<u>**Solutions**</u>

1) Let $f(x)=\begin{cases} \dfrac{x^3-x^2-16x+20}{(x-2)^2}, & x \ne 0 \\ k, & x=2 \end{cases}$, if f(x) is continuous for all x, then k?

(A) 3 (B) 5 (C) 7 (D) 9

<u>Sol</u>: Option (C)

<u>Explanation</u>:
Given f(x) is continuous

$$\boxed{\operatorname*{lt}_{x\to a} f(x)=f(a)}$$

k = f(2)

$$\Rightarrow \operatorname*{lt}_{x\to 2} \frac{x^3+x^2-16x+20}{(x-2)^2}=k$$

Applying L- Hospital rule

$$\Rightarrow \operatorname*{lt}_{x\to 2} \frac{3x^2+2x-16}{2(x-2)} \qquad \left[\frac{0}{0}\text{form}\right]$$

$$\Rightarrow \operatorname*{lt}_{x\to 2} \frac{6x+2}{2}=k$$

$$\Rightarrow \frac{14}{2}=k$$

$$\boxed{k=7}$$

2) Let $f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0 \end{cases}$, if f(x) is continuous at x =0, then find a, b, c?

(A) a + c = 0, b = 1
(B) a + c = 1, b ϵ R
(C) a + c = -1, b ϵ R
(D) a + c = -1, b = -1

<u>Sol</u>: Option (C)

<u>Explanation</u>:
Given f(x) is continuous

$$\boxed{\lim_{x\to a} f(x)=f(a)}$$

<u>L.H</u>:

$$\lim_{x\to 0}\frac{\sin(a+1)x+\sin x}{x} \quad \left[\frac{0}{0}\text{form}\right]$$

Applying L-Hospital rule

$$\Rightarrow \lim_{x\to 0}(a+1)\cos(a+1)x+\cos x$$

$\Rightarrow$ a + 2

<u>R.H:</u>

$$\lim_{x\to 0}\frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}$$

$$\Rightarrow \lim_{x\to 0}\frac{\sqrt{x}\left(\sqrt{1+bx}-1\right)}{bx\sqrt{x}}$$

$$\Rightarrow \lim_{x\to 0}\frac{\sqrt{1+bx}-1}{bx}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sqrt{1+bx}-1}{bx} \times \frac{\sqrt{1+bx}+1}{\sqrt{1+bx}+1}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sqrt{1+bx}-1}{bx} \times \frac{\sqrt{1+bx}+1}{\sqrt{1+bx}+1}$$

$$\Rightarrow \lim_{x\to 0} \frac{1+bx-1}{bx\left(\sqrt{1+bx}+1\right)}$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\sqrt{1+bx}+1}$$

$$= \frac{1}{2}$$

Since f(x) is continuous

L.H = R.H = f(x)

$a+2 = \frac{1}{2} = c$

$a + c = -1$

$\boxed{c = \frac{1}{2}}$, $\boxed{a = -\frac{3}{2}}$ $b \in R$

3) If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2)$, $[\cdot]$ denotes greatest integer function, is continuous & differentiable in (4,6) then find 'a' range?

(A) a ϵ (-∞,∞)

(B) a ϵ [64, ∞)

(C) a ϵ [128, ∞)

(D) Not defined

<u>Sol</u>: Option (B)

<u>Explanation:</u>

Since $\left[\frac{(x-2)^3}{a}\right]$ is not continuous and differentiable at integral point (because [ ] functions are not continuous)

Given function f(x) is continuous and differentiable in (4,6) if $\left[\frac{(x-2)^3}{a}\right] = 0$

$\Rightarrow$ a $\geq$ 64 [since if we have a value less than 64, we will get 1 as step function, but we need step function to be'0']

4) If the derivative of the function $f(x) = \begin{cases} bx^2 + ax + 4 \,;\, x \geq -1 \\ ax^2 + b \quad ;\, x < -1 \end{cases}$ is continuous everywhere, then a, b values?

