Ankita Nag

1. Summary:
Here Static analysis of thick cylinder under internal Pressure (Lame's problem) is done , using ANSYS Workbench we found the total deformation , Xand Y directional deformation, X ,Y and Z normal stress , X and Y normal stress along path for three cases – Plane stress , Plane strain and Axisymmetric. We verify all these result with analytical solution , we can find the errors.

2. Introduction:
A cylinder is considered to be 'thick' if the ratio of the inner diameter to the thickness of the walls is < 20. Analytically we used Lame's problem to get values and by FEA in ANSYS we solve the problem.

3. Objectives:
To determine the radial displacement and radial & hoop stresses of thick cylinder under internal pressure.
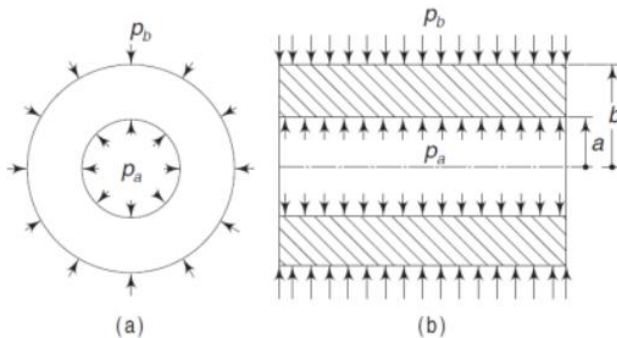Data given,
Material = steel, Young's Modulus is 200 GPa , Poison's ratio v =0.3
Geometry: a =0.1 m, b =0.2 m
Internal pressure: p =100 MPa

4. Analytical solutions (if available):
Thick cylinder under internal and external pressure,

(a)   (b)

$$u_r = \frac{1-v}{E}\frac{p_a a^2 - p_b b^2}{b^2 - a^2}r + \frac{1+v}{E}\frac{a^2 b^2}{r}\frac{p_a - p_b}{b^2 - a^2}$$

[Derived from plane stress condition]

$$\sigma_r = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} - \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

$$\sigma_\theta = \frac{p_a a^2 - p_b b^2}{b^2 - a^2} + \frac{a^2 b^2}{r^2}\frac{p_a - p_b}{b^2 - a^2}$$

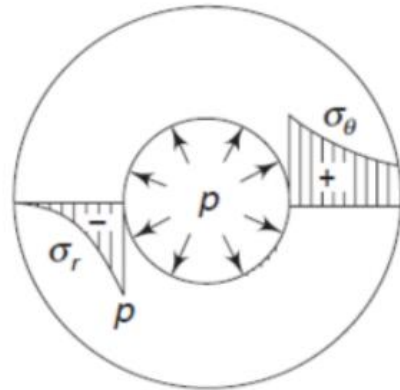Applicable for both plane stress/strain

Thick cylinder under internal pressure.

$$u_r = \frac{1-v}{E}\frac{p_a a^2}{b^2-a^2}r + \frac{1+v}{E}\frac{b^2}{r}\frac{p_a a^2}{b^2-a^2}$$

[Derived from plane stress condition]

$$\sigma_r = \frac{pa^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right)$$

$$\sigma_\theta = \frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right)$$

Applicable for both plane stress/strain

For analytical an excel sheet is used ,

E	200	Gpa
nu	0.3	
a	100	mm
b	200	mm
p	100	Mpa
sigma_ra	-100	MPa
sigma_rb	0	MPa
sigma_Φa	166.6667	MPa
sigma_Φb	66.66667	MPa
ura	0.098333	mm
urb	0.066667	mm

