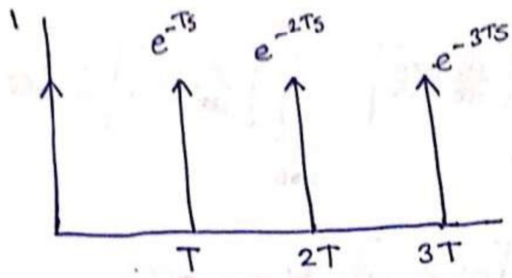
- Find LT of the following—



Laplace Transform of 1st Cycle = 1.

$\therefore$ using periodic signal formula

$$= 1 \cdot \frac{1}{1-e^{-Ts}}$$

$$= \frac{1}{1-e^{-Ts}}$$

Initial Value and Final Value Theorem

$f(0) \rightarrow$ Value at 0 (Initial Value)

$f(\infty) \rightarrow$ Final Value

If given in s domain convert to time domain using Inverse Laplace.

Then apply above method.

This is the classical method.

But it can be directly found as well.

$$L\left[f(t)\right] = F(s)$$

$$\lim_{s\to\infty} L\left[\frac{d}{dt}f(t)\right] = \lim_{s\to\infty}\int_0^\infty \left[\frac{d}{dt}f(t)\right]e^{-st}\,dt$$

$$\Rightarrow \lim_{s\to\infty}\left[sF(s)-f(0)\right] = 0$$

$$\Rightarrow \boxed{\lim_{s\to\infty} sF(s) = \lim_{s\to\infty} f(0) = f(0)}$$

$\rightarrow$ Initial Value Theorem

$$\underset{s\to 0}{Lt}\, L\left[\frac{d}{dt}f(t)\right] = \underset{s\to 0}{Lt}\int_0^\infty \left[\frac{d}{dt}f(t)\right]e^{-st}\,dt$$

$$\Rightarrow \underset{s\to 0}{Lt}\left[sF(s) - f(0)\right] = f(\infty) - f(0)$$

$$\Rightarrow \boxed{\underset{s\to 0}{Lt}\, sF(s) = f(\infty)}$$

$\longrightarrow$ Final Value Theorem

1. $F(s) = \dfrac{1}{s(s+2)}$

Find out the initial value and final value of the equation.

$\therefore$ final value

$= \underset{s\to 0}{Lt}\, sF(s)$

$= \underset{s\to 0}{Lt}\, \dfrac{1}{(s+2)}$

$= \dfrac{1}{2}$ (Ans)

$\therefore$ initial value

$= \underset{s\to \infty}{Lt}\, sF(s)$

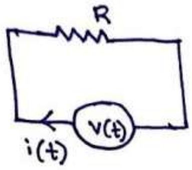
$= \underset{s\to \infty}{Lt}\, \dfrac{1}{(s+2)}$

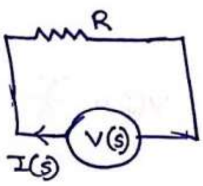
$= 0$ (Ans)

~~Laplace~~ S- Domain Representation of R, L, C

t | s

1) R

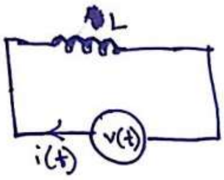


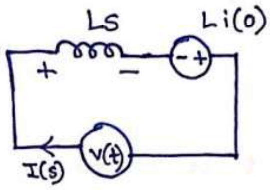


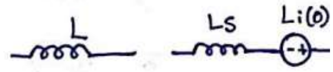
$$V(t) = R\,i(t)$$

$$V(s) = R\,I(s)$$

2) L







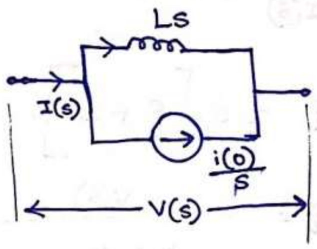
$$V(t) = L\,\frac{di(t)}{dt}$$

$$V(s) = L\left[sI(s) - i(0)\right]$$

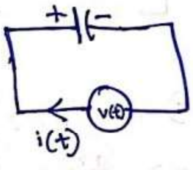
$$= LsI(s) - Li(0)$$

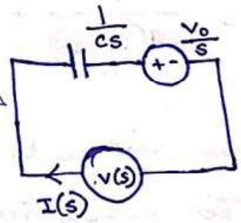
Current Source

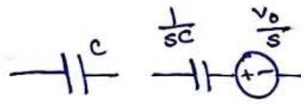
$$I(s) = \frac{V(s)}{Ls} + \frac{\cancel{L}\,i(0)}{\cancel{L}s}$$



