# DISTRIBUTION ASSIGNMENT

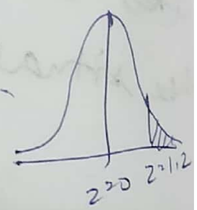
1) Answer shared in 'age distribution' ipynb file on github.

2) $\mu = 38000$, $\sigma = 10{,}000$

a) $X = 50{,}000$

$$Z = \frac{50{,}000 - 38000}{10000}$$

$$= 1.2$$



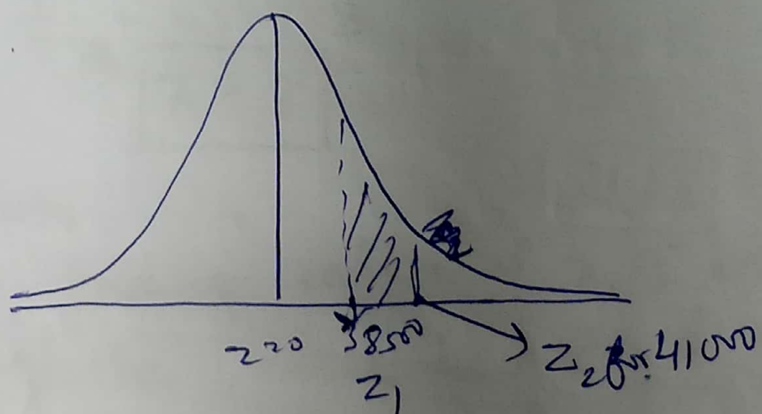

Z score gives area $= 0.8849$

$$P(X > 50000) = P(Z > 1.2)$$
$$= 1 - P(Z < 1.2)$$
$$= 1 - 0.8849 = 0.11$$

i.e 11% of firms have sales over 50,000

$\therefore$ No. of firms $= 0.11 \times 2000 = \boxed{220}$ firms

b) $P(38500 < X < 41{,}000)$




$$Z_1 = \frac{38500 - 38000}{10{,}000} = 0.05$$

$$Z_2 = \frac{41000 - 38000}{10000} = 0.3$$

$P(Z < 0.3) = 0.6179$

$P(Z < 0.05) = 0.5199$

$\therefore P(0.05 < Z < 0.3) = 0.6179 - 0.5199$

$= 0.098 \approx 10\%$

c) $X_1 = 30,000$

$X_2 = 50,000$

$Z_1 = \frac{30000 - 38000}{10000} = -0.8$

$Z_2 = \frac{50000 - 38000}{10000} = 1.2$

$\therefore P(Z_1 < Z < Z_2)$

$= P(-0.8 < Z < 1.2)$

$= P(Z < 1.2) - P(Z < -0.8)$

$= P(Z < 1.2) - [1 - P(Z < 0.8)]$

$= P(Z < 1.2) - 1 + P(Z < 0.8)$

$= 0.88493 - 1 + 0.78814$

$= 0.673$

No. of firms among 2000 firms

$= 0.673 \times 2000 = \boxed{1346 \text{ firms}}$

3) let success be correct answer

Probability of right answer $= \frac{1}{4}$

Probability of wrong answer $= \frac{3}{4}$

$\therefore$ Prob of 5 wrong answers = Prob of 15 right ans

$$P(X=15) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= {}^{20}C_{15} \left(\frac{1}{4}\right)^{15} \left(\frac{3}{4}\right)^{5}$$

$$= 15504 \times (9.313 \times 10^{-10}) \times 0.2373$$

$$= \boxed{3.43 \times 10^{-6}}$$

4) $\mu = \lambda = 4$ photons per second

$$P(X) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

~~P(X=0)~~ Prob that no photon reaches the telescope in a given second:

$$P(X=0) = \frac{e^{-4} 4^{0}}{0!} = \frac{e^{-4} \times 1}{1} = e^{-4}$$

$$= 0.0183$$

$$= \boxed{1.83\%}$$

5) $\mu = \lambda = 3 =$ No. of calls coming per minute

a) $P(X=0) =$ Prob of no calls coming in per minute:

$$= \frac{e^{-3} \times 3^{0}}{0!} = e^{-3} = 0.0498$$

$$= \boxed{4.98\%}$$

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b) Poisson mean = 3 / min.

