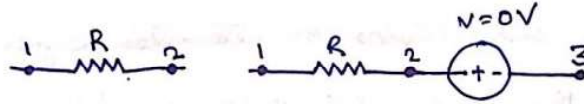
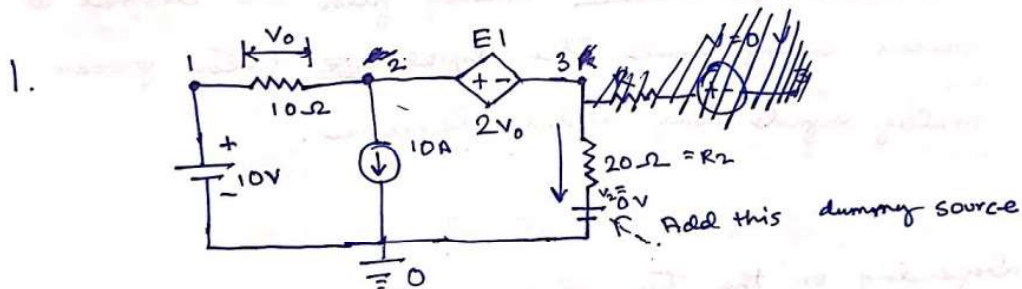
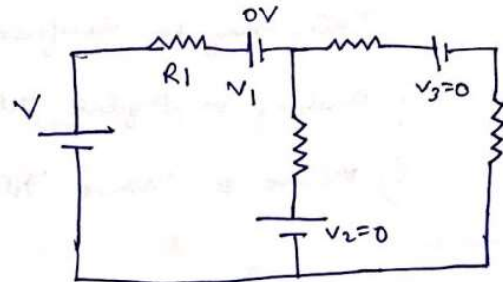
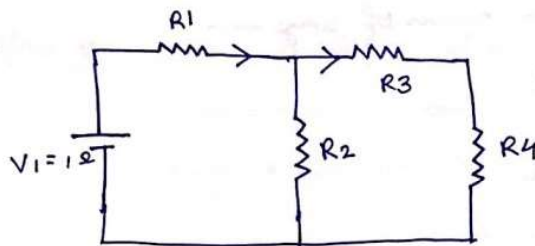


## Dummy Source



The objective of this dummy source along with .OP command is to get the value of current through the branch where the dummy source is present.



Find the current through  $20\Omega$  resistance.

Title: ~~Find~~

V1 1 0 DC 10

R1 1 2 10

I1 2 0 DC 10

E1 2 3 1 2 2

R2 3 0 20

V2 4 0 DC 0

.OP

.END

## Filter

A filter is a frequency selective circuit that passes a specified band of frequencies and blocks or attenuates signal of frequencies outside this band. That means a filter is an electrical network that can transmit signals within a specified frequency range.

### Classification of filter:

Filter may be classified in number of ways —

- 1) Analog or Digital Filter
- 2) Active or Passive Filter

Depending on the signal the filter may be classified as —

Analog or Digital Filter: Analog filters are designed to process analog signals. The digital ~~filter~~ filters process analog signals using digital techniques.

Depending on the type of component the filter may be classified as active or passive filter.

Passive Filter: Passive filters ~~are~~ consist passive elements like resistor, inductor, capacitor.

Active Filters: The filters which consist active element like transistor or OPAMP in addition with resistor and capacitor are called active filter. They do not contain inductor.

## Active Filter

- 1) Low Pass Filter
- 2) High Pass Filter
- 3) Band Pass Filter
- 4) Band Reject Filter
- 5) All Pass Filter

Pass Band: The band in which ideal filters have to pass all frequencies without reduction in magnitude, is referred to as Pass Band. That means the band of frequencies transmitted through this filter is known as Pass Band.

Stop Band: The band in which ideal filters have to attenuate or stop frequency are referred to as stop band. That means the band of frequencies which is attenuated by the filter is known as Stop Band.

Cut-Off Frequency: Frequency which separates the Pass Band and Stop Band is known as cut-off frequency or corner frequency.

The cut-off frequency is a particular frequency at which the gain of the system is  $\frac{1}{\sqrt{2}}$  times of its maximum gain.

2nd Defn: The particular frequency at which the deviation is 3dB.



