CL202: Fluid Mechanics

Introductory Concepts

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What has been covered till now?

Let us list down

- Definition of a fluid
- Applications
- Three approaches to analyse fluid flow problems
- Continuum hypothesis
- No-slip condition
- Eulerian and Lagrangian descriptions
- System and control volume

Velocity field

- Field: A quantity that has a value at each point in time and space (continuum)
- Velocity $\vec{v}(x, y, z, t)$ or $\boldsymbol{v}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$ is a vector quantity

$$\boldsymbol{v}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t}) = u(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})\hat{\boldsymbol{\imath}} + v(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})\hat{\boldsymbol{\jmath}} + w(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})\widehat{\boldsymbol{k}}$$

  - **Indicates the velocity of a fluid particle that is at point P(x, y, z) at time t in Eulerian description**
    - P(x,y,z) are coordinate of a point fixed in space not the position of the fluid particle
    - Steady flow:

$$\frac{\partial \boldsymbol{v}}{\partial t} = 0$$

Stress Field

The force acting on a fluid particle can be:

- Body forces e.g. gravitational force, electromagnetic force
  - Force acts on an element of fluid volume

    Gravitational force $=\rho \boldsymbol{g} d\forall$

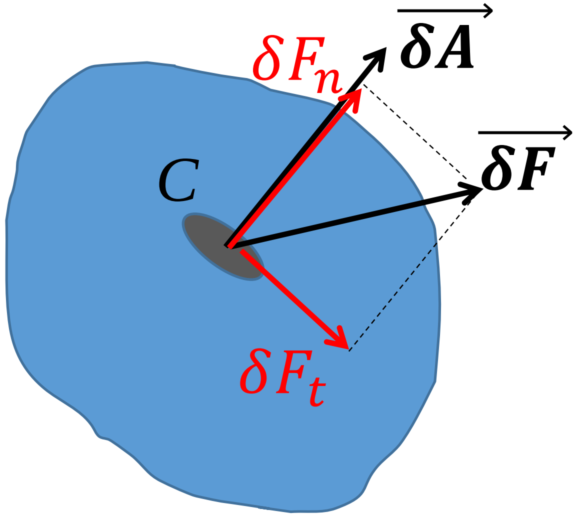
    Gravitational force per unit volume $=\rho \boldsymbol{g}$

    Gravitational force per unit mass $=\boldsymbol{g}$

- Surface forces e.g. pressure, friction
  - Force acts on the boundary of the fluid medium
    - The force is transmitted throughout the medium
      - Stress is developed in the fluid medium

Stress Field

- Stress: defined as force per unit area
- Consider a small area $\boldsymbol{\delta A}$ on the surface of a fluid particle which is in contact with other fluid particles
  - Note that area is a vector and its direction is along the outward normal with respect to the particle
- We can resolve the force in components tangential and normal to the surface



$$\text{Normal stress } \sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$$