In 1 min, 3 calls arrive on average

$\therefore$ In 2 min, 6 calls arrive on average

$\therefore \lambda' = 6$

$$\therefore P(X \geq 2) = \sum_{i=2}^{\infty} \frac{e^{-\lambda'} \lambda'^{x_i}}{x_i!}$$

$$= \sum_{i=2}^{\infty} \frac{e^{-6} 6^{x_i}}{x_i!}$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\frac{e^{-6} 6^0}{0!} + \frac{e^{-6} 6^1}{1!}\right]$$

$$= 1 - [e^{-6} + 6e^{-6}]$$

$$= 1 - 7e^{-6}$$

$$= \boxed{0.983}$$

6) ~~Hyper~~geometric distribution problem

$p = 0.20$ = probability of defective product

Probability of obtaining first defected part after three good parts

$$= (1 - 0.20)^3 \times 0.20$$

$$= \boxed{0.1024} \approx \boxed{0.1}$$

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Let X be the number of trials before the first success occurs

Avg number of inspections (trials) = E(X)

$= \frac{1}{p} = \frac{1}{0.2} = \boxed{5}$

$= \frac{1}{0.1024} = 9.76 \approx \boxed{10}$

7) Probability that student is accepted in college = 0.3 = p

n = 5

Binomial distribution

$P(x) = {}^nC_x p^x q^{n-x}$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= {}^5C_0\, 0.3^0 (0.7)^5 + {}^5C_1\, 0.3^1 (0.7)^4 + {}^5C_2\, 0.3^2 (0.7)^3$

$= 0.16807 + 0.36015 + 0.3087$

$= \boxed{0.837}$

8) $\mu = 70$, $\sigma^2 = 200$, $n = 10$

Let X be the weight of ~~anpo~~ adult

$X \sim N(\mu, \sigma^2)$

$\Sigma X \sim N(n\mu, n\sigma^2)$

Standard normal $Z = \frac{X-\mu}{\sigma}$

$$P(\Sigma X \leq 800) = P\left(Z \leq \frac{800 - n\mu}{\sqrt{n\sigma^2}}\right)$$

$$= P\left(Z \leq \frac{800 - 10 \times 70}{\sqrt{10 \times 200}}\right)$$

$$= P\left(Z \leq \frac{100}{\sqrt{2000}}\right)$$

$$= P(Z \leq 2.236)$$

$$= \boxed{0.987} = 98.7\%$$

If number of adults are 12, then

$$\cancel{P(\Sigma X \leq)} = P\left(Z \leq \frac{800 - 12 \times 70}{\sqrt{12 \times 200}}\right)$$

$$= P\left(Z \leq \frac{-40}{\sqrt{2400}}\right)$$

$$= P(Z \leq -0.816)$$

$$= 1 - P(Z \leq 0.816)$$

If number of adults are 12, then ~~average~~ weight ~~of that~~ the lift can take becomes $12 \times 70 = 840$. This is above the lifts capacity. So lift will not be able to sustain this weight

9) Let $p$ be the probability that the student ~~answers out passes the exam~~ answers correctly by chosing correct option

$n = 50$

$p = \frac{1}{2}$

Here, $X \sim \text{Binomial}(50, \frac{1}{2})$

Probability that he will pass where $X$ is the number of questions he answers correctly

$= P(X \geq 20)$

$= 1 - P(X < 19)$

As $n$ is large, we approximate $X$ to normal distribution

ie, $X \sim Bin(50, \frac{1}{2})$ is approximated to

$X \overset{\cdot}{\sim} N(np, np(1-p))$

$X \sim N(50 \times \frac{1}{2}, 50 \times \frac{1}{2} \times \frac{1}{2})$

$X \sim N(25, 12.5)$

$P(X \geq 20)$

$= 1 - P(X < 19)$

$= 1 - P\left(Z < \frac{19 - 25}{\sqrt{12.5}}\right)$

$= 1 - P\left(Z < \frac{-6}{\sqrt{12.5}}\right)$

$= 1 - P(Z < -1.697) = 1 - [1 - P(Z < 1.697)]$

$= 1 - (1 - 0.95543)$

$= 1 - 0.045 = \boxed{0.96}$

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when 4 options are given for a question, then $p = \frac{1}{4}$