### Advantages of Active Filter over passive filter:

- 1) Less cost due to the absence of inductor, active element are less costly to the passive filter.
- 2) Smaller size and weight due to the absence of bulky inductor, active filter are small in size and weight.
- 3) Excellent Impedance Matching Property: Active filters provide an excellent impedance matching property due to ~~that means i.e.~~ high input impedance and low ~~input~~ <sup>output</sup> impedance.
- 4) No Loading Problem: ~~Active filters do not~~ ~~cause loading~~ from the source or load.
- 5) High Range of Q factor (Quality Factor)

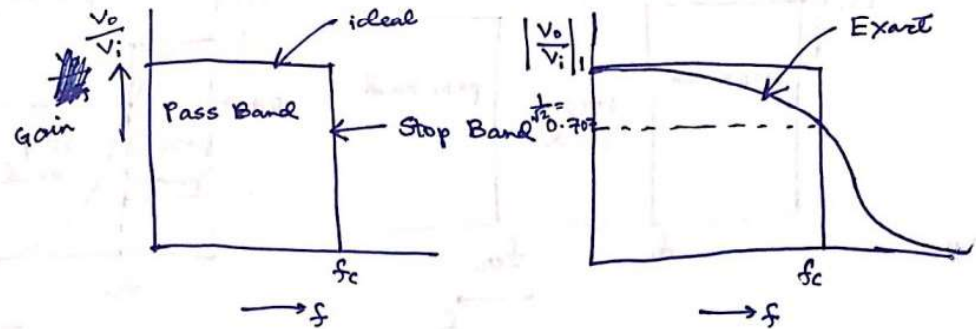
### Application of Active Filter:

The filter is a frequency selective ~~circuit~~ device. It can be used for cementing a particular band of frequency from a wide range of frequency spectrum.

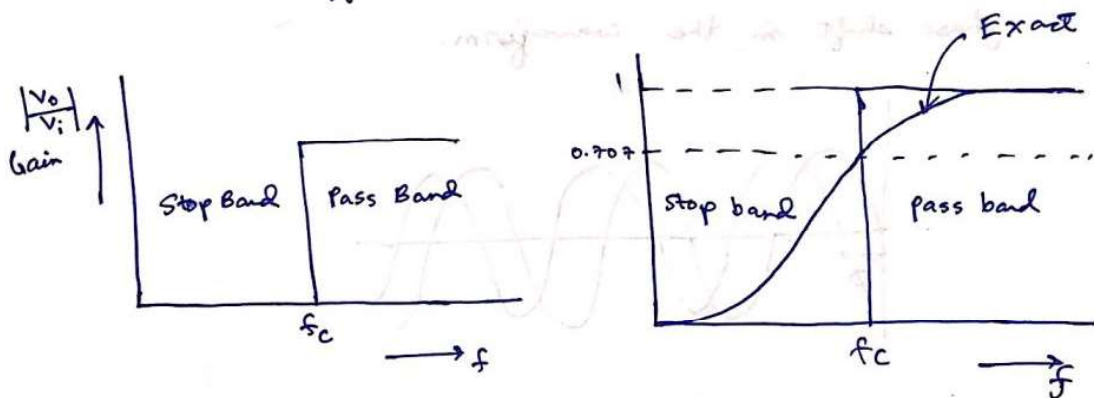
- 1) In the field of communication and signal processing.
- 2) In All electronic system such as radio, TV, radar, space ~~satellites~~ <sup>satellites</sup>, bio medical equipment.
- 3) In regulated power supply, filters are used to provide smooth ~~a~~ dc output from ac input.

## Active Filter

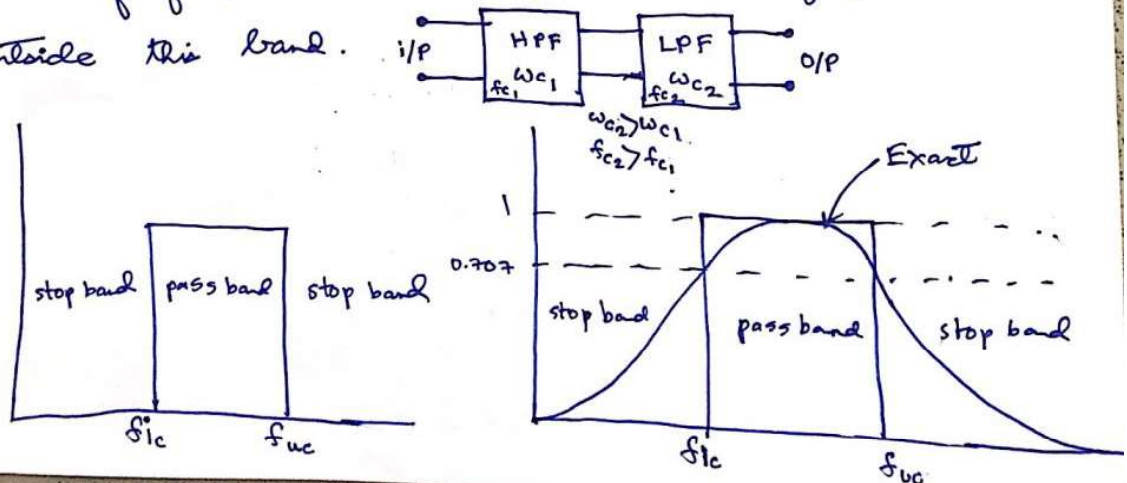
- 1) Low Pass Filter: This type of filter rejects all the frequency above cut off frequency and pass all the frequency below cut off frequency.



