).

The literature on portfolio optimization has focused on refining the mean-variance framework through three primary strategies. First, significant research has been dedicated to improving estimation techniques for model inputs. Many studies employ shrinkage estimators, which enhance stability by adjusting sample estimates towards a reliable target. For example, Jorion (1986) developed a shrinkage estimator for mean returns using a Bayes–Stein framework, while other researchers have adapted this method to the covariance matrix, combining sample covariance with a target estimator like a single-factor covariance (Ledoit & Wolf, 2003) or identity matrix (Ledoit & Wolf, 2004). These methods yield more stable estimations and help reduce the impact of estimation errors on portfolio performance.

A second approach involves adding constraints to the model to reduce volatility in portfolio weights, which can arise from estimation inaccuracies. One popular constraint is the no-short-selling restriction, which prevents the assignment of highly negative weights to portfolio assets (Jagannathan & Ma, 2003). Building on this, De Miguel et al. (2009) introduced penalties on portfolio weights, which help control risks by limiting extreme weight allocations. Further, Ban et al. (2018) proposed a regularization approach that adjusts based on sample variance, thereby enhancing risk control within the portfolio.

The third major approach in the literature focuses on integrating different portfolio models to create a more resilient combined portfolio. Rather than refining parameter estimations alone, this strategy blends a mean-variance portfolio with another model to mitigate risk. For instance, Kan and Zhou (2007) advocate using the minimum-variance portfolio as a stable reference point, while Tu and Zhou (2011) suggest the simpler 1/N portfolio, which distributes weights equally across assets. This approach often yields strong out-of-sample performance and reduces reliance on precise input estimations. By combining the mean-variance portfolio with a more stable alternative, this strategy can effectively manage estimation uncertainty.

Recent studies have explored innovative methods that diverge from traditional utility-based approaches, introducing distance-based portfolio optimization as an alternative. These approaches use metrics like the Euclidean distance from an optimal reference portfolio to guide portfolio adjustments, thus shifting the focus from maximizing returns to minimizing estimation errors. This distance-based perspective reduces reliance on assumptions about returns, making it particularly suitable for situations with significant transaction costs or market instability. By adopting nonparametric methods, these models gain adaptability, better accommodating diverse market conditions without the rigid assumptions often present in traditional models.

Through these advancements, portfolio theory continues to evolve, refining the tools available for investors. The ongoing development of estimation methods, constraints, and combination strategies reflects a growing understanding of portfolio optimization in complex markets. Each approach contributes valuable insights for creating portfolios that are both theoretically sound and practically effective, ultimately moving beyond the limitations of the original mean-variance framework to meet the demands of contemporary investing.

3 Gap in Literature

While portfolio optimization has seen considerable advancements, significant gaps remain in the current literature. One major limitation lies in the reliance on estimation techniques that can still be vulnerable to errors despite improvements. Although shrinkage estimators and regularization techniques have helped stabilize estimates, they still rely on historical data assumptions that may not fully capture future market behaviors. This is especially problematic in rapidly changing or volatile markets, where historical relationships among assets may shift unpredictably, reducing the effectiveness of traditional parameter estimation methods. Thus, there is a clear need for approaches that can adapt to real-time market changes with minimal reliance on historical data assumptions.

Moreover, although constraints like no-short-selling and penalty functions on portfolio weights have proven useful, they may still impose restrictions that limit flexibility and adaptability, especially in environments with high transaction costs or limited liquidity. These constraints often lead to models that work well under ideal conditions but fail to perform optimally in real-world applications where flexibility in weight allocation can be crucial. The literature would benefit from new methodologies that can offer flexibility while ensuring stability, particularly in practical, transaction-heavy environments.

A further gap is evident in the area of portfolio combination strategies. While blending mean-variance portfolios with alternative models has shown promise, the theoretical frameworks supporting these strategies are typically based on assumptions that may not hold in real-world data, such as independent and identically distributed returns. Many existing combination methods, such as those using minimum-variance or equally weighted portfolios, aim to provide stability but do not address the risk of model overfitting to historical data. Addressing this gap requires exploring nonparametric approaches that can accommodate complex data patterns and adapt to diverse market conditions without requiring restrictive assumptions.

Additionally, most studies do not account for distance-based approaches in portfolio optimization, which could provide a more dynamic measure of portfolio performance. By focusing on minimizing Euclidean distance or other distance metrics between portfolios and optimal reference portfolios, researchers could move toward models that are less sensitive to estimation errors. This shift from traditional utility maximization to error minimization is underexplored but has potential, particularly in enhancing out-of-sample performance. A systematic exploration of distance-based metrics could thus provide a new dimension to portfolio optimization, addressing limitations in stability and adaptability that current methods face.

Finally, there is a lack of unified frameworks that leverage past portfolio weights, rather than returns, as inputs for adjustment. Most optimization strategies rely heavily on historical returns, but weights-based models could potentially capture more nuanced trends in investor behavior and market dynamics over time. This alternative approach remains underrepresented in the literature, despite its potential to yield more flexible and accurate portfolio adjustments.

In conclusion, while the existing literature provides various techniques to improve portfolio stability and performance, gaps remain in adapting these models to real-world scenarios. Addressing these issues—particularly through adaptive, nonparametric, distance-based methods and flexible weight-based frameworks—could significantly enhance the robustness and resilience of portfolio models, helping investors better navigate the complexities of modern financial markets. These gaps underscore the need for continued research that moves beyond traditional optimization techniques to address the unique challenges of contemporary investing.

4 Research Questions

In light of the gaps identified in the literature, this report aims to address a series of research questions that delve into the evolving requirements and complexities of portfolio optimization in modern financial markets. These questions are designed to guide the investigation towards developing robust and adaptable portfolio optimization models, incorporating innovative methodologies that address current limitations. The research questions are as follows:

1. How can portfolio optimization methodologies be enhanced to reduce reliance on traditional estimation techniques, thus improving adaptability and robustness in volatile or rapidly changing market environments?

This question seeks to explore alternative optimization approaches that reduce the dependency on historical data, which can be unreliable in dynamic market conditions. It considers whether innovative methodologies could create models that adjust to real-time market shifts, making them more resilient to the estimation errors that can arise when relying solely on historical data patterns.

2. What types of flexible constraints could be introduced to portfolio optimization models to better address real-world challenges, particularly in environments where high transaction costs and liquidity constraints are significant factors?

This question examines the potential for new constraint frameworks within portfolio optimization models, aiming to strike a balance between stability and flexibility. The goal is to identify constraints that maintain performance and adaptability even under real-world limitations like transaction costs and liquidity shortages, which are often overlooked in traditional portfolio models.

