Ankita Nag

Title: Stress concentration problem for a circular/elliptical hole

1. Summary:

In this report, the nominal and maximum stress variation is shown for stress concentration problem for a circular or elliptical hole for both finite and infinite plate. Parametric study shows the variation of these parameters for variation of radius, length, thickness etc. Mesh sensitivity study is used to lessen the error as much as possible.

2. Introduction:

Information given in datasheet and some are self-chosen ,
For circular hole,
Length of plate L = 50 mm
Breath of plate D= 50 mm
Radius of hole r = 2.5 mm
Thickness t = 1 mm
For elliptical hole finite plate, L, D and t are same
Semimajor axis length a =10 mm
Semi minor axis length b = 6 mm For
elliptical hole infinite plate ,
a=6 mm b=10
mm L=500
mm
D=500 mm
T=1 mm

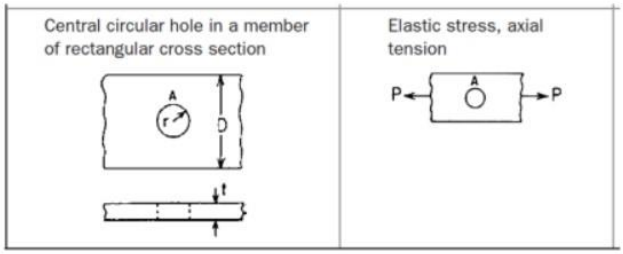
3. Objectives:

We have to calculate the value of Sigma nominal and maximum and also kt , we will obtain this from the analytical solution and also from the ANSYS WB.

4. Analytical solutions:

Circular hole:

(a)



$$\sigma_{max} = \sigma_A = K_t \sigma_{nom}$$

where $\sigma_{nom} = \dfrac{P}{t(D-2r)}$

$$K_t = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

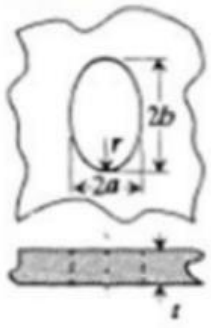
I used an excel sheet for analytical solution , the data of that sheet is shown below,

Stress conc for circular hole						
E	200	Gpa		E	200	Gpa
nu	0.3			nu	0.3	
L	40	mm		L	40	mm
D	40	mm		D	40	mm
r	2.5	mm		r	5	mm
t	1	mm		t	1	mm
pressure	10	Mpa		pressure	10	Mpa
2r/D	0.125			2r/D	0.25	
kt	2.662949	mm^3		kt	2.422344	mm^3
Sigma_nor	11.42857	MPa		Sigma_nor	13.33333	MPa
sigma_ma:	30.43371	MPa		sigma_ma:	32.29792	MPa
	30.986				35.688	
Sigma FE	-10.758			Sigma FE	-14.626	
kt FEA	2.711275			kt FEA	2.6766	
error kt	1.814747			error kt	10.49629	

Elliptical hole infinite plate:

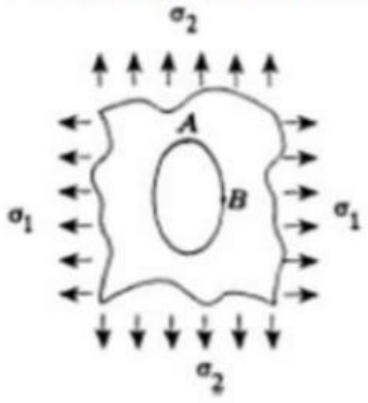
(b)

Single elliptical hole in infinite plate



$$r = \frac{a^2}{b}$$

a. In-plane normal stress



Uniaxial tension ($\sigma_1 = \sigma,\ \sigma_2 = 0$):

$\sigma_A = K_t\sigma$

$K_t = 1 + \frac{2a}{b} = 1 + 2\sqrt{\frac{a}{r}}$ for $0 < a/b < 10$

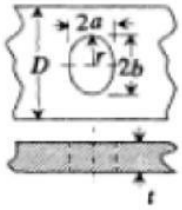
and $\sigma_B = -\sigma$

1	Stress conc for infinite plate elliptocal hole			
2				
3				
4				
5	E	200	Gpa	
6	nu	0.3		
7	L	500	mm	
8	D	500	mm	
9	a	6	mm	
10	b	10	mm	
11	t	1	mm	
12	P	10	Mpa	
13	b/a	1.666667		
14	kt	4.333333		
15	sigma_nom	10.246		
16	sigma_max	44.39933		
17				
18				
19	Sigma max FE	42.829		
20	error	-3.53684		

