Inference Project

Extreme Value Distribution and Convergence of the Maximum Ordee Statistic

M.Sc. First Year, 2nd Semester

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GROUP – C

PROJECT on Extreme Value Distribution and Convergence of Maximum Order Statistic

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"There will always be one (or more) value that will exceed all others"

-Emil J. Gumbel

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Introduction :

Probabilistic extreme value theory is a curious and fascinating blend of enormous variety of applications involving natural phenomena such as rainfall, floods, wind guests, air pollution, corrosion and delicate advance mathematical results on point process and regularity varying functions. This area of research attracted the interests of the theoretical Probabilists, Engineers, Hydrologists and mainstream Statistician.
Probabilistic extreme value theory deals with stochastic behaviour continuity of maximum and minimum of $i.i.d$ random variables. The Distribution Properties of maximum,minimum,intermediate order statistics exceedances over(below) high(low) thresholds are determined by upper and lower tails of underlying distribution.
Extreme value theory has originated mainly from the needs of astronomers in utilizing or rejecting outlying observations Practical application of extreme value theory started with- Humans life times, Radioactive emissions [Gumble(1997)], Strength of materials[Weibull(1939)], Flood analysis, Seismic Analysis[Nord quist(1945)] and Rainfall Analysis [Potter(1949)]. Gumble was the first to call the attention of the Engineers and Statisticians to possible Application of the formal "extreme value" to certain distributions which had previously been treated empirically.

Comparison Between Extreme Value Theory And CLT:
Extreme Value theory is very similar to Central Limit Theorem (CLT). Both theories involve limiting behaviours of distributions of i.i.d. random variables as n $\to \infty$, but there is a difference :
The CLT deals with the behaviour of the entire distribution of random variables, where the Extreme value theory only concerns the behaviour of the tails of those distribution.
To put the difference between the EVT and CLT a little more precisely , the CLT describes the limiting behaviour of $X_1, X_2, \cdots, X_n$ while the extreme value theory describes the limiting behaviour of the extremes $\max\{X_1, X_2, \cdots, X_n\}$ or $\min\{X_1, X_2, \cdots, X_n\}$.

General Extreme Value Distribution:
The General Extreme Value Distribution(GEV) is a family of continuous Probability Distribution's developed within extreme value theory to combine the Gumble,Frechet and Weibull families also known as type I ,II and III ex-

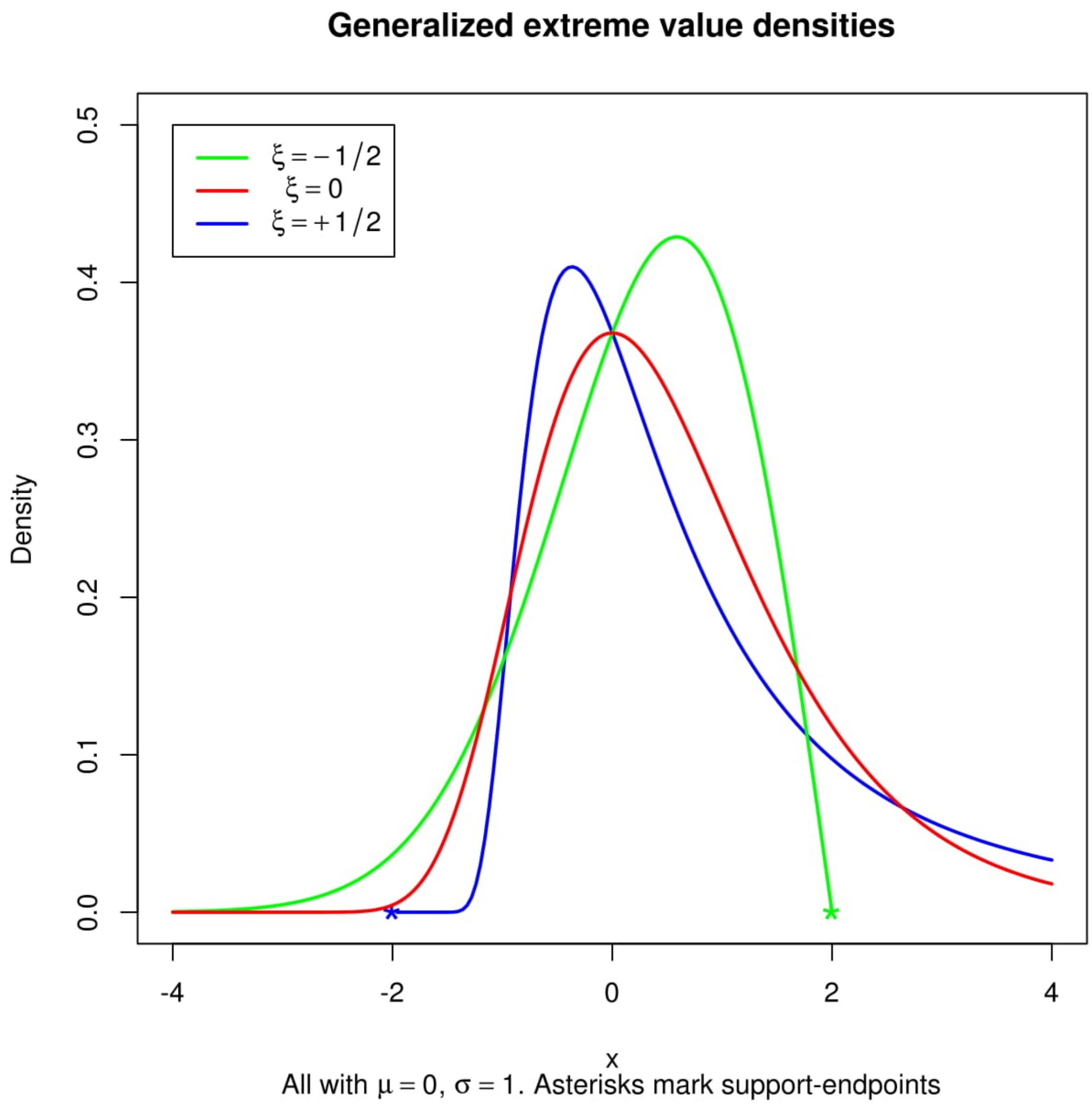
treme value distributions. By Extreme Value theorem the GEV distribution is the only possible limit distribution of properly normalized maxima of a sequence of $i.i.d$ Random Variables, the GEV distribution was first introduced by Jenkinson (1955).
The Cumulative Distribution of the Generalized extreme value Distributions is given by -

