Central limit theorem Assignment

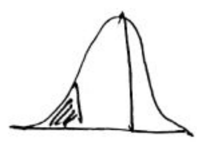
1) . $\mu_{pop} = 10 = \mu_{SD}$

$\sigma_{pop} = 4$

$\sigma_{SD} = \dfrac{\sigma_{pop}}{\sqrt{n}} = \dfrac{4}{\sqrt{100}} = 0.4$

$\cancel{P(\bar{X}<9)}$

$Z_{9} = \dfrac{x-\mu}{\sigma}$



$= \dfrac{9-10}{.4/\sqrt{100}} = \dfrac{-1}{0.4} = -2.5$

$P(\bar{X}<9) = P(Z<-2.5)$

$= 1 - P(Z<2.5)$

$= 1 - 0.99379$

$= \boxed{6.21 \times 10^{-3}}$

5) Let X be the head breadths of male.

$X \sim N(6, 1)$ $\mu_{pop} = 6$, $\sigma_{pop} = 1$

a) $P(X<6.2) =$ where $Z = \dfrac{X-\mu}{\sigma}$

$= P\left(Z < \dfrac{6.2-6}{1}\right)$

$= P(Z<0.2) = \boxed{0.57926} \simeq \boxed{0.58}$

b) $n = 100$, $\mu_{SD} = \mu_{pop} = 6$

Let $\overline{X}$ be the mean breadth of male head.

$$SD = \frac{\sigma_{pop}}{\sqrt{n}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = 0.1$$

$$\therefore P(\overline{X} < 6.2) = P\left(Z < \frac{6.2-6}{0.1}\right)$$

where $Z = \frac{X-\mu}{\sigma/\sqrt{n}}$

$$= P(Z < 2)$$

$$= 0.97725 \approx \boxed{0.98}$$

2) Let X be weight of students

$X \sim N(\mu, \sigma^2)$

$\approx N(50, 15^2)$

$n = 10$

Probability that all 10 students will safely reach 8th floor $= \cancel{P(\Sigma X \leq 550)}$

$$= P\left(\sum_{x=1}^{10} X \leq 550\right)$$

when $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$$\Sigma X_i \sim N(n\mu, n\sigma^2)$$

$$P(\Sigma X \leq 550) = P\left(Z \leq \frac{550 - 10\times 50}{\sqrt{10 \times 15^2}}\right)$$

$$= P\left(Z \leq \frac{50}{15\times\sqrt{10}}\right)$$

$= P(Z \leqslant 1.0541)$

$= 0.85314$

$\simeq \boxed{85.3\%}$

3) Let X be number of tickets purchased by a passenger

$X \sim N(2.4, 2^2)$

$n = 100$

Total no. of tickets remaining = 250

$\therefore P(\Sigma X \leqslant 250)$.

$= P\left(Z \leqslant \frac{250 - 2.4 \times 100}{\sqrt{100 \times 2^2}}\right)$

$= P\left(Z \leqslant \frac{10}{10 \times 2}\right)$

$= P(Z \leqslant 0.5)$

$= 0.69146$

$\simeq \boxed{0.69}$

4) Avg IQ of a soldier $= \mu = 96$

$\sigma = 16$

$n = 35$

Probability that officer will get what he wants.

$P(\cancel{\bar{x}>98})$: $P(X>98)$

$Z = \frac{X-\mu}{\sigma/\sqrt{n}}$

$= P\left(Z > \frac{98-96}{16/\sqrt{35}}\right)$

$= P\left(Z > \frac{2}{16/\sqrt{35}}\right)$

$= P(Z > 1.972)$

$= 1 - P(Z<1.972)$

$= 1 - 0.97558$

$= 0.0244$

$\boxed{\approx 2.4\%}$

6) The wrong with his reasoning is that he is concluding the result on the basis of just 100 samples. He should take more ~~sa~~ samples into consideration

7) Let X be length of pregnancy

~~so~~ $X \sim N(268, 15^2)$, $n = 25$

$P(\bar{X}<260) = P\left(Z < \frac{260-268}{15/\sqrt{25}}\right)$ $\quad Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$= P\left(Z < \frac{-8}{15/5}\right)$

$= P(Z < -2.667)$

$= 1 - P(Z < 2.667)$

$= 1 - 0.99621$

$= \boxed{0.379\%}$

8) ~~The diet~~ This does not conclude that diet has effect on length of pregnancy, because the sample size is just of 25 women and chances of length of pregnancy being less than 260 days is just 0.38%. So this diet ~~has~~ plan cannot be generalised.