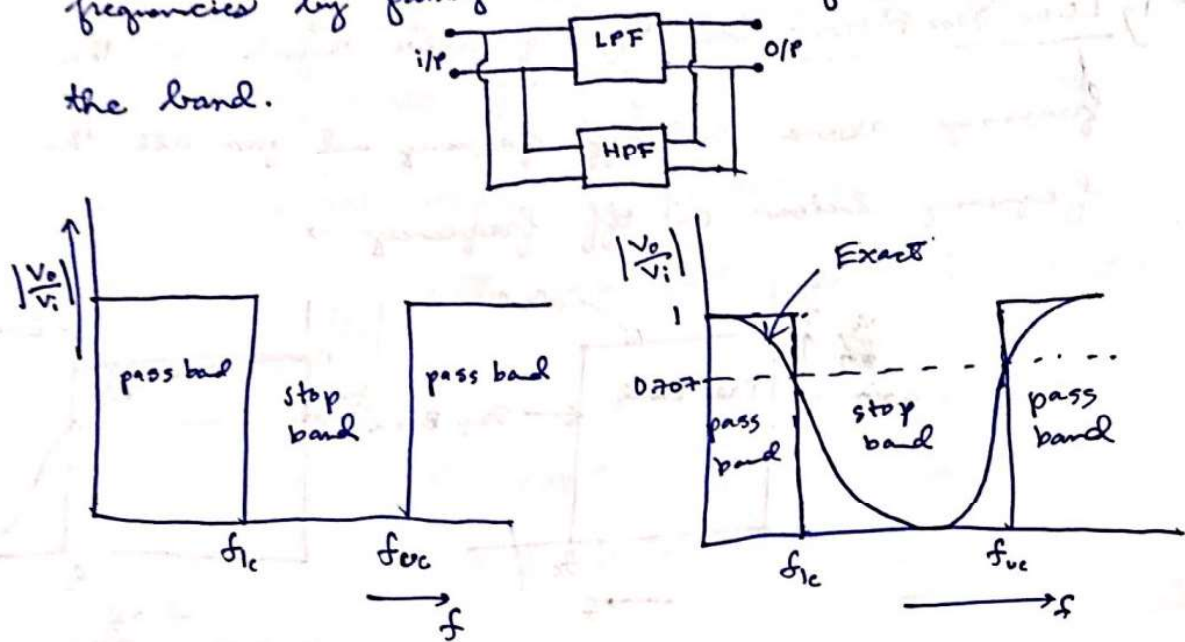
- 2) High Pass Filter: This type of filter rejects all the frequency below cut off frequency and ~~rejects~~ <sup>pass</sup> all the frequency above cut off frequency.



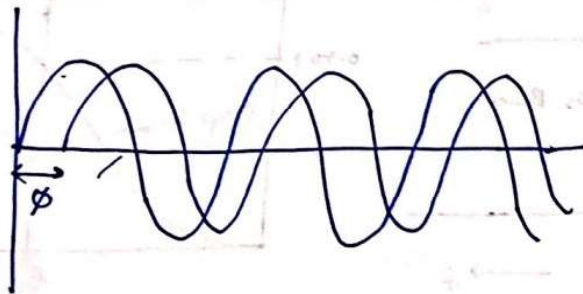
- 3) Band Pass Filter: It is a circuit that passes a particular band of frequencies and rejects all the frequencies outside this band.



4) Band Reject Filter: Rejects a specified band of frequencies by passing all the other frequencies outside the band.



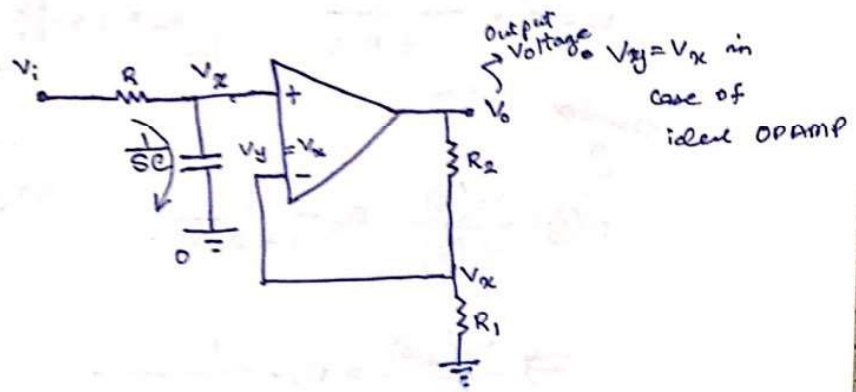
5) All Pass Filter: It passes all frequencies, i.e. output and input voltages are equal in magnitude. But there is a phase shift in the waveform.