$$F_X(x) = \begin{cases} exp\left(-(1+\xi\frac{(x-\mu)}{\sigma})^{-\frac{1}{\xi}}\right), & \text{if } -\infty < x \leqslant \mu - \frac{\sigma}{\xi} \text{ for } \xi < 0 \\ & \text{or } \mu - \frac{\sigma}{\xi} \leqslant x < -\infty \text{ for } \xi > 0 \\ exp\left(-exp(-\frac{x-\mu}{\sigma})\right), & \text{if } -\infty < x < \infty \text{ for } \xi = 0 \end{cases}$$

where $\mu(\in R)$ is **location parameter**
σ (>0) is a **scale parameter**
$\xi(\in R)$is a **shape parameter**
The distribution is also referred to as the Von Mises type extreme value distribution or the Von Mises-Jenkinson type distribution.
The term "extreme value" is attached to these distribution because they can be obtained as limiting distributions of highest order statistics among **n** independent random variable each having the same continuous distribution
The density function corresponding to above C.D.F

$$f_X(x) = \begin{cases} exp\left(-(1+\xi\frac{x-\mu}{\sigma})^{-\frac{1}{\xi}}\right)\frac{1}{\sigma}\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}-1}, & \text{if } -\infty < x \leqslant \mu - \frac{\sigma}{\xi} \text{ for } \xi < 0 \\ & \text{or } \mu - \frac{\sigma}{\xi} \leqslant x < -\infty \text{ for } \xi > 0 \\ exp\left(-exp(-\frac{x-\mu}{\sigma})\right)\frac{1}{\sigma}exp\left(-\frac{x-\mu}{\sigma}\right), & \text{if } -\infty < x < \infty \text{ for } \xi = 0 \end{cases}$$

where $\mu(\in R)$ is **location parameter**
σ (>0) is a **scale parameter**
$\xi(\in R)$is a **shape parameter**
Graph of Generalized extreme value densities are given below at some specific value of the Parameter.



Type I , II , III Distribution :

The behaviour of maximum and minimum value of any distribution can be discribe through type I , II and III distribution. extreme value distribution are comprized of fallowing three families with cdf :

Gumble Type or Type I:

Gumble distribution is the most common distribution and is define entire range of real number it is used for both type of data(maximum and minimum).GEV fallows Gamble if ξ tend to zero.

