Mod 1

Network Theorem | NC

Mod 2

Laplace Transform and its Electrical Application | ED

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Laplace Transform and its Electrical Application

$$V(t) = i(t)R + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt}$$

Converting to s domain,

$$V(s) = RI(s) + \frac{I(s)}{sC} + LsI(s)$$

Let $F(0)=0$

$$= I(s)\left[R + \frac{1}{Cs} + Ls\right]$$

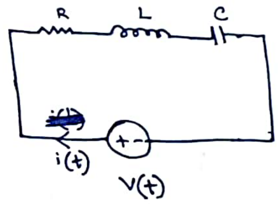
$$\Rightarrow I(s) = \frac{V(s)}{\left[R + \frac{1}{Cs} + Ls\right]}$$

$$= \frac{V(s)Cs}{RCs + 1 + LCs^2}$$

$$= \frac{V(s)\cancel{C}s}{L\cancel{C}\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}$$

$$L[f(t)] = F(s)$$

Time domain to s domain



$$V(t) = V_R(t) + V_C(t) + V_L(t)$$

$$= i(t)R + \frac{1}{C}\int i(t)\,dt + L\frac{di(t)}{dt}$$

$$L\left[\int F(t)\,dt\right] = \frac{F(s)}{s}$$

$$L\left[\frac{d}{dt}f(t)\right] = \left[sF(s) - F(0)\right]$$

Step I: Convert the given circuit in s domain → Write the circuit KVL/KCL equation in s domain

Using Laplace Transform

Circuit given in time domain

Using Classical method ✗ Not Needed

Solving Algebraic Equation

Desired quantity in time domain

Inverse Laplace Transform

Find desired quantity in s-domain

To be studied—

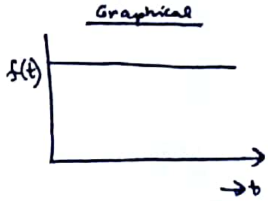
- Various types of signals and their laplace transforms
- s domain representation of R, L, C.
- Transient Response
- Initial and Final Value Theorem

Various Types of Signals and their Laplace Transforms

$$L[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s)$$

–ve time cannot be solved by Laplace Transform.

i) Step Function:

t	s	Graphical
$Au(t)$: $f(t) = A$ (Const) $t > 0$; $= 0$ $t \leq 0$	$F(s) = \frac{A}{s}$	

ii) Unit Step Function:

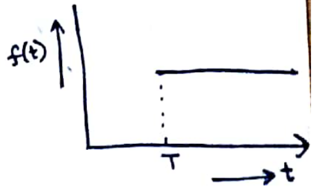
$u(t)$: $f(t) = 1$ $t > 0$; $= 0$ $t \leq 0$

$F(s) = \frac{1}{s}$

iii) Delayed Unit Step Function:

$U(t-T)$: $f(t) = 1$ $t > T$; $= 0$ $t \leq T$

$F(s) = \frac{e^{-Ts}}{s}$



Physical Significance of Step Function:

Any DC voltage source or battery can be considered as a step function.

2)

i) Ramp Function:

$$r(t)\quad \boxed{\begin{aligned} f(t) &= t, \quad t > 0 \\ &= 0 \quad t \leq 0 \end{aligned}}$$

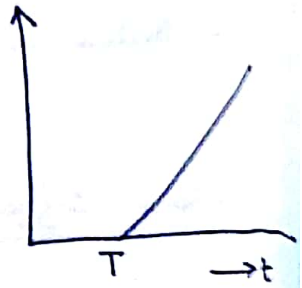
= At → slope = A

$$F(s) = \frac{1}{s^2}$$

ii) Delayed Ramp Function:

$$r(t-T)\quad \boxed{\begin{aligned} f(t) &= t, \quad t > T \\ &= 0 \quad t \leq T \end{aligned}}$$

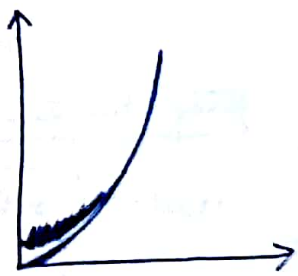
$$F(s) = \frac{e^{-Ts}}{s^2}$$



3) Parabolic Function:

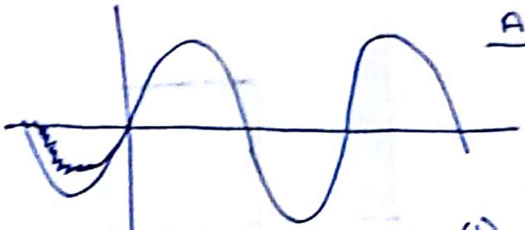
$$\boxed{\begin{aligned} f(t) &= At^2 \quad t > 0 \\ &= 0 \quad t \leq 0 \end{aligned}}$$

$$F(s) = \frac{2}{s^3}$$



4) General Functions:

i)



A sin ωt

i) For $t<0$, laplace Transform cannot be found.