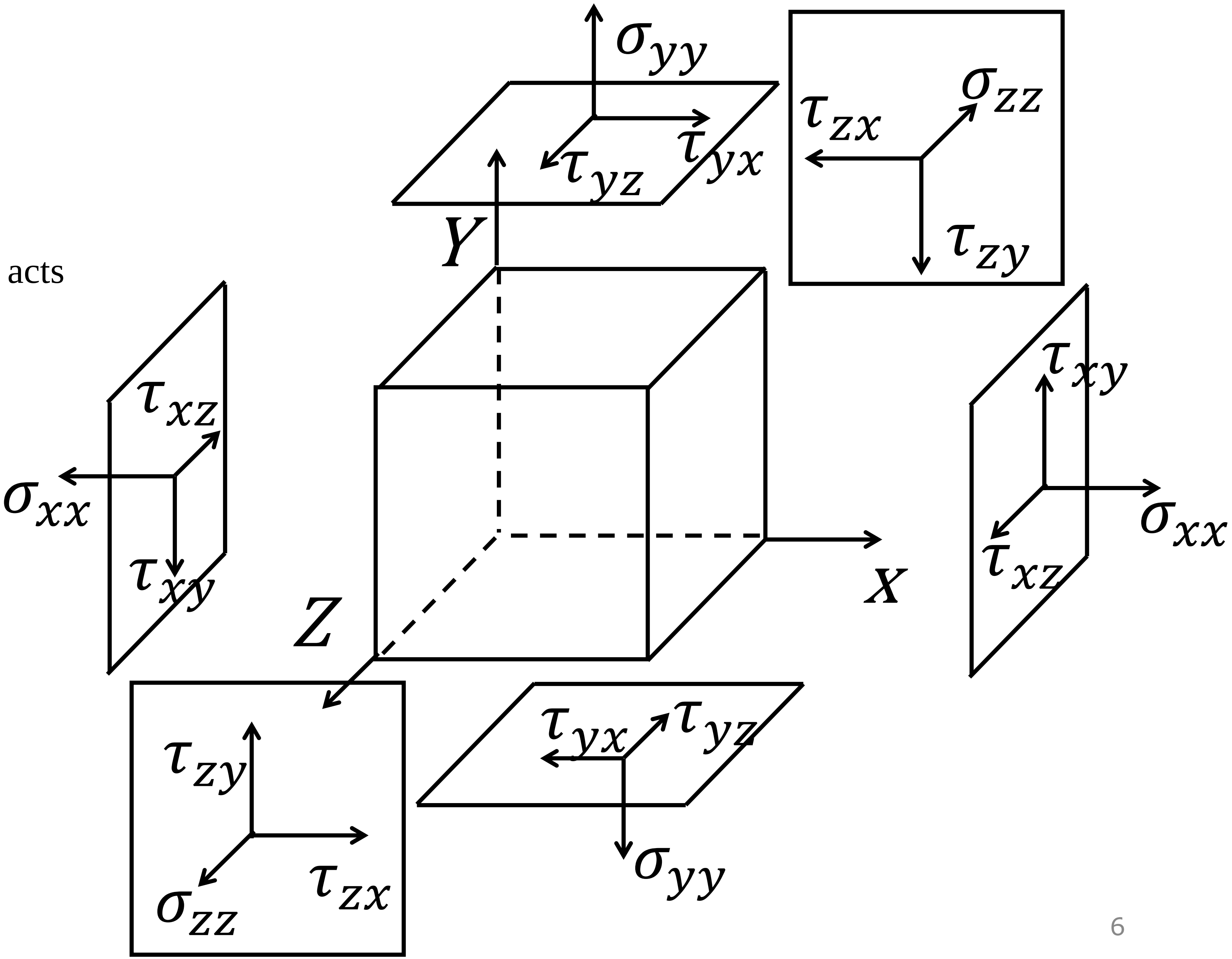
$$\text{Shear (tangential) stress } \tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$$

