F-test assignment

1) $H_0: \sigma_H^2 = \sigma_M^2$

$H_1: \sigma_H^2 \neq \sigma_M^2$

Hydb.	Mumbai
$n = 11$	$n = 9$
$\bar{h} = 192.8$	$\bar{m} = 350.2$

$$F_{n-1, n-2} = \frac{S_1^2}{S_2^2}$$

$$\therefore F_{8,10} = \frac{S_M^2}{S_H^2}$$

$$S_M^2 = \frac{1}{8}\left[\Sigma x_M^2 - 9 \times (350.2)^2\right]$$

$$= \frac{1}{8}\left[6501.64\right] = 812.71$$

$$S_{H}^2 = \frac{1}{10}\left[\Sigma x_H^2 - 11 \times 192.8^2\right]$$

$$= \frac{1}{10}\left[18362.76\right] = 1836.28$$

$$\therefore F_{8,10} = \frac{812.71}{1836.28}$$

$$= 0.4426$$

$$F_{10,8} = \frac{1}{0.4426} = 2.259$$

$F_{10,8}$ critical value @ 2.5% = 3.855

As F statistic is less than critical value, we accept null hypothesis, hence we conclude that there is no variance in both offices.

3) $F_{statistic}$: $F_{n_1-1, n_2-1} = \dfrac{S_1^2/S_2^2}{\sigma_1^2/\sigma_2^2}$

$F_{statistic}$ if women in numerator $= F_{6,11} = \dfrac{35^2/45^2}{30^2/50^2}$

$= 1.68$

$F_{11,6} = \dfrac{1}{F_{6,11}} = \boxed{0.596}$

4) $P(F_{11,6} > 3.881) = 2.5\%$

$\Rightarrow P\left(\dfrac{1}{F_{11,6}} > \dfrac{1}{3.881}\right) = 2.5\%$

$\Rightarrow P(F_{6,11} < 0.258) = 2.5\%$

Cumulative probability of F statistic is greater than 10%.

5) $H_0: \mu_{M} = \mu_{SUV} = \cancel{\mu_{Pick}} = \mu_{pick}$
We assume population means are equal ~~to~~ ie, avg highway gas mileage is equal across the three types of cars

H_1: At least one mean mileage is not statistically equal.

Test statistic ANOVA $= \dfrac{MSTR}{MSE}$ $\dfrac{\text{(avg between variation)}}{\text{(avg within variation)}}$

Grand mean $\bar{\bar{X}} = \dfrac{\sum x}{N} = \dfrac{31\times 25.8 + 31\times 22.68 + 21.29\times 14}{31+31+14}$

$= 23.697$

SSTR: Treatment sum of squares i.e. Variation between samples $\sum n_j(\bar{X}_j - \bar{\bar{X}})^2$

$= \left[31\times(25.8-23.7)^2\right] + \left[31\times(22.68-23.7)^2\right] + \left[14\times(21.29-23.7)^2\right]$

$= 136.71 + 32.25 + 81.3 = 250.27$

Error sum of squares (SSE) $= \sum\sum(x_{ij} - \bar{x}_j)^2$
= variation in the data

$= \cancel{\left[\sum(\bar{x}_i n - \bar{x}_i)^2\right]} = \left[\sum \bar{x}^2 (n-1)^2\right]$

$= \cancel{\sum}$ = weighted avg of sample variance

$= \left[(31-1)\times 2.56^2\right] + \left[(31-1)\times(3.67)^2\right] + \left[(14-1)\times 2.76^2\right]$

$= 699.7$

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Mean Square Treatment (MSTR) $= \frac{SSTR}{col - 1} = \frac{250.27}{2} = 125.14$

$$MSE = \frac{SSE}{N-C} = \frac{699.7}{(31+31+14-3)} = 9.585$$

$$F_{dof} = \frac{MSTR}{MSE} \text{ ie, } F_{2,73} = \frac{125.14}{9.585} = 13.06$$

$f_{2,73}$ critical value @ 1% $= 4.91$ (from calculator online)

As Fstatistic value (13.06) > $F_{critical}$ value, we reject the null hypothesis at 1% C.I. we conclude that avg gas mileage is not statistically equal for the vehicles