$\therefore$ Laplace Transform cannot be formed here.

ii)

$A \sin \omega t\, u(t)$

ii) $F(s) = A \dfrac{\omega}{s^2+\omega^2}$

iii)

$A \sin \omega t\, u(t-T)$ $\rightarrow A \sin \omega(t-T+T)\, u(t-T)$

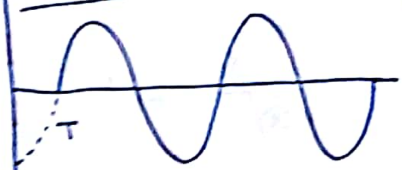
$\cancel{F(s)=} \mathcal{L}\{A[\sin \omega(t-T) \cos \omega T + \cos \omega(t-T) \sin \omega T]$

$\mathcal{L}\{A[\cos \omega T \sin \omega(t-T)\, u(t-T) + \sin \omega T \cos \omega(t-T)\, u(t-T)]$

$F(s) = A \cos \omega T \dfrac{\omega e^{-Ts}}{s^2+\omega^2} + A \sin \omega T \dfrac{s e^{-Ts}}{s^2+\omega^2}$

iv)

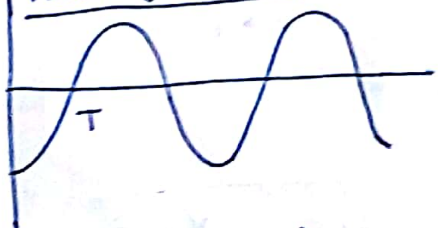
$A \sin \omega(t-T)\, u(t-T)$



iv) $F(s) = A e^{-Ts} \dfrac{\omega}{s^2+\omega^2}$

v)

$A \sin \omega(t-T)\, u(t)$



v) $A \sin \omega(t-T)\, u(t)$

$= A[\sin \omega t \cos \omega T - \sin \omega T \cos \omega t]\, u(t)$

$= A[\cos \omega T \sin \omega t\, u(t) - \sin \omega T \cos \omega t\, u(t)]$

$= A \cos \omega T \dfrac{\omega}{s^2+\omega^2} - A \sin \omega T \dfrac{s}{s^2+\omega^2}$

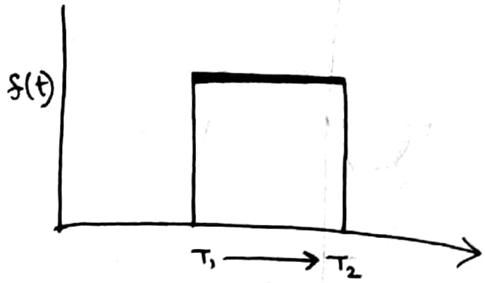
Gate Signal

$G_{T_1,T_2}(t) / G(T_1,T_2) \rightarrow$ signifies Gate Function

$f(t) = 1 \quad T_1 < t < T_2$

$= 0 \quad t \geq T_2$

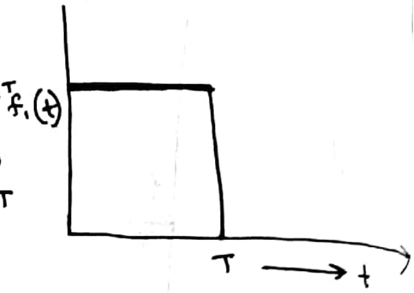
$\quad\;\; t \leq T_1$



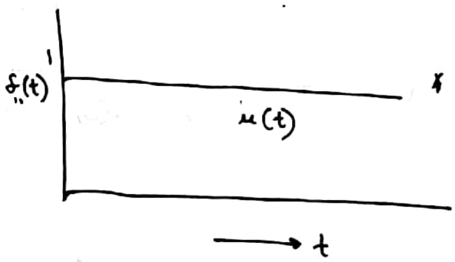
$f_1(t) = 1 \quad 0 < t < T$

$= 0 \quad t \leq 0$

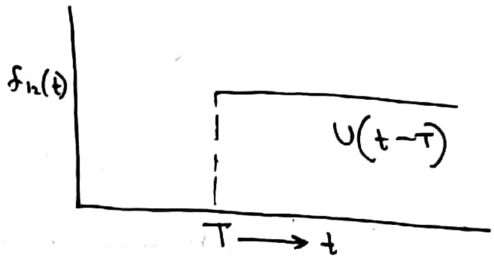
$\quad\;\; t \geq T$



- How to find the Laplace Transform of Gate Signal ?



$L[f(t)] = \frac{1}{s}$



$L[f(t)] = \frac{e^{-Ts}}{s}$

$f_1(t) = f_{11}(t) - f_{12}(t)$

$F_1(s) = F_{11}(s) - F_{12}(s)$

$= \frac{1}{s}\left(1 - e^{-Ts}\right)$

$\therefore \cancel{G(T_1,T_2)}\; G(0,T)$

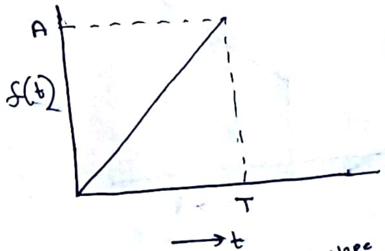
$= U(t) - U(t-T)$

$G_{T_1,T_2}(t)$

$= U(t-T_1) - U(t-T_2)$

$= \frac{1}{s}\left(e^{-T_1 s} - e^{-T_2 s}\right)$

Ramp Function upto T



$$f(t) = \overset{\to m,\ \text{i.e. slope}}{\frac{A}{T}}\, t\ \overset{\to}{G_{0,T}(t)}$$

