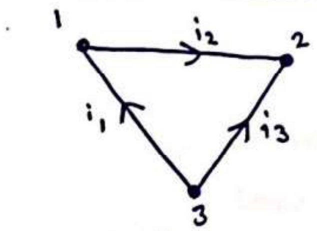
Network Theorems in Terms of Graph Theory

KCL:	KVL:	Relation between branch current and loop current :
$A_a I_b = 0$ $A I_b = 0$ $Q I_b = 0$	$B V_b = 0$ Relation between branch voltage and node voltage $V_b = A^T V_n$ V_b ↓ branch voltage V_n ↓ node voltage	$I_b = B^T I_L$ I_b ↓ branch current I_L ↓ loop current



$-i_1 + i_2 = 0$ ..... (1)

$-i_2 - i_3 = 0$ ..... (2)

$i_1 + i_3 = 0$ ..... (3)

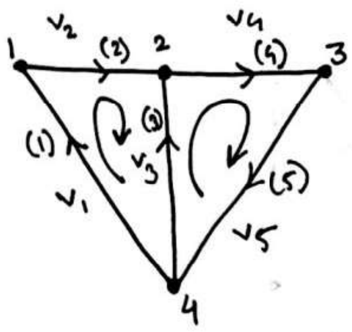
Complete Incidence matrix

$$A_a = \begin{array}{c|ccc} & (1) & (2) & (3) \\ \hline 1 & -1 & 1 & 0 \\ 2 & 0 & -1 & -1 \\ 3 & 0 & 0 & 1 \end{array}$$

$$A I_b = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = 0$$ [According to KCL of graph theory]

$$= \begin{bmatrix} -i_1 + i_2 \\ -i_2 - i_3 \\ i_1 + i_3 \end{bmatrix}$$

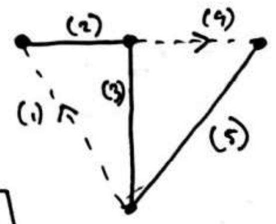
$= 0$ [Proved]



$$v_1 + v_2 - v_3 = 0 \text{ ----- } (1)$$

$$v_3 + v_4 + v_5 = 0 \text{ --- } (2)$$

Consider the Tree —



Tie set 1 [1 2 3]

Tie set 4 [4 5 3]

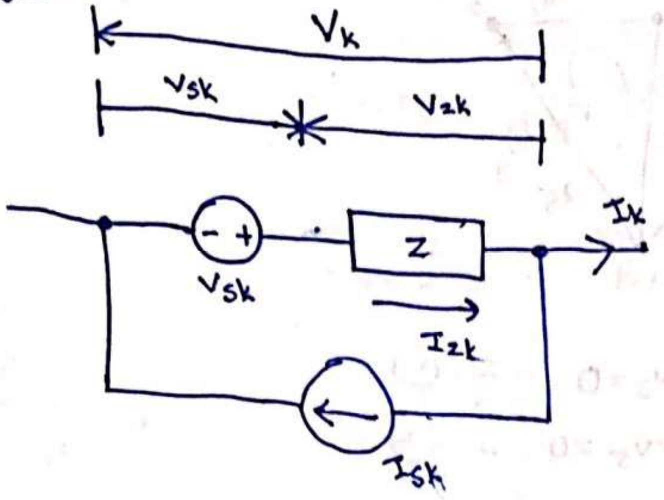
$$B = \begin{array}{c} \\ TS1 \\ TS4 \end{array} \begin{array}{c} \begin{array}{ccccc} (1) & (2) & (3) & (4) & (5) \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

According to KVL of graph theory $\rightarrow BV_b = 0 = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$

$$\therefore \begin{bmatrix} v_1 + v_2 - v_3 \\ v_3 + v_4 + v_5 \end{bmatrix} = 0 \text{ [Proved]}$$

Mesh Law

Kth branch



$$V_k = V_{zk} - V_{sk}$$

or, $$V_{sk} + V_k = V_{zk} = (Z_k)(I_{zk}) = Z_k(I_k + I_{sk})$$

for branch 1,

$$V_{s1} + V_1 = (Z_1)(I_1 + I_{s1})$$

for branch 2,

$$V_{s2} + V_2 = (Z_2)(I_2 + I_{s2})$$

$\vdots$

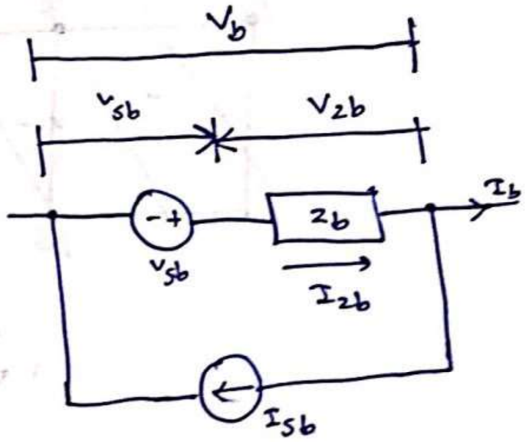
for branch ~~3~~, b,

$$V_{sb} + V_b = (Z_b)(I_b + I_{sb})$$

source voltage matrix, branch voltage matrix, Impedance matrix

$$\begin{bmatrix} V_{s1} \\ V_{s2} \\ V_{s3} \\ \vdots \\ V_{sb} \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_b \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 & \cdots & \\ 0 & Z_2 & \cdots & & \\ \cdots & & Z_3 & \cdots & \\ \cdots & & & & \\ 0 & 0 & \cdots & & Z_b \end{bmatrix} \left(\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_b \end{bmatrix} + \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_{s3} \\ \vdots \\ I_{sb} \end{bmatrix} \right)$$