3. Can nonparametric and distance-based approaches, such as Euclidean distance metrics, be applied effectively to portfolio optimization to minimize estimation errors and enhance out-of-sample performance?

By shifting focus from maximizing returns to minimizing errors, this question investigates the viability of using nonparametric, distance-based methods as an alternative to conventional optimization techniques. It explores whether metrics like Euclidean distance could provide more stable portfolios that are less sensitive to estimation inaccuracies, ultimately improving performance outside the original data sample.

4. How can portfolio combination strategies be designed to incorporate historical portfolio weights, rather than relying solely on historical returns, to more accurately capture evolving market trends and investor behavior?

This question looks at the potential advantages of using weight-based frameworks over traditional return-based models in portfolio optimization. By examining whether past portfolio weights can better reflect investor responses and market dynamics, this question aims to identify more nuanced and flexible ways of adjusting portfolios over time.

5. What are the practical implications of implementing adaptive, distance-based optimization methods in terms of portfolio turnover, stability, and transaction costs in real-world investment scenarios?

This question seeks to understand how these innovative optimization methods affect practical investment considerations. Specifically, it aims to evaluate the impact of adaptive, distance-based approaches on factors like turnover rates, portfolio stability, and transaction costs, providing insight into their applicability and effectiveness in real investment contexts.

Together, these research questions guide the report's investigation, aiming to build a comprehensive understanding of how portfolio optimization models can be adapted to meet the evolving demands of financial markets. By addressing these questions, this study hopes to develop insights into creating portfolios that are more resilient, flexible, and practical in the face of real-world challenges

5. Research Objectives

In response to the gaps identified in the literature and the questions guiding this study, this research aims to develop and evaluate innovative portfolio optimization methods that address the challenges of modern financial markets. By pursuing alternative approaches to traditional estimation-dependent models, this study seeks to create adaptable, resilient, and practical portfolio strategies that are better suited for dynamic market conditions. The specific objectives of this research are as follows:

1. To develop portfolio optimization methodologies that minimize dependency on historical data, enhancing adaptability in volatile and rapidly changing market conditions.

This objective focuses on creating optimization models that reduce the need for precise historical estimates, which are often unreliable in fast-changing markets. By seeking methods that adapt to current market shifts, the study aims to enhance portfolio resilience against estimation errors, thus improving real-time applicability and performance.

2. To introduce and test flexible constraints within portfolio optimization models that can account for real-world challenges, such as high transaction costs and liquidity restrictions.

This objective seeks to design constraints that allow for flexible weight adjustments in portfolios, helping investors manage risks in contexts where high transaction costs and limited liquidity are significant concerns. By testing these constraints, the study aims to balance model stability with adaptability, ensuring that portfolios remain viable under various market conditions.

3. To apply and evaluate nonparametric, distance-based metrics—specifically Euclidean distance—as a criterion for portfolio optimization, aiming to reduce estimation errors and enhance out-of-sample performance.

This objective aims to explore how distance-based approaches, like Euclidean distance minimization, can be used to build portfolios that are less sensitive to traditional estimation inaccuracies. By testing these methods, the study will assess their potential to deliver stronger out-of-sample results and provide more stable performance across varying market scenarios.

4. To investigate the potential of using historical portfolio weights, rather than returns, as inputs for portfolio adjustments, thereby capturing more nuanced trends in investor behavior and market dynamics.

This objective is centered on developing weight-based portfolio combination strategies, which could offer a more accurate reflection of investor responses and shifting market conditions. By evaluating weight-based models, the study aims to identify innovative ways to adjust portfolios in response to market trends without over-reliance on past returns.

5. To assess the practical impact of adaptive, distance-based optimization approaches on portfolio turnover, stability, and transaction costs within real-world investment frameworks.

This objective examines how the proposed methods influence practical considerations for investors, including the rate of portfolio adjustments, overall stability, and associated transaction costs. By understanding these practical impacts, the study will evaluate the feasibility of implementing distance-based optimization in real investment scenarios, assessing its benefits and limitations.

Through these objectives, the study aims to advance portfolio optimization techniques, contributing methodologies that are not only theoretically sound but also practical for modern financial markets. By achieving these objectives, this research seeks to develop portfolio strategies that provide enhanced flexibility, stability, and adaptability, helping investors navigate the complexities of today’s investment landscape.

6 Theoretical Framework

This project investigates advanced portfolio optimization by integrating traditional financial models with machine learning-driven approaches, specifically through reinforcement learning. The objective is to balance risk and return in a dynamic, data-driven way, allowing for adaptive portfolio weight adjustments in response to forecasted asset data. Using a custom environment, PortfolioOptimizationEnv, built with OpenAI's Gym library, this project seeks to optimize portfolio weights based on forecasted distances to an ideal allocation.

1. **Traditional Portfolio Models**
   The foundational models used in this project include the Minimum Variance (Min-Var) and Equally Weighted portfolios. These strategies have been widely adopted in portfolio optimization and serve as baselines for comparison against more complex techniques:

   - **Minimum Variance Portfolio (Min-Var)**: This model focuses on minimizing portfolio risk by optimizing asset weights to achieve the lowest possible variance. By concentrating on variance reduction, the Minimum Variance Portfolio aims to offer investors the least risky allocation, given certain market conditions. This approach relies heavily on the covariance matrix of returns, allowing for the diversification of risk but with limitations when returns and covariances are highly volatile or difficult to predict accurately.

   - **Equally Weighted Portfolio**: This strategy is straightforward, assigning equal weights to each asset in the portfolio. The Equally Weighted Portfolio distributes risk evenly among all assets, providing broad diversification without requiring extensive data on asset returns or volatilities. Although it offers simplicity, it may not achieve the same level of risk-adjusted return as optimized portfolios, making it an effective benchmark but often suboptimal in risk-heavy environments.

2. **Hierarchical Portfolio Models**
   To enhance portfolio construction, this project leverages sophisticated hierarchical clustering techniques. Specifically, the Hierarchical Risk Parity (HRP) and Hierarchical Equal Risk Contribution (HERC) models are incorporated to improve risk management by allocating weights according to asset correlations. These models use machine learning to create clusters of assets, with each cluster's risk contribution balanced through recursive bisection:

   - **Hierarchical Risk Parity (HRP)**: The HRP approach applies clustering to segment assets based on correlations, using a hierarchical structure to manage risk allocation within clusters. This method begins by grouping assets into clusters that reflect their underlying relationships, such as industry sector or market behaviour similarities. Once grouped, HRP assigns weights recursively, from the top clusters down to individual assets, balancing the risk contribution of each cluster. This approach is known for its stability and effectiveness in reducing sensitivity to estimation errors.