Multiplication by this Gate Function makes Laplace Transformation possible, because otherwise time is not mentioned (whether +ve or −ve or any range)

$$= \frac{A}{T}\left[t\right]\left[U(t) - U(t-T)\right]$$

$$= \frac{A}{T}\left[tU(t) - t\,U(t-T)\right]$$

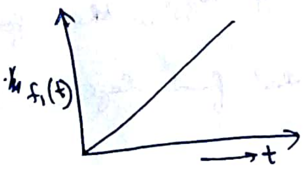
→ Time scale is different hence Laplace Transform can't be applied

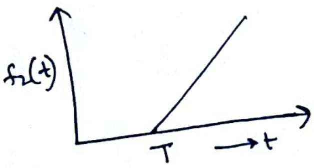
$$= \frac{A}{T}\left[t\,U(t) - (t-T+T)\,U(t-T)\right]$$

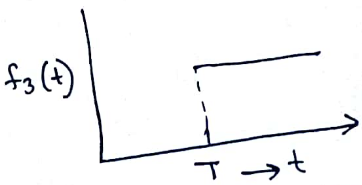
$$= \frac{A}{T}\left[tU(t) - (t-T)\,U(t-T) - T\,U(t-T)\right]$$

Taking Laplace Transforms,

$$F(s) = \frac{A}{T}\left[\underbrace{\frac{1}{s^2}}_{\text{Ramp}} - \underbrace{\frac{e^{-Ts}}{s^2}}_{\text{Delayed Ramp}} - \underbrace{\frac{Te^{-Ts}}{s}}_{\text{Delayed Step}}\right]$$







Gate Signal

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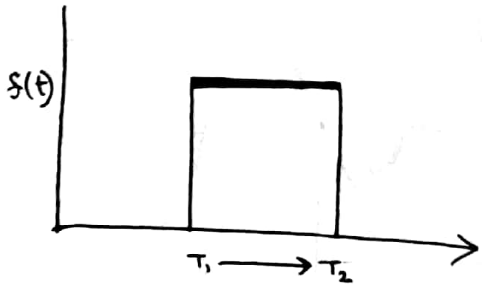
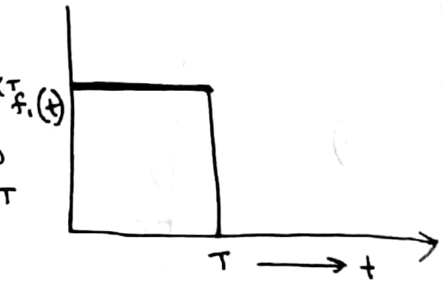


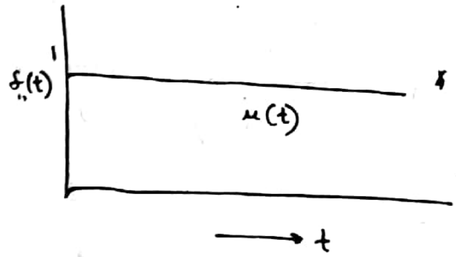
$f_1(t) = 1 \quad 0 < t < T$

$\quad = 0 \quad t \leq 0$

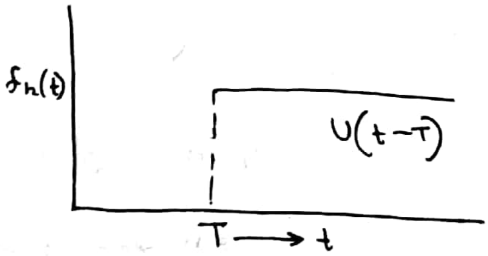
$\quad\quad\quad t \geq T$



- How to find the Laplace Transform of Gate Signal?



$L[f(t)] = \frac{1}{s}$



$L[f(t)] = \frac{e^{-Ts}}{s}$

$f_1(t) = f_{11}(t) - f_{12}(t)$

$F_1(s) = F_{11}(s) - F_{12}(s)$

$\quad = \frac{1}{s}\left(1 - e^{-Ts}\right)$

$\therefore \cancel{G(T_1,T_2)}\ G(0,T)$

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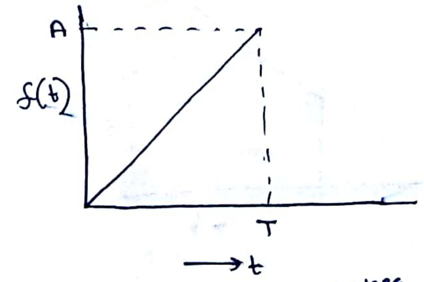
$G_{T_1,T_2}(t)$