c.d.f of Gumble distribution is

$\mathbf{F_X(x)} = \exp\left(-\exp(-\frac{x-\mu}{\sigma})\right)$

$p.d.f.$ of Gumble distribution is

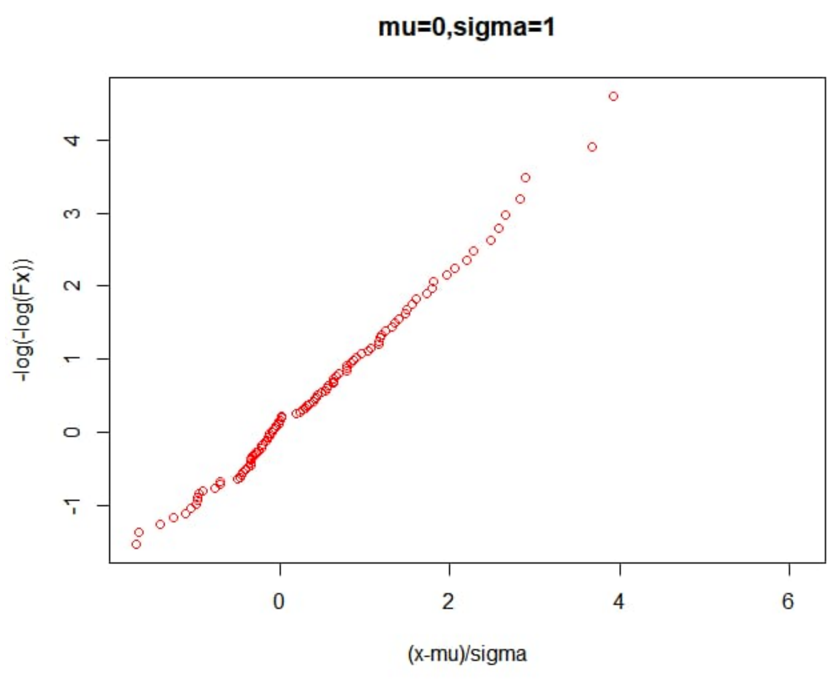
$\mathbf{f}(\mathbf{x}) = \frac{1}{\sigma} exp\left(-y - exp(-y)\right)$

where $y = \frac{x-\mu}{\sigma}$
$\mu(\in R)$ is **location parameter**
σ (>0) is a **scale parameter**

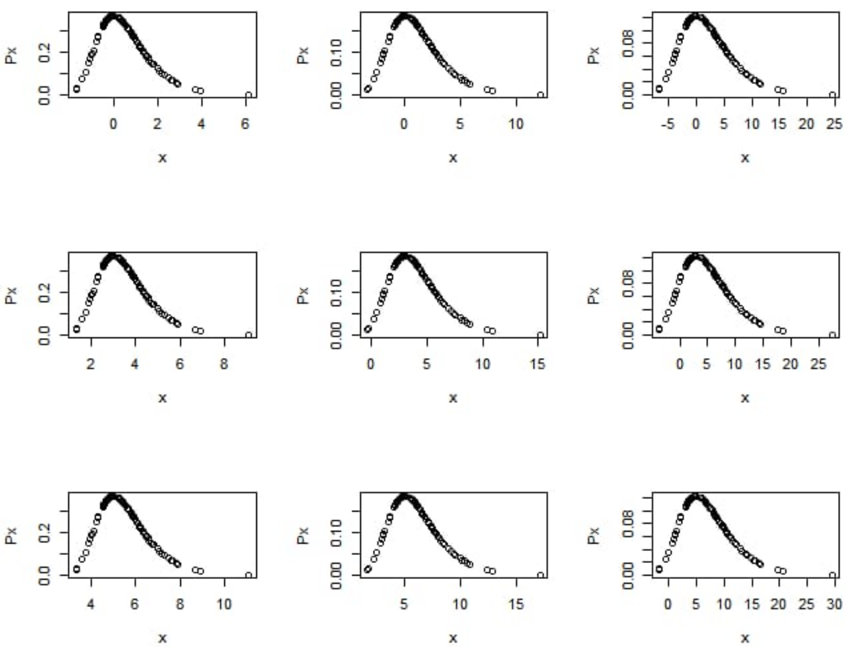
Random no generation from distribution and Graphical representation.

(1) Gumbel type distribution

It is clear from Gumbel type ($type\ I$), it has not any shape parameter so shape of probability curve does not vary on varying parameters
as we know $F(x) = exp(-exp\frac{-(x-\mu)}{\sigma})$
$\Rightarrow \qquad -log\left(-logF(x)\right) = \frac{x-\mu}{\sigma}$
This imply graph between $-log\left(-logF(x)\right)$ and $\frac{x-\mu}{\sigma}$ is approximately linearly related
if F(x)=$\frac{no\ of\ observation\ less\ then\ x}{total\ no\ of\ observation}$ and if we plot $-log\left(-logF(x)\right)$ against x it would be straight line with slope $\frac{1}{\sigma}$ and intercept of $x = \mu$ on x axis



This part has been shown through R programming
which is attached at last and graph shown below is approximately linear. Now as we have x (the random sample generated by gumbel distribution) and $P(x) = \frac{1}{\sigma} exp\left(-y - exp(-y)\right)$ where $y = \frac{x-\mu}{\sigma}$ and we have plotted P(x) against x for different parameter (μ,σ)



Here μ= $(0, 3, 5)$ and σ= $(1, 2, 4)$ and first row of figure corresponding to μ=0 and σ=(1,2,4), second row is corresponding to $\mu = 3$ and so on. so it can be seen that how graphs changes on changing parameters.