   - **Hierarchical Equal Risk Contribution (HERC)**: Expanding on HRP, the HERC model introduces additional refinements to improve risk diversification. While HRP forms clusters based solely on asset correlations, HERC adjusts for the optimal number of clusters using the Gap Index, ensuring that each cluster's risk contribution is more evenly distributed. By determining the right cluster structure, HERC achieves a more balanced risk contribution across assets, offering enhanced diversification compared to HRP alone.

3. **Reinforcement Learning for Dynamic Weight Optimization**

   Reinforcement learning (RL) is employed to optimize the allocation of portfolio weights as a dynamic, sequential decision-making problem. Unlike static models, RL adapts continuously based on observed data, allowing the agent to learn from each allocation decision. In this project, the custom Gym environment PortfolioOptimizationEnv models this interaction, with an RL agent tasked with adjusting portfolio weights to minimize the forecasted distance from the optimal allocation. Each time step provides the agent with updated data (forecasted distances for different portfolio strategies), guiding it to assign weights that progressively bring the portfolio closer to an ideal configuration.

4. **Environment Design and Reward Structure**

The PortfolioOptimizationEnv environment provides a structured setting where the RL agent makes weight allocation decisions based on observations of forecasted distances across various portfolio strategies (e.g., Min-Var, Equal-weight, HRP, HERC). Within this environment:

- **Action Space**: The action space allows the agent to choose weights for each asset, divided into 20 discrete segments for precise control. This discretization enables the agent to explore numerous combinations of weights across different assets.
- **Observation Space**: The agent's observations consist of forecasted distance values, which indicate how close each portfolio strategy is to the optimal configuration. Observing these values enables the agent to recognize the effectiveness of various strategies at each time step.
- **Reward Structure**: To drive optimal decision-making, the reward function is defined as the negative of the portfolio's weighted distance from the optimal configuration. A lower distance indicates better alignment with the target allocation, so the agent is incentivized to adjust weights to minimize this value. This reward design encourages the agent to continuously refine its weight assignments, optimizing portfolio performance over time.

5. **Sequential Decision-Making Process and Learning**

The RL framework in this project operates as a sequential decision process, where each episode represents a cycle through forecasted data. During each episode, the agent receives observations, adjusts the portfolio weights, and receives a reward, learning over time how each action affects portfolio alignment with the optimal distance. By iteratively refining its strategy through repeated episodes, the agent learns the best allocation strategy, allowing it to adapt to changing data patterns and improve portfolio performance.

6. **Application and Evaluation of Portfolio Models**

Through simulation, this project evaluates the effectiveness of each portfolio model under various market scenarios. The reinforcement learning agent continuously adjusts weights across episodes, accumulating knowledge of effective strategies to achieve optimal weight allocation. By examining the performance of Min_Var, Equally Weighted, HRP, and HERC approaches, the project assesses each strategy's impact on portfolio distance minimization and overall risk reduction. The adaptive, data-driven nature of this approach highlights the advantages of integrating traditional models with machine learning-based optimization, resulting in a portfolio that is both resilient and responsive to market shifts.

This theoretical framework combines traditional portfolio strategies with advanced reinforcement learning and clustering techniques, offering a robust foundation for portfolio optimization. The approach aims to deliver a portfolio that not only aligns with optimal risk-return profiles but also adapts dynamically to changing market conditions, providing practical and strategic benefits for managing investment portfolios.

7 Data and Variables

This study utilizes data from the Nifty 50, a prominent index of the Indian stock market, to conduct a thorough analysis of portfolio optimization. The dataset, spanning recent data points from October 2024, offers a comprehensive look at various stocks included in the Nifty 50 index. By focusing on this index, which reflects the performance of the top 50 companies in India by market capitalization, the study captures the dynamics of a representative sample of the Indian equity market, offering a relevant foundation for portfolio modeling and analysis.

The dataset includes key variables essential for evaluating portfolio performance and constructing optimization models. Below are the primary variables used in the analysis:

1. Stock Prices: Historical stock prices form the foundation for calculating returns, which are central to portfolio optimization. Both opening and closing prices, along with high and low values, are available for each company listed in the Nifty 50

index, offering insights into price volatility and market trends over the analyzed period.

2. Returns: Calculated based on daily price changes, returns represent the percentage change in stock prices over time. These returns are crucial for estimating the mean return of each stock, a key parameter in portfolio optimization. Daily, weekly, or monthly returns can be calculated depending on the intended frequency of analysis, with daily returns typically offering higher granularity.

3. Volatility (Standard Deviation of Returns): Volatility is measured through the standard deviation of returns, providing an indication of the risk associated with each stock. High volatility generally signals a higher level of risk, and thus, volatility is a critical variable when assessing the risk-return profile of the portfolio.

4 Covariance and Correlation Matrix of Returns: The relationships among the returns of various stocks are captured by the covariance and correlation matrices. The covariance matrix, representing the extent to which stock prices move together, serves as an input for constructing diversified portfolios. Meanwhile, the correlation matrix helps in understanding the degree of co-movement between stocks, which is essential for risk management in portfolio construction.

5 Market Capitalization: As an indicator of a company’s size, market capitalization aids in weighting stocks within the portfolio. Large-cap stocks may be given different weights compared to mid-cap or small-cap stocks based on specific investment strategies, adding a layer of flexibility to the portfolio optimization process.

6 Sharpe Ratio: This variable is calculated as the ratio of excess returns over the risk-free rate to the portfolio's volatility. The Sharpe ratio serves as a measure of risk-adjusted return and helps to evaluate the performance of the optimized portfolio relative to a risk-free investment.

7 Sector Classification: Sector information for each stock provides a basis for analyzing industry diversification within the portfolio. Sector-level analysis enables the study to assess how portfolios perform across various economic sectors and offers insights into sector-specific risk and return.

By utilizing these variables, the study is equipped to construct and evaluate diverse portfolio models, emphasizing stability, risk, and return metrics. The combination of stock prices, returns, volatility, and market capitalization allows for a holistic approach to portfolio optimization, while the inclusion of sector classification aids in achieving balanced diversification. These data points, when analyzed together, provide the necessary information for developing and testing the portfolio strategies and optimization methods explored in this research.