Stress Field

τ_{xy}

Direction in which the stress acts

Plane on which the stress acts



Stress Field

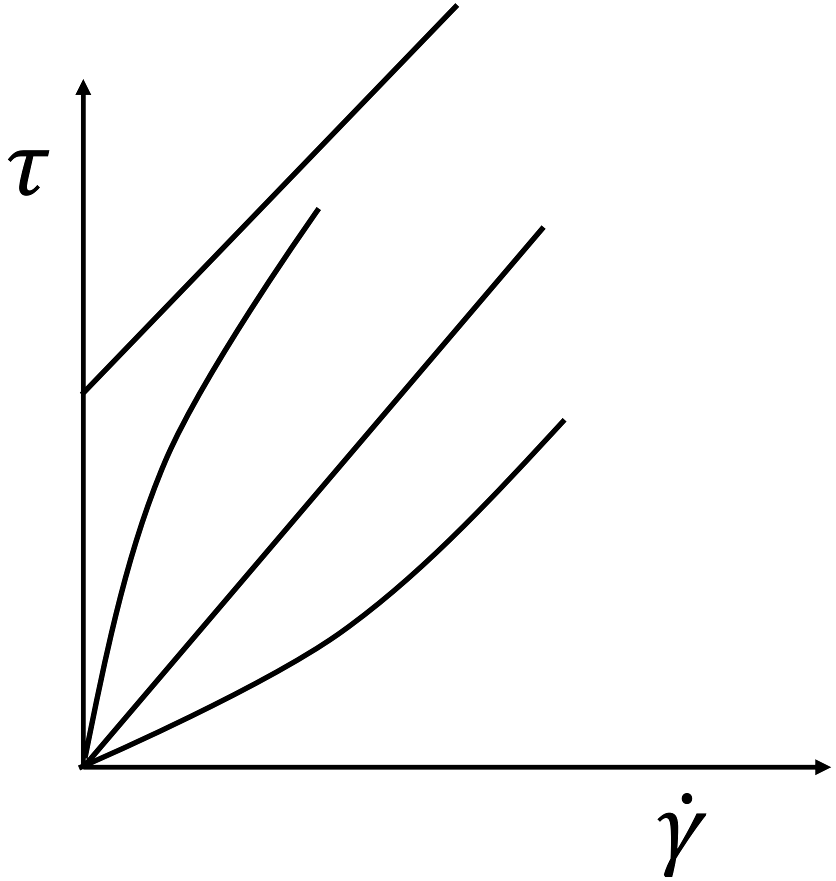
- The state of stress at a point can be described by the stresses acting on any three mutually perpendicular planes.
- The stress at a point has nine components

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

- Sign convention for stress:
  - Positive when the direction of stress component and that of the normal on the plane both are positive or both are negative.

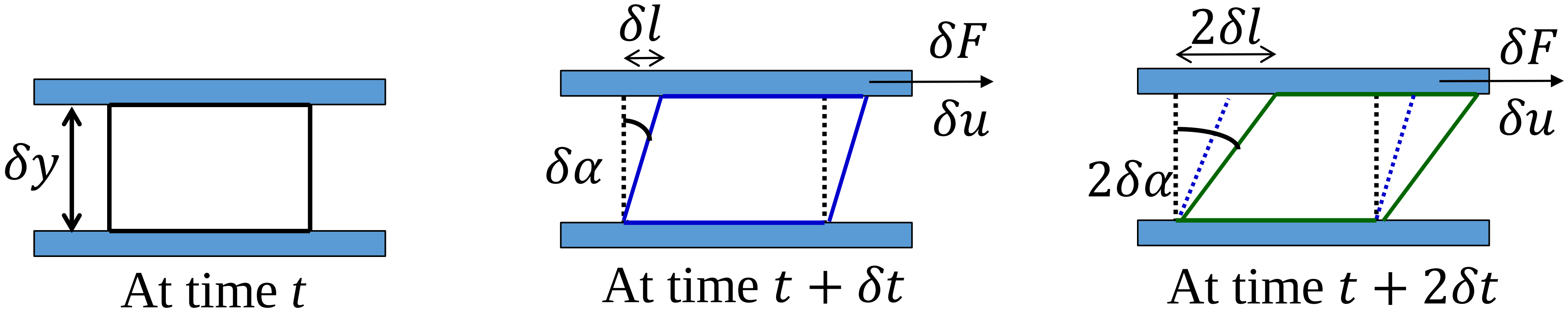
Rheology

- Fluids are characterized by the relationship between shear stress (τ) and rate of deformation/ rate of strain ($\dot{\gamma}$) in a fluid
- A constitute equation gives the relationship between stress and rate of strain.
- Rheology
- Rheometers



Rate of strain

- What is $\dot{\gamma}$?
  - Rate of deformation or shear rate or rate of strain
  - Shear strain: Relative displacement of particles in the body
- Consider flow between two parallel plates with the upper plate moving