Note:-Scaling for x and y axis is different for different graphs

LEAST SQUARE ESTIMATE OF PARAMETERS:-

We have $-log\left(-logF(x)\right) = \frac{x-\mu}{\sigma}$
if $y = -log\left(-logF(x)\right)$
$\frac{1}{\sigma}= \sigma^*$
and ....(1)
$\frac{\mu}{\sigma} = \mu^*$ then
$y = \mu^* + \sigma^*$ X if Y= a + bX then least square estimate of b and a is given as
$\hat{b} = \frac{\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)\left(X_i-\bar{X}\right)}{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}$
and
$\hat{a} = \bar{Y} - \hat{b}\bar{X}$

so in this case $\hat{\sigma^*} = \frac{\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)\left(X_i-\bar{X}\right)}{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2} and \hat{\mu^*} = \bar{Y} - \hat{\sigma^*}\bar{X}$
....(2)

from (1) and (2) we have
$\hat{\sigma}=\frac{1}{\hat{\sigma^*}}$ and $\hat{\mu}$ = - $\hat{\sigma}\hat{\mu^*}$
these are estimate of σ and μ

Frechet Type or Type II:
Frechet distribution is used to model maximum values in a data set. For example horse racing maximum rainfalls etc. GEV fallows Frechet distribution if shape parameter ξ is greater than zero.
c.d.f of Frechet distribution is

$$F_X(x) = \begin{cases} 0, & \text{if} x < \mu \\ exp\left(-(\frac{x-\mu}{\sigma})^{-\xi}\right), & \text{if} x \geq \mu \end{cases}$$

p.d.f of Frechet distribution is
$\mathbf{f(x)} = \frac{\xi}{\sigma} y^{-1-\xi} exp\left(-y^{-\xi}\right)$

where $y = \frac{x-\mu}{\sigma}$
$\mu(\in R)$ is **location parameter**
σ (>0) is a **scale parameter**
$\xi(\in R)$is a **shape parameter**

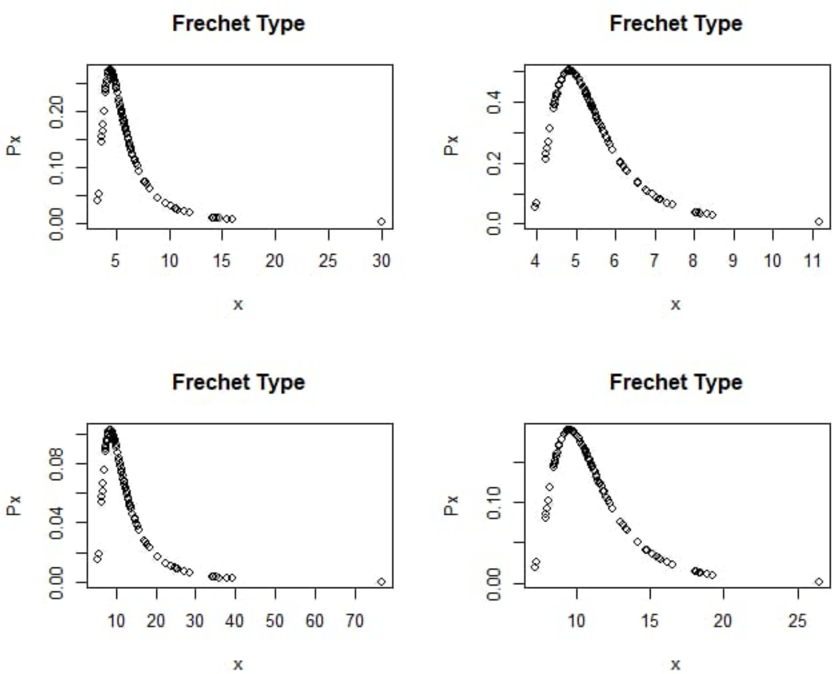
Random no generation from distribution and Graphical representation.

(2)Frechet type distribution

First we generate random no of this distribution using CDF.
As $F(x) = exp\left(-\frac{(x-\mu)}{\sigma}^{-\xi}\right)$ when x is greater than μ
and if k is a random no between 0 and one then on solving
$k = exp\left(-\frac{(x-\mu)}{\sigma}^{-\xi}\right)$ we will get
$x = \sigma\left(-log(k)^{\frac{-1}{\xi}}\right) + \mu$ and here x is random no generated by frechet distribution.

as we know probability distribution which is given as
$P(x) = \frac{\xi}{\sigma}(y)^{-\xi-1}exp\left(-(y)^{-\xi}\right)$ where y=$\frac{(x-\mu)}{\sigma}$
Now we will draw probability curve for different parameter and will see how graphs changes on changing the parameter

The graphs of P(x) against x are shown below



here $\mu = (2)$ $\sigma = (3, 8)$, ξ=(2,4) first row is corresponding to $\sigma = 3$ and second corresponding to $\sigma = 8$ for $\xi = (2, 4)$ respectively
Note:- Scaling on the x and y axix are different