KVL Equation for bth branch,



$$V_{sb} + V_b = (z_b)(I_b + I_{sb}) \quad \text{----} \; (1)$$

$$B V_{sb} + \underset{0}{B V_b} = B Z_b (I_b + I_{sb})$$

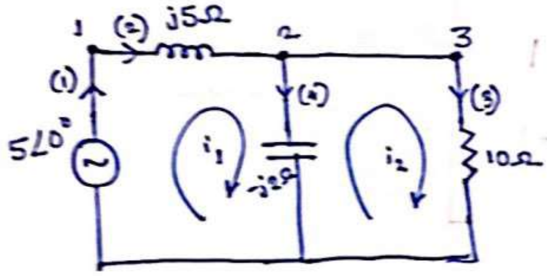
$$B V_{sb} = B Z_b I_b + B Z_b I_{sb}$$

or, $B Z_b I_b = B(V_{sb} - Z_b I_{sb})$

or, $\boxed{B Z_b B^T I_L = B(V_{sb} - Z_b I_{sb})}$ → Mesh Law for Graph Theory

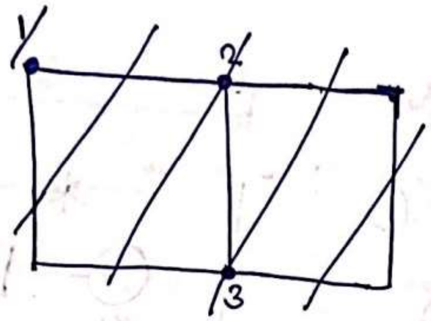
$$\textcircled{I_L} \rightarrow \textcircled{I_b} \rightarrow V_{2b} \rightarrow V_b$$

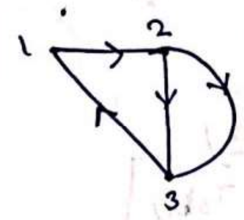
<u>Problem:</u> Derive the Mesh Equation in term of Graph Theory for the circuit shown below.



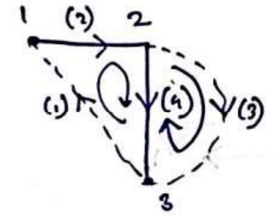
$$V_{sb} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad I_{sb} = 0$$

$$Z_b = \begin{array}{c} \begin{array}{cccc} (1) & (2) & (3) & (4) \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & j5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -j2 \end{bmatrix} \end{array}$$





Take tree as ——



Tie set 1 [1 2 4]

Tie set 3 [3 2]

$$B = \begin{array}{c} \\ TS_1 \\ TS_3 \end{array} \begin{array}{cccc} (1) & (2) & (3) & (4) \\ \hline \end{array}$$

$$B = \begin{array}{c|cccc} & (1) & (2) & (3) & (4) \\ TS_1 & 1 & 1 & 0 & 1 \\ TS_3 & 0 & 0 & 1 & -1 \end{array}$$

$$\therefore \quad RHS = BV_{sb} = \underset{2\times4}{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}} = \underset{2\times1}{\begin{bmatrix} 5 \\ 0 \end{bmatrix}}.$$

$$LHS = \underset{2\times4}{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}} \underset{4\times4}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & j5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -j2 \end{bmatrix}} \underset{4\times2}{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}}$$

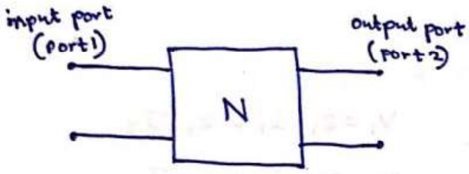
$$= \begin{bmatrix} j3 & j2 \\ j2 & 10-j2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

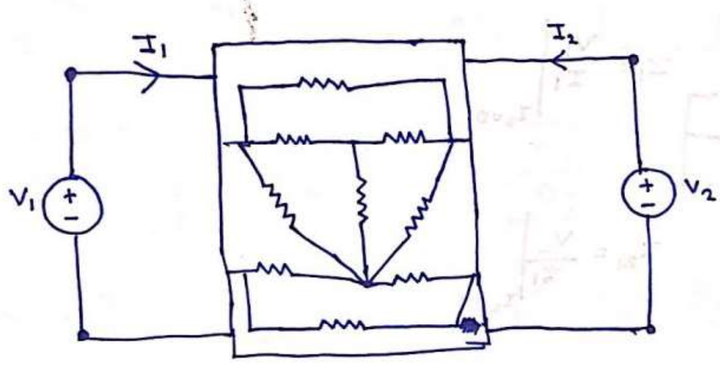
∴ Mesh Equation :

$$\begin{bmatrix} j3 & j2 \\ j2 & 10-j2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

∴ LHS = RHS $\boxed{\text{Proved}}$

Two Port Networks





Dependent Variable	Independent Variable
V_1, V_2	I_1, I_2
I_1, I_2	V_1, V_2
V_1, I_1	V_2, I_2
V_1, I_2	I_1, V_2
V_2, I_2	V_1, I_1
I_1, V_2	V_1, I_2