(A) a = 2, b = 3

(B) a = -2, b = -3

(C) a = -2, b = 3

(D) a = 2, b = -3

Sol: Option (A)

Explanation :

$$f(x) = \begin{cases} bx^2 + ax + 4 \,;\, x \geq -1 \\ ax^2 + b \quad ;\, x < -1 \end{cases}$$

Given derivative of f(x0 is continuous everywhere that means f(x) is continuous at '-1'

$\Rightarrow \lim\limits_{x\to -1-} f(x) = \lim\limits_{x\to -1+} f(x)$

$\Rightarrow$ a + b = b(-1)^2 + a(-1) +4

$\Rightarrow$ a + b = b – a + 4

$\Rightarrow$ $\boxed{a = 2}$

Given derivative of f(x) continuous

$$f'(x) = \begin{cases} 2bx + a \,;\, x \geq -1 \\ 2ax \quad ;\, x < -1 \end{cases}$$

$\Rightarrow \lim\limits_{x\to -1-} f'(x) = \lim\limits_{x\to -1+} f'(x)$

$\Rightarrow$ 2a (-1) = 2b(-1) + a

$\Rightarrow$ -2a = -2b + a

$\Rightarrow$ 3a = 2b

b = 3

$\boxed{\therefore a = 2, b = 3}$

5) If the function $f(x) = \begin{cases} (1+|\sin x|)^{a/|\sin x|} & , -\frac{\pi}{6} x < 0 \\ b & , x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}} & , 0 < x < \frac{\pi}{6} \end{cases}$ is continuous at x = 0, then a, b?

Given f(x) is continuous at 0

$$\boxed{\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)} \qquad \text{---------- (1)}$$

LHL:

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$$

$$= \lim_{h \to 0} \left(1 + (\sin(-h))\right)^{a/|\sin(-h)|}$$

$$= \lim_{h \to 0} \left(1 + \sin h\right)^{a/\sin h}$$

$$= e^{\lim_{h \to 0} (1 + \sin h - 1)^{a/\sin h}} \qquad \left[\because \lim_{x \to a} f(x)^{g(x)} = \lim_{x \to a} (f(x) - 1) g(x)\right]$$

$$= e^{\lim_{h \to 0} \sin h \cdot a/\sin h}$$

$$= e^{a}$$

RHL:

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(h)$$

$$= \lim_{h \to 0} e^{\frac{\tan 2h}{\tan 3h}}$$

$$= e^{\underset{h\to 0}{\text{lt}} \frac{2}{3}\cdot\frac{\tan 2h}{2}\cdot\frac{3h}{\tan 3h}}$$

$$= e^{\frac{2}{3}} \qquad \left[\underset{h\to 0}{\text{lt}} \frac{\tanh}{h} = 1\right]$$

From eq (1)

$$b = e^{\frac{2}{3}} = e^{a}$$

$$\boxed{\therefore a = \frac{2}{3},\ b = e^{\frac{2}{3}}}$$

6) The value of P for which the function $f(x) = \begin{cases} \dfrac{(4^x - 1)^3}{\sin\left(\frac{x}{p}\right) ln\left(1+\frac{x^2}{3}\right)}, & x \neq 0 \\ k, & x = 0 \end{cases}$, is continuous at x = 0

(A) 1 (B) 2 (C) 4 (D) 8

Sol: Option (C)

Explanation:

Given f(x) is continuous

f(0) = RHL

$$\Rightarrow \underset{x\to 0^+}{\text{lt}} f(x) = \underset{h\to 0}{\text{lt}} f(h)$$

$$\Rightarrow 12(ln\ 4)^3 = \underset{h\to 0}{\text{lt}} \frac{(4^h - 1)^3}{\sin\left(\frac{h}{p}\right) ln\left(1+\frac{h^2}{3}\right)}$$

$$= \lim_{h\to 0} \frac{\left(\frac{4^h - 1}{h}\right)^3}{\frac{\sin\left(\frac{h}{p}\right)}{\left(\frac{h}{p}\right)} \cdot \frac{ln\left(1+\frac{h^2}{3}\right)}{\left(\frac{h^2}{3}\right)} \cdot \frac{1}{3p}}$$

$$= \frac{(ln\,4)^3 \cdot 3p}{1 \cdot 1}$$

$\Rightarrow$ f(0) = RHL

$12(ln\,4)^3 = (ln\,4)^3\, 3p$

$$\boxed{p = 4}$$

7) The value of f(0), so that the function $f(x) = \frac{1-\cos(1-x)}{x^4}$ is continuous everywhere is _____?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

Sol: Option (C)

Explanation:

Given f(x) is continuous every where

At x = 0

$$f(0) = \text{RHL} = \lim_{x\to 0^+} f(x)$$

$$= \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0} \frac{1-\cos(1-\cosh)}{h^4}$$

$$= \lim_{h\to 0} \frac{1-\cos(1-\cosh)}{h^4} \times \frac{1+\cos(1-\cosh)}{1+\cos(1-\cosh)}$$

$$= \lim_{h\to 0} \frac{\sin^2(1-\cos h)}{(1+\cos(1-\cos h))} \times \frac{(1-\cos h)^2}{(1-\cos h)^2} \cdot \frac{1}{h^4}$$

$$= \lim_{h\to 0}\left[\frac{\sin(1-\cos h)}{(1-\cos h)}\right]^2 \times \lim_{h\to 0}\left(\frac{1-\cos h}{h}\right)^2 \times \lim_{h\to 0}\frac{1}{1+\cos(1-\cos h)}$$

$$= (1)^2 \times \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

8) The function $f(x) = (\sin 3x)^{\tan^2 3x}$ is not defined at x = $\frac{\pi}{6}$. The value of $f\left(\frac{\pi}{6}\right)$, so that the f is continuous at x= $\frac{\pi}{6}$, is?

(A) e (B) $e^{-\frac{1}{2}}$ (C) $e^{\frac{1}{2}}$ (D) e^2

<u>Sol</u>: Option (B)

<u>Explanation</u>:

For f(x) to be continuous at x = $\frac{\pi}{6}$

$$f\left(\frac{\pi}{6}\right) = \lim_{x\to\frac{\pi}{6}} f(x)$$

$$f\left(\frac{\pi}{6}\right) = \lim_{x\to\frac{\pi}{6}} (\sin 3x)^{\tan^2 3x}$$

$$= \lim_{x\to\frac{\pi}{6}} \left\{(\sin^2 3x)^{1/2}\right\}^{\tan^2 3x}$$

$$= \operatorname*{lt}_{x \to \frac{\pi}{6}} \left\{ (1 - \cos^2 3x)^{1/2} \right\}^{\tan^2 3x}$$

$$= \operatorname*{lt}_{x \to \frac{\pi}{6}} \left\{ (1 - \cos^2 3x)^{1/2} \right\}^{\frac{\sin^2 3x}{\cos^2 3x}}$$

$$= \operatorname*{lt}_{x \to \frac{\pi}{6}} \left\{ (1 - \cos^2 3x)^{\frac{1}{\cos^2 3x}} \right\}^{\sin^2 3x}$$

$$= e^{\operatorname*{lt}_{x \to \frac{\pi}{6}} \left\{ (1 - \cos^2 3x - 1) \times \frac{1}{\cos^2 3x} \right\}^{\frac{\sin^2 3x}{2}}}$$

$$\left[\because \operatorname*{lt}_{x \to a} f(x)^{g(x)} = e^{\operatorname*{lt}_{x \to a} (f(x) - 1) g(x)} \right]$$

