Continuity & Differentiability

Practice questions:

1) Let $f(x)=\begin{cases}\dfrac{x^3-x^2-16x+20}{(x-2)^2}, & x\neq 0\\ k, & x=2\end{cases}$, if f(x) is continuous for all x, then k?

(A) 3 (B) 5 (C) 7 (D) 9

2) Let $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0\end{cases}$, if f(x) is continuous at x =0, then find a, b, c?

(A) a + c = 0, b = 1
(B) a + c = 1, b ϵ R
(C) a + c = -1, b ϵ R
(D) a + c = -1, b = -1

3) If the function $f(x)=\left[\dfrac{(x-2)^3}{a}\right]\sin(x-2)+a\cos(x-2)$, [·] denotes greatest integer function, is continuous & differentiable in (4,6) then find 'a' range?

(A) a ϵ (-∞,∞)
(B) a ϵ [64, ∞)
(C) a ϵ [128, ∞)
(D) Not defined

4) If the derivative of the function $f(x)=\begin{cases}bx^2+ax+4; & x\geq -1\\ ax^2+b; & x<-1\end{cases}$ is continuous everywhere, then a, b values?

(A) a = 2, b = 3
(B) a = -2, b = -3
(C) a = -2, b = 3
(D) a = 2, b = -3

5) If the function $f(x)=\begin{cases}(1+|\sin x|)^{a/|\sin x|}, & -\dfrac{\pi}{6}<x<0\\ b, & x=0\\ e^{\frac{\tan 2x}{\tan 3x}}, & 0<x<\dfrac{\pi}{6}\end{cases}$ is continuous at x = 0, then a, b?

6) The value of P for which the function $f(x)=\begin{cases}\dfrac{(4^x-1)^3}{\sin\left(\dfrac{x}{p}\right)ln\left(1+\dfrac{x^2}{3}\right)}, & x\neq 0\\ k, & x=0\end{cases}$, is continuous at x = 0

(A) 1 (B) 2 (C) 4 (D) 8

7) The value of f(0), so that the function $f(x)=\dfrac{1-\cos(1-x)}{x^4}$ is continuous everywhere is _____?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

8) The function $f(x)=(\sin 3x)^{\tan^2 3x}$ is not defined at x = $\frac{\pi}{6}$. The value of $f\left(\frac{\pi}{6}\right)$, so that the f is continuous at x= $\frac{\pi}{6}$, is?

(A) e (B) $e^{-\frac{1}{2}}$ (C) $e^{\frac{1}{2}}$ (D) e^2

9) The function $f(x)=\begin{cases}ax^2-bx+2; & x<3\\ bx^2-3; & x\geq 3\end{cases}$ is differentiable everywhere then find a, b?

10) Given $f(x)=\begin{cases}\dfrac{1-\cos ax}{x\sin x}, & x\neq 0\\ \dfrac{1}{2}, & x=0\end{cases}$, if f is continuous at x = 0, then the value of a^2 must be ____

(A) 1 (B) -1 (C) 0 (D) 2

11) Let $f(x)=\begin{cases}\dfrac{1-\tan x}{4x-\pi}, & x\neq\dfrac{\pi}{4}\\ \lambda, & x=\dfrac{\pi}{4}\end{cases}$, $x\in\left(0,\dfrac{\pi}{2}\right]$

if f(x) is continuous in $\left(0,\frac{\pi}{2}\right]$ then λ is?

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

12) Which one of the following is continuous at x = 3?

(a) $f(x) = \begin{cases} 2 & ,\text{if } x = 3 \\ x - 1 & ,\text{if } x > 3 \\ \dfrac{x+3}{3} & ,\text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4 \ , & \text{if } x = 3 \\ 8 - x \ , & \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 3 \ , & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3 - 27}$, if $x \neq 3$

Maxima & Minima

13) What is the maximum value of the function $f(x) = x^2 - 2x + 6$ in the interval [0,2]?

(A) -9 (B) 9 (C) 7 (D) -7

14) A point on a curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is __?

(A) 0 (B) 1 (C) 2 (D) 3

15) Find the points of local maxima and minima, of the function $f(x) = x^3 - 6x^2 + 9x + 15$ in [0, 5].

(A) (1, 3) (B) (1, -3) (C) (-1, 3) (D) (-1, -3)

16) What is the local minimum value of $f(x) = x^3(x + 4)$?

(A) -189 (B) -27 (C) 27 (D) 189

17) Find the local maximum and local minimum if any, for the function f(x) = sin x + cos x, $0 < x < \dfrac{\pi}{2}$

(A) $\left(\dfrac{\pi}{4}, -\sqrt{2}\right)$ (B) $\left(\dfrac{\pi}{2}, \sqrt{2}\right)$ (C) $\left(\sqrt{2}, \dfrac{\pi}{2}\right)$ (D) $\left(\dfrac{\pi}{4}, \sqrt{2}\right)$

18) What is the maximum (or) minimum point for curve $f(x) = 4x - x^4$?

(A) A minimum at (-1, -3)

(B) A maximum at (-1, -3)

(C) A minimum at (1, 3)

(D) A maximum at (1, 3)

19) Find the local maxima and minima for function f(x) = cos 4x; $0 < x < \frac{\pi}{2}$

(A) -1 (B) 1 (C) -2 (D) 2

20) Find the local maximum and local minimum for function $f(x) = \frac{x}{1+x^2}$

(A) Minimum value at x = 1
(B) Minimum value at x= -1
(C) Maximum value at x = 1
(D) Maximum value at x = -1