Z-Parameter

$$V_1 = f(I_1, I_2)$$

$$V_2 = f(I_1, I_2)$$

~~z-parameter~~ input driving point impedance

backward/reverse ~~forward~~ transfer impedance

Definition Equation of z-parameter

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{------ (1)}$$

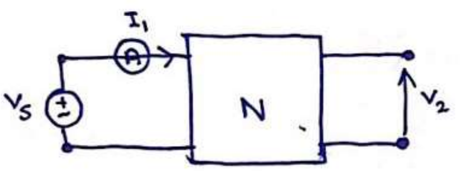
$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{..... (2)}$$

forward transfer impedance

output driving point impedance

$$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

- Finding the z-parameters —



$$V_1 = z_{11} I_1 + z_{12} I_2$$

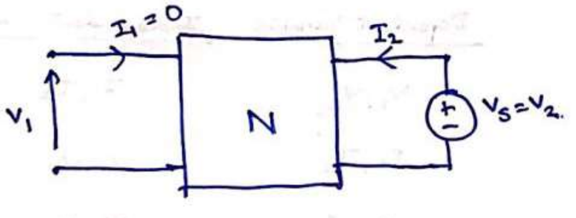
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$V_1 = V_s$

V_1	V_2	I_1	I_2	
V	V	V	0	

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

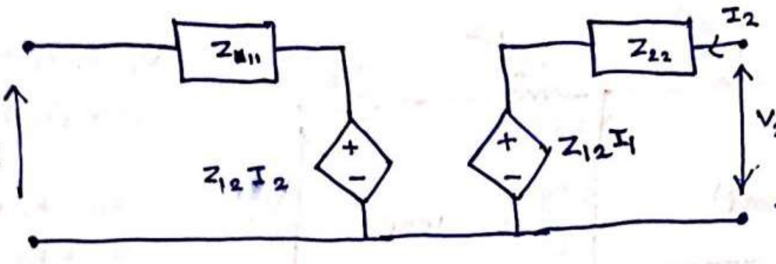


- open circuit impedence parameter → symbol: z

V_1	V_2	I_1	I_2		
V	V	0	V		

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



Generator Circuit of Z-parameter

Y- Parameter

$I_1 = f(V_1, V_2)$

$I_2 = f(V_1, V_2)$

Same as Z (admittance in place of impedance)

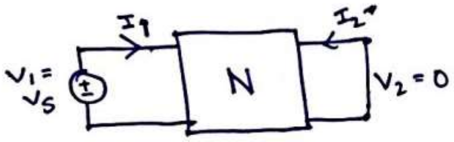
~~Short~~

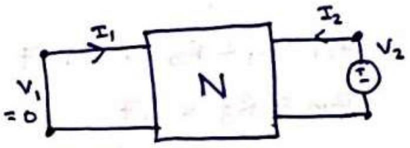
$$\begin{cases} I_1 = Y_{11}V_1 + Y_{12}V_2 \dots \text{(1)} \\ I_2 = Y_{21}V_1 + Y_{22}V_2 \dots \text{(2)} \end{cases}$$

$Y_{11}, Y_{12}, Y_{21}, Y_{22} \rightarrow$ Admittance

$Y \rightarrow$ Short Circuit Admittance parameter

$\rightarrow$ Y parameter





V_1	V_2	I_1	I_2	
	0			

V_1	V_2	I_1	I_2	
0				

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0}$$

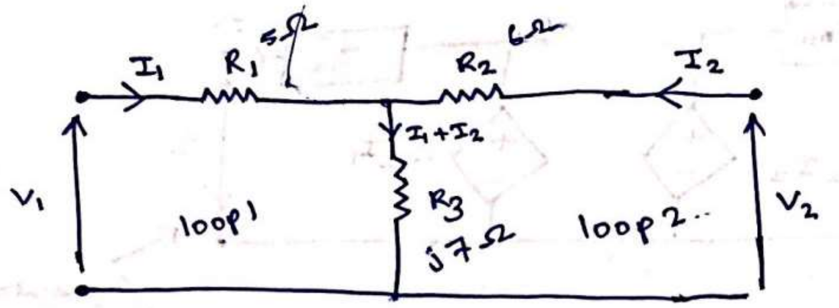
$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

Problem :

1.



$$V_1 = Z_{11} I_1 + Z_{12} I_2 \ldots$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \ldots$$

$$Z_{11} = R_1 + R_3$$
$$Z_{12} = R_3$$

$$Z_{11} = 5 + j7$$
$$Z_{12} = j7$$

for loop1

$$V_1 = I_1 R_1 + R_3 (I_1 + I_2)$$

or, $V_1 = (R_1 + R_3) I_1 + R_3 I_2 \ldots\ldots (1)$

for loop 2

$$V_2 = I_2 R_2 + (I_1 + I_2) R_3$$

or, $V_2 = R_3 I_1 + (R_2 + R_3) I_2 \ldots (2)$

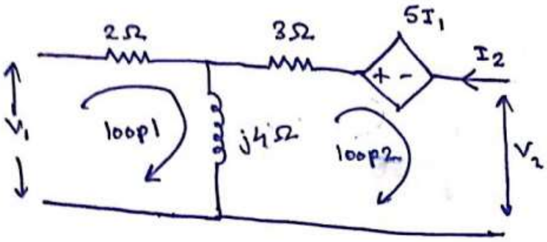
$\therefore Z_{11} = R_1 + R_3 = 5 + j7$

$Z_{12} = R_3 = j7$

$Z_{21} = R_3 = j7$

$Z_{22} = R_2 + R_3 = 6 + j7$ (Ans)

2.