$$= e^{\operatorname*{lt}_{x \to \frac{\pi}{6}} \left\{ (-\cos^2 3x) \times \frac{1}{\cos^2 3x} \right\}^{\frac{\sin^2 3x}{2}}}$$

$$\left[\because \sin^2\left(3 \cdot \frac{\pi}{6}\right) = \sin^2 \frac{\pi}{2} = 1 \right]$$

$$= e^{\frac{-1}{2}}$$

9) The function $f(x)=\begin{cases} ax^2 - bx + 2\,; x<3 \\ bx^2 - 3 \quad ; x \geq 3 \end{cases}$ is differentiable everywhere then find a, b?

Given f(x) is differentiable, which means f(x) is continuous.

$$\therefore f(3) = \lim_{x\to 3^-} f(x)$$

$$b(9) - 3 = \lim_{h\to 3} f(3-h)$$

$$= \lim_{h\to 3} a(3-h)^2 - b(3-h) + 2$$

$$= \lim_{h\to 3} a(9+h^2-6h) - 3b + bh + 2$$

$$9b - 3 = 9a - 3b + 2$$

$$\Rightarrow \boxed{9a - 12b = -5} \quad \text{----------} \ (1)$$

Given f(x) is differentiable,

$$f'(x)=\begin{cases} 2ax - b\,; x<3 \\ 2bx \quad ; x \geq 3 \end{cases}$$

$$f'(3) = \lim_{x\to 3^-} f'(x)$$

$$= \lim_{h\to 0} f'(3-h)$$

$$= \lim_{h\to 0} 2a(3-h) - b$$

$$= 6a - b$$

$$f'(3) = 6b$$

$$\Rightarrow 6a - b = 6b$$

$$\Rightarrow \boxed{6a = 7b} \quad \text{-----------} \ (2)$$

By solving (1) & (2), we get

$a = \frac{35}{9}$, $b = \frac{10}{3}$

10) Given $f(x) = \begin{cases} \frac{1-\cos ax}{x \sin x} & , \quad x \neq 0 \\ \frac{1}{2} & , \quad x = 0 \end{cases}$, if f is continuous at x = 0, then the value of a^2 must be ____

(A) 1 (B) -1 (C) 0 (D) 2

Sol: Option (A)

Explanation:

Since f is continuous at x = 0, then

f(0) = RHL

$$\frac{1}{2} = \underset{x \to 0^+}{\text{lt}} f(x) = \underset{h \to 0}{\text{lt}} f(0+h)$$

$$\Rightarrow \frac{1}{2} = \underset{h \to 0}{\text{lt}} \frac{1-\cos ah}{h \sin h}$$

$$\Rightarrow \underset{h \to 0}{\text{lt}} \frac{1-\cos ah}{h^2 \left(\frac{\sin h}{h}\right)} = \frac{a^2}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2}{2}$$

$$\boxed{a^2 = 1}$$

11) Let $f(x)=\begin{cases}\dfrac{1-\tan x}{4x-\pi} & , \ x\neq\dfrac{\pi}{4} \\ \lambda & , \ x=\dfrac{\pi}{4}\end{cases}, \ x\in\left(0,\dfrac{\pi}{2}\right]$

if f(x) is continuous in $\left(0,\dfrac{\pi}{2}\right]$ then λ is?

(A) $\dfrac{3}{2}$ (B) $-\dfrac{3}{2}$ (C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{2}$

Sol: Option (C)

