Step 1:- solve
$LY = B$

$AX = B$

$L(UX) = B$

put

let $UX = Y$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 14/9 & 1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

$y_1 = 14$

Step 2:- $UX = Y$

$$\begin{bmatrix} 1 & 5 & 3 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -15 \\ -5/3 \end{bmatrix}$$

$y_2 = -15 \leftarrow 2y_1 + y_2 = 13 \Rightarrow 13 - 28 = y_2$

$3(14) + \frac{14}{9}(-15) + y_3 = 17 \leftarrow 3y_1 + \frac{14}{9}y_2 + y_3 = 17$

$y_3 = (-5/3)$

$x_1 + 5y + 2 = 14$

$-9y + z = -15$

$-5/9 z = -5/3$

$x = 1$, $y = 2$, $z = 3$ - check to see out as Ans

10th April 2024 Wednes

Cholesky method

↳ it is LU decom only but

यहाँ पर यह method आएगा

(i) LU method

(ii) Cholesky method

$AX = B$ —(i)

let $A = LL^T$ —(ii)

where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$

T = transpose

find $l_{11}, l_{21}, l_{22}, l_{31}, l_{32}, l_{33}$

By ① & ② $LL^T = B$ —③

put $L^T X = Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times3}$

put

$\boxed{LY = B}$ solve and find Y

$L^T X = Y$

↳ solve find X Ans

Que. Solve the system by Cholesky meth.

$x + 2y + 3z = 5$

$2x + 8y + 22z = 6$

$3x + 22y + 82z = -10$

Soln:- let the given system is

$AX = B$, where —(i)

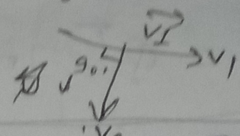
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

let $A = LL^T$ i.e. —(ii)

↳ constant

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{11}l_{21} & l_{21}^2 + l_{22}^2 & l_{21}l_{31} + l_{22}l_{32} \\ l_{31}l_{11} & l_{31}l_{21} + l_{32}l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{bmatrix}$$

$u_2 = a_2 - \cancel{(a_2 \cdot e_1)} e_1 = a_2$ — orthogonal vector — perpendicular



$\Rightarrow u_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad e_2 = \dfrac{u_2}{\|u_2\|}$

$\|u_2\| = \sqrt{(1)^2 + (-1)^2 + 1^2} = \sqrt{3}$

$e_2 = \text{length} = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

Reason: $a_2 \cdot e_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}_{3\times1} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}_{3\times1} = 0$

dot product: $\frac{1\times1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 0 = 0$

Also,

~~$u_3 = a_3 - (a_3 \cdot e_1) e_1 \cdot a_3$~~

$\Rightarrow u_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Reason: $a_3 \cdot e_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{3\times1} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}_{3\times1} \neq 0$

$a_3 \cdot e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$ vector

dot product $= -1/\sqrt{3}$

$u_3 = a_3 - (a_3 \cdot e_1) e_1 - (a_3 \cdot e_2) e_2$

$= a_3 - (1/\sqrt{2}) e_1 - \left(-\frac{1}{\sqrt{3}}\right) e_2$

$a_3 \cdot e_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = 1/\sqrt{2}$

$\Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{\sqrt{2}} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}_{3\times1} - \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0 - 1/2 + 1/3 \\ 1 - 1/2 - 1/3 \\ 1/3 \end{bmatrix}_{3\times1} = \begin{bmatrix} -1/6 \\ 1/6 \\ 1/3 \end{bmatrix}$ or $\begin{bmatrix} -1/6 \\ 1/6 \\ 2/6 \end{bmatrix}$

$0 - \frac{1}{2} + \frac{1}{3} = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

$1 - \frac{1}{2} - \frac{1}{3} = \frac{6-3-2}{6} = \frac{1}{6}$

$e_3 = \dfrac{u_3}{\|u_3\|} = \dfrac{u_3}{1/\sqrt{6}} = \sqrt{6}\, u_3$

magnitude $u_3 = \sqrt{\left(\frac{-1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{2}{6}\right)^2} = \frac{1}{\sqrt{6}}$