This filter is known as phase shift filter or time delay filter.



# 1st Order Active Low Pass Filter



$$\frac{V_i - V_x}{R} = \frac{V_x - 0}{1/CS}$$

$$\Rightarrow \frac{V_i}{R} - \frac{V_x}{R} = \frac{V_x}{1/CS}$$

$$\Rightarrow V_x \left( \frac{1}{R} + CS \right) = \frac{V_i}{R}$$

$$\Rightarrow V_x \left( \frac{1 + RCS}{R} \right) = \frac{V_i}{R}$$

$$\Rightarrow V_x = \frac{V_i}{1 + RCS} \dots \dots \textcircled{1}$$

In case of ideal ~~OP~~ OPAMP

$$V_y = V_x$$

$$\text{So, } \frac{V_o - V_x}{R_2} = \frac{V_x - 0}{R_1}$$

$$\Rightarrow \frac{V_o}{R_2} = V_x \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{V_o}{R_2} = V_x \left( \frac{R_1 + R_2}{R_1 R_2} \right)$$

$$\Rightarrow V_o = V_x \left( \frac{R_1 + R_2}{R_1} \right)$$

$$\Rightarrow V_x = V_o \times \frac{R_1}{R_1 + R_2} \dots \dots \textcircled{2}$$

Equating  $\textcircled{1}$  and  $\textcircled{2}$ ,

$$\frac{V_i}{1 + RCS} = V_o \times \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow V_o = \left( \frac{R_1 + R_2}{R_1} \right) \times \frac{V_i}{1 + RCS}$$

$$\Rightarrow V_o = \left( 1 + \frac{R_2}{R_1} \right) \times \frac{V_i}{1 + RCS}$$

$\downarrow$   
 $A_o$

$$V_o = A_o \times \frac{V_i}{1 + RCs}$$

$$\left[ A_o = 1 + \frac{R_2}{R_1} \right]$$

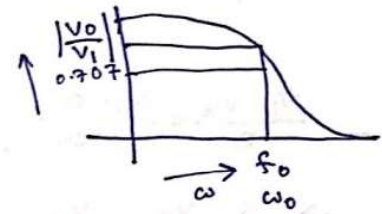
Transfer Function

$$H(s) = \frac{V_o(s)}{V_i(s)} = A_o \frac{1}{1 + RCs}$$

$$s = j\omega$$

$$\omega_o = \frac{1}{RC} \rightarrow f_o$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = A_o \frac{1}{1 + RCj\omega}$$



$$\Rightarrow \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = |A_o| \times \frac{1}{\sqrt{1 + (RC\omega)^2}}$$

$$= |A_o| \times \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}} \quad \left[ \because RC = \frac{1}{\omega_o} \right]$$

Case 1:  $\omega \ll \omega_o$

$$\Rightarrow \frac{\omega}{\omega_o} \ll 1$$

$$\Rightarrow \left(\frac{\omega}{\omega_o}\right)^2 \ll 1$$

$$\left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = |A_o| \times \frac{1}{1} = |A_o|$$

Case 2:  $\omega \gg \omega_o$

$$\Rightarrow \frac{\omega}{\omega_o} \gg 1$$

$$\Rightarrow \left(\frac{\omega}{\omega_o}\right)^2 \gg 1$$

$$\therefore \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = |A_o| \times \frac{1}{\text{high value}} = |A_o| \times 0 = 0$$



Case 3:  $\omega = \omega_0$

$$\Rightarrow \frac{\omega}{\omega_0} = 1$$

$$\Rightarrow \left(\frac{\omega}{\omega_0}\right)^2 = 1$$

$$\therefore \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = \frac{|A_0|}{\sqrt{2}} = 0.707 |A_0|$$

### Physical Significance of Active Low Pass Filter

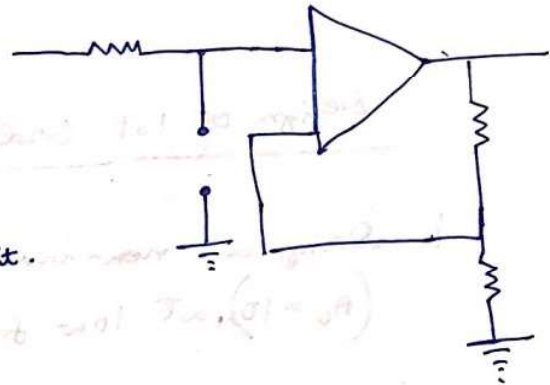
Case 1:

For a low frequency

$$X_C = \frac{1}{2\pi f C} \rightarrow 0$$

$$X_C = \infty$$

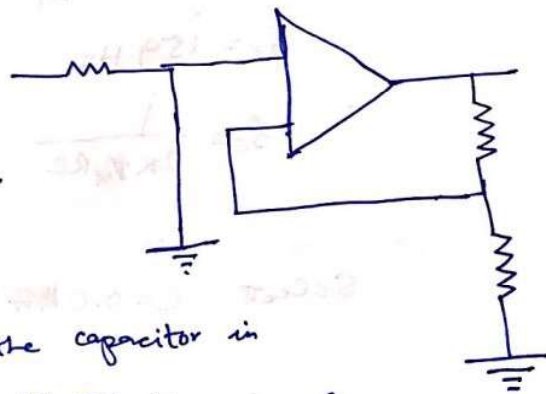
$\therefore$  It acts as an open circuit.



For High Frequency

$$X_C = 0$$

$\therefore$  It acts a short circuit.



For a low frequency signal, the capacitor in the circuit acts as open path to the signal to reach the ground potential.

Low frequency input signal  $V_i$  reaches the non-inverting terminal, and hence gets amplified at the output of

the OPAMP. Hence low frequency <sup>signal is available at</sup>  $f$  is available at the output of the OPAMP.

For high frequency, the capacitor acts as a ~~good~~ ~~of potential~~ short path so that the non-inverting terminal is at ground potential 0. So this makes the output voltage <sup>became</sup>  $V_u = \frac{R_1}{R_1 + R_2} V_o$ .  $V_u = 0$ .

$\therefore V_o = 0$ . So, it can be said that high frequency signal to be attenuated at the output of the OPAMP.

### Design of 1st Order Active Low Pass Filter

1. Design a non-inverting low pass filter, gain = 10, ( $A_0 = 10$ ), at low frequency <sup>with cutoff</sup> ~~at~~  $f_c$   $f_{req.} = 159 \text{ Hz}$ .

$$A_0 = 10 = 1 + \frac{R_2}{R_1}$$

$$f_c = 159 \text{ Hz}$$

$$f_c = \frac{1}{2\pi RC}$$

Select  $C = 0.01 \mu\text{F}$ ,  $0.047 \mu\text{F}$ ,  $100 \text{ nF}$

$$\therefore 159 = \frac{1}{2\pi R(0.047 \times 10^{-6})}$$

$$\Rightarrow R = 21.308 \text{ k}\Omega$$

$$\frac{R_2}{R_1} = 10 - 1 = 9$$

$$\Rightarrow R_2 = 9R_1$$

$$\text{Let } R_1 = 10 \text{ k}\Omega$$

$$\therefore R_2 = 90 \text{ k}\Omega$$

2. Design a non-inverting low pass filter with  $g_{ai} = 10$ ,  $f_c = 159 \text{ Hz}$ , with a input impedance<sup>(R)</sup> of  $10 \text{ k}\Omega$ .

$$\therefore f_c = \frac{1}{2\pi RC}$$

$$\Rightarrow 159 = \frac{1}{2\pi(10 \times 10^3)C}$$

$$\Rightarrow C = 100 \text{ nF}$$

$$\therefore \frac{R_2}{R_1} = 10 - 1 = 9$$

$$\Rightarrow R_2 = 9R_1$$

$$\text{Let } R_1 = 10 \text{ k}\Omega$$

$$\therefore R_2 = 90 \text{ k}\Omega$$



# 1st Order Active High Pass Filter

An ideal OPAMP,

$$V_x = V_y$$

$$\frac{V_i(s) - V_x(s)}{1/sC} = \frac{V_x(s) - 0}{R}$$

$$\Rightarrow \frac{V_i}{1/s} = \frac{V_x}{1/sC} + \frac{V_x}{R}$$

$$\Rightarrow sC V_i = sC V_x + \frac{V_x}{R}$$

$$\Rightarrow sC V_i = V_x \left( \frac{1}{R} + sC \right)$$

$$\Rightarrow sC V_i = V_x \frac{1 + RCS}{R}$$

$$\Rightarrow V_x = \frac{RCS}{RCS + 1} V_i \dots \dots \dots (1)$$

$$\frac{V_o - V_x}{R_2} = \frac{V_x - 0}{R_1}$$

$$\Rightarrow \frac{V_o}{R_2} = V_x \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{V_o}{R_2} = V_x \left( \frac{R_1 + R_2}{R_1 R_2} \right)$$

$$\Rightarrow V_x = \frac{R_1}{R_1 + R_2} V_o \dots \dots \dots (2)$$

Equating (1) and (2),

$$\left( \frac{R_1}{R_1 + R_2} \right) V_o = \left( \frac{RCS}{RCS + 1} \right) V_i$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \left( \frac{RCS}{RCS + 1} \right) \left( \frac{R_1 + R_2}{R_1} \right)$$