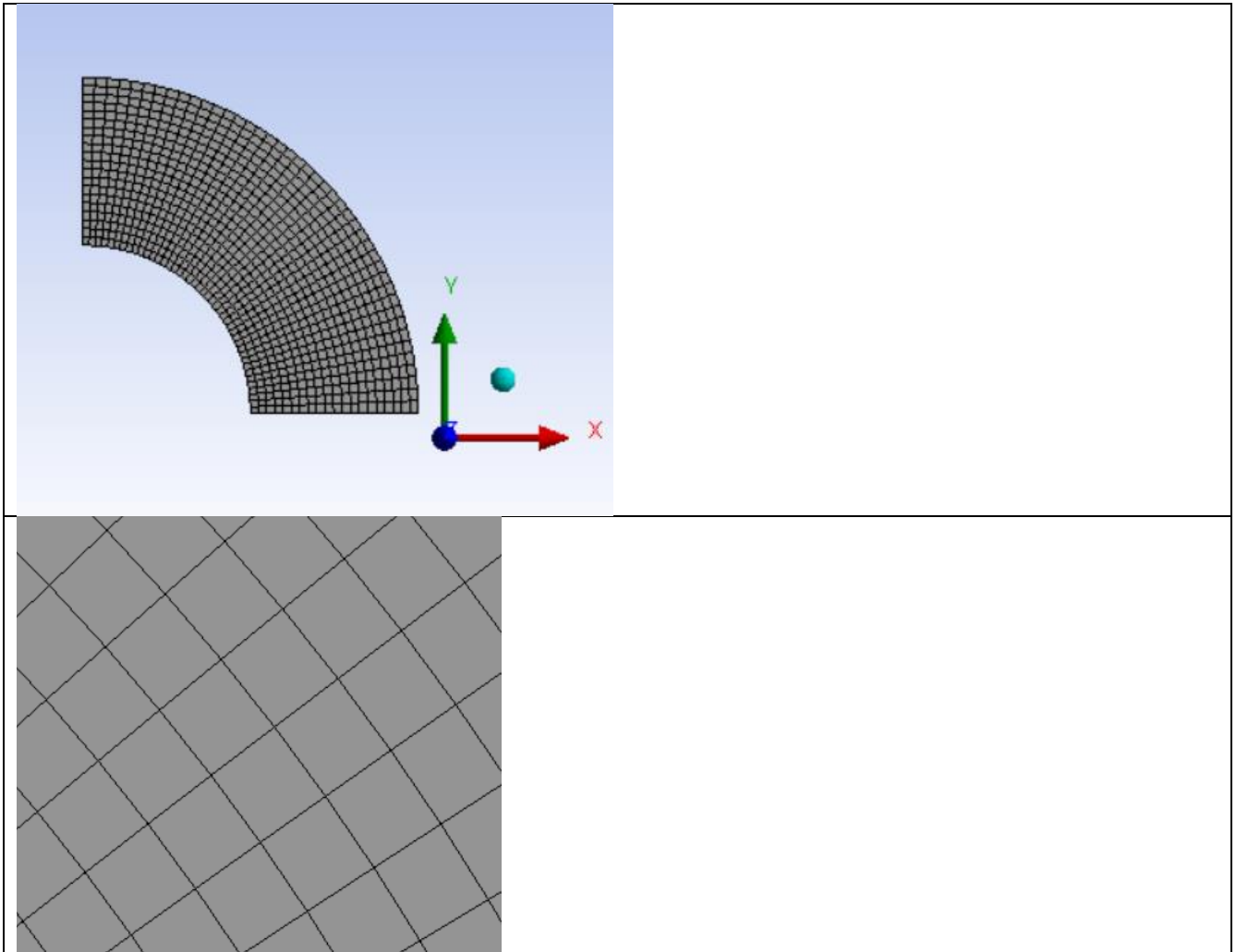# 5. Model Details:

Model geometry	
Inner Radius	100 mm
Outer Radius	200 mm

5.1 Element details:

Number of nodes: 2707
Number and type of elements: 903 , contact and solid element

Show the models in figure after meshing: if required give multiple figures with zoomed view etc (can use more no. of column/rows).

5.2 Type of Analysis:

Static Structural
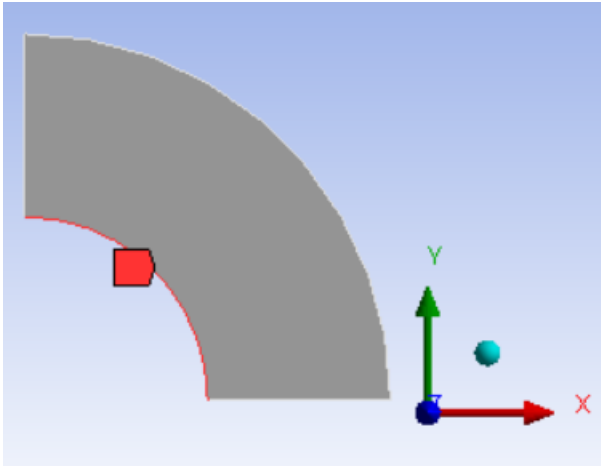
5.3 Material Data:

Young's Modulus: 200 Gpa
Poisson's Ratio:0.3
Density: N/A

5.4 Loads:

Model 100 MPa internal pressure, shown below by figure,

5.5 Boundary conditions:

Model is provided two frictionless support at the ends as shown below,
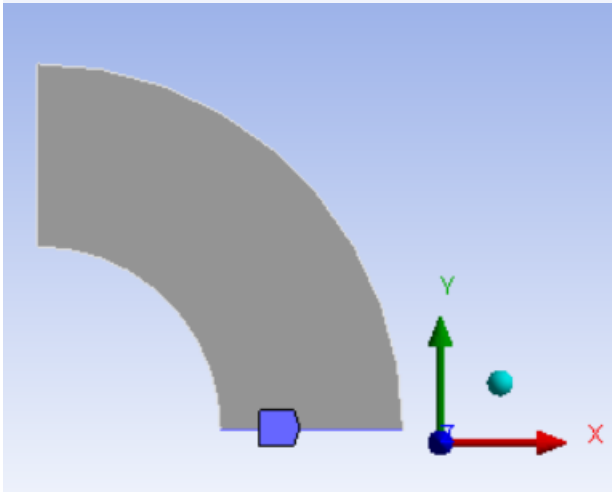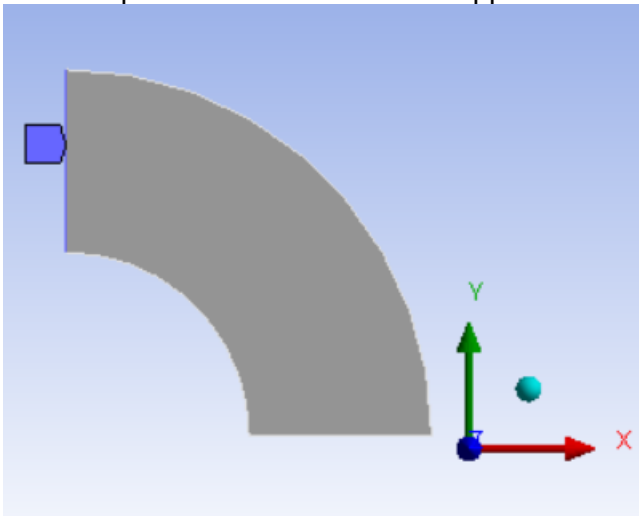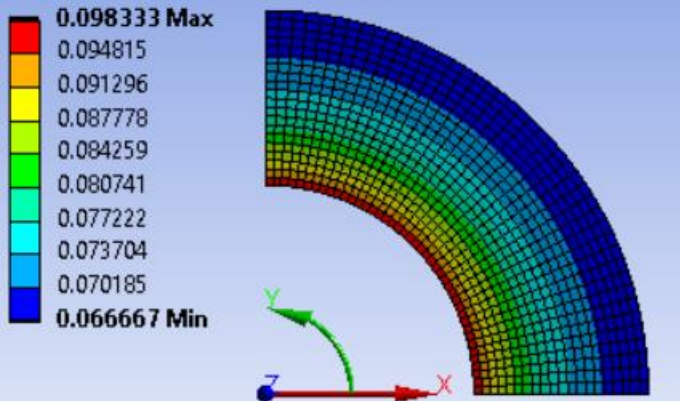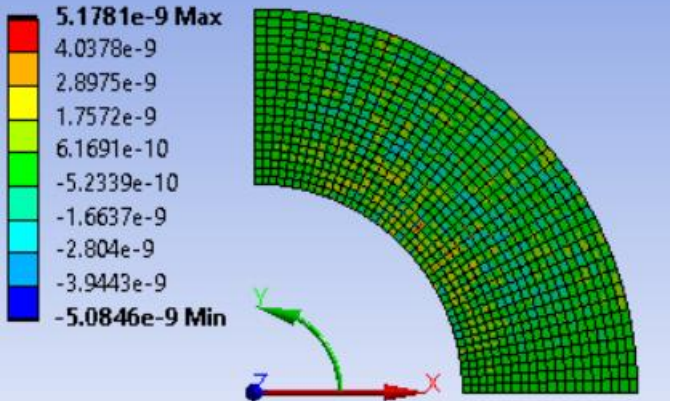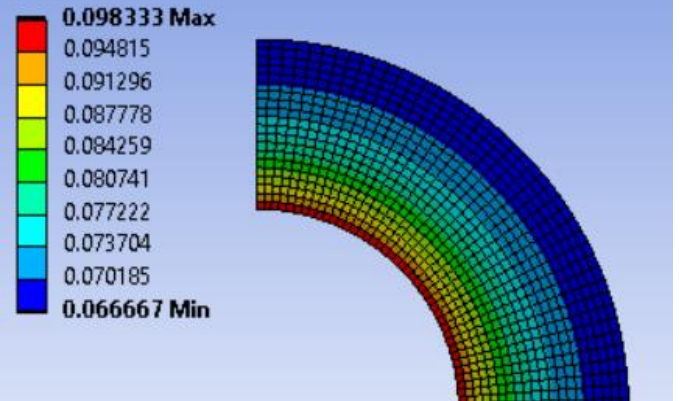
6. Calculations using Analytical solutions (if available):

Analytical solution for plane stress.

sigma_ra	-100	MPa
sigma_rb	0	MPa
sigma_Φa	166.6667	MPa
sigma_Φb	66.66667	MPa
ura	0.098333	mm
urb	0.066667	mm

7. Results:

Include pictures of key stress and displacement plots. It is useful to tabulate or graph results when reporting a high number of load cases (can use more no. of column/rows).