- Rate of deformation $\dot{\gamma}=\lim\limits_{\delta t\to 0}\dfrac{\delta\alpha}{\delta t}$
- $\delta l = \delta u\ \delta t = \delta\alpha\ \delta y$ $\longrightarrow$ $\dfrac{\delta\alpha}{\delta t}=\dfrac{\delta u}{\delta y}$

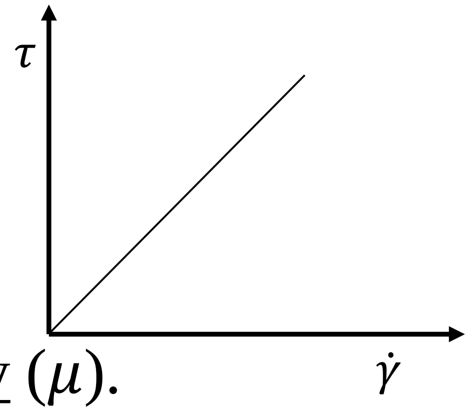
$$\dot{\gamma}=\frac{d\alpha}{dt}=\frac{du}{dy}$$

Viscosity

- For most of the common fluids, shear stress is directly proportional to the rate of strain

$$\tau \propto \dot{\gamma}$$

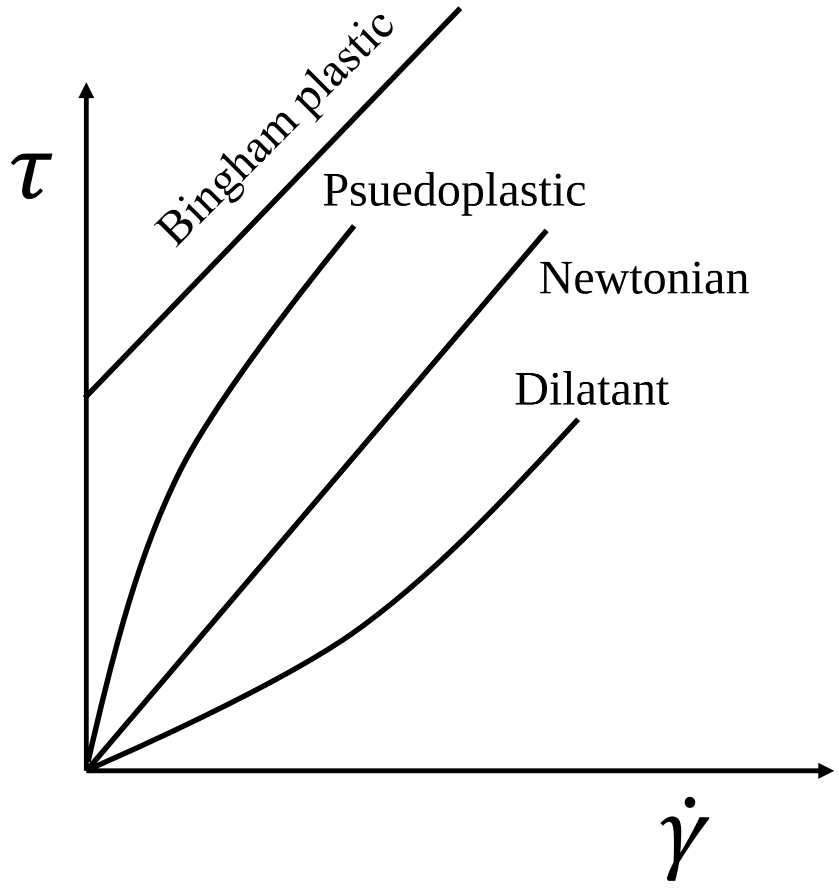
$$\tau_{yx} = \mu\dot{\gamma}$$



- This is known as Newton's law of viscosity.
- The proportionality constant is known as dynamic or absolute viscosity (μ).
- The fluids following Newton's law of viscosity are known as Newtonian fluids.
- Unit of viscosity- kg $m^{-1}s^{-1}$, Pa.s, Poise in CGS.
- Kinematic viscosity (ν): ratio of dynamic viscosity and density (μ/ρ)
  - Unit: m^2/s or Stokes
- Viscosity of gases increases with increase in temperature whereas those of liquid decreases.