Explanation:

$$f(x)=\frac{1-\tan x}{4x-\pi}$$

Since f(x) is continuous in $\left(0,\dfrac{\pi}{2}\right]$

$$\text{RHL} = \lim_{x\to\frac{\pi}{4}^{+}} f(x)$$

$$= \lim_{h\to 0} f\left(\frac{\pi}{4}+h\right)$$

$$= \lim_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}+h\right)}{4\left(\frac{\pi}{4}+h\right)-\pi} \qquad \left[\because \text{Tan}(A+B)=\frac{\text{TanA}+\text{TanB}}{1-\text{TanATanB}}\right]$$

$$= \lim_{h\to 0}\frac{1-\left(\frac{1+\tan h}{1-\tan h}\right)}{4h}$$

$$= \lim_{h\to 0}\frac{-2\tan h}{(1-\tan h)4h}$$

$$= \lim_{h\to 0}\frac{-2\left(\frac{\tan h}{h}\right)}{(1-\tan h)4}$$

$$= \frac{-1}{2} \qquad \left[\because \underset{h\to 0}{\mathrm{lt}} \frac{\tanh}{h} = 1\right]$$

$\mathrm{RHL} = \lambda$

$$\boxed{\lambda = \frac{-1}{2}}$$

12) Which one of the following is continuous at x = 3?

(a) $f(x) = \begin{cases} 2 & ,\text{if } x = 3 \\ x - 1 & ,\text{if } x > 3 \\ \dfrac{x+3}{3} & ,\text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4 & ,\ \text{if } x = 3 \\ 8 - x & ,\ \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 3, & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3 - 27}$, if $x \neq 3$

<u>Sol</u>: Option (A)

<u>Explanation</u>:

Given function is continuous at x = 3, which means

$$\boxed{\text{LHL} = \text{RHL} = f(x)}$$

$$\underset{x\to 3}{\mathrm{lt}}\ f(x) = f(3)$$

<u>A:</u>

$$f(x) = \begin{cases} 2 & ,\text{if } x = 3 \\ x - 1 & ,\text{if } x > 3 \\ \dfrac{x+3}{3} & ,\text{if } x < 3 \end{cases}$$

f(3)= 2

LHL

$\lim\limits_{x\to 3^-} f(3-h)$

$\Rightarrow \lim\limits_{h\to 0} \frac{3-h+3}{3} = \frac{6}{3} = 2$

RHL

$\lim\limits_{x\to 3^+} f(3+h)$

$\Rightarrow \lim\limits_{h\to 0} 3+h-1 = 2$

$\boxed{\therefore \text{LHL} = \text{RHL} = f(3)}$

f(x) is continuous

B:

$$f(x) = \begin{cases} 4 \ , & \text{if } x = 3 \\ 8 - x \ , & \text{if } x \neq 3 \end{cases}$$

LHL

$\lim\limits_{x\to 3^-} f(3-h) = 8 - (3 - h)$

$= 5 + h$

$= 5$

LHL $\neq$ f(3)

$\therefore$ f(x) is not continuous

C:

$$f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 3 \ , & \text{if } x > 3 \end{cases}$$

RHL

$\lim\limits_{x\to 3^+} f(3+h) = (3 + h) - 4$

$= h - 1$

$= -1$

RHL $\neq$ f(3)

$\therefore$ f(x) is not continuous

D:

Given $f(x) = \dfrac{1}{x^3 - 27}$ is not defined when x= 3, so f(x) is not continuous.

Maxima & Minima

13) What is the maximum value of the function $f(x) = x^2 - 2x + 6$ in the interval [0,2]?

(A) -9 (B) 9 (C) 7 (D) -7

Sol: Option (B)

Explanation:

Given $f(x) = x^2 - 2x + 6$

In order to find max (or) min value, we need to find f(x) = 0

$f'(x) = 2x - 2$

$\Rightarrow 2x = 2$

$\boxed{x = 1}$

$f''(x) = 2$

$\therefore f''(x) > 0$

f(x) has minimum at 'x = 1'

f(0) = 6 f(3) = 9 – 6 + 6 = 9 f(1) = 1 – 2 + 6 = 5

Maximum value of f(x) in given interval [0, 3] is 9

14) A point on a curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is __?