So, $X \sim N\left(50 \times \frac{1}{4},\ 50 \times \frac{1}{4} \times \frac{3}{4}\right)$

$\sim N(12.5, 18.75)$

$\therefore 1 - P(X < 19)$

$= 1 - P\left(Z < \frac{19 - 12.5}{\sqrt{18.75}}\right)$

$= 1 - P(Z < 1.5)$

$= 1 - 0.93319$

$= \boxed{0.067} = \boxed{6.7\%}$ = probability of passing exam

10) Probability of faulty LED bulbs = 30%.

$n = 6$.

So, $P(X = 2)$ where $X$ is the number of faulty bulb

$= {}^nC_x\, p^x q^{n-x}$

$= {}^6C_2 (0.3)^2 (0.7)^4$

$= 0.324$

$= \boxed{\cancel{3.24\%}} = \boxed{32.4\%}$

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12) Probability that old sawmill sites contain soil residuals = 0.05

$n = 20$

a) It is a binomial distribution.
Let X be number of sites exceeding recommended level of dioxin

$P(X<1) = P(X=0) = {}^{n}C_x\, p^x q^{n-x}$

$= {}^{20}C_0 (0.05)^0 (0.95)^{20}$

$= \boxed{0.358}$

b) $P(X\leq 1) = P(X=0) + P(X=1)$

$= 0.358 + {}^{20}C_1 (0.05)^1 (0.95)^{19}$

$= \boxed{0.735}$

c) $P(X\leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.7354 + P(X=2)$

$= 0.7354 + {}^{20}C_2 (0.05)^2 (0.95)^{18}$

$\cancel{= 0.7354\ \boxed{0.189}}$

$= 0.7354 + 0.189$

$= \cancel{0.294}\ \boxed{0.924}$

13) a) $p = 0.05$, $n = 5$, $X = 2$

$\therefore P(X=2) = {}^5C_2\ 0.05^2 \times 0.95^3$

$= \boxed{0.0214} \approx 2\%$.

b) P( 2 in two years)

$= {}^2C_2 \times 0.05^2 \times 0.95^0$

$= \boxed{0.0025} \approx 0.25\%$.

c) $P(X \geq 1)$ = Prob of at least once in 4 year

$= 1 - P(X=0)$

$= 1 - ({}^4C_0\ 0.05^0\ 0.95^4)$

$= 1 - (0.8145)$

$= \boxed{0.1855}$

14 a) $p = 0.2$, $n = 15$

$P(X=2) = {}^{15}C_2\ 0.2^2 \times 0.8^{13}$

$= 0.2309 \approx \boxed{23\%}$

b) $P(X \geq 1)$

$= 1 - P(X < 1)$

$= 1 - P(X=0)$

$= 1 - {}^{15}C_0 \times 0.2^0 \times 0.8^{15}$

$= \boxed{0.9648} \approx \boxed{97\%}$

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11) No. of words entered = 77 words/min

No. of errors ~~/~~ hr = 6

No. of errors in 1 min = $\frac{6}{60}$ = 0.1 /min

To find

Probability of 2 errors in 322 word

Time taken to enter 322 words = $\frac{322}{77}$

= 4.18 min

Applying Poisson distribution

$$P(X=2) = \frac{e^{-(0.418)} \times (4.18 \times 0.1)^2}{2!}$$

$$= \frac{0.658 \times 0.174}{2}$$

$$= 0.057$$

$$\simeq 5.7\%$$

$$\boxed{\simeq 6\%}$$

~~[illegible]~~

~~[illegible]~~

$1 \to 0.$

$4.18$