Weibull Type or Type III:
Weibull distribution is generally used in assessing product reliability to model failure times and life data analysis. GEV follows Weibull distribution if ξ less than zero.

c.d.f of Weibull distribution is

$$F_X(x) = \begin{cases} exp\left(-(-y)^{\xi}\right) & \text{if} x \leq \mu \\ 0 & \text{if} x > \mu \end{cases}$$

$p.d.f$ of Weibull distribution is

$$\mathbf{f(x)} = \frac{\xi}{\sigma}\left(-y\right)^{\xi-1} exp\left(-(-y)^{\xi}\right)$$

where $y = \frac{x-\mu}{\sigma}$
$\mu(\in R)$ is **location parameter**
σ (>0) is a **scale parameter**
$\xi(\in R)$is a **shape parameter**

The corresponding distribution of **-X** are also called extreme value distributions.Frechet and Weibull distribution are related by a simple change of sign of these families of distributions,type I can be obtained from type II and III by a simple transformation $Z = log(X-\mu)$, $Z = -log(\mu-X)$ respectively.

Random no generation from distribution and Graphical representation.

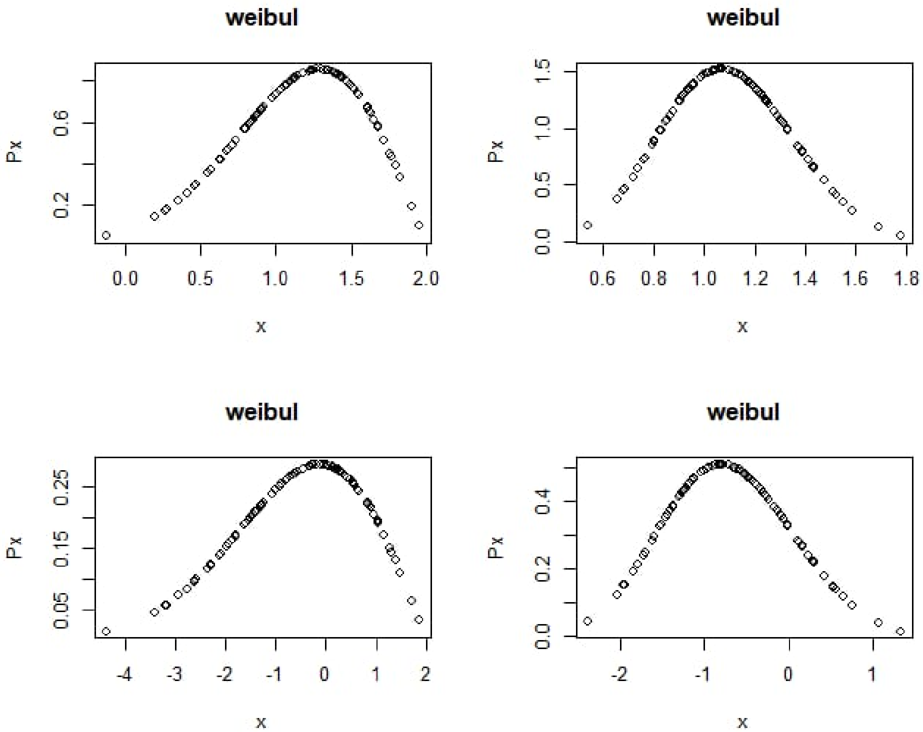
(3) Weibull type distribution

In similar manner first we generate random no of this distribution using CDF.
As $F(x) = exp\left(-\frac{(\mu-x)}{\sigma}^{\xi}\right)$ when x is less than μ
and if k is a random no between 0 and 1 then on solving
$k = exp\left(-\frac{(x-\mu)}{\sigma}^{-\xi}\right)$ we will get
$x = \mu - \sigma\left(-log(k)^{\frac{1}{\xi}}\right)$ and here x is random no generated by weibull distribution.
as we know probability distribution is given as

$$P(x) = \frac{\xi}{\sigma}(\frac{\mu-x}{\sigma})^{\xi-1}exp\left(-(\frac{\mu-x}{\sigma})^{\xi}\right)$$

Now we will draw probability curve for diiferent parameter and will see how graphs changes on changing the parameter

The graphs of P(x) against x are shown below



here μ = (2) $\sigma = (1,3)$, ξ=(2,4) first row is corresponding to $\sigma = 1$ and second corresponding to $\sigma = 3$ for $\xi = (2,4)$ respectively
Note:- Scaling on the x and y axix are different

The graphs shown above are drown
using R software. Coreespondig code for graphs is attached at last