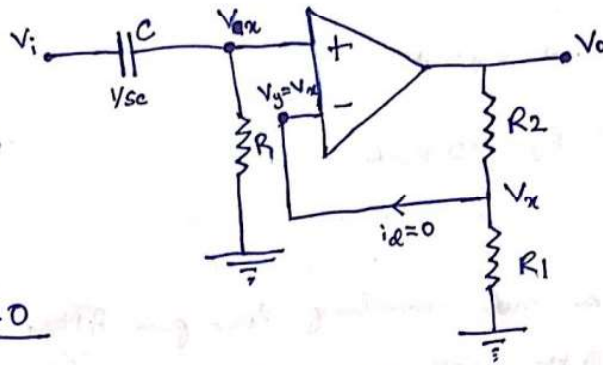


fig (i)

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \left( \frac{RCS}{RCS+1} \right) \left( 1 + \frac{R_2}{R_1} \right) \rightarrow A_o$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = A_o \left( \frac{RCS}{RCS+1} \right)$$

$$\Rightarrow H(s) = A_o \left( \frac{RCS}{RCS+1} \right)$$

For RC Circuit, time constant

$$T = RC$$

$$\omega_o = \frac{1}{T} = \frac{1}{RC}$$

$$f_o = \frac{1}{2\pi RC}$$

$$s = j\omega$$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = A_o \left( \frac{j\omega/\omega_o}{j\omega/\omega_o + 1} \right)$$

$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = A_o \left( \frac{j\omega}{j\omega + \omega_o} \right)$$

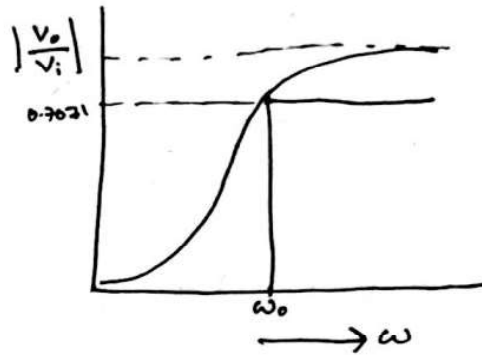
$$RC = \frac{1}{\omega_o}$$

Finding magnitude,

$$\left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = |A_o| \left( \frac{\sqrt{\omega^2}}{\sqrt{\omega_o^2 + \omega^2}} \right)$$

$$= |A_o| \frac{1}{\sqrt{1 + \left( \frac{\omega_o}{\omega} \right)^2}}$$

$$\Rightarrow \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right| = |A_o| \frac{1}{\sqrt{1 + \left( \frac{\omega_o}{\omega} \right)^2}}$$



Case 1:  $\omega$  is very small ( $\omega \ll \omega_0$ )  $\Rightarrow \omega_0 \gg \omega$

$$\Rightarrow \frac{\omega_0}{\omega} \gg 1$$

$$\Rightarrow \left(\frac{\omega_0}{\omega}\right)^2 \gg \gg 1$$

$$\frac{V_o}{V_i} = |A_o| \times \frac{1}{\infty}$$

$$\Rightarrow \frac{V_o}{V_i} \rightarrow 0$$

$$\Rightarrow V_o \rightarrow 0$$

Case 2:  $\omega \gg \omega_0$

$$\omega_0 \ll \omega$$

$$\frac{\omega_0}{\omega} \ll 1$$

$$\left(\frac{\omega_0}{\omega}\right)^2 \ll \ll 1$$

$$\frac{V_o}{V_i} = A_o \times \frac{1}{1}$$

$$= A_o$$

$$\Rightarrow \frac{V_o}{V_i} = A_o$$

$$\Rightarrow V_o = A_o V_i$$



Case 3:  $\omega = \omega_0$

$$\frac{\omega_0}{\omega_0} = 1$$

$$\Rightarrow \left(\frac{\omega_0}{\omega}\right)^2 = 1$$

$$\frac{V_o}{V_i} = A_o \times \frac{1}{\sqrt{1+1}}$$

$$\Rightarrow \frac{V_o}{V_i} = A_o \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{V_o}{V_i} = 0.707 \times A_o$$

Case 1:

For low frequency signal, the capacitor in the circuit acts as an open gate to the non-inverting terminal of OPAMP. So the input signal  $V_i$  cannot reach to the non-inverting ~~signal~~ terminal. So  $V_o = 0$ . That means the low frequency signal is not available at the output of the OPAMP.

Case 2:

For high frequency signal, the capacitor acts as a shorted gate to the non-inverting terminal of the OPAMP and gets amplified at the OUTPUT of the OPAMP. So it can be said that the high frequency signal is available at the output of the OPAMP in amplified form.