$e_3 = \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}_{3\times1} = \sqrt{6} \begin{bmatrix} -1/6 \\ 1/6 \\ 2/6 \end{bmatrix}$

$\therefore Q = [e_1 \;\; e_2 \;\; e_3] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \end{bmatrix}_{3\times3}$

length of $e_1 = 1$

$\frac{1}{2} + \frac{1}{2} + 0 = 1$

Orthogonal होने के लिए $\text{Column}_1 \times \text{Column}_2 + \text{Column}_3 = 0$ $(e_1 \cdot e_2)$

किन्हीं दो corresponding column का गुणा $= 0$ होना चाहिए

$(\text{Column}_1 \times \text{Column}_2 + \text{Column}_3 = 0)$ same

Hence e_1, e_2, e_3 are pairwise ortho Normal

Q is orthonormal

$\therefore Q^{-1} = Q^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{6} & 2/\sqrt{6} \end{bmatrix}_{3\times3}$

Step 2:- To find R (upper triangular matrix)

$$\boxed{R = Q^T A}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{6} & 2/\sqrt{6} \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{3\times3} = \begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & \sqrt{3} & -1/\sqrt{3} \\ 0 & 0 & 1/\sqrt{6} \end{bmatrix}_{3\times3}$$

So, $A = QR = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & \sqrt{3} & -1/\sqrt{3} \\ 0 & 0 & 1/\sqrt{6} \end{bmatrix}$ ~~Answer~~ Answer

by Norm

Kappa of $A = K(A) = \|A\| \, \|A^{-1}\|$

$A = QR$, where Q = Ortho Normal set of vec = $m \times n$

R = Upper invertible matrix $n \times n$

Let $S = \{x_1, x_2, \dots x_n\}$ then S is called orthogonal, if $\langle x_i, x_j \rangle = 0 \;\; \forall i \neq j$

Norm: S is called orthoNorm if

(i) S is orthogonal

(ii) Norm of each vector in $S = 1$

$\langle x_i, x_j \rangle = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$

Q How to find QR calc

- (i) Gram-Schmidt orthogonalization
- (ii) Householder
- (iii) Givens man

Some Decomposition

(i) LU

(ii) PLU

(iii) Cholesky $\longrightarrow$ Positive Definite Matrix

(iv) QR $\longrightarrow$ invertible matrix, Ortho Normal matrix $= Q$, R = Upper matrix

(v) SVD (Singular value Decomp)

Symmetric ($A^T = A$)

Positive Definite

Non-zero

$$(A \vdots B) = \begin{bmatrix} 1 & 4 & 9 & \vdots & 16 \\ 2 & 1 & 1 & \vdots & 10 \\ 3 & 2 & 3 & \vdots & 18 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 4 & 9 & \vdots & 16 \\ 0 & -7 & -17 & \vdots & -32 \\ 0 & -10 & -24 & \vdots & -30 \end{bmatrix} \quad R_3 \to 7R_3 - 10R_2 \quad \begin{bmatrix} 1 & 4 & 9 & \vdots & 16 \\ 0 & -7 & -17 & \vdots & -22 \\ 0 & 0 & 2 & \vdots & 10 \end{bmatrix}$$

$x = 7$ ← $x + 4y + 9z = 16$

$-7y - 35 = -22$ ← $-7y - 17z = -22$

$y = -9$

$2z = 10 \to z = 5$

$x = 7$
$y = -9$
$z = 5$
Ans

Method 2 :- Gauss Jordan

diagonal — Gauss — Jordan

Same Gauss

$$\begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & -7 & -17 & -22 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

1st step :-

Upper RHS

Step :- 2

Same

1st diagonal make it zero

$$\begin{bmatrix} 1 & 4 & 9 & \vdots & 16 \\ 0 & -7 & -17 & \vdots & -22 \\ 0 & 0 & 2 & \vdots & 10 \end{bmatrix} = \begin{bmatrix} 7 & 0 & -5 & 24 \\ 0 & -7 & -17 & -22 \\ 0 & 0 & 2 & 10 \end{bmatrix} \to \begin{matrix} R_1 \to 2R_1 + 5R_3 \\ R_2 \to 2R_2 + 7R_3 \end{matrix}$$