$= U(t-T_1) - U(t-T_2)$

$= \frac{1}{s}\left(e^{-T_1 s} - e^{-T_2 s}\right)$

Ramp Function upto T



$$f(t) = \overset{m,\text{ i.e. slope}}{\frac{A}{T}}\, t\; G_{0,T}(t)$$

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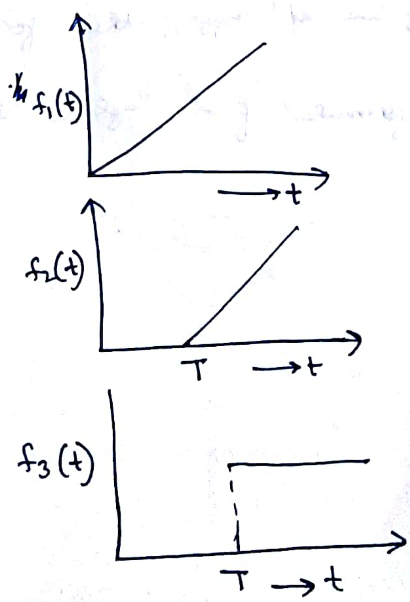
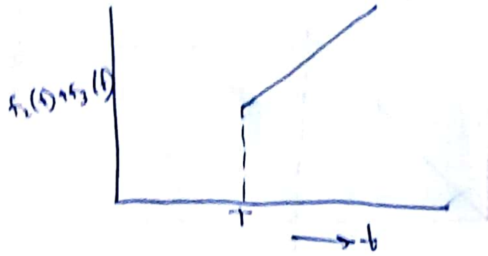
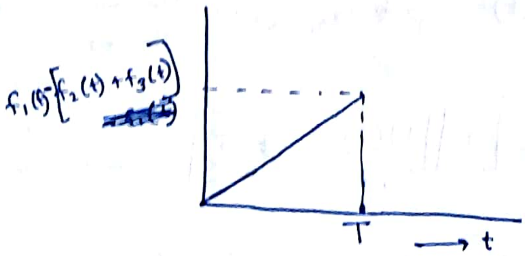
$$= \frac{A}{T}\Big[tU(t) - (t-T)\,U(t-T) - T\,U(t-T)\Big]$$

Taking Laplace Transforms,

$$F(s) = \frac{A}{T}\Bigg[\underbrace{\frac{1}{s^2}}_{\text{Ramp}} - \underbrace{\frac{e^{-Ts}}{s^2}}_{\text{Delayed Ramp}} - \underbrace{\frac{Te^{-Ts}}{s}}_{\text{Delayed Step}}\Bigg]$$







- How to find the Laplace Transform of any Signal ?

<u>If the function is aperiodic</u>

Step 1 : Write the equation of the given function

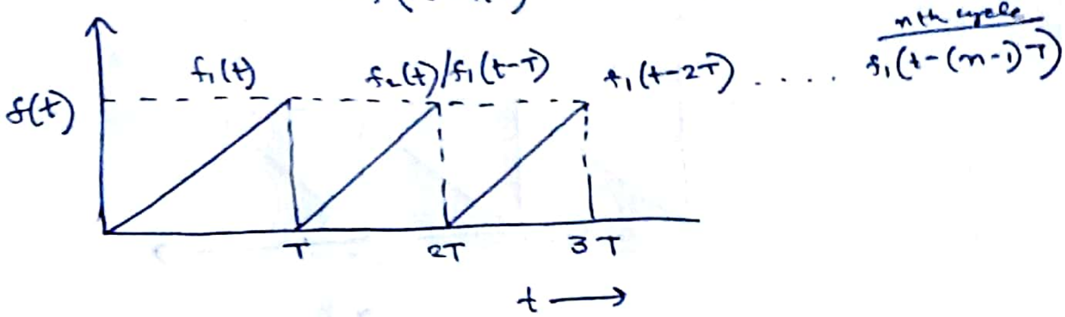
Step 2 : Multiply unit step function $(0,\infty)$ / Delayed Unit Step function (T,∞) / Gate Function $(0,T)$ / Delayed Gate Function (T,T_2) with the main function depending upon the time range.

<u>Step 3</u> : ~~We are in the form~~ Now rearrange the terms in such a way that directly we can apply the formula.

<u>Step 4</u> : Now using formula, find Laplace Transform of the main function.

Periodic Function

$f = f(t \pm nT) \rightarrow$ Periodic Function



$$f(t) = f_1(t) + f_1(t-T) + f_1(t-2T) + \ldots\ldots + f_1(t-(n-1)$$

$$F(s) = F_1(s) + F_1(s)e^{-Ts} + F_1(s)e^{-2Ts} + \ldots$$

$$= F_1(s)\left[1 + e^{-Ts} + e^{-2Ts} + \ldots\ldots\right]$$

$$= F_1(s)\frac{1}{1-e^{-Ts}}$$

$\downarrow$

Solved Before

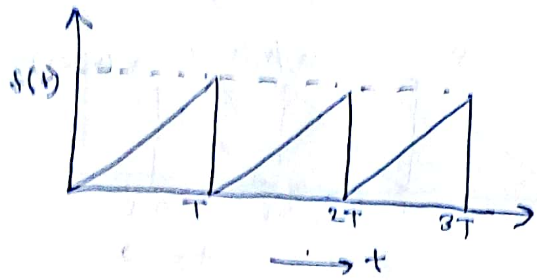
- How to find out the Laplace Transform of Periodic Function?

Step 1: Choose the 1st cycle and time period

Step 2: Find Laplace Transform of the 1st cycle using the Method written before.

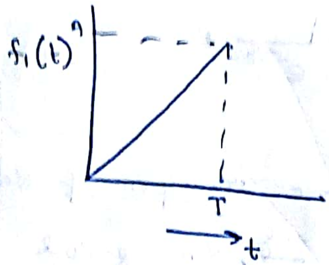
Step 3: The Laplace Transform of the total periodic function will be Laplace Transform of the 1st cycle $\times \frac{1}{1-}$

1.



The function is periodic.
The Time period of the function is T.