## Design of Active high pass filter

1. Design a non-inverting high pass filter for a passband gain = 5 and cut-off frequency = 500 Hz.

Select a  $C = 0.01 \mu F$ .

$$f_c = \frac{1}{2\pi RC}$$

$$\Rightarrow 500 = \frac{1}{2\pi R \times 0.01 \times 10^{-6}}$$

$$\Rightarrow R = 3.185 \text{ K}\Omega$$

$$A_0 = 5 = \left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow \frac{R_2}{R_1} = 4$$

$\therefore$  Select ~~any~~  $R_2 = 4R_1$

$$R_1 = 10 \text{ K}\Omega$$

$$R_2 = 40 \text{ K}\Omega.$$

Draw sig(i) and put these values.

2. Design a non-inverting high pass filter of gain = 10, cut off frequency 159 Hz, cal <sup>input</sup> ~~put~~ impedance of  $10 \text{ K}\Omega$ .

$$C = \frac{1}{2\pi f_c R}$$

$$= \frac{1}{2\pi \times 159 \times 10 \times 10^3}$$

$$A_B = 10 = \left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow \frac{R_2}{R_1} = 9$$

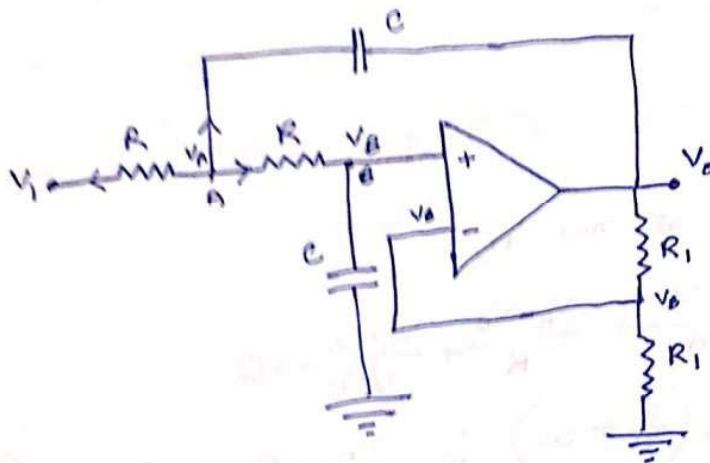
$$\Rightarrow R_2 = 9R_1$$

$$\text{Select } R_1 = 10 \text{ K}\Omega$$

$$R_2 = 90 \text{ K}\Omega$$

Draw sig (1) and put these values.

### 2nd Order Low Pass Filter



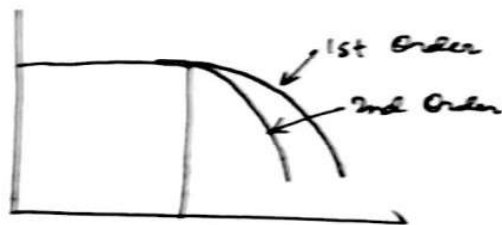
2nd Order LPF contains two storage element like capacitor along with the resistive element. So the transfer function of this filter will be such that its denominator we have maximum power of  $s$  as 2.

- 1) 2nd Order LPF has very high and constant gain when the signal frequency is low in comparison



For the 1st order LPF.

- 2) At high frequency the gain of the filter is very low compared to the 1st order LPF.
- 3) The attenuation of high frequency signal is more in 2nd order LPF than 1st order.
- 4) The frequency response of the 2nd order filter is better than that of the 1st order.



### Analysis

Apply KCL at node A,

$$\frac{V_A - V_i}{R} + \frac{V_A - V_B}{R} + \frac{V_A - V_o}{1/sC} = 0$$
$$\Rightarrow V_A \left( \frac{2}{R} + sC \right) - \frac{V_i}{R} - \frac{V_o}{R} - sC V_o = 0 \dots \dots (1)$$

$$\frac{V_o - V_B}{R_1} = \frac{V_B - 0}{R_1}$$

$$\Rightarrow \frac{V_o}{R_1} = V_B \times \frac{2}{R_1}$$

$$\Rightarrow V_B = \frac{V_o}{2}$$

$$\frac{V_A - V_B}{R} = \frac{V_B}{1/sC} = I$$

$$\Rightarrow \frac{V_A}{R} = V_B \left( \frac{1}{R} + sC \right)$$

$$\Rightarrow \frac{V_A}{R} = \frac{V_B}{R} (1 + sCR)$$

$$\Rightarrow V_A = V_B (1 + sCR)$$

$$\Rightarrow V_A = \frac{V_o}{2} (1 + sCR) \quad \left[ \because V_B = \frac{V_o}{2} \right]$$