Non-Newtonian Fluids

- Fluids that do not follow Newton's law of viscosity.
- Bingham plastic fluids:
  - Yield stress
  - Toothpaste, drilling mud, clay suspensions
- Psuedoplastic (shear thinning)
  - Apparent viscosity decreases with increasing shear rate
  - Polymer solutions, colloidal suspension
- Dilatant (shear thickening)
  - Apparent viscosity increases with increasing shear rate
  - Suspension of starch, sand

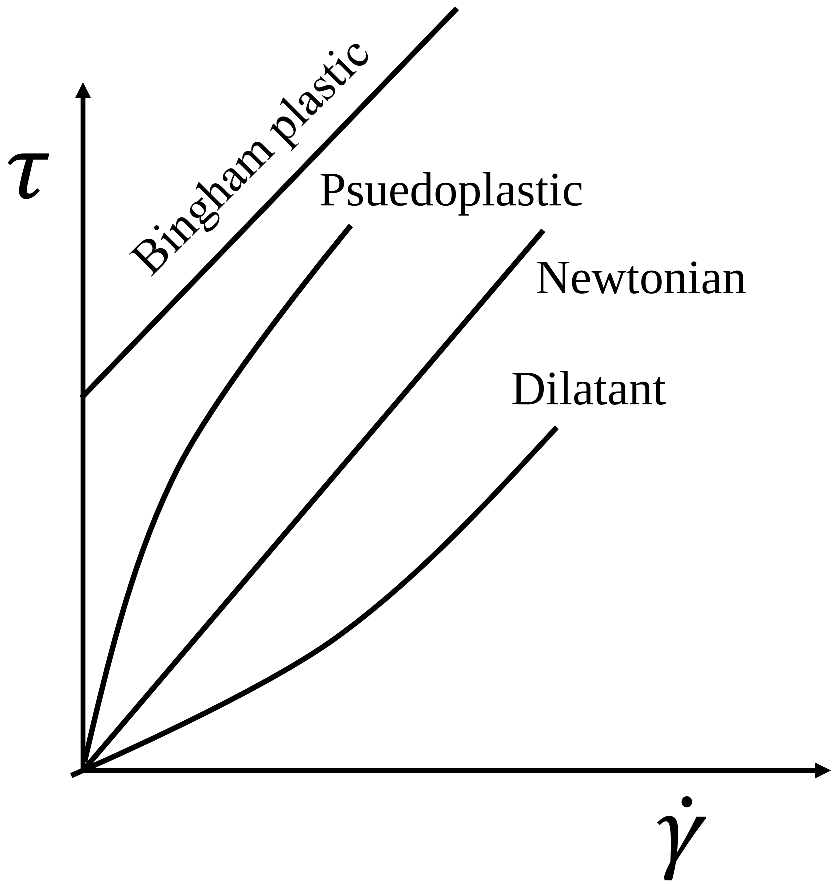


Non-Newtonian Fluids

- Herschel-Bulkley model:

$$\tau_{yx} = \tau_y + k\dot{\gamma}^n$$

- τ_y: Yield stress
- k: Consistency index
- n: Flow behaviour index
- For Bingham plastic fluid
  - Yield stress (τ_y) non-zero, n =1
- Psuedoplastic fluids:
  - Yield stress zero, $n < 1$
- Dilatant fluid
  - Yield stress zero, $n > 1$



Non-Newtonian Fluids

- The apparent viscosity may be time-dependent
- Thixotropic fluids:
  - Decrease in apparent viscosity with time
- Rheopectic fluids:
  - Increase in apparent viscosity with time
- Viscoelastic fluids:
  - Partially regain the original shape when the applied stress is released