8 Methodology

This study employs a multi-step methodology that integrates forecasting, portfolio construction techniques, and reinforcement learning (RL) to create an adaptive and robust investment strategy. The methodology is detailed in the following sections:

1. Data Forecasting Using Exponential Smoothing

The initial step involves data forecasting using an Exponential Smoothing model to generate short-term predictions for various portfolio metrics. Exponential smoothing is particularly suitable for capturing trends in financial data while being adaptable to recent changes. The model is implemented using the Exponential Smoothing function from the stats models, library, with a smoothing parameter (alpha) that controls the weight assigned to recent observations.

The forecast output from this model is used to generate projections for four primary portfolio strategies: Minimum Variance (Min-Var), Equally Weighted, Hierarchical Risk Parity (HRP), and Hierarchical Equal Risk Contribution (HERC). These forecasts form the basis for the next stages of portfolio construction and reinforcement learning.

2. Portfolio Construction

This study adopts a combination of the Minimum Variance Portfolio, Equally Weighted Portfolio, and advanced portfolio construction techniques like Hierarchical Risk Parity (HRP) and Hierarchical Equal Risk Contribution (HERC) to construct and evaluate optimized portfolios.

Minimum Variance Portfolio:

The minimum variance portfolio is a core concept within modern portfolio theory, aimed at minimizing risk for a specified level of expected return or maximizing returns for a given risk level. This approach seeks to construct a portfolio with the lowest possible variance, thereby reducing overall risk exposure. The minimum variance portfolio is formulated by using the covariance matrix of asset returns to assign optimal weights to each asset. These weights are selected to minimize the portfolio’s total variance, subject to constraints like the sum of asset weights equalling one and meeting desired target returns. By leveraging the historical correlations and volatilities of assets, this method allows investors to diversify their holdings effectively, thereby achieving risk reduction across the portfolio.

To determine the Minimum Variance Portfolio (MVP) for a two-asset portfolio, we aim to minimize the portfolio variance, which is calculated as follows:

$$\text{Minimize } \sigma_p^2 = (\sigma_1 w_1)^2 + (\sigma_2 w_2)^2 + 2\rho(\sigma_1 w_1)(\sigma_2 w_2)$$

where:

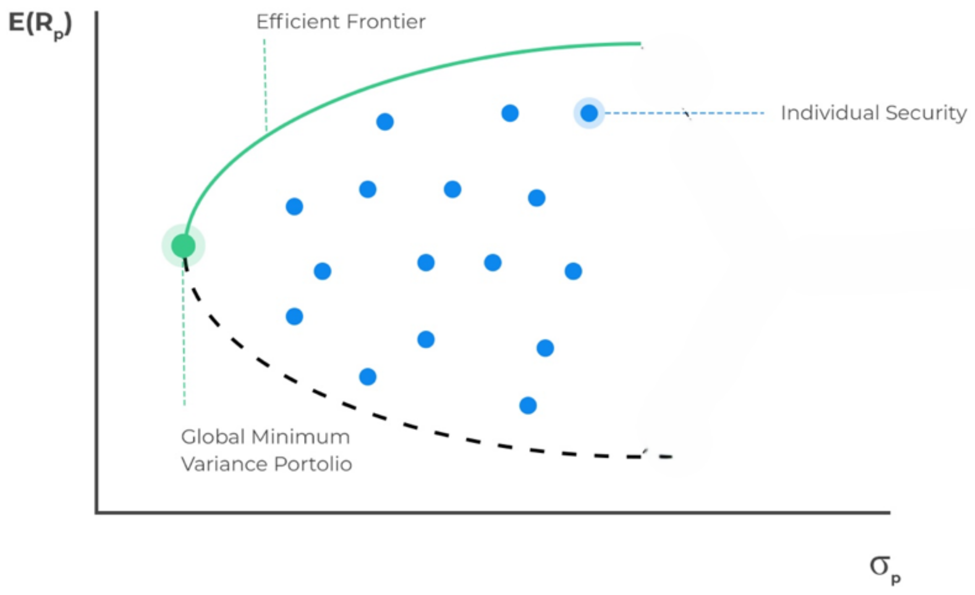
- σp^2 is the variance of the portfolio,
- σ1 and σ2 are the standard deviations of assets 1 and 2, respectively,
- w1 and w2 are the weights of assets 1 and 2 in the portfolio,
- ρ is the correlation coefficient between the returns of the two assets.

The optimal weights w1 and w2 for the assets can be calculated as:

$$w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

$$w_2 = 1 - w_1$$

These formulas provide the weights that minimize the overall portfolio variance by considering the individual asset volatilities and their correlation.



Equally Weighted Portfolio:

An equally weighted portfolio is an investment approach where each asset in the portfolio is assigned the same weight, distributing the total investment evenly across all assets. Mathematically, for a portfolio containing N assets, the weight Wi of each asset i is calculated as:

$$W_i = \frac{1}{N}$$

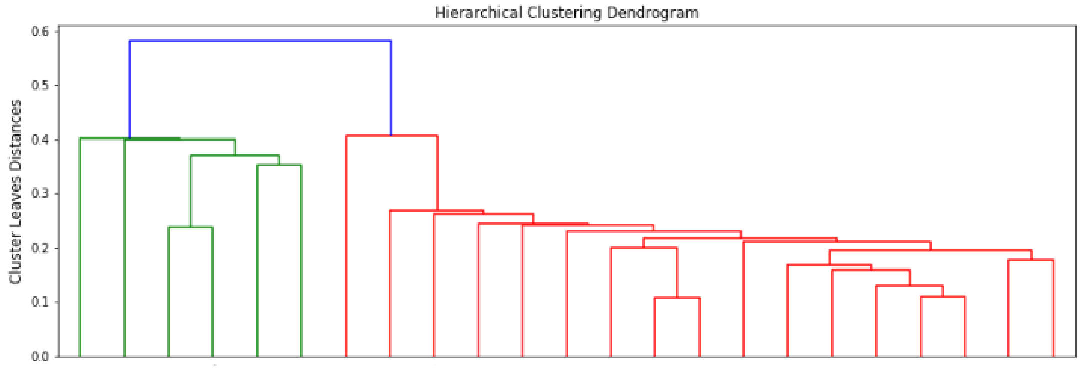
This allocation means that every asset, regardless of its individual characteristics, receives an equal share of the investment. For instance, in a portfolio comprising 5 assets, each asset would be allocated a weight of 20%.

This method provides simplicity and broad diversification by evenly spreading investment, which helps reduce concentration risk. While it may not achieve the optimal balance that can be found through other weighting strategies—such as those based on market capitalization or risk-adjusted measures—it remains a popular choice among investors. Equally weighted portfolios appeal particularly to those seeking straightforward exposure to a range of assets without requiring extensive analysis or forecasting.