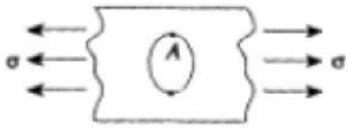
Elliptical hole finite plate:

(c)

Single elliptical hole in finite-width plate



a. Axial tension



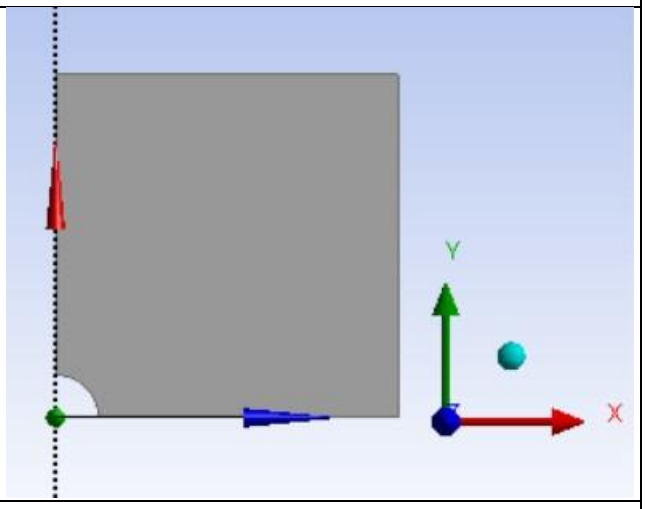
$$\sigma_{max} = \sigma_A = K_t \sigma_{nom}, \qquad \sigma_{nom} = \sigma/(1 - 2a/D)$$

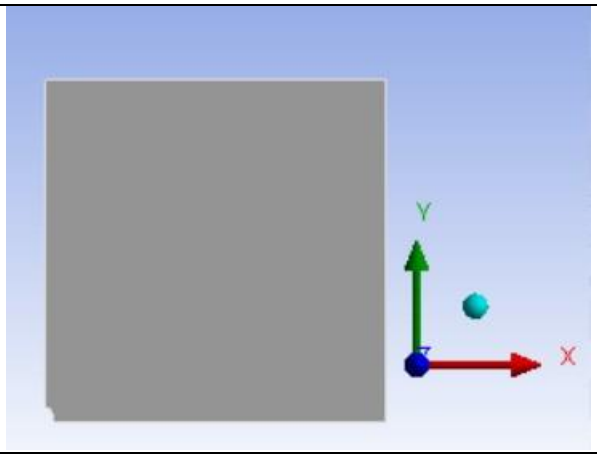
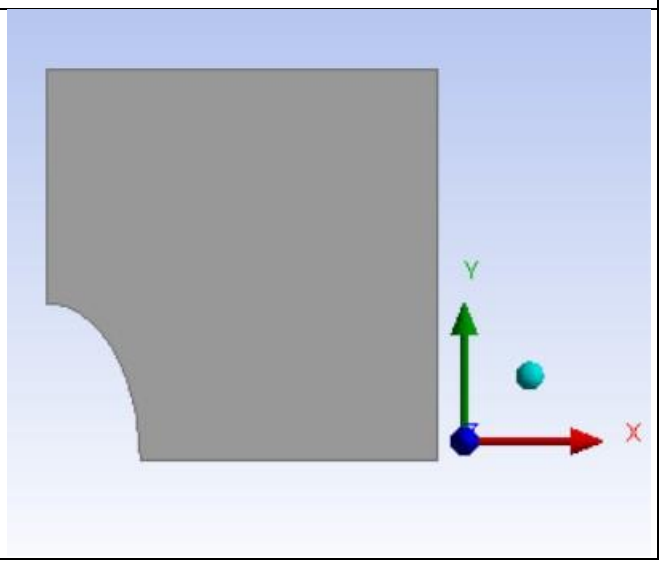
$$K_t = C_1 + C_2\frac{2a}{D} + C_3\left(\frac{2a}{D}\right)^2 + C_4\left(\frac{2a}{D}\right)^3,$$

	$1.0 \le a/b \le 8.0$
C_1	$1.109 - 0.188\sqrt{a/b} + 2.086a/b$
C_2	$-0.486 + 0.213\sqrt{a/b} - 2.588a/b$
C_3	$3.816 - 5.510\sqrt{a/b} + 4.638a/b$
C_4	$-2.438 + 5.485\sqrt{a/b} - 4.126a/b$