The 1st cycle is —



Using Laplace Transform,

$$\therefore\ F_1(s) = \frac{A}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{e^{-Ts}}{s}\right]$$

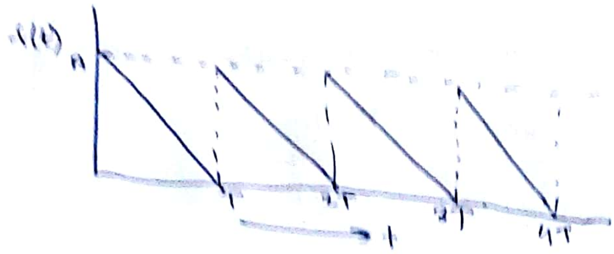
Using the periodic function formula,

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

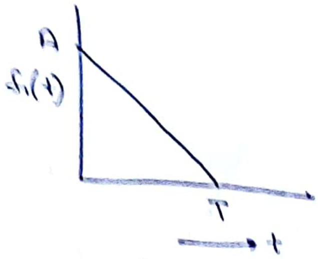
$$= \frac{1}{1-e^{-Ts}} \cdot \frac{A}{T} \cdot \left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{e^{-Ts}}{s}\right]$$

Simplify

2.



The function is periodic and the time period is T.
The 1st cycle —



$$\frac{f_1(t)}{A} + \frac{t}{T} = 1$$

$$\Rightarrow \quad f_1(t) = \left(1 - \frac{t}{T}\right) A$$

$$= \frac{A}{T}(T - t)$$

$$= \underbrace{-\frac{A}{T}}_{\text{slope}}(t - T)$$

multiplying with gate function,

$$f_1(t) = -\frac{A}{T}(t - T)\, G_{0,T}(t)$$

$$= -\frac{A}{T}(t - T)\left[u(t) - u(t - T)\right]$$

$$= -\frac{A}{T}\left[t\,u(t) - T\,u(t) - (t - T)\,u(t - T)\right]$$

$$\Rightarrow \quad F_1(s) = -\frac{A}{T}\left[\frac{1}{s^2} - \frac{T}{s} - \frac{e^{-Ts}}{s^2}\right]$$

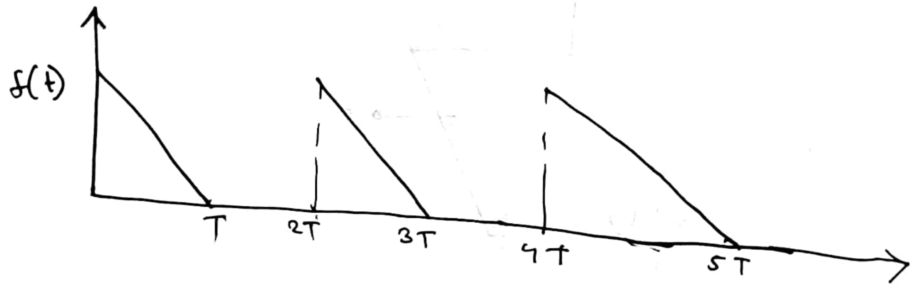
Using the periodic function formula,

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}}$$

$$= \frac{-1}{1-e^{-Ts}} \cdot \frac{A}{T}\left[\frac{1}{s^2} - \frac{T}{s} - \frac{e^{-Ts}}{s^2}\right]$$

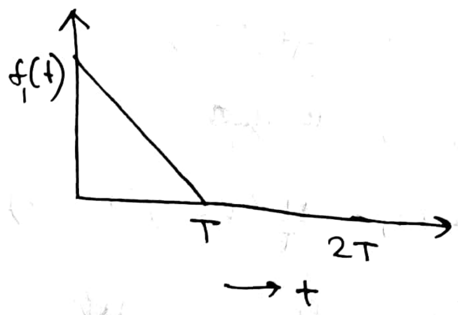
Simplify

3.



The function is periodic with period 2T.

1st cycle —

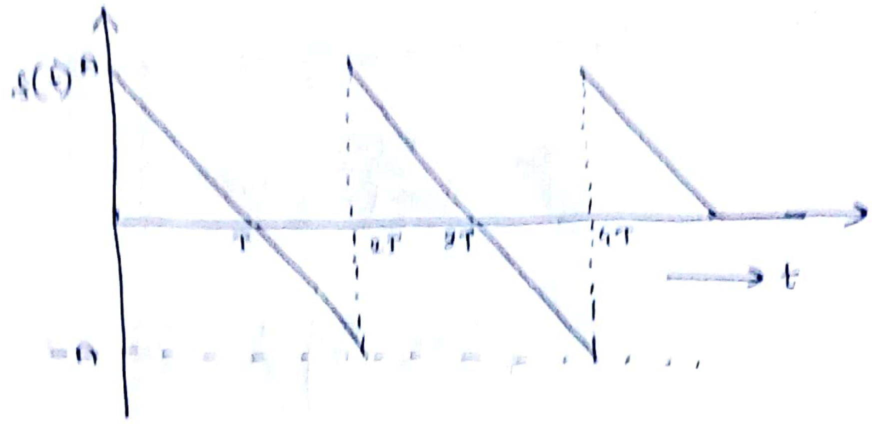


$$F_1(s) = -\frac{A}{T}\left[\frac{1}{s^2} - \frac{T}{s} - \frac{e^{-Ts}}{s^2}\right]$$ Same as (2)

∴ Using the periodic function formula,

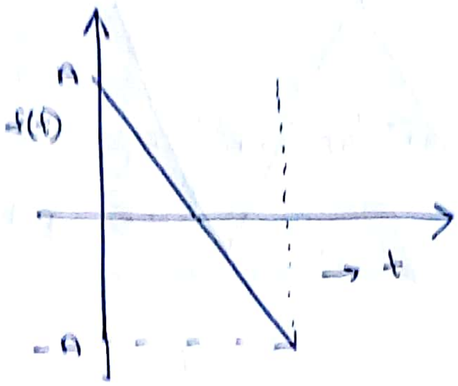
$$F(s) = \frac{F_1(s)}{1-e^{-2Ts}}$$

4.