Hierarchical Risk Parity (HRP) Portfolio:

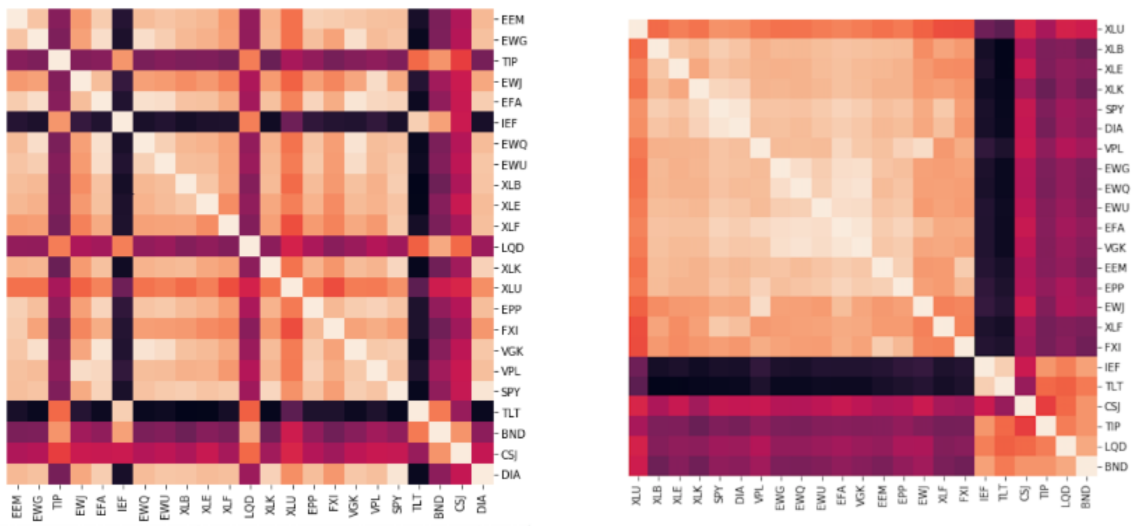
The Hierarchical Risk Parity (HRP) portfolio is a modern approach to portfolio optimization that combines elements of machine learning with traditional portfolio management principles. Known for its simplicity, the HRP algorithm offers notable stability, often outperforming traditional mean-variance optimization techniques, especially in terms of managing risk.

Step 1: Hierarchical Tree Clustering
In this first step, assets within the portfolio are organized into hierarchical clusters using the Hierarchical Tree Clustering algorithm. This clustering approach groups assets based on their relationships, simulating real-world interactions between different assets in the portfolio. For example, certain stocks may be more correlated with one another, allowing them to be clustered together. The result is a hierarchical tree structure, or dendrogram, that visualizes these relationships and sets the stage for optimized asset allocation.



Step 2: Matrix Seriation

Matrix seriation is a well-established statistical technique used to reorganize data, making inherent clusters more visually distinct. In this step, the hierarchical order obtained from the previous clustering process is applied to rearrange the rows and columns of the covariance matrix. By organizing similar assets closer together and positioning less correlated assets further apart, this rearrangement enhances the clarity of relationships within the portfolio. This structure is essential for optimizing asset allocation in line with the risk parity approach, as it groups similar investments, aiding in effective diversification and risk management.



Unclustered Correlations

Clustered Correlations

Step 3: Recursive Bisection

In this final, critical step, the HRP algorithm assigns weights to assets using a top-down recursive approach. Guided by the hierarchical tree (or dendrogram) created in the first step, weights are distributed progressively through the branches of the tree. Starting from the top, the algorithm recursively splits clusters and allocates weights according to the structure, allowing the weights to "trickle down" and reach each individual asset in the portfolio. This method ensures that weight assignments align with the hierarchical clustering, promoting balanced risk distribution across the portfolio.

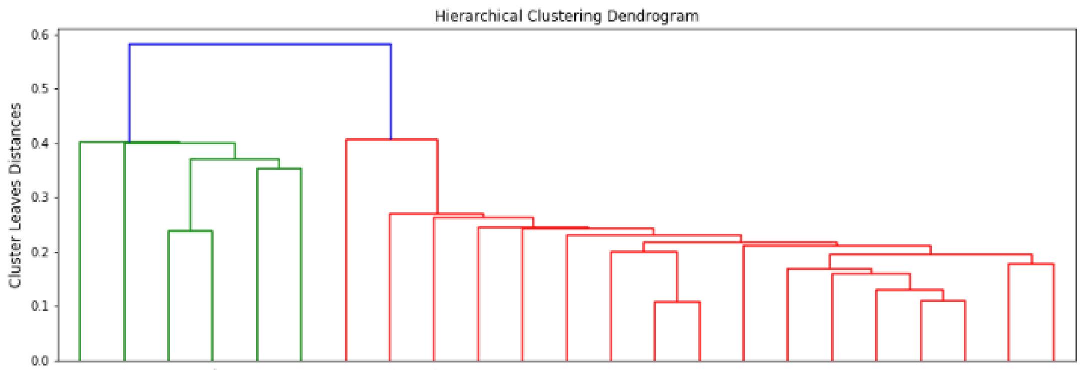
Hierarchical Equal Risk Contribution (HERC) Portfolio:

The Hierarchical Equal Risk Contribution (HERC) method builds upon concepts from the Hierarchical Risk Parity (HRP) algorithm and Hierarchical Clustering-based Asset Allocation (HCAA). This method employs machine learning techniques to achieve efficient weight allocation. Both HERC and HRP utilize hierarchical tree clustering to determine asset weights, though they differ in certain aspects of their application.