Stress conc for elliptical hole finite plate							
E	200	Gpa			E	200	Gpa
nu	0.3				nu	0.3	
L	50	mm			L	500	mm
D	50	mm			D	500	mm
a	10	mm			a	10	mm
b	6	mm			b	6	mm
t	1	mm			t	1	mm
pressure	10	Mpa			pressure	10	Mpa
a/b	1.666667				a/b	1.666667	
2a/D	0.4				2a/D	0.04	
Sigma_nom	16.66667	MPa			Sigma_nom	10.41667	MPa
c1	4.34296				c1	4.34296	
c2	-4.52435				c2	-4.52435	
c3	4.432621				c3	4.432621	
c4	-2.23356				c4	-2.23356	
kt	3.09949				kt	4.168935	
Sigma_max ana	51.65817				Sigma_max ana	43.42641	
Sigma_maxFE	60.145				Sigma_maxFE	42.829	
Error	16.42882				Error	-1.37567	

5. Model Details:

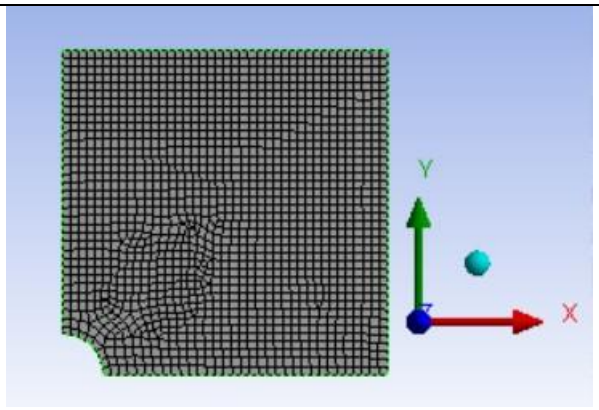
Circular Hole	
Length of plate	50 mm
Breadth of plate	50 mm
Radius of hole	2.5 mm
Thickness	1 mm
Diagram of 1/4th model	
Elliptical Hole infinite plate	

Length of plate	500 mm
Breadth of plate	500 mm
a	6 mm
b	10 mm
t	1 mm
Diagram of 1/4th model	
Elliptical Hole finite plate	
L	50 mm
D	50 mm
a	10mm
b	6 mm
t	1 mm
Diagram of 1/4th model	

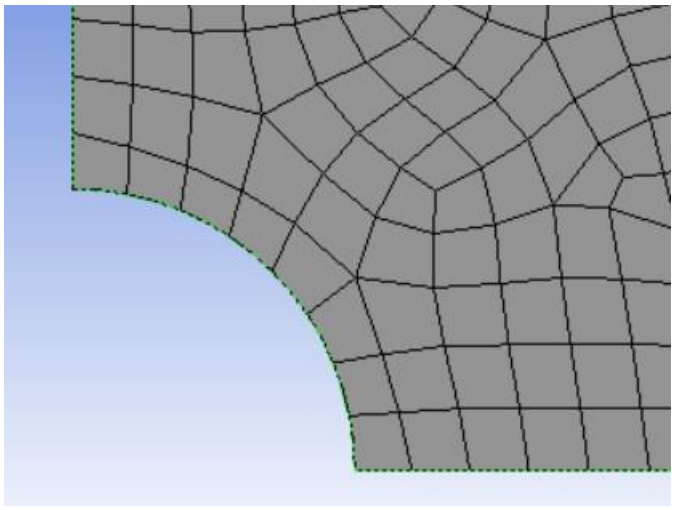
5.1 Element details:

1. Number of nodes: For infinite plate elliptical hole 46213
Number and type of elements: Contact & Solid element 15397
2. Number of nodes: For finite plate elliptical hole 7042
Number and type of elements: Contact & Solid element 2336
3. Number of nodes: For finite plate circular hole 4836
Number and type of elements: Contact & Solid element 1601

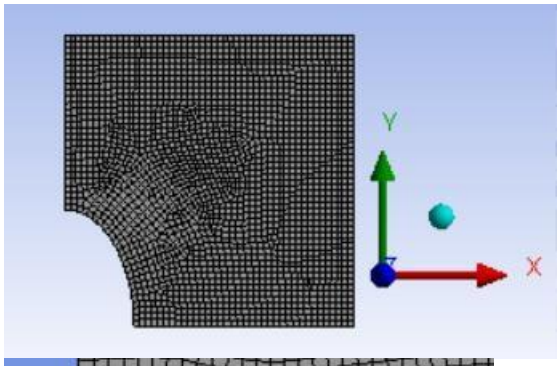
Show the models in figure after meshing: if required give multiple figures with zoomed view etc (can use more no. of column/rows).