The signal is periodic, with time period 2T.

1st Cycle —



$$f_1(t) = \left(1 - \frac{t}{T}\right)A = \frac{A}{T}(T-t) = -\frac{A}{T}(t-T)\, G_{0,2T}(t)$$

∴ Using Laplace Transform,

$$f_1(t) = -\frac{A}{T}(t-T)\left[u(t) - u(t-2T)\right]$$

$$= -\frac{A}{T}\left[t\,u(t) - T\,u(t) - (t-2T)\,u(t-2T) - T\,u(t-2T)\right]$$

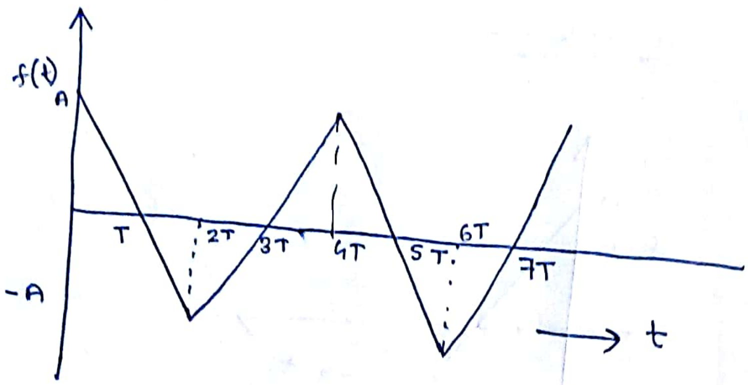
$$\Rightarrow F_1(s) = -\frac{A}{T}\left[\frac{1}{s^2} - \frac{T}{s} - \frac{e^{-2Ts}}{s^2} - \frac{T e^{-2Ts}}{s}\right]$$

∴ Using the periodic function formula —

$$F(s) = \frac{F_1(s)}{1 - e^{-2Ts}}$$

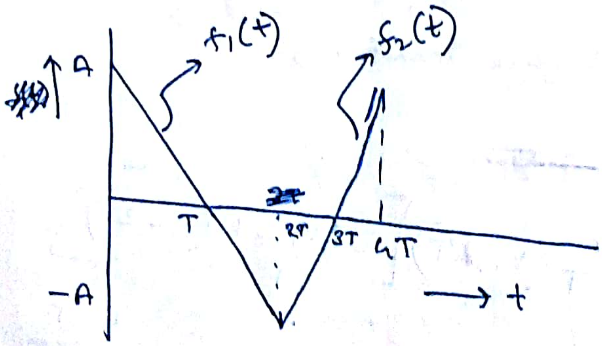
$$= \frac{-1}{1-e^{-2Ts}} \cdot \frac{A}{T}\left[\frac{1}{s^2} - \frac{T}{s} - \frac{e^{-2Ts}}{s^2} - \frac{T e^{\dots}}{s}\right.$$

5.



The signal is periodic, with time period 4T.

1st Cycle —



6.

$$\frac{f_1(t)}{A} + \frac{t}{T} = 1$$

$$\Rightarrow f_1(t) = -\frac{A}{T}(t-T)\,G_{0,2T}(t)$$

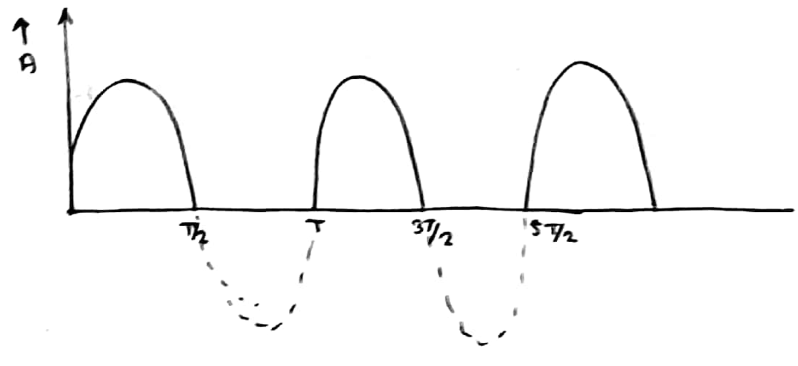
$$\frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2}$$

$$\Rightarrow \frac{f_2(t)+A}{t-2T} = \frac{-A-A}{2T-4T}$$

$$\Rightarrow \frac{f_2(t)+A}{t-2T} = \frac{\cancel{2}A}{\cancel{2}T}$$

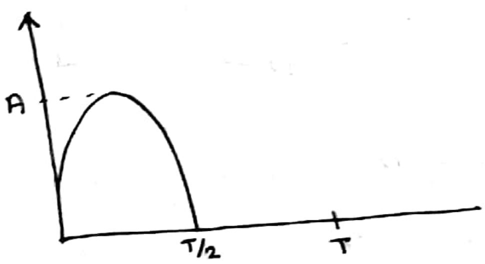
$$\Rightarrow f_2(t) = \left[\frac{A}{T}(t-2T)-A\right]G_{2T,4T}(t)$$

6. Find out the Laplace Transform of half wave rectified sine wave.



The function is periodic, where time period is T.