Step- 01 Hierarchical Tree Clustering

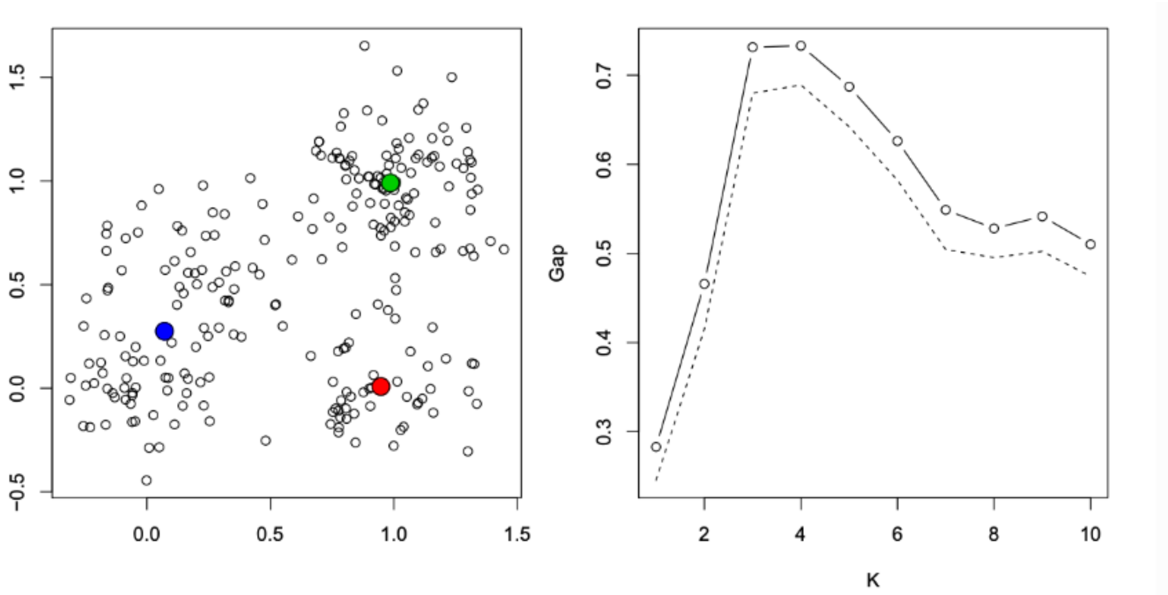
In this initial step, assets in the portfolio are divided into hierarchical clusters through the Hierarchical Tree Clustering algorithm, specifically using agglomerative clustering. This approach organizes assets into clusters that reflect their real-world relationships

such as similarities or correlations among certain stocks. By grouping related assets, the clustering mirrors market interactions and dependencies. The result is a dendrogram, a tree structure that visually represents these clusters, forming the foundation for further weight allocation in the HERC approach.



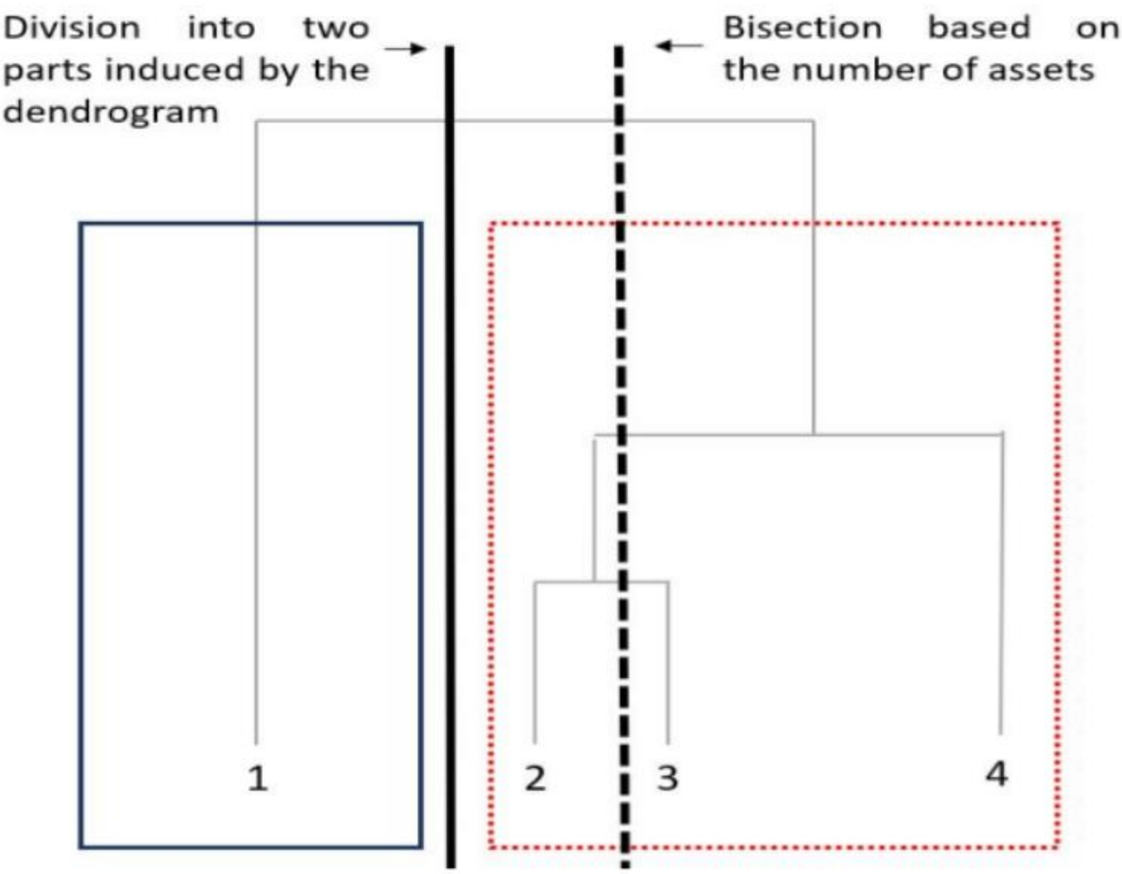
Step- 02 Determining the Optimal Number of Clusters

In this step, the HERC method diverges from the standard HRP approach. Unlike HRP, which applies single linkage and extends the tree to its maximum depth, HERC calculates the optimal number of clusters to achieve better performance. Maximally extending the tree can sometimes result in too many clusters, potentially leading to less effective weight distribution. To address this, HERC determines the most suitable number of clusters by trimming the hierarchical tree created in Step 1 to an optimal height. Currently, the Gap Index is applied as a criterion to identify the ideal number of clusters, ensuring that weight allocations are based on a balanced and effective cluster structure.



Step- 03) Top-Down Recursive Bisection

This is the step where weights for the clusters are calculated. If you are familiar with how the hierarchical risk parity algorithm works, then you know this is similar to how HRP also allocates its weights. However, there is a fundamental difference between the recursive bisections of the two algorithms.



As seen in the above image, at each step, the weights in HRP trickle down the tree by breaking it down the middle based on the number of assets. Although, this uses the hierarchical tree identified in Step-1, it does not make use of the exact structure of the dendrogram while calculating the cluster contributions. This is a fundamental disadvantage of HRP which is improved upon by HERC by dividing the tree, at each step, based on the structure induced by the dendrogram.