PLANE STRESS	
X axis directional deformation	
Y axis directional deformation	
Total deformation	
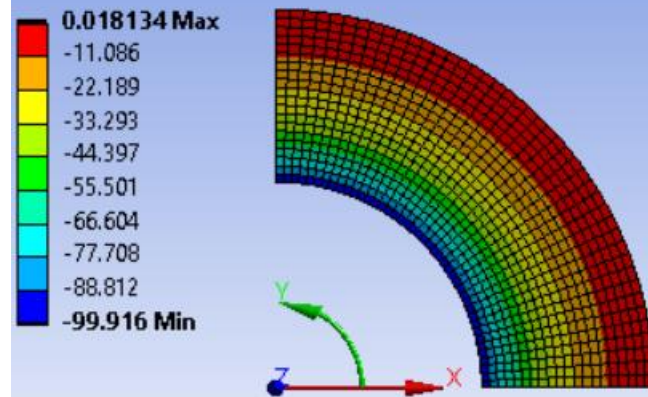
X axis normal stress	
Y axis normal stress	
X axis normal stress along path	
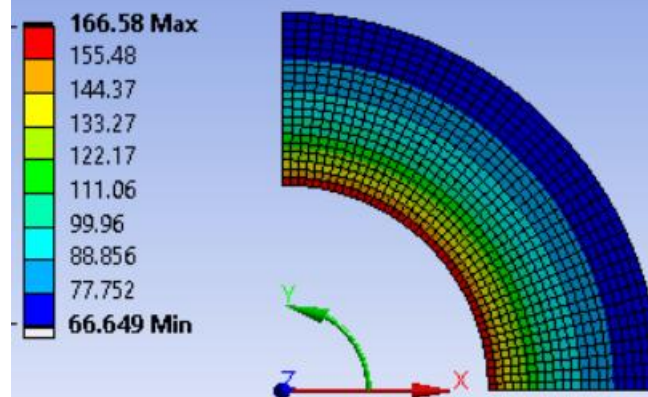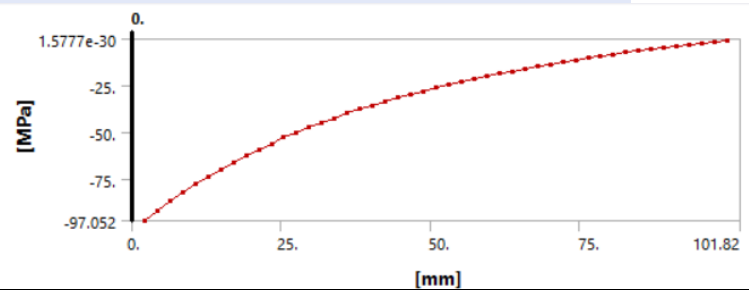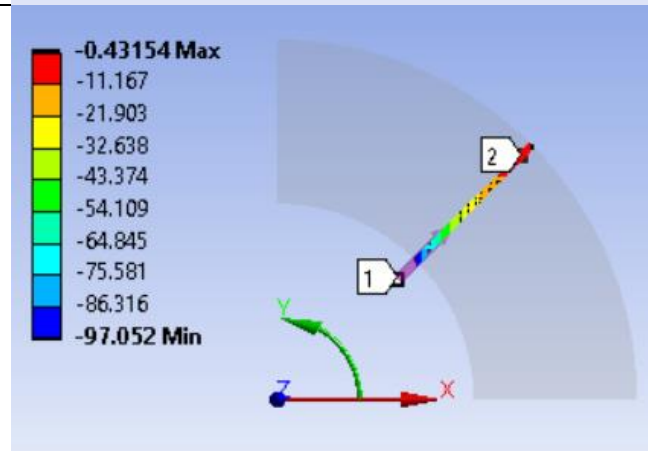
Y axis normal stress along path	
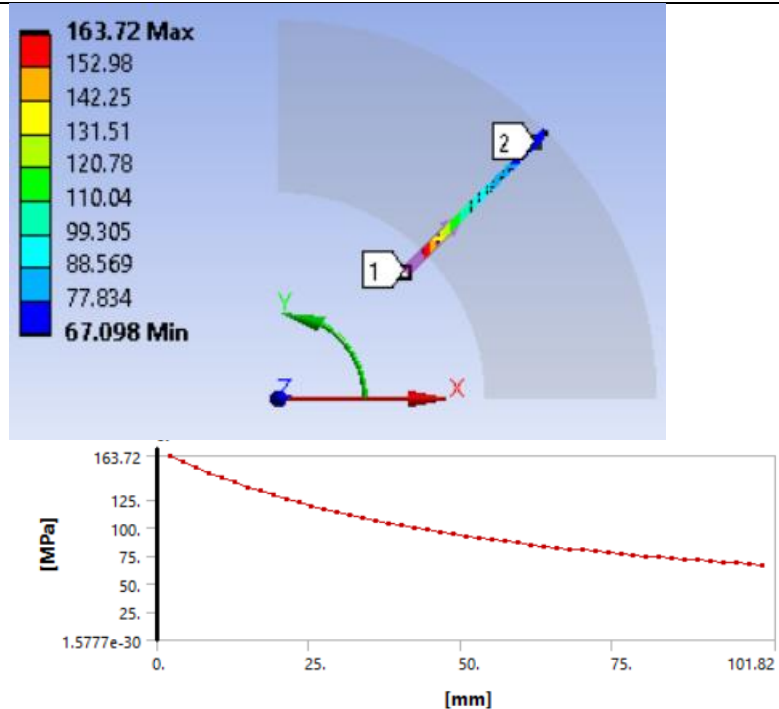
Plane stress.