Defn of Z parameter:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \cdots (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \cdots (2)$$

For loop1 by applying KVL,

$$V_1 = 2I_1 + j4(I_1 + I_2)$$

$$V_1 = (2+j4)I_1 + j4 I_2 \cdots (i)$$

$$Z_{11} = (2+j4)\Omega$$

$$Z_{12} = j4\Omega$$

$$V_2 \pm 5I_1 = 3I_2 + j4(I_1 + I_2)$$

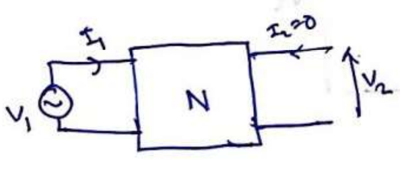
$$V_2 = (-5+j4)I_1 + (3+j4)I_2 \cdots (ii)$$

$$Z_{21} = (-5+j4)\Omega$$

$$Z_{22} = (3+j4)\Omega$$

$$[Z] = \begin{bmatrix} 2+j4 & j4 \\ -5+j4 & 3+j4 \end{bmatrix} \text{ (Ans)}$$

ABCD Parameter or Transmission Parameter or T-Parameter



defn of T-parameter:

$$V_1 = AV_2 + B(-I_2) \cdots (1)$$

$$I_1 = CV_2 + D(-I_2) \cdots (2)$$

from equation (1),

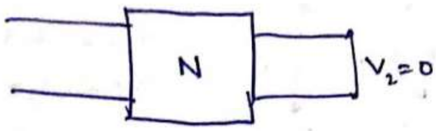
$$V_1 = AV_2$$

$$\therefore A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$ Unit of A → Unitless

from equation (2),

$$I_1 = CV_2$$

$$\Rightarrow C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$ Unit of C → ℧



from equation ①,

$$V_1 = B(-I_2)$$

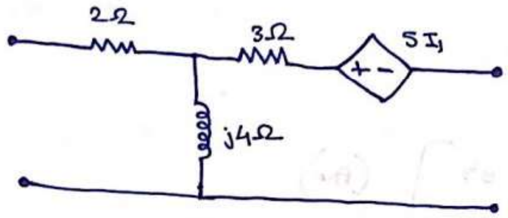
$$\Rightarrow B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \qquad \text{Unit of } B \rightarrow \Omega$$

from equation ②,

$$I_1 = D(-I_2)$$

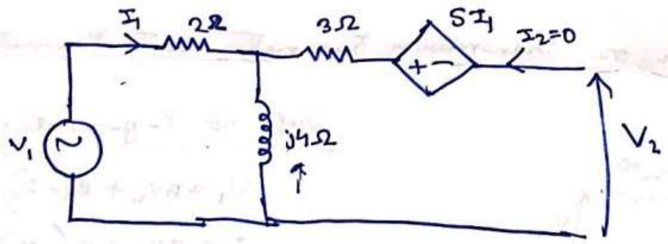
$$\therefore D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \qquad \text{Unit of } D \rightarrow \text{Unitless}$$

Find out ABCD Parameter —



$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{2+j4}{-5+j4} & \end{bmatrix}$$

Step 1:



$$\cancel{V_1 - 2I_1 - (I_1 - 5I_1)\,j4 = 0}$$

$$\cancel{\Rightarrow}\; V_1 = (2+j4)I_1 + j4I_2 \;\ldots\; ①$$

$$V_2 = (-5+j4)I_1 + (3+j4)I_2 = 0$$

$\underline{I_2 = 0}$

$$V_1 = (2+j4)I_1 \quad \cancel{\Rightarrow \frac{I_1}{V_1} = \frac{1}{2+j4}}$$

$$V_2 = (-5+j4)I_1 \quad \Rightarrow \frac{I_1}{V_2} = C = \frac{1}{-5+j4} \text{ (Ans)}$$

$\therefore A = \frac{V_1}{V_2} = \frac{2+j4}{-5+j4}$ (Ans)

$\therefore$ $\cancel{I_1}$ $\underline{V_2 = 0}$

$$0 = (-5+j4) I_1 + (3+j4) I_2$$

$$\Rightarrow \cancel{\frac{I_1}{-I_2}} = \cancel{\frac{3+j4}{5-j4}}$$

$$\Rightarrow (5-j4) I_1 = (3+j4) I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{3+j4}{5-j4} \quad \Rightarrow \frac{I_1}{-I_2} = \frac{3+j4}{-5+j4}$$