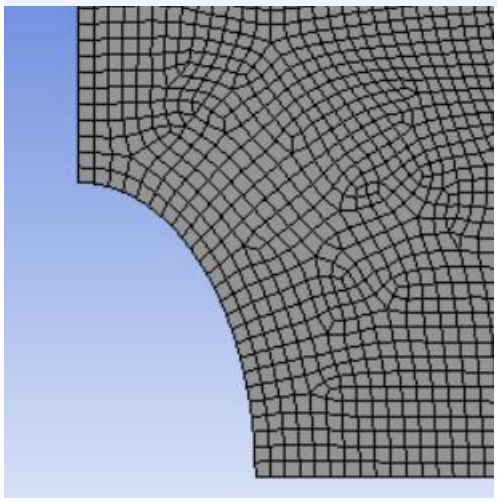


Circular hole

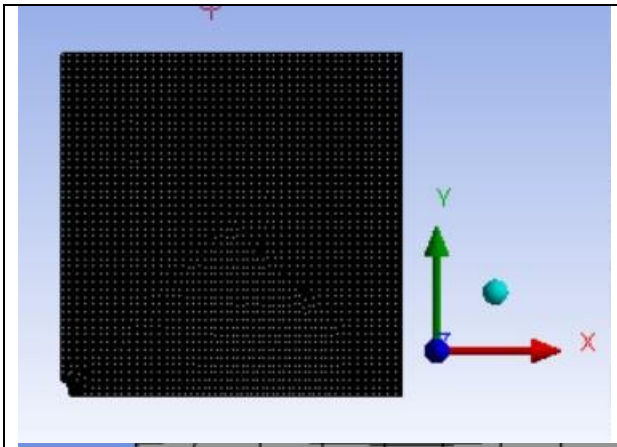


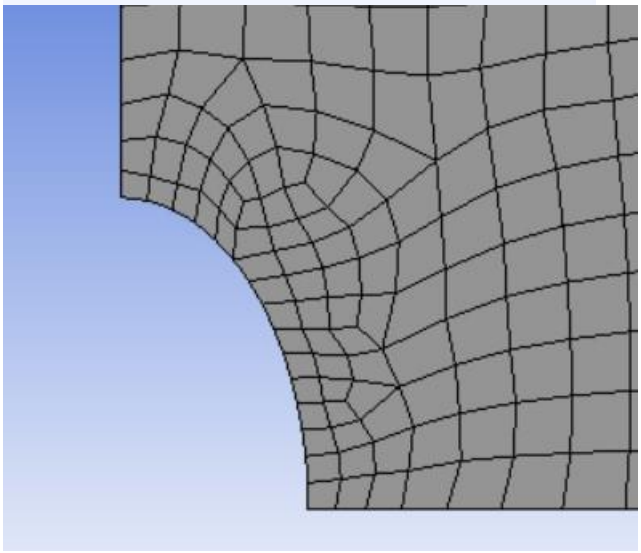
Elliptical hole finite plate





Elliptical hole infinite plate





5.2 Type of Analysis:

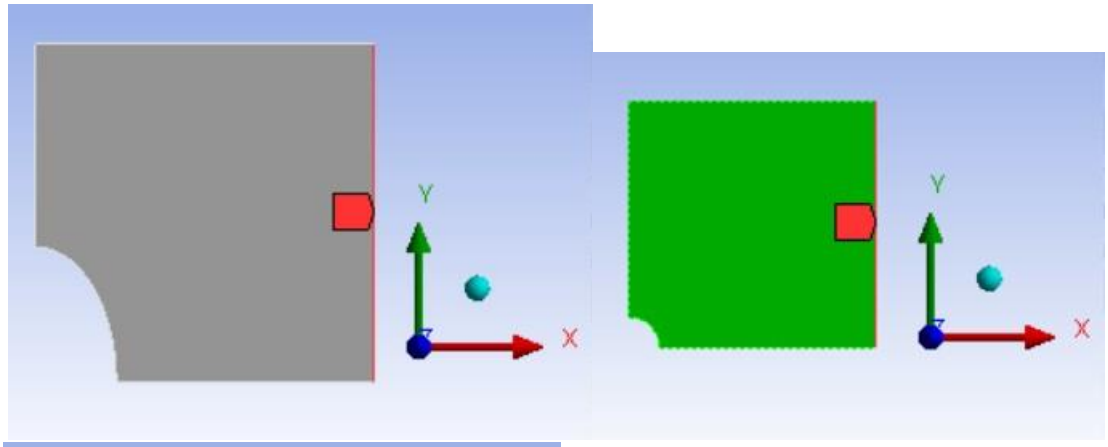
Static structural

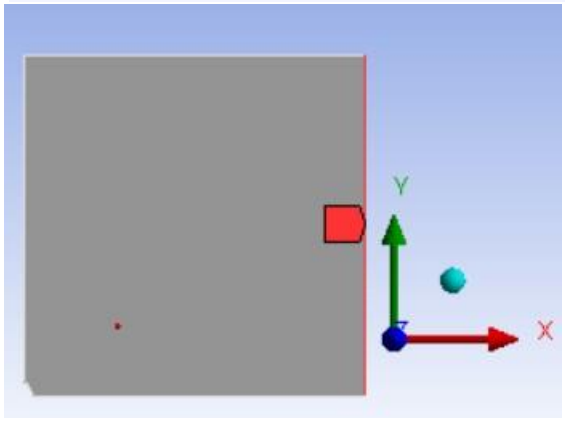
5.3 Material Data:

Young's Modulus: 200 GPa
Poisson's Ratio: 0.3
Density: N/A

5.4 Loads:

For every case pressure of 10 MPa tensile is applied





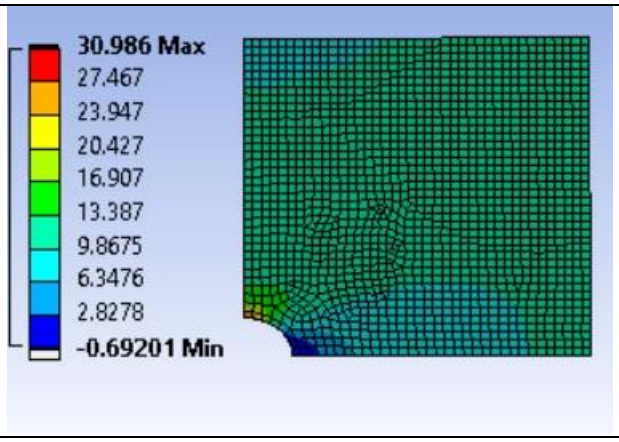
5.5 Boundary conditions:

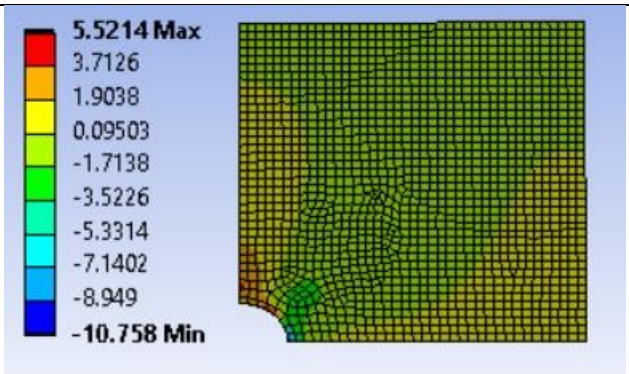
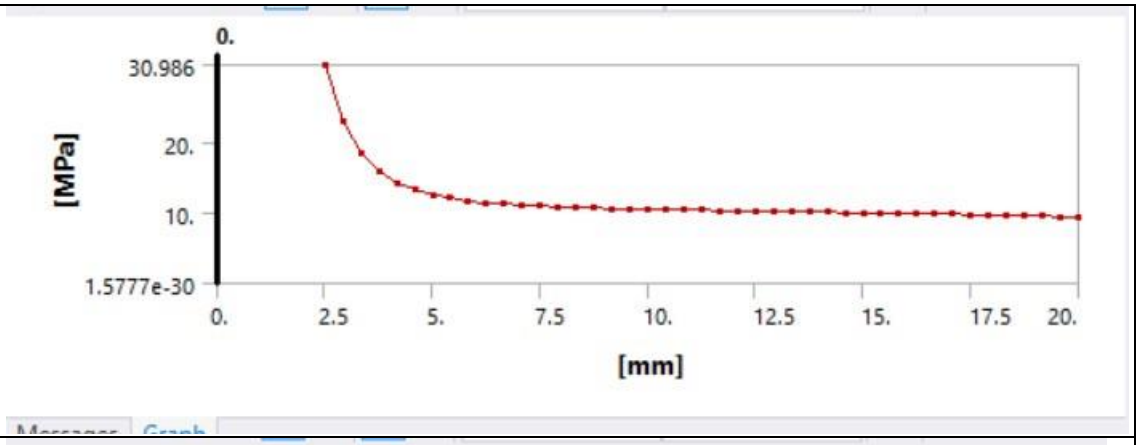
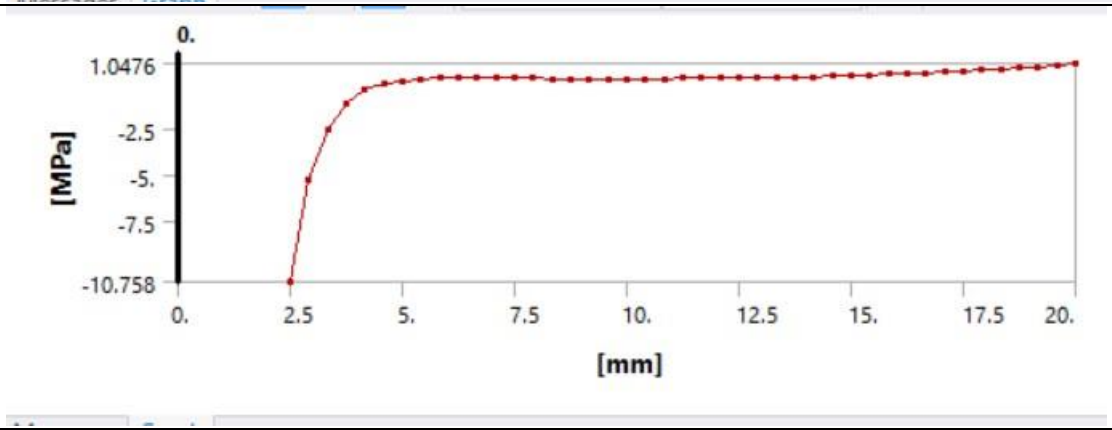
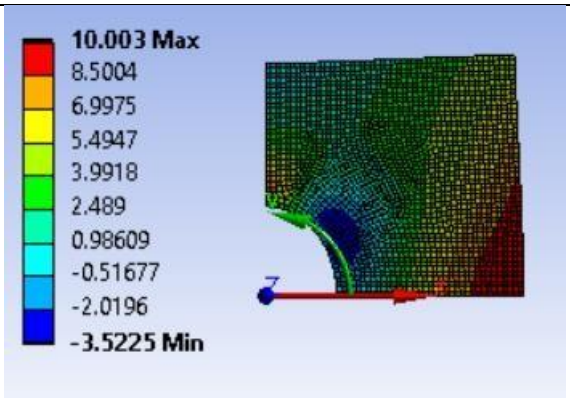
No such Boundary condition.