At each level of the tree, an Equal Risk Contribution allocation is used i.e. the weights are:

$$\alpha_1 = \frac{RC_1}{RC_1 + RC_2}$$

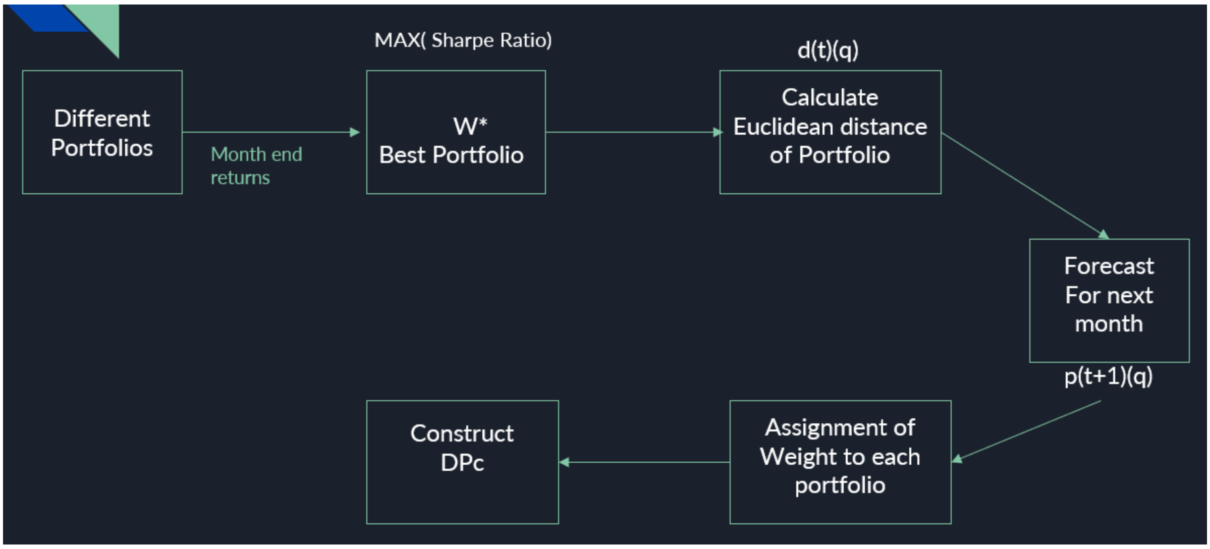
$$\alpha_2 = 1 - \alpha_1$$

Here:

- RC1 and RC2 represent the risk contributions of assets 1 and 2, respectively.
- α1 is the weight assigned to asset 1, while α2 is the weight for asset 2.

These formulas ensure that the weights are proportional to each asset's risk contribution, supporting balanced risk distribution across the portfolio.

DPc Model Flow



3. Reinforcement Learning Environment Setup

To adapt portfolio weights dynamically, a custom reinforcement learning environment (PortfolioOptimizationEnv) is built using OpenAI's Gym library. This environment serves as the platform for training the RL agent and is structured as follows:

- **Action Space**: The RL agent's action space is a multidimensional discrete space where each action corresponds to a specific allocation of portfolio weights across the strategies. Weights are discretized into 20 possible segments for each strategy, enabling granular control over weight allocation.
- **Observation Space**: The agent's observations consist of forecasted distance values for each portfolio strategy, representing their relative effectiveness in approaching an optimal configuration.
- **Reward Structure**: The reward function is designed as the negative weighted distance from an optimal configuration. By minimizing this distance, the RL agent is incentivized to find a balanced allocation that maximizes risk-adjusted returns.

4. Reinforcement Learning with Q-Learning

The Q-learning algorithm trains the RL agent over multiple episodes, where each episode represents a cycle of weight adjustment based on forecasted data. Key components of the Q-learning setup include:

- **Exploration-Exploitation Strategy**: An ε-greedy policy guides the agent in exploring the action space, gradually shifting from exploration to exploitation as it learns optimal weight configurations. The exploration rate decays over time, enabling the agent to refine its strategy based on accumulated experience.
- **Q-Table Update**: For each action, the Q-table is updated using the Bellman equation, with adjustments based on observed rewards and expected future rewards. This update process is influenced by the learning rate and discount factor, which control the balance between immediate and future rewards.
- **Reward Maximization**: The agent's primary objective is to maximize cumulative rewards by minimizing the distance from the ideal configuration across episodes, thus achieving an optimized allocation strategy.

5. Monthly Portfolio Rebalancing and Distance-Based Weight Calculation

The portfolio is rebalanced monthly based on the forecasts and optimal weights derived from the Q-learning process. For each month:

- **Data Collection**: Historical price data for each asset is gathered, and the weights of each strategy are computed based on the forecasted values.
- **Distance-Based Weighting**: Weights are calculated based on the Euclidean distance of each strategy from the optimal configuration, ensuring that closer strategies receive higher weights. This approach aligns the allocation with

strategies that perform closer to the ideal, promoting a diversified and balanced portfolio.

- **Portfolio Value Calculation**: The final portfolio value is computed by applying the weighted allocations to the assets. This calculation helps track the portfolio's performance over time and ensures alignment with risk-return objectives.

6. Performance Evaluation

The Sharpe ratio, a key metric for assessing risk-adjusted returns, is calculated for each portfolio strategy (Min_Var, Equal_weight, HRP, HERC) and the combined portfolio. By comparing these Sharpe ratios, the study assesses the relative effectiveness of each strategy, with the combined portfolio constructed by weighting the four strategies based on the Q-learning optimization.

9 Results and Discussions

The analysis of Sharpe Ratios for various portfolio strategies provides valuable insights into their relative performance in terms of risk-adjusted returns.