3) C





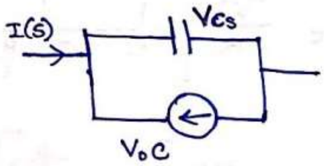


$$v(t) = \frac{1}{C}\int i(t)\,dt + V_0$$

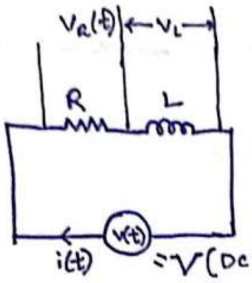
$$V(s) = \frac{1}{C}\,\frac{I(s)}{s} + \frac{V_0}{s}$$

Current Source

$$I(s) = Cs\,V(s) - V_0 C$$



Transient Response



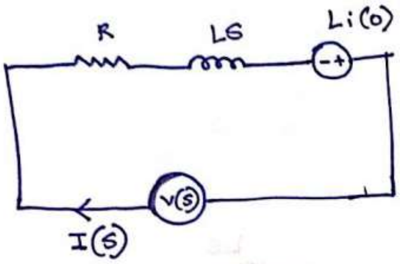
$V(s) = \frac{V}{s}$

$= V$ (DC voltage)

O/P

1) $i(t)$
2) $V_R(t) = i(t)R$
3) $V_L(t)$

Convert from t to s domain:



Using KVL,

$$V(s) = I(s)\left[R + LS\right] - Li(0)$$

$$\Rightarrow I(s) = \frac{V(s)}{R+LS} + \frac{Li(0)}{R+LS}$$

$$= \frac{V}{LS\left(\frac{R}{L}+S\right)} + \frac{\cancel{L}i(0)}{\cancel{L}\left(S+\frac{R}{L}\right)} \qquad \left[\because V(s) = \frac{V}{s}\right]$$

Use inverse Laplace to find ~~I(t)~~ $i(t)$. Find these values

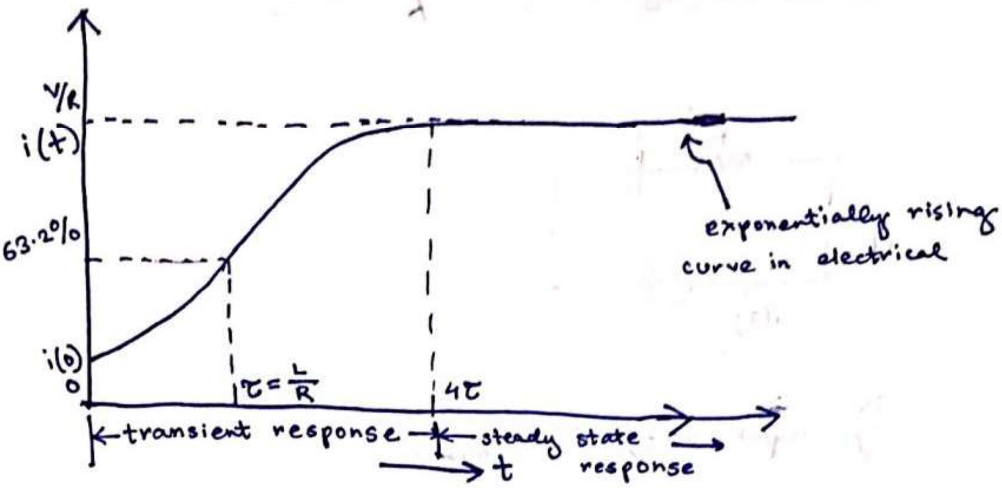
Convert to partial fraction form. $\rightarrow \frac{A}{s} + \frac{B}{s+\frac{R}{L}}$

$$A = \frac{V}{R} \quad B = -\frac{V}{R}$$

$$I(s) = \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right] + \frac{i(0)}{s+\frac{R}{L}}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right] + i(0)\,e^{-\frac{R}{L}t}$$