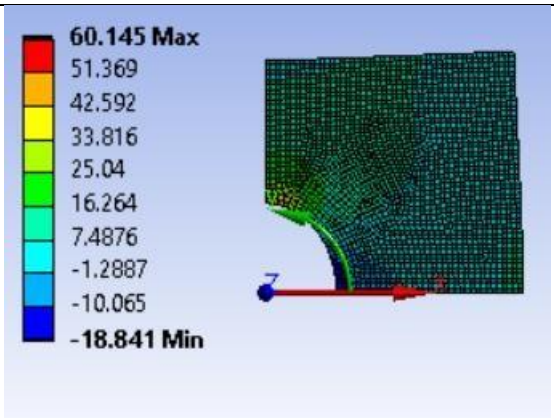
Analysis type: Plane stress , 1/4th model

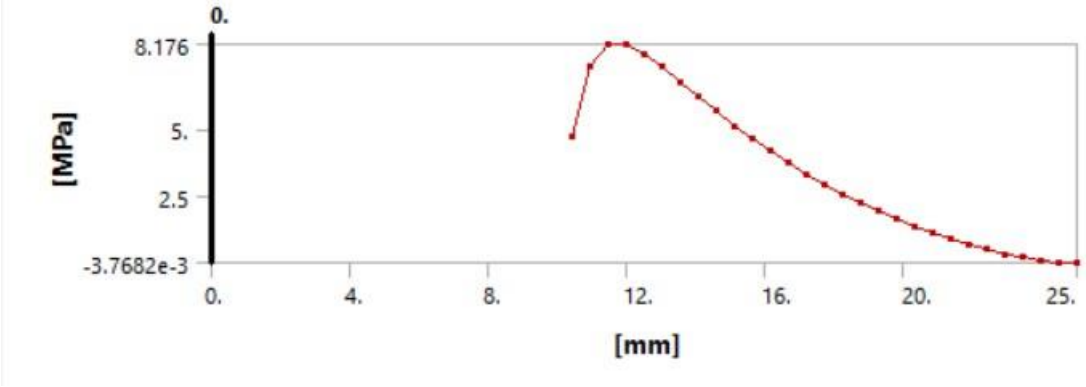
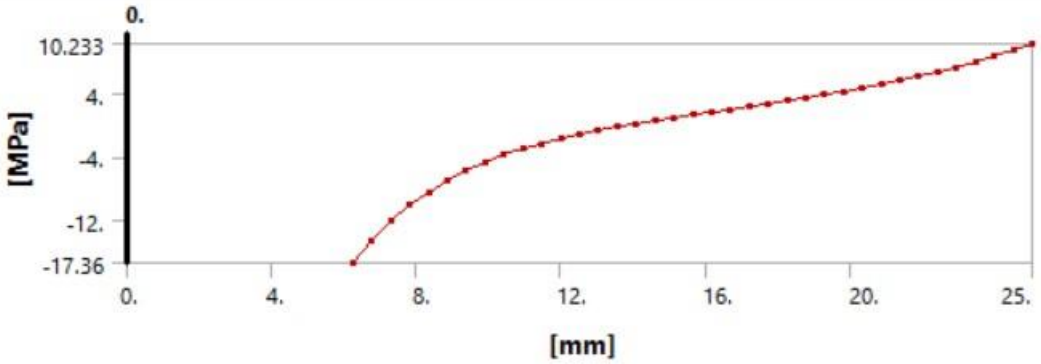
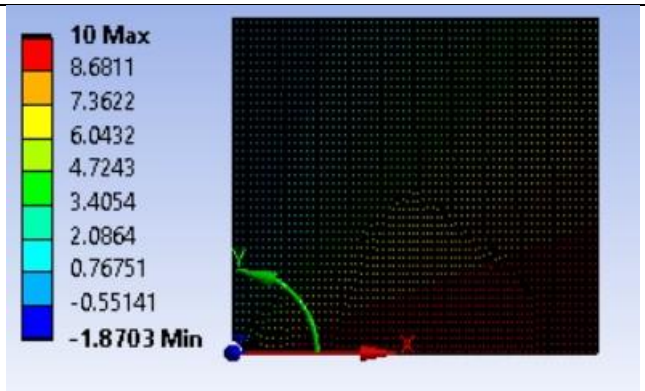
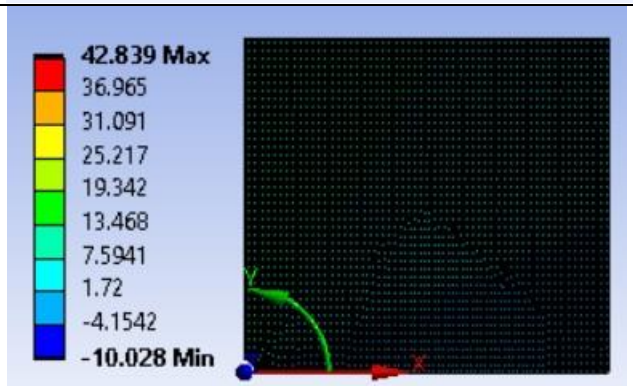