~~$R_3 \to 7R_3 - 10R_2$~~

$R_1 \to 7R_1 + 4R_2$

$$\begin{bmatrix} 14 & 0 & 0 & 98 \\ 0 & -14 & 0 & 126 \\ 0 & 0 & 2 & 10 \end{bmatrix}$$

$14x = 98 \to x = 7$

$-14y = 126 \to y = -9$

$2z = 10 \to z = 5$

8th April '24
Monday

Today Lec

(i) LU Decomposition method
a factorisation

L – Lower triangular, U → Upper

(ii) factorisation method

(iii) Cholesky's method

Ans

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$
$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$
$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = LU$$

$AX = B$

$L(UX) = B$

$UX = Y$

$LY = B$ ← solve

$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$, $U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$ $A = LU$

Step: $LUX = B$ तरीका L और U को खोजें

(i) first row of U i.e. U_{12}, U_{21}, U_{23}

(ii) first column of L i.e. l_{11}, l_{21}, l_{31}

(iii) second row of U i.e. U_{22}, U_{23}

(iv) " " L i.e. l_{21}

(v) third row of U_{33}

Q Solve the system of eqn by LU decomp method?

$x + 5y + z = 14$

$2x + y + 3z = 13$

$3x + y + 4z = 17$

$$\underbrace{\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

Soln:-

$$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = LU$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & (l_{21}U_{12} + U_{22}) & (l_{21}U_{13} + U_{23}) \\ l_{31}U_{11} & (l_{31}U_{12} + l_{32}U_{22}) & (l_{31}U_{13} + l_{32}U_{23} + U_{33}) \end{bmatrix}$$

say

On compa

$U_{11} = 1$

$U_{12} = 5$

$U_{13} = 1$

$l_{21}U_{11} = 2$

$\boxed{l_{21} = 2}$

$l_{21}U_{12} + U_{22} = 1$

$2 \times 5 + U_{22} = 1$

$\boxed{U_{22} = -9}$

$l_{31}U_{11} = 3$

$\boxed{l_{31} = 3}$

$l_{21}U_{13} + U_{23} = 3$

$2 \cdot 1 + U_{23} = 3$

$\boxed{U_{23} = 1}$

$l_{21}U_{12} + U_{22} = 1$

$2 \cdot 5 + -9 = 1$

$l_{31}U_{12} + l_{32}U_{22} = 1$

$3 \cdot 5 + l_{32}(-9) = 1$

$15 + l_{32}(-9) = 1$

$l_{32} = \frac{+14}{+9}$

$U_{31}U_{13} + l_{32}U_{23} + U_{33} = 4$

$3 \cdot 1 + \frac{14}{9}$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{14}{9} & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -9 & 1 \\ 0 & 0 & -5/9 \end{bmatrix}$$

By Gram-Schmidt process, हम orthogonal करते हैं

$$V^T = \begin{bmatrix} 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 1/\sqrt{30} & \frac{2}{\sqrt{30}} & \frac{-5}{\sqrt{30}} \end{bmatrix}, \quad \mathcal{E} = \begin{bmatrix} \sqrt{12} & 0 & 0 \\ 0 & \sqrt{10} & 0 \end{bmatrix}_{2\times3}$$

higher singular

Ans

$A_{2\times3} = \mathcal{E}_{2\times3}$

$A = U.\mathcal{E}.V^T$ where $\mathcal{E}$ = diag matrix with singular value diagonal (singular) eigen

Recommendation system

A	B	C
1	0	0
0	5	1

AP I S C — Complex Eigen

Recommendation system Matrix factorization

पहला step eigen $\lambda_1 = 10, \lambda_2 = 12$

दूसरा $\lambda_1 = 0, \lambda_2 = 10, \lambda_3 = 12$

हैं unique = $\lambda_1 = 12, \lambda_1 = 10$ $\mathcal{E}$

Singular matrix को हम लिखते हैं increasing to decreasing order को

is diagonal in nature

Hint matrix $A = [\quad]_{2\times}$

Step 1:- $A.A^T$ निकालो यहाँ eigen value find U निकालो

Step 2:- $A^T.A$ निकालो इससे जो eigen value, इससे हम आएंगे $\mathcal{E} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \end{bmatrix}_{2\times3}$