$\therefore$  in ①,

$$\frac{V_o}{2} \left( \cancel{1} + \frac{2}{R} + sC \right) (1 + sCR) - \frac{V_i}{R} - \frac{V_o}{2R} - sCV_o = 0$$

$$\Rightarrow \frac{V_o}{2R} (2 + sCR) (1 + sCR) - \frac{V_i}{R} - \frac{V_o}{2R} - sCV_o = 0$$

$$\Rightarrow V_o (2 + sCR) (1 + sCR) - 2V_i - V_o - 2sCRV_o = 0$$

$$\Rightarrow V_o (2 + 2sCR + sCR + s^2 C^2 R^2) - 2V_i - V_o - 2sCRV_o = 0$$

$$\Rightarrow V_o (2 + 3sCR + s^2 C^2 R^2) - 2V_i - V_o - 2sCRV_o = 0$$

$$\Rightarrow V_o (1 + sCR + s^2 C^2 R^2) - 2V_i = 0$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{2}{s^2 C^2 R^2 + sCR + 1}$$

$$= \frac{2}{s^2 C^2 R^2 \left( s^2 + \frac{s}{RC} + \frac{1}{R^2 C^2} \right)}$$

$$= \frac{2/R^2 C^2}{s^2 + \frac{s}{RC} + \frac{1}{R^2 C^2}}$$

$$= \frac{2\omega_o}{s^2 + s\omega_o + \omega_o^2}$$

$$RC = T = \frac{1}{\omega_o}$$

$$\omega_o = \frac{1}{RC}$$

$$\Rightarrow \frac{V_o}{V_i} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right) + 1}$$

Transfer function

$$\Rightarrow H(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \left(\frac{s}{\omega_0}\right) + 1}$$

$$\text{Let } K=2, Q=1$$

$$\therefore H(s) = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1} \dots \dots (4)$$

At low frequency,

$$\omega \ll \omega_0$$

$$\frac{\omega}{\omega_0} \ll 1$$

$$\left(\frac{\omega}{\omega_0}\right)^2 \ll \ll 1$$

$$H(s) = \frac{K}{\underbrace{\left(\frac{s}{\omega_0}\right)^2}_{\ll 1} + \underbrace{\frac{1}{Q}\left(\frac{s}{\omega_0}\right)}_{\ll 1} + 1}$$

$$\Rightarrow H(s) = K \quad \text{constant.}$$

At high frequency,

$$\omega \gg \omega_0$$

$$\frac{\omega}{\omega_0} \gg 1$$

$$\left(\frac{\omega}{\omega_0}\right)^2 \gg \gg 1$$

$$\frac{V_o}{V_i} \rightarrow 0$$

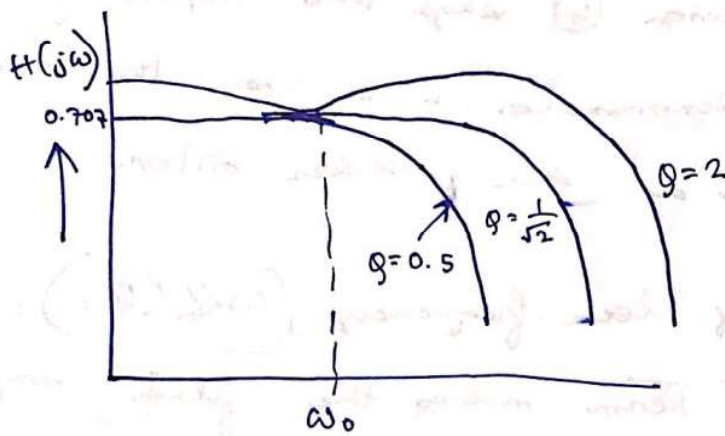
$$V_o \rightarrow 0$$



## Conclusion

1. This expression (4) says that highest power of  $s$  in the denominator is 2. i.e. this circuit behaves as a 2nd Order filter.
2. At very low frequency ( $\omega \ll \omega_0$ ), the presence of  $s^2$  term makes the filter very high and constant gain.
3. At high frequency ( $\omega \gg \omega_0$ ), the gain of the filter tends to 0. i.e.,  $\frac{V_o}{V_i} \rightarrow 0, \Rightarrow V_o \rightarrow 0$ .  
i.e. the filter blocks the high frequency signal to reach at the output.
4. The cut off frequency  $\omega_0$  depends on  $R$  and  $C$ .  
So by arranging the value of  $R$  and  $C$ , the value can be changed.
5. The value of  $Q$  is equal to 1.  
because here the resistor and capacitor is identical.  
If the  $R$  and  $C$  value are different,  
 ~~$Q$~~   $Q$  value may be different.

## Pattern of Response



By adjusting different component of R and C in the OPAMP circuit, the value of  $Q$  can be adjusted to give different pattern of response.

By adjusting  $Q=0.707$ , the horizontal portion of the response curve towards the cut off frequency is maximum flat which is most realistic response of the low pass filter.