1st cycle —



$f(t) = A \sin \omega t$ $\qquad \omega = \frac{2\pi}{T} \rightarrow$ Time Period

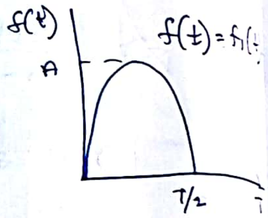
$= A \sin\left(\frac{2\pi}{T}t\right) G_{0,T/2}(t)$

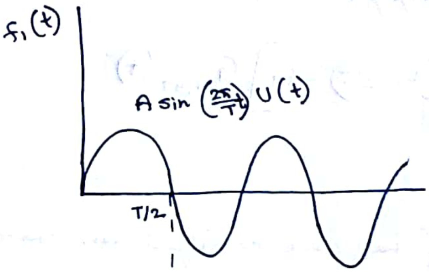
$= A \sin\left(\frac{2\pi}{T}t\right)\left[U(t) - U\left(t-\frac{T}{2}\right)\right]$

$= A \sin\left(\frac{2\pi}{T}t\right) U(t) - A \sin\left[\frac{2\pi}{T}\left(t - \frac{T}{2} + \frac{T}{2}\right) U\left(t - \frac{T}{2}\right)\right]$

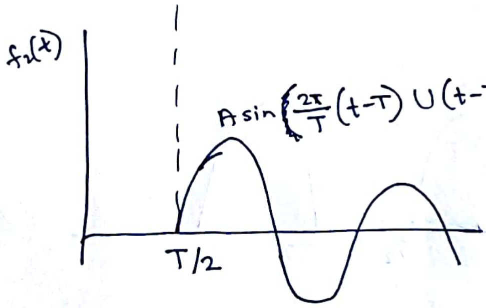
$=$

<u>Graphical Method</u>





$$A \frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2}$$



$$A \frac{\frac{2\pi}{T} e^{-\frac{T}{2}s}}{s^2 + \left(\frac{2\pi}{T}\right)^2}$$

<u>For 1 cycle:</u>

$\therefore F(s) = F_1(s) + F_2(s)$

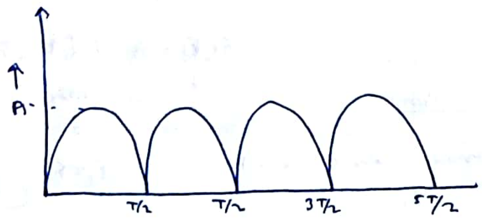
$$= A \frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2}\left[1 + e^{-\frac{T}{2}s}\right]$$

$\therefore$ ~~Lat~~ using the periodic function formula, the laplace transform of the whole signal will be

$$A \frac{\frac{2\pi}{T}}{s^2 + \left(\frac{2\pi}{T}\right)^2}\left[1 + e^{-\frac{T}{2}s}\right] \frac{1}{1 - e^{-Ts}}$$

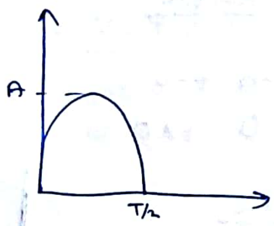
$$= \frac{A\,\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2} \cdot \frac{1}{1-e^{-T/2 s}} \quad \text{(Ans)}$$

7. Laplace Transform of Full-Wave Rectified Sine Wave.

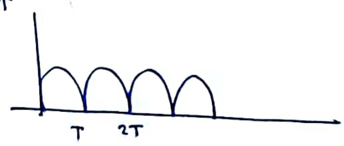


The signal is periodic.
The time period is T/2.

1st cycle:



Note



$\omega = \frac{2\pi}{2T} \rightarrow$ Time period of sine wave

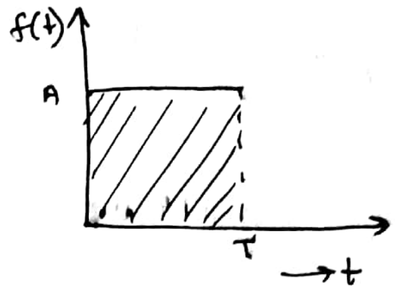
$= \frac{\pi}{T}$

$A\sin\left(\frac{\pi}{T}t\right)u(t)$

$$\therefore \; F(s) = \frac{A\,\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}\left[1+e^{-T/2 s}\right]$$

∴ Laplace Transform of whole cycle —

$$\frac{A\,\frac{2\pi}{T}}{s^2+\left(\frac{2\pi}{T}\right)^2}\left[1+e^{-T/2 s}\right]\frac{1}{1-e^{-\frac{T}{2}s}} \quad \text{(Ans)}$$



$f(t) = A \quad 0 < t < T$

$\quad = 0 \quad$ otherwise

Consider:

1) Area under curve = 1

$\downarrow$

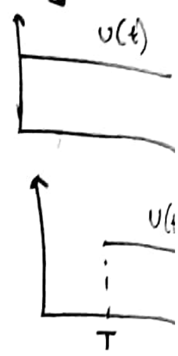
(AT)

$AT = 1$

$A = \frac{1}{T}$

$\therefore f(t) = A\left[U(t) - U(t-T)\right]$

$= \underset{T\to 0}{Lt} \frac{1}{T}\left[U(t) - U(t-T)\right]$



2) $Lt\, T \to 0$

$\delta(t) = \frac{d}{dt}\left[U(t)\right]$

$\downarrow$

Impulse Function

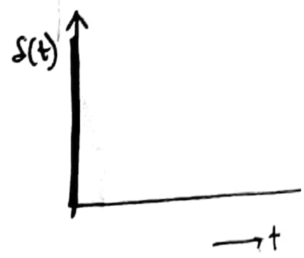
$\delta(t) = \infty \quad t = 0$

$\quad = 0 \quad t \neq 0$

Unit Impulse

$\delta(t) = 1 \quad t = 0$

$\quad = 0 \quad t \neq 0$



$\delta(s) = sU(s) - U(0)$

$= 1$

- $L\left[\delta(t)\right] = 1$ • laplace transform of impulse function = 1.

- Find LT of the following—

Laplace Transform of 1st Cycle = 1.

$\therefore$ using periodic signal formula

$$= 1 . \frac{1}{1-e^{-Ts}}$$

$$= \frac{1}{1-e^{-Ts}}$$

Initial Value and Final Value Theorem

$f(0) \rightarrow$ Value at 0 (Initial Value)

$f(\infty) \rightarrow$ Final Value

If given in s domain convert to time domain using Inverse Laplace.

Then apply above method.

This is the classical method.

But it can be directly found as well.

$$L[f(t)] = F(s)$$

$$\underset{s\to\infty}{Lt} L\left[\frac{d}{dt}f(t)\right] = \underset{s\to\infty}{Lt}\int_0^\infty \left[\frac{d}{dt}f(t)\right] e^{-st}\,dt$$

$$\Rightarrow \underset{s\to\infty}{Lt}\left[sF(s)-f(0)\right] = 0$$

$$\Rightarrow \boxed{\underset{s\to\infty}{Lt}\, sF(s) = \underset{s\to\infty}{Lt} f(0) = f(0)} \rightarrow \text{Initial Value Theorem}$$

$$\lim_{s\to 0} L\left[\frac{d}{dt}f(t)\right] = \lim_{s\to 0}\int_0^\infty \left[\frac{d}{dt}f(t)\right]e^{-st}\,dt$$

$$\Rightarrow \lim_{s\to 0}\left[sF(s)-f(0)\right] = f(\infty)-f(0)$$

$$\Rightarrow \boxed{\lim_{s\to 0} sF(s) = f(\infty)} \rightarrow \underline{\text{Final Value Theorem}}$$

1. $F(s) = \dfrac{1}{s(s+2)}$

Find out the initial value and final value of the equ

$\therefore$ final value

$= \lim_{s\to 0} sF(s)$

$= \lim_{s\to 0} \dfrac{1}{(s+2)}$

$= \dfrac{1}{2}$ (Ans)

$\therefore$ initial value

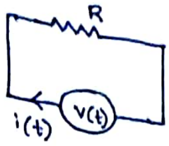
$= \lim_{s\to\infty} sF(s)$

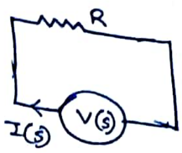
$= \lim_{s\to\infty} \dfrac{1}{(s+2)}$

$= 0$ (Ans)

~~Laplace~~ S-Domain Representation of R, L, C

1) R

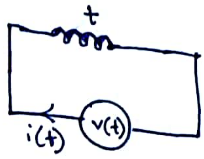


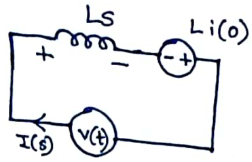


$$v(t) = R\,i(t)$$

$$V(s) = R\,I(s)$$

2) L





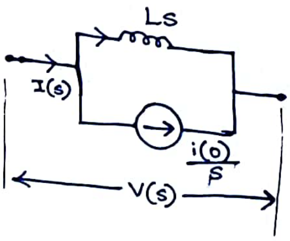
$$v(t) = L\,\frac{di(t)}{dt}$$

$$V(s) = L\left[sI(s) - i(0)\right]$$

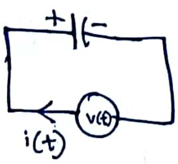
$$= LsI(s) - Li(0)$$

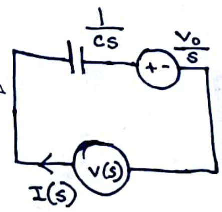
Current Source

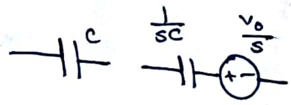
$$I(s) = \frac{V(s)}{Ls} + \frac{\cancel{L}\,i(0)}{\cancel{L}s}$$



3) C







$$v(t) = \frac{1}{C}\int i(t)\,dt + V_0$$

$$V(s) = \frac{1}{C}\,\frac{I(s)}{s} + \frac{V_0}{s}$$

Current Source

$$I(s) = Cs\,V(s) - V_0 C$$