singular value matrix

Eigen vector find

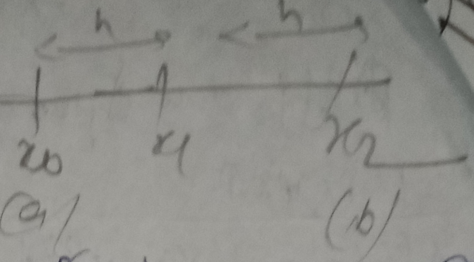
$V = \begin{bmatrix} & & \end{bmatrix}_{\alpha_1\ \alpha_2\ \alpha_3} \longrightarrow V^T = [\quad]$

finally

$\boxed{A = U.\mathcal{E}.V^T}$

deep learning में

Simpson's 1/3 Rule :→



$$I = \int_a^b f(x)\,dx$$

$$= \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right] \quad \cdots\; (1)$$

Composite form ... in ...

$$I = \frac{h}{3}\Big[f(x_0) + 4\{f(x_1) + f(x_3) + \cdots + f(x_{2N-1})\}$$
$$+ 2\{f(x_2) + f(x_4) + \cdots + f(x_{2N})\}\Big]$$

Que find $\int_0^1 \frac{1}{1+x}\,dx$ → 2 interval → $N = 1$; → 8 sub int → $N = 4$

अब Gauss को दूंढीये

Gauss Legendre Integr Rule :—

$$I = \int_{-1}^{1} f(x)\,dx \qquad \text{limit } [-1, 1] \qquad \omega(x) = 1 \text{ (weight fun)}$$

two point $\int_{-1}^{1} f(x)\,dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

remark · $\frac{1}{\sqrt{3}}$ =

अब limit $[-1, 1]$ नहीं है तो transform!!

$$\int_a^b f(x)\,dx \xrightarrow[\text{to}]{\text{transform}} \int_{-1}^{1} f(t)\,dt$$

$$x = \frac{1}{2}\left[(b-a)t + (b+a)\right] \qquad dx = \left(\frac{b-a}{2}\right)dt$$

value रखा है

$f(x)\,dx$ $f(x)\,\cancel{dx} = f(t)$

remark $\frac{1}{\sqrt{3}} = 0.5773$

$\boxed{\sqrt{3} = 1.7320}$

Three point

$$\int_{-1}^{1} f(x)dx = \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

Remark $\frac{5}{9} = 0.55$, $\sqrt{\frac{3}{5}} = 0.7741$, $\frac{8}{9} = 0.888$

Gauss Hermite

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx$$

(i) two point

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = \frac{\sqrt{\pi}}{2}\left[f\left(-\frac{1}{\sqrt{2}}\right) + f\left(\frac{1}{\sqrt{2}}\right)\right]$$

(ii) Gauss three point

Remark $\frac{\sqrt{\pi}}{2} = 0.8862$, $\frac{\sqrt{\pi}}{6} = 0.2954$, $\frac{1}{\sqrt{2}} = 0.7071$

$$\int_{-\infty}^{\infty} e^{-x^2} f(x)dx = \frac{\sqrt{\pi}}{6}\left[f\left(-\frac{\sqrt{6}}{2}\right) + 4f(0) + f\left(\frac{\sqrt{6}}{2}\right)\right]$$

Remark:- $\frac{\sqrt{\pi}}{2} = 0.8862$, $\frac{\sqrt{\pi}}{6} = 0.2954$

$\frac{1}{\sqrt{2}} = 0.7071$, $\frac{\sqrt{6}}{2} = 1.2247$

For Differentiation using Newton forward Δ

$y = f(x)$

$$f''(x) = \frac{d^2(y)}{d(x)^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \right]$$

$$f'(x) = \frac{d(y)}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \dots\right]$$

Note:-

by backward diff par $\frac{2}{n}$ term ko के आगे positive हो, बस ∇

$$f''(x) = \frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \frac{5}{6}\nabla^5 y_0\right]$$

$$f'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\Delta^2 f(x_0) + \frac{1}{3}\Delta^3 f(x_0) - \frac{1}{4}\Delta^4\right]$$