(A) 0 (B) 1 (C) 2 (D) 3

Sol: Option (B)

Explanation:

Given $f(x) = 3x^4 - 16x^3 + 24x^2 + 37$

$$f'(x) = 12x^3 - 48x^2 + 48x$$

$$f''(x) = 36x^2 - 96x + 48$$

In order to find stationary points we need to equal $f'(x) = 0$

$$\Rightarrow x\left(12x^2 - 48x + 48\right) = 0$$

$x = 0$, $x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x = 0, 2$

At x = 0

$$f''(0) > 0$$

$\therefore$ f(x) has minimum value at x = 0

$f(0) = 0 + 0 + 0 + 37 = 37$

At x =2

$f''(2) = 0$ and $f'''(2) \neq 0$

$\therefore$ f(x) has no extremum at x = 2

Number of distinct extrema for curve is '1'

15) Find the points of local maxima and minima, of the function $f(x) = x^3 - 6x^2 + 9x + 15$ in [0, 5].

(A) (1, 3) (B) (1, -3) (C) (-1, 3) (D) (-1, -3)

Sol: Option (A)

Explanation:

Let $f(x) = x^3 - 6x^2 + 9x + 15$

$$f'(x) = 3x^2 - 12x + 9$$

$$f''(x) = 6x - 12$$

Equating $f'(x)$ to 0, we get stationary points

$$f'(x) = 0$$

$\Rightarrow 3x^2 - 12x + 9 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

At x = 1

$$f''(x) < 0$$

$$f''(1) < 0$$

$-6 < 0$

$\therefore$ f(x) has maximum a x = 1

$f(1) = 1 - 6 + 9 + 15 = 19$

At x = 3

$$f''(x) > 0$$

$18 - 12 > 0$

$$6 > 0$$

$\therefore$ f(x) has minimum at x = 3

$$f(3) = 27 - 54 + 27 + 15 = 15$$

16) What is the local minimum value of $f(x) = x^3(x+4)$?

(A) -189 (B) -27 (C) 27 (D) 189

Sol: Option (B)

Explanation:

$$f(x) = x^3(x+4) = x^4 + 4x^3$$

$$f'(x) = 4x^3 + 12x^2$$

$$f''(x) = 12x^2 + 24x$$

For finding maximum (or) minimum put $f'(x) = 0$

$$\Rightarrow 4x^3 + 12x^2 = 0$$

$$\Rightarrow 4x^2(x+3) = 0$$

$$x = 0 \text{ or } -3$$

$$f''(0) = 0$$

$$f''(-3) = 12(-3)^2 + 24 \times (-3) = 108 - 72 = 36 > 0$$

$\therefore$ f(x) has local minimum at x = 3

$$\boxed{f(-3) = (-3)^2(-3+4) = -27}$$

17) Find the local maximum and local minimum if any, for the function f(x) = sin x + cos x, $0 < x < \frac{\pi}{2}$

(A) $\left(\frac{\pi}{4}, -\sqrt{2}\right)$ (B) $\left(\frac{\pi}{2}, \sqrt{2}\right)$ (C) $\left(\sqrt{2}, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{4}, \sqrt{2}\right)$

<u>Sol</u>: Option (D)

<u>Explanation</u>:

We have f(x) = sin x + cos x

$$f'(x) = \cos x - \sin x$$

For finding local minima / maxima, $f'(x) = 0$

$\Rightarrow \cos x - \sin x = 0$

$\Rightarrow \cos x = \sin x$

$\Rightarrow \tan x = 1$ (or) $x = \frac{\pi}{4}$ in $\left(0, \frac{\pi}{2}\right)$

At $x = \frac{\pi}{4}$

$f''(x) = -\sin x - \cos x$

$f''\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= 1 - \sqrt{2}$

$f''(x) > 0$

$\therefore$ f(x) has local maxima at $x = \frac{\pi}{4}$

$$f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ Point of local maxima is $\left(\frac{\pi}{4}, \sqrt{2}\right)$

