ASSIGNMENT ON PROBABILITY

BASIC PROBABILITY

1. ~~Prob~~ Total outcome = $6 \times 6 = 36$

If 1st dice has 6, then even outcomes are: (6,2), (6,4), (6,6)

If 2nd dice shows 6, then outcomes are same.

$\therefore$ Possible outcomes $= \frac{6}{36} = \boxed{\frac{1}{6}}$ ← Ans ✓

2. Total outcome when 2 dies are rolled = 36

Outcomes where sum of the numbers will be either 7 or greater than 7

(1,6), (2,5), (2,6), (3,4), (3,5), (3,6), (4,3), (4,4), (4,5)
(4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2),
(6,3) . . . . (6,6)

$\therefore$ Probability of sum of numbers greater than 7 $= \frac{21}{36}$

$\therefore$ Prob of sum of numbers less than 7 $= 1 - \frac{21}{36} = \frac{15}{36} = \boxed{\frac{5}{12}}$ Ans ✓

3. Total outcome of tossing a coin 3 times $= 2^3 = 8$.

Possibilities

For 2 heads = (H,H,T), (H,T,H), (T,H,H)

For 3 heads = (H,H,H)

Total possible outcome = 4

$\therefore$ Probability of observing more than or equal to 2 heads (A_2)
$= \frac{4}{8} = \boxed{\frac{1}{2}}$

4) Probability of at least one head (A_1) = 1 − Prob of all tails
$= 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

$$\therefore P(A_2 | A_1) = \frac{P(A_2 \cap A_1)}{P(A_1)}$$

$$= \frac{P(A_2)}{P(A_1)}$$

$$= \frac{4}{8} / \frac{7}{8}$$

$$= \boxed{\frac{4}{7}} \quad \text{Ans}$$

4) Let A be event that both children are girls

$$A = \{G, G\}$$

Let B be event that atleast one of them is a girl

$$B = \{(G,G), (G,B), (B,G)\}$$

Total possibilities $= \{(G,G), (G,B), (B,G), (B,B)\}$

So, $P(A) = \frac{1}{4}$

So, $P(B) = \frac{3}{4}$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A subset of B

$$= \frac{P(A)}{P(B)} \quad \text{as } A \subset B$$

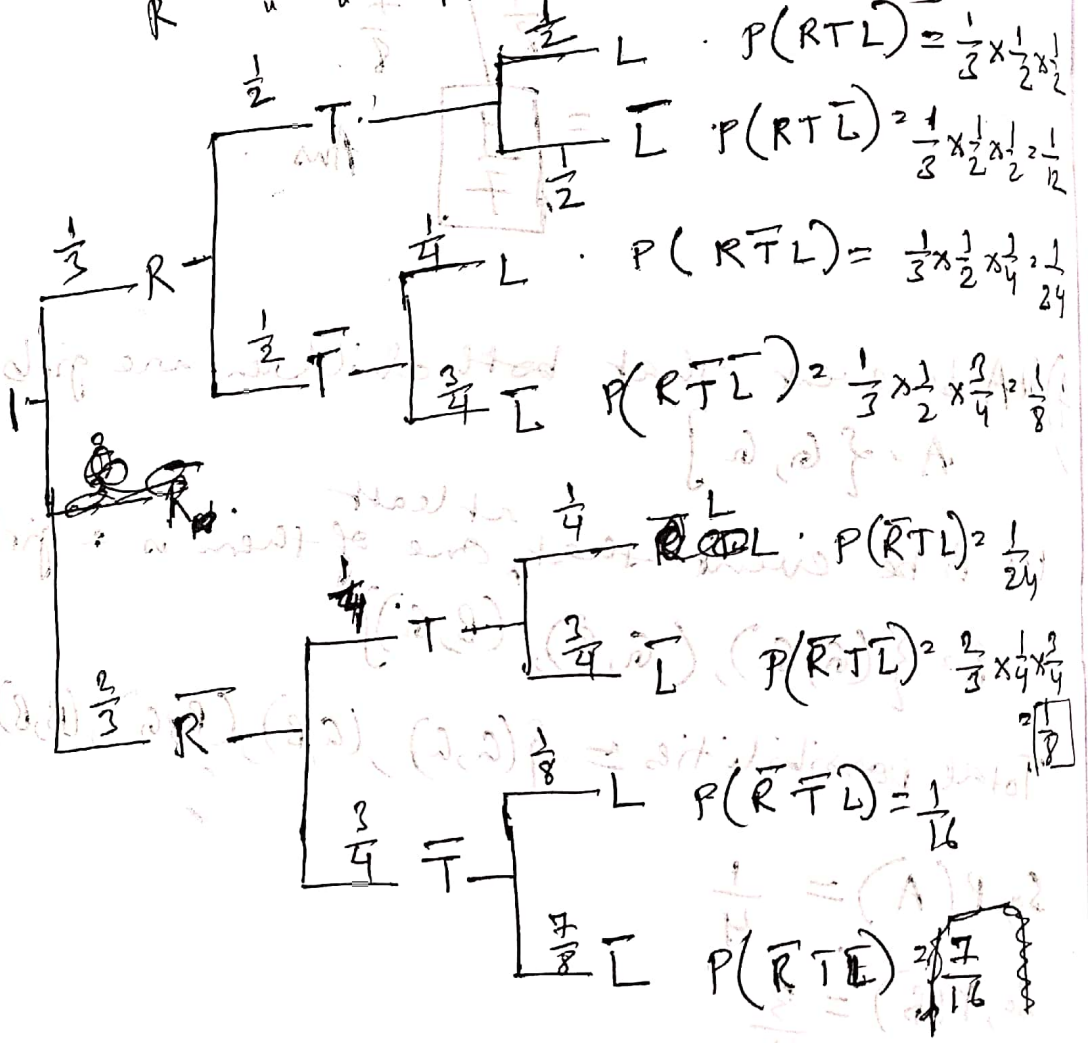
$$= \frac{1}{4} / \frac{3}{4} = \boxed{\frac{1}{3}} \quad \text{Ans}$$