22nd April 2024 Mon

Singular Value Decomposition (SVD) of matrix

$$A_{m\times n} = U_{m\times n}\,\Sigma_{m\times n}\,V^T$$

Dim Compress

Divide करता है unitary matrix which orthogonal in a rectangular diag. matrix of singular values

$\Sigma_{m\times n}$: Single unity row

$V^T_{m\times n} \rightarrow$ orthogonal

$U_{m\times n} \rightarrow$ orthogonal

$U \cdot U^T = I$

$V \cdot V^T =$ Identity matrix

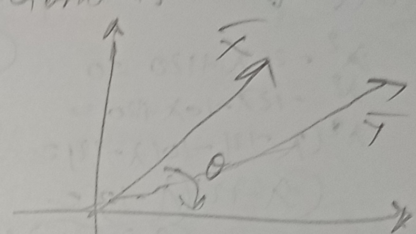
$$A^T \cdot A = (V\,\Sigma^T\,U^T)\,U\cdot\Sigma\cdot V^T$$

$$\boxed{A^T A = V\,\Sigma^T\,\Sigma\,V^T}$$

$$A\cdot A^T = (U\Sigma V^T)\cdot(V\cdot\Sigma^T U^T)$$

$$\boxed{A\cdot A^T = U\,\Sigma\Sigma^T\,U^T}$$

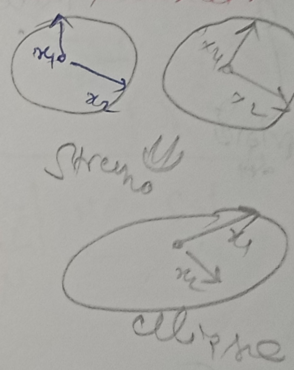
Rotation and Stretching



rotation . stretch . rotation

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \rightarrow$ Unitary transform

$\begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}$ scale axis parameter



Que $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}_{2\times 3}$

U find evaluate val and vector

$A^T = \begin{bmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}_{3\times 2}$

$A^T \cdot A = V$

$A A^T = U$

$A\cdot A^T = \begin{bmatrix} 11 & 1 \\ 1 & 11 \end{bmatrix}_{2\times 2}$

$|A - \lambda I| = 0$

$\begin{bmatrix} (11-\lambda) & 1 \\ 1 & (11-\lambda) \end{bmatrix} = 0$

$(11-\lambda)^2 - 1^2 = 0 \rightarrow (A+B)(A-B) = 0$

~~$121 + \lambda^2 - 22\lambda - 1 = 0$~~

$(11-\lambda+1)(11-\lambda-1) = 0$

$(12-\lambda)(10-\lambda) = 0$

eigen value $\lambda_1 = 10$, $\lambda_2 = 12$

for $\lambda_1 = 10$ की eigen vector.

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\sqrt{1^2+1^2} = \sqrt{2}$

$\sqrt{1^2 - 1^2}$

$\begin{bmatrix} 1 & 1 \end{bmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (1, 1)$

Gram-Schmidt

orthogonal

$A = U\cdot\Sigma\cdot U^T$

$\begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \end{bmatrix} = U$

Difference Operator :-

FORWARD

$$\Delta f(x_i) = f(x_{i+1}) - f(x_i)$$

$\Delta^2 f(x_i)$,

$\Delta^3 f(x_i)$

BACKWARD

$\nabla f(x_i)$

Central

$\delta f(x_i)$

$h = (x_1 - x_0)$ equispaced, common difference

Numerical Integration :-

Limit Difference

equispace — finite — NEWTON COTES

Trapezium Rule

Simpson's 1/3 Rule

unequispace — Unequal dist — GAUSS QUADRATURE

Gauss Legendre Rule

Gauss Hermite Rule

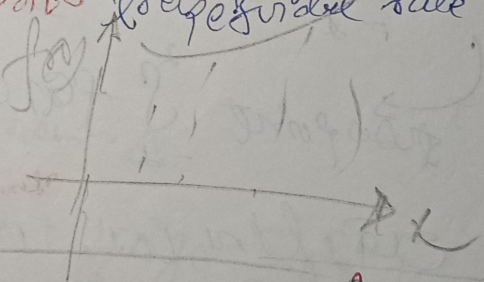
- Trapezium Rule :→

$$h = \frac{b-a}{N}$$

N for intervals

Composite trapezoidal rule :-

$$I = \int_a^b f(x)\,dx$$



$$\boxed{I = \frac{h}{2}\left[f(x_0) + 2\{f(x_1) + f(x_2) + \dots + f(x_{N-1})\} + f(x_N)\right]}$$

Que find $I = \int_0^1 \frac{1}{1+x}\,dx$ & $N = 8$.