Derivation of CDF:
Extreme Value distributions were obtained as limiting distributions of greatest(or least)Value in random samples of increasing size.So,the distributions will be degenerate. To get rid of this degeneracy it is necessary to reduced the actual greatest value by applying a linear transformation with coefficients which depends on the sample size **n**. Let X_1,X_2......,X_n are independent random variables with common p.d.f
$f_X(x) = f(x)$
then c.d.f of$X_{(n)}$=max$\{X_1$,X_2......,$X_n\}$ is,
$F_n(x) = (F(x))^n$

where $F(x) = \int_{-\infty}^{x} f(x)dx$
and we know $0 \leq F(x) \leq 1 \quad \forall x$

as n tends to infinity ,it is clear that for any fixed value of x

$$\lim_{n\to\infty} F_n(x) = \begin{cases} 1 & \text{if} F(x) = 1 \\ 0 & \text{if} F(x) \neq 1 \end{cases}$$

which is the degenerate distribution. So, for non degenerate distribution we need transformation.
Let the transformation be $X_{(n)} \longrightarrow \frac{X_{(n)}-bn}{a_n}$ where a_n ,b_n depend on n but not on x.
So, $P\left(\frac{X_{(n)}-bn}{a_n} \leq x\right) = P\left[X_{(n)}<a_n x + b_n\right] = (F(a_n x + b_n))^n$

$$\longrightarrow G(x)(say) \;\; as \;\; n \longrightarrow \infty \quad \text{for any } a_n>0,\ b_n \in \mathbb{R}$$

Then$(F(a_n x + b_n))^{nN} \longrightarrow (G(x))^N$ as n $\longrightarrow \infty$ for any $N \in \mathbb{N}$
Again $(F(a_{nN} x + b_{nN}))^{nN} \longrightarrow (G(x))^N$ as n $\longrightarrow \infty$ for any $N \in \mathbb{N}$
Now Convergence to Type Theorem,
Let $\lim_{n\to\infty} \frac{a_n}{a_{nN}} = c(N)>0$
$\lim_{n\to\infty} \frac{b_n - b_{nN}}{a_{nN}} = d(N)>0$
and $(G(x))^N = G\left(c(N)x + d(N)\right)$ ...(1)
This equation some times called Stability Postulate. Type-I distributions are obtained by taking c(N)=1,types-II and types-III by taking c(N)≠1 .In latter case x-c(N)x=$d(N)$ if x=$\frac{d(N)}{1-c(N)}$and from equation (1) it is evident that G$\left(\frac{d(N)}{1-c(N)}\right)$ must equal to 1 or 0. Type-II corresponds to 1 and type-III corresponds to 0.
Now, for c(N)=1,
$(G(x))^N = G\left(x + d(N)\right)$ ....(2)
Since G$(x + d(N))$ must also satisfy
$[G(x)]^{NM} = G\left(x + d(N)\right)^M = G\left(x + d(N) + d(M)\right)$ ....(3)
Also from equation(1)
$[G(x)]^{NM} = G\left(x + d(NM)\right)$ ......(4)
from equation(3) and (4) we have,
d(N)+d(M)=d(NM)
where d(N)=$\sigma log(N)$ where σ is a constant
Taking logarithm of (2) twice and inserting value of $b(N) = \sigma log(N)$, We have
$log(N) + log\left(-log(G(x))\right) = log\left(-log(G(x + \sigma log(N)))\right)$ .....(5)

Let,
h(x)=$log\left(-log(G(x))\right)$ ....(6)
From equation(6) we have
h(x)=h(0)-$\frac{x}{\sigma}$
As h(x) decreases as x increases,$\sigma>0$
from equation(6),We have
$-log\left(G(x)\right)=exp\left[-\frac{x-\sigma h(0)}{\sigma}\right]=exp\left(-\frac{x-\mu}{\sigma}\right)$
where $\mu=\sigma log\left(-log(G(0))\right)$
Hence G(x)=exp$\left[-exp^{-\frac{x-\mu}{\sigma}}\right]$
Which is type-I distribution

Conditions:
Gnedendko(1943) established certain correspondences between the parent distribution F(x) and the type to which the limiting distribution belongs. The conditions established by Gnedendko can be summarized as follows:
For the type I:

$$\text{Define } F(X_\alpha)=\alpha\ ,$$

the condition is

$$\lim_{n\to\infty} n\left[1-F\left(X_{1-\frac{1}{n}}+y(X_{1-\frac{1}{ne}}-X_{1-\frac{1}{n}})\right)\right]=e^{-y}$$

For the type II:

$$\lim_{x\to\infty}\frac{1-F(x)}{1-F(cx)}=c^k,c>0,k>0,$$

For the type III:

$$\lim_{x\to\ 0^-}\frac{1-F(cx+w)}{1-F(x+w)}=c^k,c>0,k>0,$$

where F$(w)=1,\ F(x)<1$ for $x<w$

Gnedendko also showed that three conditions are necessary and sufficient and there are no other distributions satisfying the stability postulate. Among distributions satisfying the type-I condition are Normal, Exponential and Logistic; the type II condition is satisfied by Cauchy and the type-III condition is satisfied by non-degenerate distributions with range of variation bounded above.