1. Portfolio Names and Sharpe Ratios

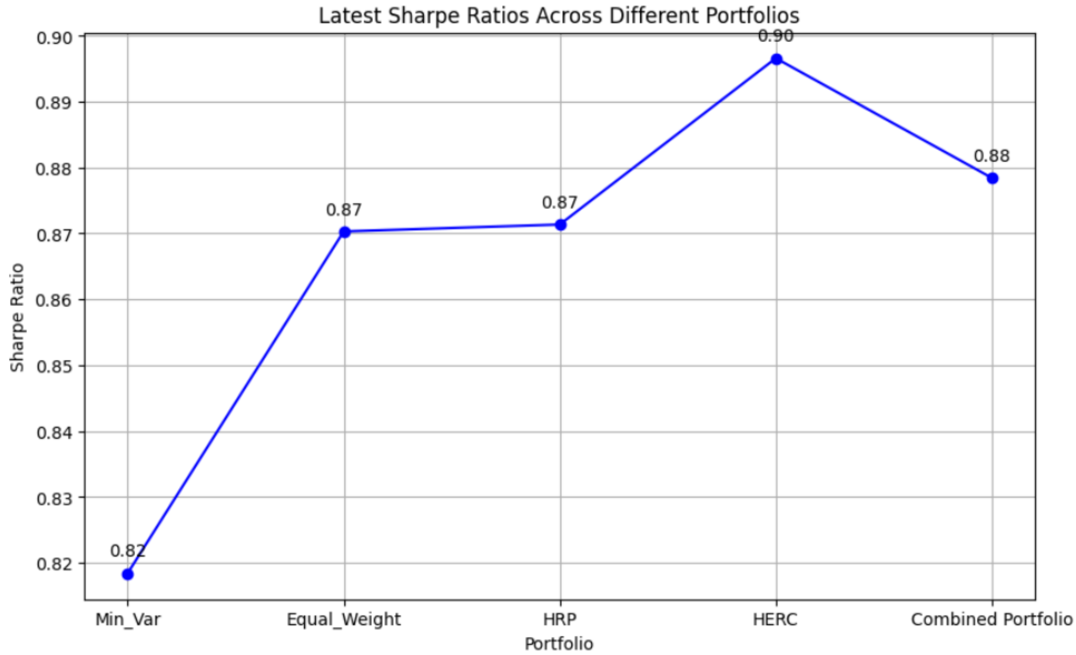
- **Min-Var (Minimum Variance)**: Starting with a Sharpe Ratio of approximately 0.82, this strategy focuses solely on minimizing portfolio variance. However, it delivers the lowest risk-adjusted return among the tested strategies, indicating limitations in risk-return optimization.

- **Equal-Weight**: Achieving a higher Sharpe Ratio of about 0.87, this approach improves risk-adjusted returns by allocating equal weights to all assets, thus providing basic diversification.

- **HRP (Hierarchical Risk Parity)**: With a Sharpe Ratio of 0.87, the HRP strategy balances risk more effectively across assets, leveraging hierarchical clustering to allocate weights and achieve better returns per unit of risk.

- **HERC (Hierarchical Equal Risk Contribution)**: This strategy peaks at a Sharpe Ratio of 0.90, demonstrating superior risk-adjusted returns by equalizing risk contribution across clusters, which leads to enhanced diversification.

- **Combined Portfolio**: Ending with a Sharpe Ratio of around 0.88, this approach integrates the strengths of all strategies but does not achieve the highest Sharpe Ratio due to the dilution of high-performing strategies when combined.



2. Insights

- **Increasing Trend from Min-Var to HERC**: The progressive increase in Sharpe Ratios from Min-Var to HERC highlights the impact of incorporating risk contribution into portfolio strategies. By focusing on equal risk contribution, HERC effectively enhances risk-adjusted returns, achieving the highest Sharpe Ratio among all portfolios.
- **Stability of HRP and HERC**: Both HRP and HERC exhibit relatively stable, high Sharpe Ratios compared to Min-Var and Equal-Weight. This stability underscores the value of risk parity approaches, which distribute risk more equitably and lead to consistent returns.
- **Slight Decline in the Combined Portfolio**: The slight drop in the Combined Portfolio's Sharpe Ratio (from HERC's 0.90 to 0.88) suggests that while combining multiple strategies diversifies risk, it may not achieve the highest possible risk-adjusted return. The combination dilutes the high performance of HERC by averaging it with the lower-performing strategies, indicating that an optimized mix should carefully consider the weight given to high-performing strategies like HERC.

10 Conclusion, Limitation, and Future Scope

The results of this study indicate that portfolio strategies focused on risk parity and equal risk contribution—specifically the HERC approach—yield the highest risk-adjusted returns among the methods analyzed. The key findings from this analysis are as follows:

1. **Effectiveness of Risk Distribution**: Strategies like HRP and HERC, which emphasize equitable risk distribution across assets, outperform those focused solely on variance minimization or equal weighting. This suggests that more advanced risk allocation approaches provide better returns per unit of risk, particularly in diverse markets such as the Nifty 50.

2. **Optimal Strategy**: Among the tested strategies, the HERC approach demonstrated the most effective balance of risk and return, achieving the highest Sharpe Ratio. This finding suggests that the HERC strategy may be the optimal choice for managing Nifty 50 portfolios, providing an enhanced risk-return trade-off.

3. **Role of Combined Strategy**: Although the Combined Portfolio does not outperform the HERC strategy individually, it still surpasses the traditional Min_Var and Equal_Weight strategies in terms of the Sharpe Ratio. This shows that a diversified approach integrating multiple strategies can achieve balanced, robust returns, even if it slightly underperforms the top individual strategy.

Limitations

While this study demonstrates the advantages of risk parity and equal risk contribution strategies in optimizing risk-adjusted returns, it has certain limitations:

1. **Reliance on Historical Data**: The models in this study are built using historical data, which may not always predict future performance accurately, especially in volatile markets. Future research could explore more adaptive models, such as machine learning algorithms, to improve forecasting accuracy.

2. **Assumption of Stationary Market Conditions**: This study assumes that market conditions remain relatively stable over time, which may not be realistic. In rapidly changing markets, the performance of these strategies may vary, so continual adjustments to the models may be necessary.

3. **Limited Portfolio Composition**: The study focuses solely on Nifty 50 stocks, limiting the generalizability of the findings. Future research could expand the scope by including diverse asset classes to examine how these strategies perform across various market segments.

Future Scope

1. **Incorporating Machine Learning Techniques**: Future studies could integrate advanced machine learning algorithms for portfolio optimization, enabling adaptive strategies that respond to real-time market changes.

2. **Risk Factor Analysis**: Analyzing specific risk factors—such as sector performance, economic indicators, or macroeconomic shocks—could provide

further insights into the resilience of these strategies under different market conditions.

3. **Global Portfolio Application**: Extending this methodology to global portfolios could reveal whether the advantages of risk parity and equal risk contribution hold in diverse markets, potentially enhancing portfolio optimization on a larger scale.

Overall, this study highlights the effectiveness of advanced risk distribution strategies, specifically HERC, in optimizing risk-adjusted returns for Nifty 50 portfolios. These findings suggest that investors and portfolio managers may benefit from incorporating such approaches to achieve a balanced and resilient portfolio, with additional opportunities for innovation through the use of adaptive and machine learning-driven techniques.

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Code-file: https://github.com/ankit-jhajhria/MTP_code