18) What is the maximum (or) minimum point for curve f(x) = $4x - x^4$?
   (A) A minimum at (-1, -3)
   (B) A maximum at (-1, -3)
   (C) A minimum at (1, 3)
   (D) A maximum at (1, 3)

Sol: Option (D)

Explanation:

Given f(x) = $4x - x^4$

$\Rightarrow f'(x) = 4 - 4x^3$

$\Rightarrow f''(x) = -12x^2$

To find maximum (or) minimum, $f'(x) = 0$

$\Rightarrow 4 - 4x^3 = 0$

$\Rightarrow 4 = 4x^3$

$\boxed{x = 1}$

At x = 1:

f(x) = 4 – 1 = 3

$\therefore$ Maximum (or) minimum at (1, 3)

To decide whether it is max (or) min, consider the sign of $f''(x)$ at x = 1

$\Rightarrow f''(x) = -12 < 0$

$\therefore$ We have maximum value at (1, 3)

19) Find the local maxima and minima for function f(x) = cos 4x; 0 < x < $\frac{\pi}{2}$

(A) -1 (B) 1 (C) -2 (D) 2

Sol: Option (A)

Explanation:

f(x) = cos 4x

$f'(x) = -4\sin 4x$

$f''(x) = -16\cos 4x$

To find stationary points, $f'(x) = 0$

$\Rightarrow$ -4 sin 4x = 0

sin 4x = 0

4x = 0, π, 2π [since sin becomes zero when x = ± nπ]

4x = π

x = $\frac{\pi}{4}$ [given x ∈ $\left(0, \frac{\pi}{2}\right)$]

At x = $\frac{\pi}{4}$

$f''\left(\frac{\pi}{4}\right) = -16\cos 4\cdot\frac{\pi}{4} = -16\cos\pi$

= -16(-1) = 16 > 0

∴ f(x) has maximum a x = $\frac{\pi}{4}$

Minimum value $f\left(\frac{\pi}{4}\right)$ = cos π = -1

20) Find the local maximum and local minimum for function $f(x) = \frac{x}{1+x^2}$

(A) Minimum value at x = 1
(B) Minimum value at x= -1
(C) Maximum value at x = 1
(D) Maximum value at x = -1

Sol: Option (B)

Explanation:

$f(x) = \frac{x}{1+x^2}$

$$f'(x) = \frac{\left(1+x^2\right)\cdot 1 - 2x(x)}{\left(1+x^2\right)^2} \qquad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot du - u\cdot dv}{v^2}\right]$$

$$= \frac{1 + x^2 - 2x^2}{\left(1 + x^2\right)^2}$$

$$= \frac{1 - x^2}{\left(1 + x^2\right)^2}$$

For finding points of local maximum (or) minimum equate $f'(x) = 0$

$$\Rightarrow \frac{1 - x^2}{\left(1 + x^2\right)^2} = 0$$

$$1 - x^2 = 0$$

$$(1 + x)\,(1 - x) = 0$$

$$x = 1, -1$$

$$f''(x) = \frac{\left(1 + x^2\right)^2(-2x) - (1 - x^2)\,(2\,(1 + x^2)\,2x)}{\left(1 + x^2\right)^4} \qquad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot du - u \cdot dv}{v^2}\right]$$

<u>At x = 1:</u>

$$f''(x) = \frac{(1 + 1)(-2) - (1 - 1)\,(2\,(1 + 1)\,2)}{(1 + 1)^4}$$

$$= \frac{-4 - 0}{16} = \frac{-1}{4} < 0$$

$\therefore$ At x = 1, f(x) has maximum value

<u>At x = -1:</u>

$$f''(x) = \frac{(1 + 1)(2) - (1 - 1)\,(2\,(1 + 1)\,(-2))}{(1 + 1)^4}$$

$$= \frac{4}{16} = \frac{1}{4} > 0$$

$\therefore$ At x = -1, f(x) has minimum value