Domain of attraction:

here $(F(a_n x + b_n))^n \to G(x)$, $a_n > 0$, $b_n \in \mathbb{R}$
Equivalently, $\dfrac{X_{(n)} - b_n}{a_n} \xrightarrow{d} Y$, where Y has cdf G.
We say that F belongs to the domain of attraction of G.
In short F $\in$ MDA(G), where MDA = Maximum domain of attraction.
Except some pathological cases, every cdf F belongs to a maximum domain of attraction.

Moments of Extreme value distribution :

$$E(X) = \begin{cases} \mu + \frac{\sigma(g_1 - 1)}{\xi}, & \text{if } \xi \neq 0, \xi < 1. \\ \mu + \sigma\gamma, & \text{if } \xi = 0 \\ \infty, & \text{if } \xi \geq 1 \end{cases}$$

where $g_k = \Gamma(1 - k\xi)$,
$\gamma = \lim_{n\to\infty}(-\ln n + 1 + \frac{1}{2} + ... + \frac{1}{n}) = 0.577215665$

$$Var(X) = \begin{cases} \frac{\sigma^2(g_2 - g_1^2)}{\xi^2}, & \text{if } \xi < \frac{1}{2}. \\ \frac{\sigma^2\pi^2}{6}, & \text{if } \xi = 0 \\ \infty, & \text{if } \xi \geq \frac{1}{2} \end{cases}$$

$$Median = \begin{cases} \mu + \frac{\sigma(\ln 2)^{-\xi} - 1}{\xi}, & \text{if } \xi \neq 0 \\ \mu - \sigma \ln\ln 2, & \text{if } \xi = 0 \end{cases}$$

$$Mode = \begin{cases} \mu + \frac{\sigma(1+\xi)^{-\xi} - 1}{\xi}, & \text{if } \xi \neq 0 \\ \mu, & \text{if } \xi = 0 \end{cases}$$

$$Skewness = \begin{cases} sgn(\xi)\frac{g_3 - 3g_2g_1 + 2g_1^3}{(g_2 - g_1^2)^{\frac{3}{2}}}, & \text{if } \xi \neq 0, \xi < \frac{1}{3} \\ \frac{12\sqrt{6}\zeta(3)}{\pi^3}, & \text{if } \xi = 0 \end{cases}$$

sgn(x) is the sign function i.e.

$$sgn(\xi) = \begin{cases} 1, & \text{if } \xi > 0 \\ -1, & \text{if } \xi < 0 \\ 0, & \text{if } \xi = 0 \end{cases}$$

$\zeta(x)$ is Riemann Zeta function $= \sum\limits_{n=1}^{\infty} \frac{1}{n^x}$

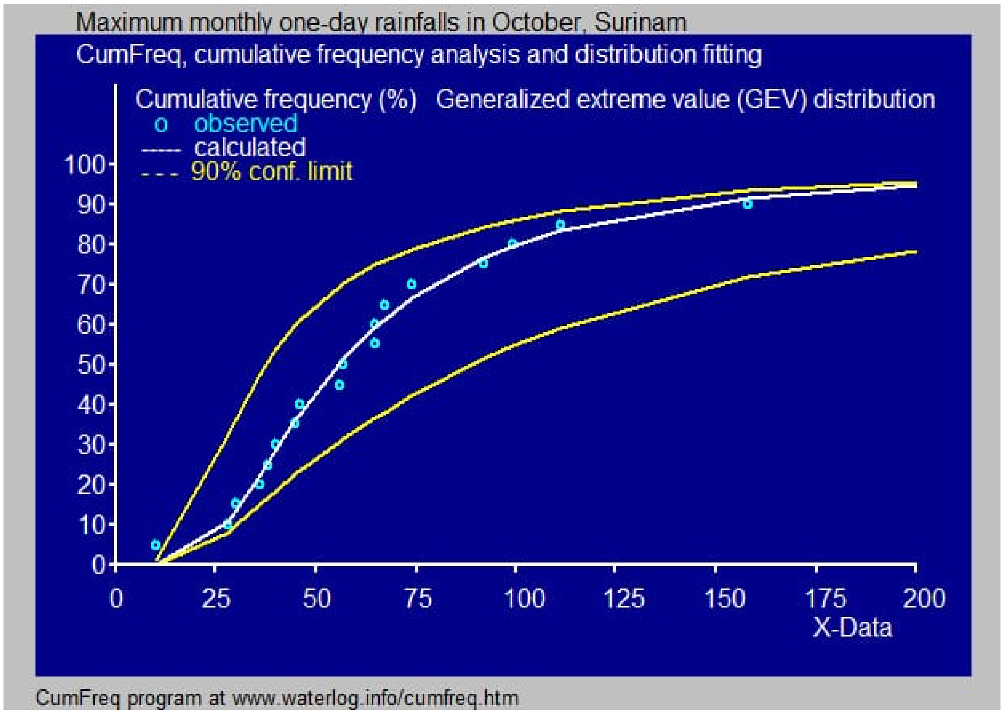
$$Kurtosis = \begin{cases} \frac{g_4 - 4g_3g_1 - 3g_2^2 + 12g_2g_1^2 - 6g_1^4}{(g_2 - g_1^2)^2} + 3, & \text{if } \xi \neq 0, \xi < \frac{1}{4} \\ \frac{27}{5}, & \text{if } \xi = 0 \end{cases}$$

Applications:

1) The EVT method can be applied to various topics in epidemiology thus contributing the public health planning for extreme events.