6. Calculations using Analytical solutions (if available): Analytical solution in shown in excel sheet mentioned above.

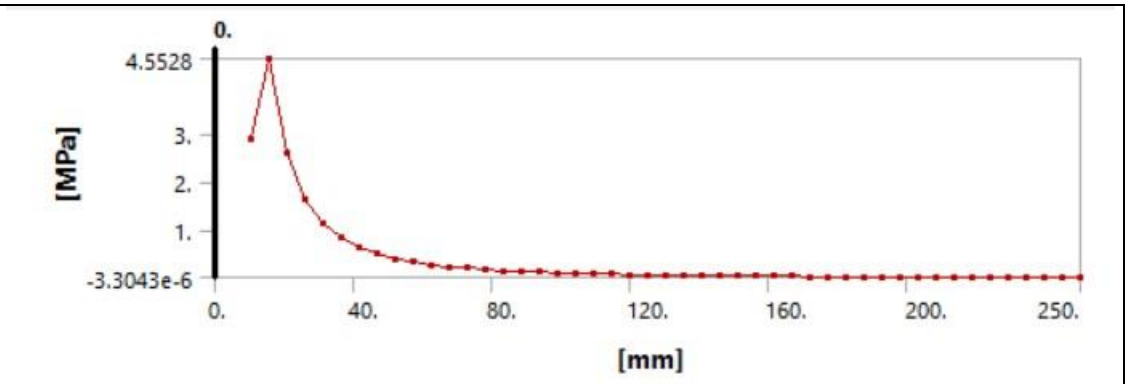
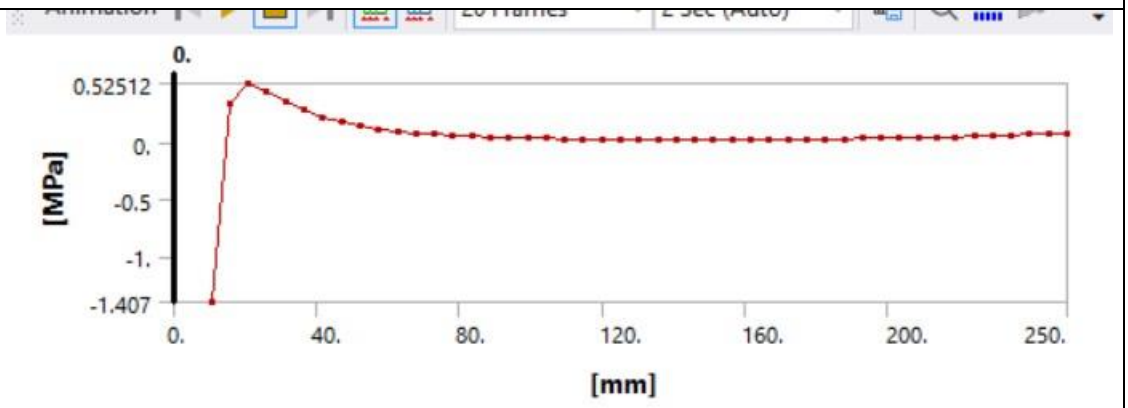
7. Results:

Circular Hole	
X axis Normal Stress	

Y axis Normal stress	
Y axis normal stress along path y axis	
Y axis normal stress along path x axis	
Elliptical hole finite plate	
X axis normal stress	

Y axis normal stress		

Y axis normal stress along path y	
Y axis normal stress along path y	
Elliptical hole infinite plate	
X axis normal stress	
Y axis normal stress	

Y axis normal stress plot along path y	
Y axis normal stress plot along path x	

Use Table for results with value/s and compare with analytical solution (if available)

Elliptical hole infinite plate				
Sigma_max_Ana	Sigma_max_FEA	Error(%)		
44.39933	42.829	-3.53684		
Elliptical hole finite plate				
Sigma_max_Ana	Sigma_max_FEA	Error(%)		
51.65817	60.145	16.42882		
Circular hole				
kt_Ana	kt_FEA	Error(%)		
2.662949	2.711275	1.814747		

Mesh sensitivity study: I here do mesh study for elliptical hole finite plate to show the perfect size of mesh with less error.

Steps	**Global mesh size**	**No. of nodes**	**No. of elements**
Step 1	**0.5 mm**	**7042**	**2336**
Step 2	**1mm**	**1844**	**610**
Step 3	**0.2 mm**	**43924**	**14612**
Step 4			

Step 5			

Steps	**Analytical solution**	**FEA results**	**% of Error**
Step 1	51.65817	60.145	16.42882
Step 2	51.65817	59.536	15.249
Step 3	51.65817	60.385	16.89341
Step 4			
Step 5			

Parametric study: The parametric result is shown below,
For infinite plate and elliptical hole

1. Vary a as 2,4,7mm

1	Name	P2 - XYPlane.L5	P3 - X Axis - Normal Stress - End Time Maximum	P4 - Y Axis - Normal Stress - End Time Maximum
2	Units	mm	MPa	MPa
3	DP 0 (Current)	6	10	42.839
4	DP 1	2	19.758	88.661
5	DP 2	4	10.786	58.404
6	DP 3	7	10	38.459
*				

2. Vary b as 15,20,25 mm

1	Name	P2 - XYPlane.L5	P5 - XYPlane.V4	P3 - X Axis - Normal Stress - End Time Maximum	P4 - Y Axis - Normal Stress - End Time Maximum
2	Units	mm	mm	MPa	MPa
3	DP 0 (Current)	6	10	10	42.839
4	DP 1	6	15	10.64	58.471
5	DP 2	6	20	15.193	71.698
6	DP 3	6	25	18.811	84.802
*					

3. Vary D as 400,600, 800 mm

	Name	P2 - XYPlane.L5	P5 - XYPlane.V4	P6 - D	P3 - X Axis - Normal Stress - End Time Maximum	P4 - Y Axis - Normal Stress - End Maximum
	Units	mm	mm	mm	MPa	MPa
	DP 0 (Current)	6	10	500	10	42.839
	DP 1	6	10	400	10	42.973
	DP 2	6	10	600	10	42.88
	DP 3	6	10	800	10	42.721

4, vary t as 5, 10,15 mm

1	Name	P2 - XYPlane.L5	P5 - XYPlane.V4	P6 - D	P7 - Surface Body Thickness	P3 - X Axis - Normal Stress - End Time Maximum	P4 - Y Axis - Normal Stress - End Time Maximum
2	Units	mm	mm	mm	mm	MPa	MPa
3	DP 0 (Current)	6	10	500	1	10	42.839
4	DP 1	6	10	500	5	10	42.839
5	DP 2	6	10	500	10	10	42.839
6	DP 3	6	10	500	15	10	42.839
*							

8. Conclusions:

All the parameters are found from the study , the variation of properties are clear by the parametric study and the error is minimised by mesh sensitivity study.

9. References:

Data are provided in assignment data sheet and for hand calculation excel sheet is used and all errors are also calculated by Excel sheet.