9) Let X be weight of adult males

$X \sim N(172, (29)^2)$

$Z = \frac{X - \mu}{\sigma}$

a) $P(X > 190)$

$= P\left(Z > \frac{190 - 172}{29}\right)$

$= P(Z > 0.621)$

$= 1 - P(Z < 0.621)$

$= 1 - 0.73237$

$= \boxed{0.268}$

$n = 25$

b) $P(\bar{X} > 190) = P\left(Z > \frac{190-172}{29/\sqrt{25}}\right)$ $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

$= P\left(Z > \frac{18}{29/5}\right)$

$= P(Z > 3.103)$

$= 1 - P(Z < 3.103)$

$= 1 - 0.99903$

$= \boxed{0.097\%}$

c) $\sum X_i \sim N(n\mu, n\sigma^2)$

$P(\sum X_i > 4750) = P\left(Z > \frac{4750 - 25 \times 172}{\sqrt{25 \times 29^2}}\right)$

$= P\left(Z > \frac{450}{5 \times 29}\right)$

$= P(Z > 3.103)$

$= 1 - P(Z < 3.103)$

$= \boxed{0.097\%}$

10) $\mu = 4$, $\sigma = 1.5$, $n = 50$

$P(3.5 < \bar{X} < 3.8)$

$= P(\bar{X} < 3.8) - P(\bar{X} < 3.5)$

$= P\left(Z < \frac{3.8-4}{1.5/\sqrt{50}}\right) - P\left(Z < \frac{3.5-4}{1.5/\sqrt{50}}\right)$

$= P(Z < -0.9428) - P(-2.357)$

$= [1 - 0.8263] - \cancel{P}[1 - 0.9908]$

$= -0.8263 + 0.9908$

$= 0.1645$

$\approx \boxed{16.45\%}$

11) $\mu = 23.1$, $\sigma = 3.1$, $n = 6$

$P(\bar{X} > 27) = P\left(Z > \frac{27-23.1}{3.1/\sqrt{6}}\right)$

$= P(Z > 3.0816)$

$= 1 - P(Z < 3.0816)$

$= 1 - 0.99896$

$= \boxed{0.104\%}$

12) $\mu = 21.50$, $\sigma = 2.22$

let X be avg amt spent on food

$P(20 < \bar{X} < 23)$

$= P(\bar{X} < 23) - P(\bar{X} < 20)$

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$$= P\left(Z < \frac{23 - 21.50}{2.22}\right) - P\left(Z < \frac{20 - 21.50}{2.22}\right)$$

$$= P(Z < 0.6757) - P(Z < -0.6757)$$

$$= 0.75175 - (1 - 0.75175)$$

$$= \boxed{0.504}$$

$$\approx \boxed{50\%}$$

13) a) $\mu = 75$, $\sigma = 5$

$$P(X \geq 83) = P\left(Z \geq \frac{83 - 75}{5}\right)$$

$$= P(Z \geq 1.6)$$

$$= 1 - P(Z < 1.6)$$

$$= 1 - 0.94520$$

$$= \boxed{0.0548}$$

$n = 5$

b) $$P(\bar{X} \geq 83) = P\left(Z \geq \frac{83 - 75}{5/\sqrt{5}}\right)$$

$$= P\left(Z \geq \frac{8}{5/\sqrt{5}}\right)$$

$$= P(Z \geq 3.578)$$

$$= 1 - P(Z < 3.578)$$

$$= 1 - 0.99984$$

$$= \boxed{0.016\%}$$

14) $\mu = 28.3$, $\sigma = 2.3$, $n = 10$

$$P(\bar{X} < 27) = P\left(Z < \frac{27 - 28.3}{2.3/\sqrt{10}}\right)$$

$$= P(Z < -1.787)$$

$$= 1 - P(Z < 1.787)$$

$$= 1 - 0.96327$$

$$= \boxed{0.0367} \approx \boxed{3.67\%}$$