2) However, the resulting shape parameters have been found to lie in the range leading to undefined means and variances, which underlies the fact that reliable data analysis is often impossible.

3) In hydrology GEV distribution is applied to extreme events such as annual maximum one-day rainfalls and rive discharge. The blue picture,made with Cumulative frequency, illustrates an example of fitting the GEV distribution to rank annually maximum one day rainfalls. Showing also 99% confidence belt based on the binomial distribution. The rainfall data are represented by plotting positions as part of the cumulative frequency analysis.



4) The GEV distribution is widely used in the treatment of "tail risks" in fields ranging from insurance to finance. In the latter case, it has been considered as a means of assessing various financial risks via metrics such as value at Risk.

#Necessary R code
#R code for black white graphs and Random Number Generation

```
#---------------------------------------------- Gumbel Type(1)------------------------------------------------------------

# generation of random var of size n between 0 and 1

n=100
mu=0
sigma=1
a=runif(n)
a=sort(a)

# we have to show that  -log(-log(Fx))= (x-mu)/sigma is straight line...............................(1)

x=mu-sigma*log(-log(a)) # this is RHS of equation one mu=0 and sigma=1
Fx=rep(0,100)
for(i in 1:n){                        #  Fx is relative frequency of vector a
  Fx[i]=(length(a[1:i]))/n
}
k=-log(-log(Fx))    # this LHS of equation 1  on tha basis of sample
length(k)
length(y)
plot(x,k,xlab="(x-mu)/sigma",ylab="-log(-log(Fx))",main="mu=0,sigma=1",col="red")

# Now we will drow probability curve for different parameters

mu=c(0,3,5)
sigma=c(1,2,4)
length(mu)
length(sigma)
par(mfrow=c(3,3))
for (i in 1:length(mu)){
  for( j in 1:length(sigma)){
    x=mu[i]-sigma[j]*log(-log(a))
    y=(x-mu[i])/sigma[j]
    Px=exp(-y-exp(-y))*(1/j)
    plot(x,Px,type="p",xlab="x",ylab="Px")
  }
}

#------------------------------------------------------------------Frechet (type 2) ------------------------------------------

# generation of sample

n=100
a=runif(n)
a=sort(a)
sigma=c(3,8)
mu=c(2)
eps=c(2,4)
par(mfrow=c(2,2))
for(i in 1:length(mu)){
  for(j in 1:length(sigma)){
    for(k in 1:length(eps)){
      x=mu[i]+sigma[j]*((-log(a))^(-1/eps[k]))
      y=(x-mu[i])/(sigma[j])
      Px=(eps[k]/sigma[j])*((y)^(-eps[k]-1))*exp(-(y)^(-eps[k]))
```

```
      plot(x,Px,main="Frechet Type")
    }
  }
}

#------------------------------------------------------------------Weibull (type 3) -----------------------------------------

# generation of sample

n=100
a=runif(n)
a=sort(a)
sigma=c(1,3)
mu=c(2)
eps=c(2,4)
par(mfrow=c(2,2))
for(i in 1:length(mu)){
  for(j in 1:length(sigma)){
    for(k in 1:length(eps)){
      x=mu[i]-sigma[j]*((-log(a))^(1/eps[k]))
      y=(x-mu[i])/(sigma[j])
      Px=(eps[k]/sigma[j])*((-y)^(eps[k]-1))*exp(-(-y)^(eps[k]))
      plot(x,Px,main = "weibul")
    }
  }
}
```

References:-

1. Samuel Kotz , Saralees Nadarajah - Extreme Value Distributions_Theory and Applications-World Scientific Publishing Company (2001)
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7. Gumble Distribution Wikipedia
8. Frechet Distribution Wikipedia
9. Weibull Distribution Wikipedia
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We would like to express our special thanks of gratitude to our teacher Prof. Debraj Das to show us various way to complete the project by suggesting several research papers & books and by clearing all doubts questioned by us.

Student's Identity

Abhilash Singh	191003
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Certificate

This is to certify that the content of this project entitled, "**Inference Project**" by

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is the bona fide work of them submitted to IIT Kanpur, Department of Mathematics & Statistics for consideration in partial fulfillment for complete of the course MTH418, 2nd semester.

The original project work was carried out by them under my supervision in the academic year 2019-20. On the basis of declaration made by them I recommend this project report for evaluation.

Teacher's Signature

....

THE END