$h = \frac{1-0}{8} = \frac{1}{8}$

Exact sol : $I = \ln 2 = 0.69147$

$$\boxed{\text{error} = \text{Exact sol} - I_{trap}}$$

Step 2 :- $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 3 & -1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$

$$A^T \cdot A = \begin{bmatrix} 10 & 0 & 2 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix}_{3\times 3}$$

$|A - \lambda I| = 0$

S_2 = minor of diag

$$= \begin{bmatrix} (10-\lambda) & 0 & 2 \\ 0 & (10-\lambda) & 4 \\ 2 & 4 & (2-\lambda) \end{bmatrix} \rightarrow$$ Shortcut, $\lambda^3 - S_1 \lambda^2 + S_2 \lambda - S_3$

S_1 = trace (A) = 22

S_3 = det(A) = 0

characteristic eqn

$\lambda^3 - 22\lambda^2 + 120\lambda = 0$

$\lambda(\lambda^2 - 22\lambda + 120) = 0$

$\lambda_1 = 0$

$\lambda^2 - 22\lambda + 120 = 0$

$\lambda^2 - 12\lambda - 10\lambda + 120 = 0$

$\lambda(\lambda - 12) - 10(\lambda - 12) = 0$

$(\lambda - 12)(\lambda - 10) = 0$

$\lambda = 12, 10$ Ans

$\lambda_2 = 12$

$\lambda_3 = 10$

S_2 = min

$\begin{vmatrix} 10 & 4 \\ 4 & 2 \end{vmatrix} = 4$, $\begin{vmatrix} 10 & 2 \\ 2 & 2 \end{vmatrix} = 16$

$\begin{vmatrix} 10 & 0 \\ 0 & 10 \end{vmatrix} = 100$

$4 + 16 + 100 = 120 = S_2$

$V^T = \begin{bmatrix} V_1^T \\ V_2^T \\ V_3^T \end{bmatrix}$ singular value

$\lambda_2 = 12$

$$\begin{bmatrix} -2 & 0 & 2 \\ 0 & -2 & 4 \\ 2 & 4 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$(A - \lambda I)X = 0$

By cramer's Rule,

$\lambda_1 = 12$

$V = \begin{bmatrix} \\ \\ \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} -2 & 4 \\ 4 & -10 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 2 \\ 2 & -10 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & -2 \\ 2 & 4 \end{vmatrix}} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

4, 4, 2

$$V = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & -5 \end{bmatrix}_{3\times 3}$$

eig val $\lambda_1 = 12$, eig $\lambda_2 = 10$

$$V^T = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 6 \\ 1 & 2 & -5 \end{bmatrix}$$

$\rightarrow \sqrt{1^2 + 2^2 + 1} = \sqrt{6}$

$\sqrt{4 + 1 + 36} = \sqrt{41}$

$\rightarrow \sqrt{1 + 4 + 25} = \sqrt{30}$

$\lambda_2 = 10$ — remove

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 4 \\ 2 & 4 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{-16} = \frac{-x_2}{4} = \frac{x_3}{0}$$

$\lambda_3 = 0$

$$\begin{bmatrix} 10 & 0 & 2 \\ 0 & 10 & 4 \\ 2 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{x_1}{4} = \frac{-x_2}{-8} = \frac{x_3}{-20}$$

On equating both sides. row-wise

$(l_{11})^2 = 1 \Rightarrow l_{11} = 1$ | $(l_{21})^2 + (l_{22})^2 = 8 \Rightarrow l_{22}^2 = 4$, $l_{22} = 2$

$l_{11} l_{21} = 2 \Rightarrow l_{21} = 2$

$l_{11} l_{31} = 3 \Rightarrow l_{31} = 3$ | $l_{21} l_{31} + l_{32} l_{22} \Rightarrow 2\times 3 + 2 l_{32}$ ...

