F-test assignment

1) $H_0: \sigma_H^2 = \sigma_M^2$

$H_1: \sigma_H^2 \neq \sigma_M^2$

Hydb.	Mumbai
$n = 11$	$n = 9$
$\bar{h} = 192.8$	$\bar{m} = 350.2$

$$F_{n-1, n-2} = \frac{S_1^2}{S_2^2}$$

$$\therefore F_{8,10} = \frac{S_M^2}{S_H^2}$$

$$S_M^2 = \frac{1}{8}\left[\Sigma x_M^2 - 9 \times (350.2)^2\right]$$

$$= \frac{1}{8}\left[6501.64\right] = 812.71$$

$$S_{H}^2 = \frac{1}{10}\left[\Sigma x_H^2 - 11 \times 192.8^2\right]$$

$$= \frac{1}{10}\left[18362.76\right] = 1836.28$$

$$\therefore F_{8,10} = \frac{812.71}{1836.28}$$

$$= 0.4426$$

$$F_{10,8} = \frac{1}{0.4426} = 2.259$$

$F_{10,8}$ critical value @ 2.5% = 3.855

As F statistic is less than critical value, we accept null hypothesis, hence we conclude that there is no variance in both offices.

3) $f_{statistic}$: $f_{n_1-1, n_2-1} = \dfrac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2}$

$f_{statistic}$ if women in numerator $= f_{6,11} = \dfrac{35^2/45^2}{30^2/50^2}$

$= 1.68$

$f_{11,6} = \dfrac{1}{f_{6,11}} = \boxed{0.596}$

4) $P(F_{11,6} > 3.881) = 2.5\%$

$\Rightarrow P\left(\dfrac{1}{F_{11,6}} > \dfrac{1}{3.881}\right) = 2.5\%$

$\Rightarrow P(F_{6,11} < 0.258) = 2.5\%$

Cumulative probability of F statistic is greater than 10%.

5) $H_0: \mu_{An} = \mu_{SUV} = \cancel{\mu_{Pick}} = \mu_{pick}$

~~We~~ We assume population means are equal ~~to~~ ie, avg highway gas mileage is equal across the three types of cars

H_1: At least one mean mileage is not statistically equal.

Test statistic ANOVA $= \dfrac{MSTR}{MSE}$ (avg between variation / avg within variation)

Ground mean $\bar{\bar{X}} = \dfrac{\Sigma x}{N} = \dfrac{31 \times 25.8 + 31 \times 22.68 + 21.29 \times 14}{31 + 31 + 14}$

$= 23.697$

SSTR: Treatment sum of squares ie, Variation between samples $\Sigma n_j (\bar{X}_j - \bar{\bar{X}})^2$

$= \left[31 \times (25.8 - 23.7)^2\right] + \left[31 \times (22.68 - 23.7)^2\right] + \left[14 \times (21.29 - 23.7)^2\right]$

$= 136.71 + 32.25 + 81.3 = 250.27$

Error sum of squares (SSE) $= \Sigma\Sigma (x_{ij} - \bar{x}_j)^2$

= variation in the data

$= \cancel{\left[\Sigma (\bar{x}_i n - \bar{x}_i)^2\right]} = \left[\Sigma \bar{x}^2 (n-1)^2\right]$

$= \cancel{\Sigma}$ = weighted avg of sample variance

$= \left[(31-1) \times 2.56^2\right] + \left[(31-1) \times (3.67)^2\right] + \left[(14-1) \times 2.76^2\right]$

$= 699.7$

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Mean Square Treatment (MSTR) $= \frac{SSTR}{col - 1} = \frac{250.27}{2} = 125.14$

$MSE = \frac{SSE}{N-C} = \frac{699.7}{(31+31+14-3)} = 9.585$

$F_{dof} = \frac{MSTR}{MSE}$ ie, $F_{2,73} = \frac{125.14}{9.585} = 13.06$

$f_{2,73}$ critical value @1% $= 4.91$ (from calculator online)

As Fstatistic value $(13.06) > F_{critical}$ value, we reject the null hypothesis at 1% CI. We conclude that avg gas mileage is not statistically equal for the vehicles

2) $H_0: \mu_1 = \mu_2 = \mu_3$

$N = 6 \times 3 = 18$, columns $= 3$, $N - col = 15$

Grand mean $\bar{\bar{X}} = 177.5$

~~SSTR~~

line 1	line 2	line 3	$(x_1-\bar{x}_1)^2$	$(x_2-\bar{x}_2)^2$	$(x_3-\bar{x}_3)^2$
210	180	145	201.64	25	277.8
215	160	170	367.24	225	69.4
205	195	165	84.03	400	11.11
180	190	160	250.7	225	2.78
175	170	155	434.03	25	44.4
190	155	175	34.03	400	177.8
Total 1175	1050	970	1370.83	1300	583.3
Mean 195.8	175	161.7			

$$SSTR = \left[6\times(195.8-177.5)^2 + 6\times(175-177.5)^2 + 6(161.7-177.5)^2\right]$$

$$= 2009.34 + 37.5 + 1497.84$$

$$= 3544.68$$

$$MSTR = \frac{3544.68}{2} = 1772.34$$

$$SSE = \sum\sum(x_{ij} - \bar{x}_i)^2 = 3254.167$$

$$MSE = \frac{3254.167}{15} = 216.94$$

$$F_{2,15} = \frac{216.94}{1772.34} = 0.122$$

$$F_{15,2} = \frac{1}{0.122} = 8.197$$

F critical value ~~at~~ at 5% CI = 19.43

As 8.197 < 19.43, we accept null hypothesis and conclude that the 3 means are equal.