$\therefore$ in (i),

$$V_1 = (2+j4)\frac{(3+j4)}{(5-j4)} I_2 + j4 I_2$$

$8j + 12j$

12
8

$$\Rightarrow \frac{V_1}{I_2} = \frac{(2+j4)(3+j4)}{(5-j4)} + j4$$

$$\Rightarrow \frac{V_1}{I_2} = \frac{6 + 2j4 + 3j4 - 16}{5-j4} + j4$$

$$= \frac{j20 - 10}{5-j4} + j4$$

$$= \frac{j20 - 10 + 20j + 16}{5-j4}$$

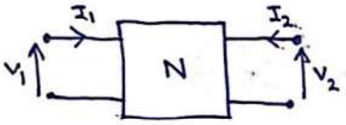
$$= \frac{j40 + 6}{5-j4}$$

h-Parameter or hybrid parameter

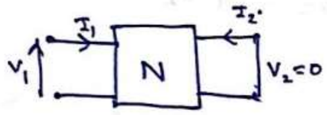
$$V_1 = f(I_1, V_2)$$
$$I_2 = f(I_1, V_2)$$

definition:

$$V_1 = h_{11} I_1 + h_{12} V_2 \ldots\ldots (1)$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \ldots\ldots (2)$$



1) Take $V_2 = 0$



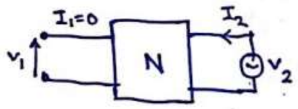
$(1) \Rightarrow V_1 = h_{11} I_1$

$\Rightarrow h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ Unit : Ω

$(2) \Rightarrow I_2 = h_{21} I_1$

$\therefore h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ Unitless

2) Take $I_1 = 0$



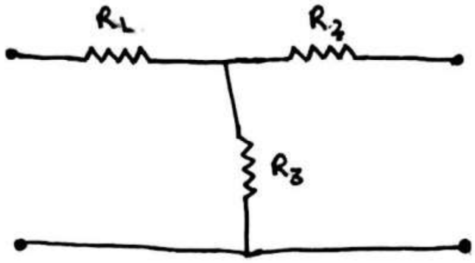
$(1) \Rightarrow V_1 = h_{12} V_2$

$\therefore h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ Unitless

$(2) \Rightarrow I_2 = h_{22} V_2$

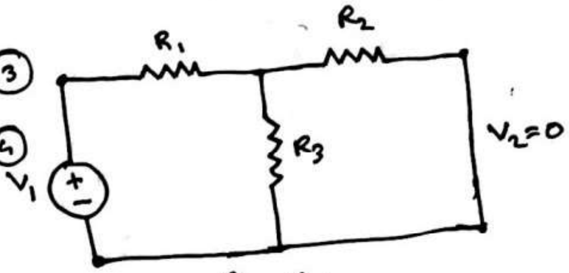
$\therefore h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ Unit : ℧

Find out h-parameter



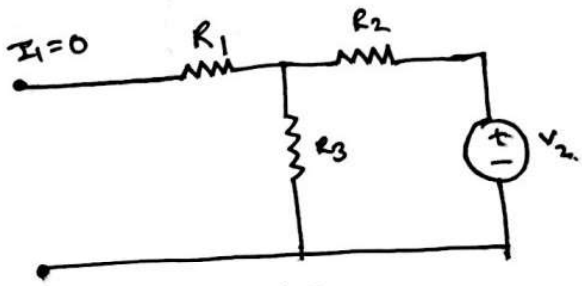
$$\therefore \quad V_1 = I_1 R_1 + (I_1 + I_2) R_3 \quad \ldots (3)$$

$$V_2 = I_2 R_2 + (I_1 + I_2) R_3 \quad \ldots (4)$$



Circuit 1

$$(3) \Rightarrow V_1 = (R_1 + R_3) I_1 +$$



Circuit 2

Inter-relationship

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$[Z] = [Y]^{-1}$$

$$= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$= \underbrace{\frac{1}{\Delta Y}}_{\text{Determinant}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

ABCD or T- Parameters

$$V_1 = AV_2 + B(-I_2) \quad \text{--- (1)}$$

$$I_1 = CV_2 + D(-I_2) \quad \text{--- (2)}$$

z-parameter

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (3)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (4)}$$

$$(2) \Rightarrow CV_2 = I_1 + DI_2$$

$$\Rightarrow V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2 \quad \text{..... (5)}$$

Comparing (5) and (4),

$$Z_{21} = \frac{1}{C} \qquad Z_{22} = \frac{D}{C}$$

Putting (5) in (1),

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) + B(-I_2)$$

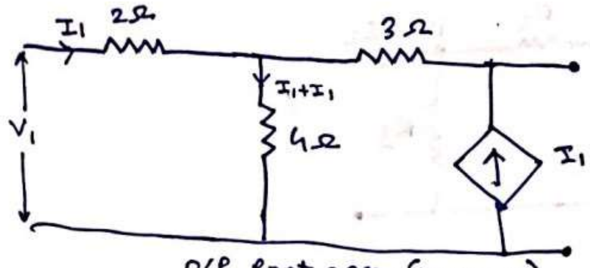
$$\Rightarrow V_1 = \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2 \quad \text{..... (6)}$$

$\therefore$ Comparing (6) and (3),

Determinant of T matrix

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD}{C} - B = \frac{AD-BC}{C} = \frac{\Delta T}{C}$$

1. Find out all the parameters.



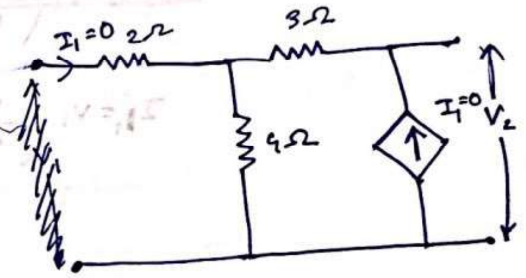
O/P Port open (1st Circuit)