	At inner radius			At outer radius		
	Analytical	FEA	Error	Analytical	FEA	Error
Deflection Mm	0.098333	0.098333	0	0.066667	0.066667	0
Radial stress Mpa	-100	-99.916	0.084	0	0.018134	0.018
Hoop stress MPa	166.6667	168.58	1.14	66.66667	66.649	0.02654

Mesh sensitivity study: For Hoop stress at inner radius

Steps	Global mesh size	No. of nodes	No. of elements
Step 1	5 mm	2707	903
Step 2	10	725	242
Step 3	2	15396	5136
Step 4			
Step 5			

After comparative studies of different mesh size on results (may be deflection and/stress etc.), final mesh size is adopted from step xx (No. of nodes elements as xxxx and xxx, respectively) and shown in section 5.1.

Steps	Analytical solution	FEA results	% of Error
Step 1	166.6667	168.58	1.147
Step 2	166.6667	**163.33**	**2.002**

Step 3	166.6667	**166.65**	0.010

It is clearly seen that for mesh size 2 mm , error can be reduce to less 1%

Parametric Study: Here I'm using the snapshots of data I got while doing parametric study,

1. Pressure variation: 100 , 10 ,50 ,200 MPa respectively,

Plane stress:

	A	B	C	D
1	Name	P10 - Pressure Magnitude	P8 - X Axis - Directional Deformation - End Time Minimum	P9 - X Axis - Directional Deformation - End Time Maximum
2	Units	MPa	mm	mm
3	DP 0 (Current)	100	0.066667	0.098333
4	DP 1	10	0.0066667	0.0098333
5	DP 2	50	0.033333	0.049167
6	DP 3	200	0.13333	0.19667

P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum	P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa	MPa
-99.916	0.018134	-97.052	-0.43154
-9.9916	0.0018134	-9.7052	-0.043154
-49.958	0.0090668	-48.526	-0.21577
-199.83	0.036267	-194.1	-0.86307

P14 - X Axis - Normal Stress - Path - End Time Maximum	P15 - Y Axis - Normal Stress - Path - End Time Minimum	P16 - Y Axis - Normal Stress - Path - End Time Maximum	P17 - Y Axis - Normal Stress - End Time Minimum	P18 - Y Axis - Normal Stress - End Time Maximum
MPa	MPa	MPa	MPa	MPa
-0.43154	67.098	163.72	66.649	166.58
-0.043154	6.7098	16.372	6.6649	16.658
-0.21577	33.549	81.859	33.324	83.291
-0.86307	134.2	327.44	133.3	333.16

Plane strain:

1	Name	P31 - Pressure Magnitude	P19 - X Axis - Directional Deformation - End Time Minimum	P20 - X Axis - Directional Deformation - End Time Maximum
2	Units	MPa	mm	mm
3	DP 0 (Current)	100	0.060667	0.095333
4	DP 1	10	0.0060667	0.0095333
5	DP 2	50	0.030333	0.047667
6	DP 3	200	0.12133	0.19067

P21 - Y Axis - Directional Deformation - End Time Minimum	P22 - Y Axis - Directional Deformation - End Time Maximum	P23 - X Axis - Normal Stress - End Time Minimum	P24 - X Axis - Normal Stress Maximum
mm	mm	MPa	MPa
-5.3954E-09	4.7744E-09	-99.916	0.018136
-4.2426E-10	4.0932E-10	-9.9916	0.0018136
-2.6977E-09	2.3872E-09	-49.958	0.0090682
-1.0791E-08	9.5488E-09	-199.83	0.036273

P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum	P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa	MPa
-99.916	0.018134	-97.052	-0.43154
-9.9916	0.0018134	-9.7052	-0.043154
-49.958	0.0090668	-48.526	-0.21577
-199.83	0.036267	-194.1	-0.86307

P29 - Y Axis - Normal Stress - Path - End Time Minimum	P30 - Y Axis - Normal Stress - Path - End Time Maximum
MPa	MPa
67.098	163.72
6.7098	16.372
33.549	81.859
134.2	327.44

Axisymmetric:

P44 - Pressure Magnitude	P32 - X Axis - Directional Deformation - End Time Minimum	P33 - X Axis - Directional Deformation - End Time Maximum	P34 - Y Axis - Directional Deformation - End Time Minimum	P35 - Y Axis - Directional Deformation - End Time Maximum
MPa	mm	mm	mm	mm
100	0.014999	0.04	-5.216E-07	1.1233E-06
10	0.0014999	0.004	-5.2161E-08	1.1232E-07
50	0.0074997	0.02	-2.608E-07	5.6165E-07
200	0.029999	0.08	-1.0432E-06	2.2466E-06

P40 - X Axis - Normal Stress - Path - End Time Minimum	P41 - X Axis - Normal Stress - Path - End Time Maximum	P42 - Y Axis - Normal Stress - Path - End Time Minimum	P43 - Y Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa	MPa
-96.253	-0.28118	21.568	69.548
-9.6253	-0.028118	2.1568	6.9548
-48.127	-0.14059	10.784	34.774
-192.51	-0.56236	43.136	139.1

P36 - X Axis - Normal Stress - End Time Minimum	P37 - X Axis - Normal Stress - End Time Maximum	P38 - Y Axis - Normal Stress - End Time Minimum	P39 - Y Axis - Normal Stress - End Time Maximum
MPa	MPa	MPa	MPa
-99.917	0.029136	21.406	71.442
-9.9917	0.0029136	2.1406	7.1442
-49.959	0.014568	10.703	35.721
-199.83	0.058272	42.812	142.88

2. Young's modulus variation: 5,10,100, 500 GPa
   Plane stress:

1	Name	P45 - Young's Modulus	P8 - X Axis - Directional Deformation - End Time Minimum	P9 - X Axis - Directional Deformation - End Time Maximum	P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum
2	Units	GPa	mm	mm	MPa	MPa
3	DP 0 (Current)	200	0.066667	0.098333	-99.916	0.018134
4	DP 1	5	2.6667	3.9333	-99.916	0.018134
5	DP 2	10	1.3333	1.9667	-99.916	0.018134
6	DP 3	100	0.13333	0.19667	-99.916	0.018134
7	DP 4	500	0.026667	0.039333	-99.916	0.018134

P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum	P15 - Y Axis - Normal Stress - Path - End Time Minimum	P16 - Y Axis - Normal Stress - Path - End Time Maximum	P17 - Y Axis - Normal Stress - End Time Minimum
MPa	MPa	MPa	MPa	MPa
-97.052	-0.43154	67.098	163.72	66.649
-97.052	-0.43154	67.098	163.72	66.649
-97.052	-0.43154	67.098	163.72	66.649
-97.052	-0.43154	67.098	163.72	66.649
-97.052	-0.43154	67.098	163.72	66.649

P17 - Y Axis - Normal Stress - End Time Minimum	P18 - Y Axis - Normal Stress - End Time Maximum
MPa	MPa
66.649	166.58
66.649	166.58
66.649	166.58
66.649	166.58
66.649	166.58

Plane strain:

1	Name	P46 - Young's Modulus	P19 - X Axis - Directional Deformation - End Time Minimum	P20 - X Axis - Directional Deformation - End Time Maximum	P21 - Y Axis - Directional Deformation - End Time Minimum	P22 - Y Axis - Directional Deformation - End Time Maximum
2	Units	GPa	mm	mm	mm	mm
3	DP 0 (Current)	200	0.060667	0.095333	-5.3954E-09	4.7744E-09
4	DP 1	5	2.4267	3.8133	-1.9672E-07	1.7251E-07
5	DP 2	10	1.2133	1.9067	-9.8359E-08	8.6257E-08
6	DP 3	100	0.12133	0.19067	-1.0791E-08	9.5488E-09
7	DP 4	500	0.024267	0.038133	-1.697E-09	1.6373E-09

1	P23 - X Axis - Normal Stress - End Time Minimum	P24 - X Axis - Normal Stress - End Time Maximum	P25 - Y Axis - Normal Stress - End Time Minimum	P26 - Y Axis - Normal Stress - End Time Maximum	P27 - X Axis - Normal Stress - Path - End Time Minimum
2	MPa	MPa	MPa	MPa	MPa
3	-99.916	0.018136	66.649	166.58	-97.052
4	-99.916	0.018136	66.649	166.58	-97.052
5	-99.916	0.018136	66.649	166.58	-97.052
6	-99.916	0.018136	66.649	166.58	-97.052
7	-99.916	0.018136	66.649	166.58	-97.052

P28 - X Axis - Normal Stress - Path - End Time Maximum	P29 - Y Axis - Normal Stress - Path - End Time Minimum	P30 - Y Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa
-0.43153	67.098	163.72
-0.43153	67.098	163.72
-0.43153	67.098	163.72
-0.43153	67.098	163.72
-0.43153	67.098	163.72

Axisymmetric:

1	Name	P47 - Young's Modulus	P32 - X Axis - Directional Deformation - End Time Minimum	P33 - X Axis - Directional Deformation - End Time Maximum	P34 - Y Axis - Directional Deformation - End Time Minimum	P35 - Y Axis - Directional Deformation - End Time Maximum
2	Units	GPa	mm	mm	mm	mm
3	DP 0 (Current)	200	0.014999	0.04	-5.216E-07	1.1233E-06
4	DP 1	5	0.59998	1.6	-2.0866E-05	4.4932E-05
5	DP 2	10	0.29999	0.8	-1.0433E-05	2.2466E-05
6	DP 3	100	0.029999	0.08	-1.0432E-06	2.2466E-06
7	DP 4	500	0.0059998	0.016	-2.0864E-07	4.4929E-07

1	P36 - X Axis - Normal Stress - End Time Minimum	P37 - X Axis - Normal Stress - End Time Maximum	P38 - Y Axis - Normal Stress - End Time Minimum	P39 - Y Axis - Normal Stress - End Time Maximum	P40 - X Axis - Normal Stress - Path - End Time Minimum
2	MPa	MPa	MPa	MPa	MPa
3	-99.917	0.029136	21.406	71.442	-96.253
4	-99.917	0.029136	21.406	71.442	-96.253
5	-99.917	0.029136	21.406	71.442	-96.253
6	-99.917	0.029136	21.406	71.442	-96.253
7	-99.917	0.029136	21.406	71.442	-96.253

P41 - X Axis - Normal Stress - Path - End Time Maximum	P42 - Y Axis - Normal Stress - Path - End Time Minimum	P43 - Y Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa
-0.28118	21.568	69.548
-0.28118	21.568	69.548
-0.28118	21.568	69.548
-0.28118	21.568	69.548
-0.28118	21.568	69.548

3. Poison ratio = 0.2, 0.25, 0.35
   Plane stress:

1	Name	P48 - Poisson's Ratio	P8 - X Axis - Directional Deformation - End Time Minimum	P9 - X Axis - Directional Deformation - End Time Maximum	P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum
2	Units		mm	mm	MPa	MPa
3	DP 0 (Current)	0.3	0.066667	0.098333	-99.916	0.018134
4	DP 1	0.2	0.066667	0.093333	-99.916	0.018132
5	DP 2	0.25	0.066667	0.095833	-99.916	0.018133
6	DP 3	0.35	0.066667	0.10083	-99.916	0.018135

	P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum	P15 - Y Axis - Normal Stress - Path - End Time Minimum	P16 - Y Axis - Normal Stress - Path - End Time Maximum	P17 - Y Axis - Normal Stress - End Time Minimum
	MPa	MPa	MPa	MPa	MPa
	-97.052	-0.43154	67.098	163.72	66.649
	-97.052	-0.43154	67.098	163.72	66.649
	-97.052	-0.43154	67.098	163.72	66.649
	-97.052	-0.43154	67.098	163.72	66.649

P18 - Y Axis - Normal Stress - End Time Maximum
MPa
166.58
166.58
166.58
166.58

Plane strain:

Name	P49 - Poisson's Ratio	P19 - X Axis - Directional Deformation - End Time Minimum	P20 - X Axis - Directional Deformation - End Time Maximum	P21 - Y Axis - Directional Deformation - End Time Minimum	P22 - Y Axis - Directional Deformation - End Time Maximum	P23 - X Axis - Normal Stress - End Time Minimum
Units		mm	mm	mm	mm	MPa
DP 0 (Current)	0.3	0.060667	0.095333	-5.3954E-09	4.7744E-09	-99.916
DP 1	0.2	0.064	0.092	-4.8167E-09	4.2169E-09	-99.916
DP 2	0.25	0.0625	0.09375	-4.4694E-09	3.4917E-09	-99.916
DP 3	0.35	0.0585	0.09675	-4.7718E-09	4.6911E-09	-99.916

P25 - Y Axis - Normal Stress - End Time Minimum	P26 - Y Axis - Normal Stress - End Time Maximum	P27 - X Axis - Normal Stress - Path - End Time Minimum	P28 - X Axis - Normal Stress - Path - End Time Maximum	P29 - Y Axis - Normal Stress - Path - End Time Minimum	P30 - Y Axis - Normal Stress - Path - End Time Maximum
MPa	MPa	MPa	MPa	MPa	MPa
66.649	166.58	-97.052	-0.43153	67.098	163.72
66.649	166.58	-97.052	-0.43154	67.098	163.72
66.649	166.58	-97.052	-0.43154	67.098	163.72
66.649	166.58	-97.052	-0.43153	67.098	163.72

Axisymmetric:

Name	P50 - Poisson's Ratio	P32 - X Axis - Directional Deformation - End Time Minimum	P33 - X Axis - Directional Deformation - End Time Maximum	P34 - Y Axis - Directional Deformation - End Time Minimum	P35 - Y Axis - Directional Deformation - End Time Maximum
Units		mm	mm	mm	mm
DP 0 (Current)	0.3	0.014999	0.04	-5.216E-07	1.1233E-06
DP 1	0.2	0.017143	0.038572	-5.7551E-07	1.2419E-06
DP 2	0.25	0.016071	0.039286	-5.5137E-07	1.19E-06
DP 3	0.35	0.013928	0.040714	-4.8551E-07	1.0397E-06

P35 - Y Axis - Directional Deformation - End Time Maximum	P36 - X Axis - Normal Stress - End Time Minimum	P37 - X Axis - Normal Stress - End Time Maximum	P38 - Y Axis - Normal Stress - End Time Minimum	P39 - Y Axis - Normal Stress - End Time Maximum
mm	MPa	MPa	MPa	MPa
1.1233E-06	-99.917	0.029136	21.406	71.442
1.2419E-06	-99.859	0.019086	21.407	71.438
1.19E-06	-99.886	0.023734	21.406	71.44
1.0397E-06	-99.953	0.035743	21.406	71.445

P40 - X Axis - Normal Stress - Path - End Time Minimum	P41 - X Axis - Normal Stress - Path - End Time Maximum	P42 - Y Axis - Normal Stress - Path - End Time Minimum	P43 - Y Axis - Normal Stress - Path - End Time Maximum	
MPa	MPa	MPa	MPa	
-96.253	-0.28118	21.568	69.548	
-96.253	-0.28117	21.568	69.548	
-96.253	-0.28118	21.568	69.548	
-96.253	-0.28118	21.568	69.549	

4. a variation 10,50,150 mm

Plane stress:

Name	P51 - a	P8 - X Axis - Directional Deformation - End Time Minimum	P9 - X Axis - Directional Deformation - End Time Maximum	P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum
Units	mm	mm	mm	MPa	MPa
DP 0 (Current)	100	0.066667	0.098333	-99.916	0.018134
DP 1	10	0.00050111	0.0065284	-95.806	0.0044366
DP 2	50	0.013333	0.035834	-99.667	0.006256
DP 3	150	0.25714	0.29036	-99.945	0.059988

P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum	P15 - Y Axis - Normal Stress - Path - End Time Minimum	P16 - Y Axis - Normal Stress - Path - End Time Maximum	P17 - Y Axis - Normal Stress - E... Minimum
MPa	MPa	MPa	MPa	MPa
-97.052	-0.43154	67.098	163.72	66.649
-0.80541	-0.0082889	0.50955	1.3067	0.49676
-20.539	-0.083527	13.417	33.873	13.327
-93.961	-1.6772	258.82	351.1	257.08

P17 - Y Axis - Normal Stress - End Time Minimum	P18 - Y Axis - Normal Stress - End Time Maximum
MPa	MPa
66.649	166.58
0.49676	96.303
13.327	113
257.08	357.09

Similarly we can do for plane strain and axisymmetric

5. b variation 150,200,300 mm

Plane stress:

1	Name	P51 - a	P52 - b	P8 - X Axis - Directional Deformation - End Time Minimum	P9 - X Axis - Directional Deformation - End Time Maximum	P11 - X Axis - Normal Stress - End Time Minimum	P12 - X Axis - Normal Stress - End Time Maximum
2	Units	mm	mm	mm	mm	MPa	MPa
3	DP 0 (Current)	100	200	0.066667	0.098333	-99.916	0.018134
4	DP 1	100	150	0.12	0.145	-99.901	0.071276
5	DP 2	100	250	0.047619	0.084048	-99.912	0.0079563
6	DP 3	100	300	0.0375	0.0775	-99.908	0.0042308
*							

	H	I	J	K	L
	P13 - X Axis - Normal Stress - Path - End Time Minimum	P14 - X Axis - Normal Stress - Path - End Time Maximum	P15 - Y Axis - Normal Stress - Path - End Time Minimum	P16 - Y Axis - Normal Stress - Path - End Time Maximum	P17 - Y Axis - Normal Stress - End Time Minimum
	MPa	MPa	MPa	MPa	MPa
	-97.052	-0.43154	67.098	163.72	66.649
	-96.173	-0.10228	160.1	256.17	159.93
	-97.443	-10.487	48.582	135.54	38.087
	-97.516	-15.381	40.381	122.52	24.996

M
P18 - Y Axis - Normal Stress - End Time Maximum
MPa
166.58
259.9
138.01
124.91

Similarly we can do for plane strain and axisymmetric.

8. Conclusions:

All the objectives are satisfied , all data are verified by this modelling process. All ideas are clearly seen by the diagrams and this kind of ANSYS simulation also clear our theoretical knowledge. Doing parametric study we can get the idea of variations of properties with the change of parameters like radius, E, poison ratio etc.

9. References:

Datasheet is provided in the assignment and an excel sheet is used for the analytical solution.