$l_{33} = 3$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix}$$

$AX = B$ ...(1) $A = LL^T$ ...(2)

eq (1) & (2)

$LL^T X = B$ ...(3)

put $L^T X = Y$ ...(4) where $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

then (3) become

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 8 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

Solve $y_1 = 5$

$2y_1 + 2y_2 = 6$

$10 + 2y_2 = 6$

$y_2 = -2$

$3y_1 + 8y_2 + 3y_3 = -10$

$3(5) + 8(-2) + 3y_3 = -10$

$15 - 16 + 3y_3 = -10$

$3y = -9$

$y_3 = -3$

from (4)

$L^T X = Y$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 8 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$$

$2z = -3$

$\Rightarrow z = -3/2$

~~$2y = -2$~~ ~~$y = 7$~~

$2y + 8z = -2 \Rightarrow y = 7$

$x + 2y + 3z = 5 \Rightarrow x = 2$ Ans

13th April 2024 Today Cp Lecture 1

QR Decomposition

any matrix $A = QR$ where Q = orthogonal matrix of normal $Q = m\times n$

R = upper triangular matrix invertible $R = n\times n$

linearly indep column

Q now to find Q?

Step (1) Let column of A be $a_1, a_2, \dots a_n$ let A be matrix of order $m\times n$

i.e. $A = [a_1 \; a_2 \; a_3 \dots a_n]$ m = row, n = column

Step 2:- Use Gram-Schmidt process to make the column of A into pairwise orthonormal as follows:

Define new vectors $u_1, u_2, \dots u_n$ using column of A

$u_1 = a_1 \Rightarrow e_1 = \frac{u_1}{\|u_1\|}$ length of u_1 ... পর্যন্ত দৈর্ঘ্য u_1

$u_2 = a_2 - (a_2 \cdot e_1)e_1 \Rightarrow e_2 = \frac{u_2}{\|u_2\|}$ = unit vector

$u_3 = a_3 - (a_3 \cdot e_1)e_1 - (a_3 \cdot e_2)e_2 \Rightarrow e_3 = \frac{u_3}{\|u_3\|}$

⋮

$u_n = a_n - (a_n \cdot e_1)e_1 - (a_n \cdot e_2)e_2 \cdots (a_n \cdot e_{n-1})e_{n-1}$

Here $e_1, e_2, \cdots e_n$ are pairwise orthoNormal when $e_n = \frac{u_n}{\|u_n\|}$

$\therefore Q = [e_1 \; e_2 \; \cdots \; e_n] \Rightarrow \boxed{Q^{-1} = Q^T}$

inverse of Q — trans of Q

OrthoNormal

$|\vec{v_1}| = 1$, $|v_2| = 1$, $(v_1) \perp (v_2)$ orthonormal

eg:- $\{e_1, e_2, \cdots e_n\}$ when $e_1 = (1, 0, 0, \ldots)$

$e_2 = (0, 1, 0, \ldots)$

$e_n = (0, 0, \ldots 1)$

length $= \sqrt{(e_1)^2 + (e_2)^2}$

$Q^{-1} \cdot Q = I$

Q How to find R :-

Soln we have $A = QR$

pre multip. on the both side Q^{-1}

$Q^{-1}(A) = Q^{-1}(QR) = \cancel{Q^{-1}Q}R$

$= IR = R$

thus $R = Q^{-1}A = \boxed{Q^TA} \rightarrow R$

Finally express the matrix A as $A = QR$

eg:- find QR decomp. using of mat $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}_{3 \times 3}$ using Gram-Schmidt process

Soln:- Step 1 let $A = [a_1 \; a_2 \; a_3]$

then, $a_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \; a_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \; a_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Step 2:- find Q using Gram Schmidt process

$A = QR$

Now, define u_1, u_2, u_3

$u_1 = a_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

magnitude $= \|u_1\|$ = Length of $a_1 = \sqrt{(1)^2 + (1)^2 + (0)^2}$

$= \sqrt{2}$

unit $\Rightarrow e_1 = \frac{u_1}{\|u_1\|} = \frac{u_1}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$