$V_1 = 2I_1 + 4(2I_1)$ ...... (1)

$\Rightarrow \frac{V_1}{I_1} = 10\Omega = Z_{11}$

2nd Circuit

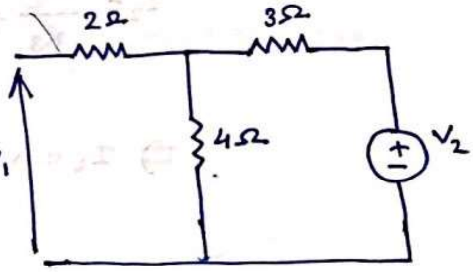
Input Port open



$V_2 = 3I_1 + 2I_1(4)$ ... (2)

$\Rightarrow \frac{V_2}{I_1} = Z_{21} = 11\Omega$

$\Downarrow$



$\therefore V_1 = 4I_2$

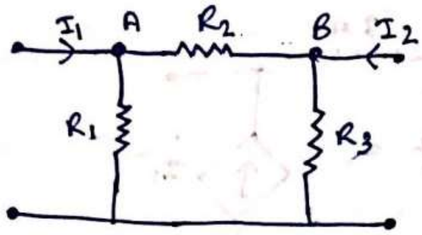
$\therefore \frac{V_1}{I_2} = 4\Omega = Z_{12}$

$V_2 = (3+4)I_2$

$\therefore \frac{V_2}{I_2} = Z_{22} = 7\Omega$

Find all the parameters

2\.



for Node A

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$\cancel{I_1 = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \left(\frac{1}{R_2}\right)V_2}$$

$$I_1 = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \left(-\frac{1}{R_2}\right)V_2$$

for Node B

$$I_2 = \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2}$$

$$\Rightarrow I_2 = V_1\left(-\frac{1}{R_2}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \quad Y_{11} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$Y_{12} = -\frac{1}{R_2}$$

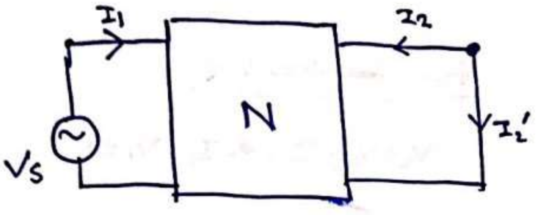
$$Y_{21} = -\frac{1}{R_2}$$

$$Y_{22} = \frac{1}{R_2} + \frac{1}{R_3}$$

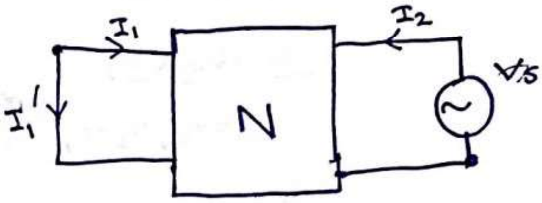
Condition of Reciprocity

~~$I' = \ldots$~~

$\boxed{I_2' = I_1'}$



Circuit 1



Circuit 2

Z parameter

for circuit 1

$V_1 = V_S\ ;\ V_2 = 0\ ;\ I_2' = -I_2$

definition of z parameter:

$V_1 = Z_{11} I_1 + Z_{12} I_2 \ldots (1)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \ldots (2)$

From equation (1);

$V_S = Z_{11} I_1 - Z_{12} I_2' \ldots (3)$

From equation (2),

$0 = Z_{21} I_1 - Z_{22} I_2' \ldots (4)$

$$I_2' = \frac{V_S Z_{21}}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

For ~~two~~ Circuit 2 :

$V_2 = V_S\ ;\ I_1' = -I_1\ ;\ V_1 = 0$

From equation (1),

$0 = -Z_{11} I_1' + Z_{12} I_2 \ldots (5)$

From equation (2),

$V_S = -Z_{21} I_1' + Z_{22} I_2 \ldots (6)$

$$I_1' = \frac{V_S Z_{12}}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

Now $I_2' = I_1'$

$\Rightarrow \boxed{Z_{21} = Z_{12}}$

Y Parameter

for circuit 1,

$$V_1 = V_S\ ;\ V_2 = 0;\ I_2' = -I_2$$

definition of Y Parameter:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \dots (1)$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \dots (2)$$

From equation (1),

$$I_1 = Y_{11}V_S \dots (3)$$

From equation (2),

$$I_2' = -Y_{12}V_S \dots (4)$$

$$\cancel{I_2'} = -\cancel{Y_{12}V_S}$$

for circuit 2,

$$V_2 = V_S\ ;\ I_1' = -I_1\ ;\ V_1 = 0$$

From equation (1),

$$I_1' = \left(-Y_{12}V_S\right)$$

$$\therefore \quad \boxed{Y_{12} = Y_{21}}$$

ABCD Parameter (T Parameter)

for circuit 1,

$$V_1 = V_S\ ;\ V_2 = 0,\ I_2' = -I_2$$

definition of T parameter:

$$V_1 = AV_2 - BI_2 \dots (1)$$

$$I_1 = CV_2 - DI_2 \dots (2)$$

From eq (1),

$$V_S = +BI_2'$$

$$\Rightarrow I_2' = \frac{V_S}{B}$$

for circuit 2,

$$V_2 = V_S;\ I_1' = -I_1;\ V_1 = 0$$

From equation (1),

$$0 = AV_S - BI_2$$

From equation (2),

$$-I_1' = CV_S - DI_2$$

$$I_1' = \frac{DAV_S}{B} - CV_S$$

$$= V_S\left(\frac{AD}{B} - C\right)$$

$$= V_S\left(\frac{AD - BC}{B}\right)$$

$$\therefore \frac{\cancel{V_S}}{\cancel{B}} = \left(\frac{AD - BC}{\cancel{B}}\right)\cancel{V_S}$$

$$\Rightarrow AD - BC = 1$$

$$\boxed{AD - BC = 1}$$

h Parameter

for circuit 1,

$V_1 = V_S$; $V_2 = 0$; $I_2' = -I_2$

definition of h parameter:

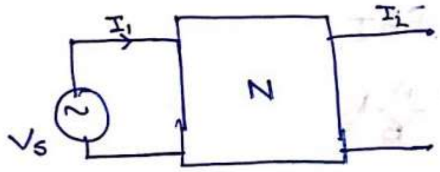
$$V_1 = h_{11} I_1 + h_{12} V_2 \text{ - - - - } (1)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \text{ - - - - } (2)$$

HW

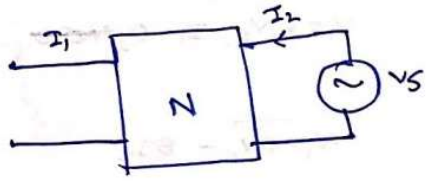
Condition of ~~Selectivity~~ Symmetry $\left(\frac{V_S}{I_1} = \frac{V_S}{I_2} \right)$

$\frac{V_S}{I_1}$



Circuit 1

$\frac{V_S}{I_2}$



Circuit 2

$V_1 = V_S$; $I_2 = 0$

Z-parameter:

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \text{ - - - } (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \text{ - - - } (2)$$

$$V_S = Z_{11} I_1$$

$$\frac{V_S}{I_1} = Z_{11}$$

$V_2 = V_S$; $I_1 = 0$

From equation (2),

$$V_S = Z_{22} I_2$$

$$\Rightarrow \frac{V_S}{I_2} = Z_{22}$$

$$\therefore \boxed{Z_{11} = Z_{22}}$$

Y- Parameter :

Circuit 1

$V_1 = V_S$; $V_2 = 0$, ~~$I_2 = I_2$~~ $I_1 = 0$

$I_1 = Y_{11}V_1 + Y_{12}V_2$ ...... (1)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ ...... (2)

$\therefore I_1 = Y_{11}V_1 = Y_{11}V_S$

$\Rightarrow \frac{V_S}{I_1} = \frac{1}{Y_{11}}$

Circuit 2

$V_2 = V_S$, $V_1 = 0$, $I_1 = 0$

$\therefore I_2 = Y_{22}V_2$

$\Rightarrow I_2 = Y_{22}V_S$

$\Rightarrow \frac{V_S}{I_2} = \frac{1}{Y_{22}}$

$$\boxed{Y_{11} = Y_{22}}$$

ABCD Parameter : $\boxed{A = D}$

Circuit 1

$V_1 = V_S$; $I_1 = 0$; ~~$V_2 = 0$~~

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$V_S = BI_2$

$I_1 = DI_2$

or, $V_S = \frac{B}{D} I_1$

$\Rightarrow \frac{V_S}{I_1} = \frac{B}{D}$

$0 = AV_S + BI_2$

$\Rightarrow I_2 = \frac{A}{B} V_S$

$\Rightarrow \frac{V_S}{I_2} = \frac{B}{A}$

$\therefore \frac{B}{D} = \frac{B}{A}$

$\Rightarrow \boxed{A = D}$

h Parameter

$$\boxed{h_{11}h_{22}-h_{21}h_{12}=1}$$

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

Circuit 1,

$V_s = h_{11}I_1 + 0$

$I_2 = h_{21}I_1 + 0$

or, $\frac{V_s}{I_1} = h_{11}$

Circuit 2

$0 = h_{11}I_1 + h_{12}V_s$

$I_2 = h_{21}I_1 + h_{22}V_s$

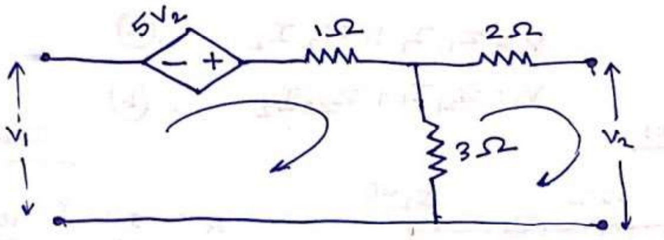
$\Rightarrow I_2 = -\frac{h_{21}h_{12}}{h_{11}} + h_{22}V_s$

$\Rightarrow I_2 = V_s\left(\frac{-h_{21}h_{12} + h_{11}h_{22}}{h_{11}}\right)V_s$