5) Let R be event that it is raining

T " " that there is heavy traffic

L " " that I am late for work

$\bar{R}$ " " it is not raining



a) $P(\bar{R} \cap T \cap \bar{L}) = P(\bar{R}T\bar{L}) = \boxed{\frac{1}{8}}$ [from tree diagram] Ans

b) $P(L) = P(R,T,L) + P(R,\bar{T},L) + P(\bar{R},T,L) + P(\bar{R},\bar{T},L)$

$= \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \boxed{\frac{11}{48}}$ Ans

c) $P(R|L) = \dfrac{P(R \cap L)}{P(L)}$

$= \dfrac{P(R,T,L) + P(R,\overline{T},L)}{P(L)}$

$= \left(\frac{1}{12} + \frac{1}{24}\right) \Big/ \frac{11}{48}$

$= \frac{1}{8} \Big/ \frac{11}{48} = \boxed{\frac{6}{11}}$ Ans.

b) Let C_1 be event that regular coin is chosen

Let C_2 be event two-headed coin is chosen

$P(H|C_1) = \frac{1}{2}$ & $P(H|C_2) = 1$

a) $P(H) = P(H|C_1) \cdot P(C_1) + P(H|C_2) \cdot P(C_2)$

$= \frac{1}{2} \times \frac{2}{3} + 1 \times \frac{1}{3} = \boxed{\frac{2}{3}}$ Ans.

b) $P(C_2|H) = \dfrac{P(H|C_2) \cdot P(C_2)}{P(H)}$

$= \dfrac{1 \times \frac{1}{3}}{2/3}$

$= \boxed{\frac{1}{2}}$ Ans.

8) Probability of white ball $= \frac{1}{9} = P(W)$

Prob of A telling truth that a white ball was drawn $= P(T|W) = \frac{5}{6}$

Bayes theorem

$$P(W|T) = \frac{P(T|W) \times P(W)}{P(T)}$$

$$= \frac{P(T|W) * P(W)}{P(T|W) \times P(W) + P(T|W^c) \times P(W^c)}$$

$$= \frac{\frac{5}{6} \times \frac{1}{9}}{\left(\frac{5}{6} \times \frac{1}{9}\right) + \left(1 - \frac{5}{6}\right) \times \left(1 - \frac{1}{9}\right)}$$

$$= \frac{\frac{5}{6 \times 9}}{\frac{5}{6 \times 9} + \left(\frac{1}{6} \times \frac{8}{9}\right)}$$

$$= \boxed{\frac{5}{13}} \rightarrow \text{Ans}$$

9) Prob of A speak truth that dice shows 6

$= P(T|six) = \frac{4}{5}$

Probability of six $= P(six) = \frac{1}{6}$

$$P(\text{Six}|T) = \frac{P(T|\text{Six}) \cdot P(\text{Six})}{P(T|\text{Six}) \cdot P(\text{Six}) + P(T|\text{Not Six}) \cdot P(\text{Not Six})}$$

$$= \frac{\frac{4}{5} \times \frac{1}{6}}{\left(\frac{4}{5} \times \frac{1}{6}\right) + \left(1 - \frac{4}{5}\right) \times \frac{5}{6}}$$

$$= \boxed{\frac{4}{9}} \rightarrow \text{Ans.}$$

10) $P(M \cap S) = 40\%$.

$P(M) = 60\%$.

$$P(S|M) = \frac{P(S \cap M)}{P(M)} = \frac{40\%}{60\%} = \boxed{\frac{2}{3}}$$

Ans.

11) a) Probability that the individual is a male & a graduate $= \frac{19}{100} = \boxed{0.19}$

$P(M \cap G)$

It is a joint probability

b) Prob that individual is a male $= \frac{60}{100}$

$P(M) = \boxed{\frac{3}{5}}$.

It is a marginal probability

c) $P(F \mid \text{Post graduate}) = P(F|PG)$

$$= \frac{P(F \cap PG)}{P(PG)} = \frac{28/100}{69/100} = \boxed{\frac{28}{69}}$$

It a conditional probability