$\Rightarrow \frac{V_s}{I_2} = \frac{h_{11}}{h_{11}h_{22}-h_{21}h_{12}}$

$\Rightarrow \boxed{h_{11}h_{22}-h_{21}h_{12}=1}$

1. Find if the network is reciprocal or not, and symmetric or not.



in Loop ①,

$V_1 + 5V_2 = I_1 + (I_1 + I_2)3$

$V_2 = 2I_2 + (I_1 + I_2)3$

or, $V_1 + 5V_2 = 4I_1 + 3I_2$

or, $V_2 = 3I_1 + 5I_2$

or, $V_1 = 4I_1 - 15I_1 + 3I_2 - 25I_2$

or, $\boxed{V_1 = -11I_1 - 22I_2}$

$$\boxed{V_2 = 3I_1 + 5I_2}$$

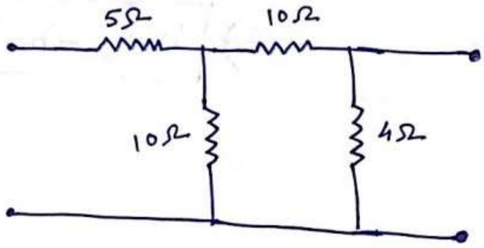
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$Z_{11} = -11$ $\quad$ $Z_{12} = -23$

$Z_{21} = 3$ $\quad$ $Z_{22} = 5$

1.



i) Determine Z parameters

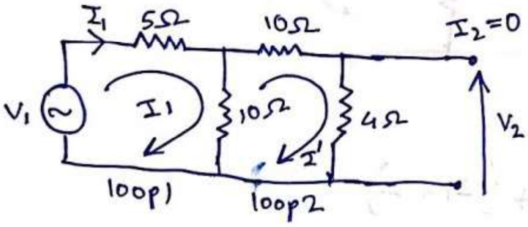
ii) Check whether the network is reciprocal.

iii) Check whether the network in Symmetric.

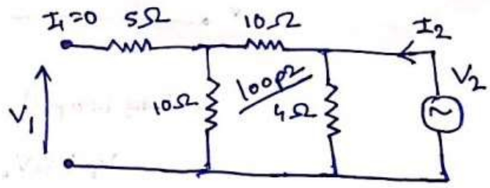
$$V_1 = Z_{11}I_1 + Z_{12}I_2 \ldots \text{(a)}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \ldots \text{(b)}$$

Circuit 1



Circuit 2



For loop 2:

$$10I' + 4I' + 10(I' - I) = 0$$

$$\Rightarrow 24I' = 10I_1 \ldots\ldots (1) \Rightarrow I' = \frac{10}{24}I_1$$

For loop 1:

$$V_1 = 5I_1 + 10(I_1 - I') \ldots\ldots (2)$$

$$\Rightarrow \frac{V_1}{I_1} = 10.83\Omega = Z_{11}$$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_1}\Big|_{I_1=0}$$

$\therefore$ Now, $V_2 = 4I' = 4\times\left(\frac{10}{24}I_1\right)$ $\left[\because I' = \frac{10}{24}I_1\right]$

$\therefore \frac{V_2}{I_1} = \frac{10}{6} = 1.67\Omega = Z_{21}$

From Circuit 2

From loop2

$10I'' + 10I'' + 4I'' - 4I_2 = 0$

$\Rightarrow 24I'' = 4I_2 \ldots\ldots (3)$

$$I_2 = \frac{V_2}{\frac{4\times 20}{4+20}}$$

$\therefore \frac{V_2}{I_2} = \frac{80}{24} = 3.33\Omega$

$V_1 = 10I'' = 10\left(\frac{4}{24}\right)I_2$

$\therefore \frac{V_1}{I_2} = 10\left(\frac{4}{24}\right)I_2$

$\therefore \frac{V_1}{I_2} = \frac{10}{6} = 1.67\Omega$

$= Z_{12}$

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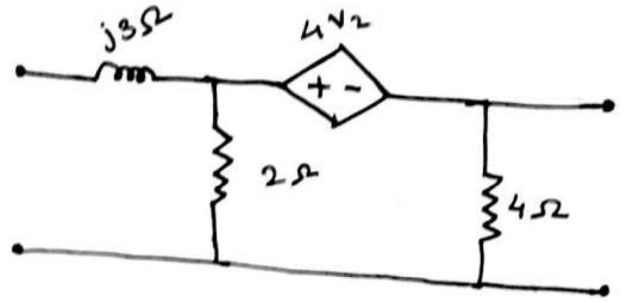
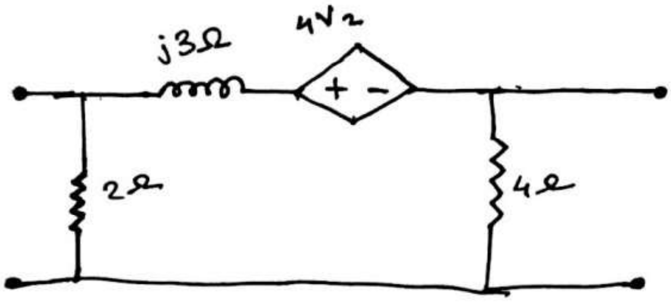
$$\therefore \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 10.83 & 1.67 \\ 1.67 & 3.33 \end{bmatrix} \quad (\text{Ans})$$

2.



1. i) Determine z parameter
   ii) Y parameter
   iii) ABCD Parameter
   iv) h-parameter
2. Check the condition of reciprocity
3. Check the condition of symmetry



$$Y_{11} = \left(\frac{1}{2} + \frac{1}{j3}\right)$$

$$Y_{12} = -\frac{5}{j3}$$

$$Y_{21} = -\frac{1}{j3}$$

$$Y_{22} = \frac{5}{j3} + \frac{1}{4}$$