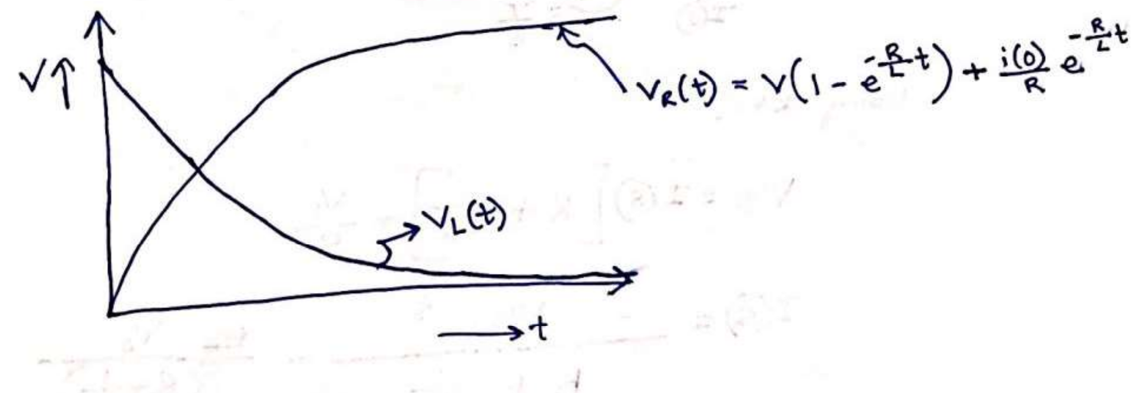
↓



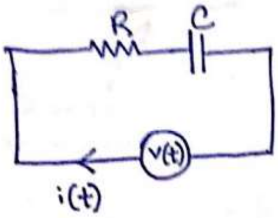
Transient response → 0 to 4τ
Beyond 4τ → Steady state response

<u>Time constant of RL Circuit:</u> At time $\tau = \frac{L}{R}$, the circuit response will be 63.2% of its steady state/final response. This time is ~~const~~ called time constant. Unit of τ = sec

Imp
- At time t=0 it acts an open circuit, at t=∞, it acts as a short circuit.



Transient Response of RC Series Circuit



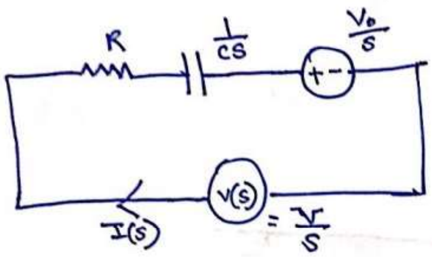
1) input $v(t) = V$ (const)

2) output i) $i(t)$

ii) $V_R(t) = i(t)R$

iii) $V_c(t) = V - V_R(t)$

Converting from t to s domain,



Using KVL,

$$V(s) = I(s)\left[R + \frac{1}{cS}\right] + \frac{V_0}{S}$$

$$I(s) = \frac{V(s)^{=V/s}}{R + \frac{1}{cs}} - \frac{V_0}{S\left(R + \frac{1}{cs}\right)}$$

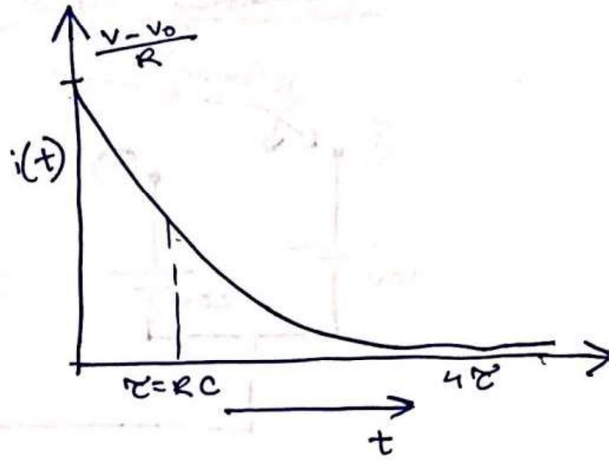
$$= \frac{V \cdot c\cancel{s}}{\cancel{s}(RCs+1)} - \frac{V_0 Cs}{s(RCs+1)}$$

$L^{-1}\left(\frac{1}{s+}\right.$

$$= \frac{V}{R}\frac{1}{\left(s + \frac{1}{RC}\right)} - \frac{V_0}{R}\frac{1}{\left(s + \frac{1}{RC}\right)}$$

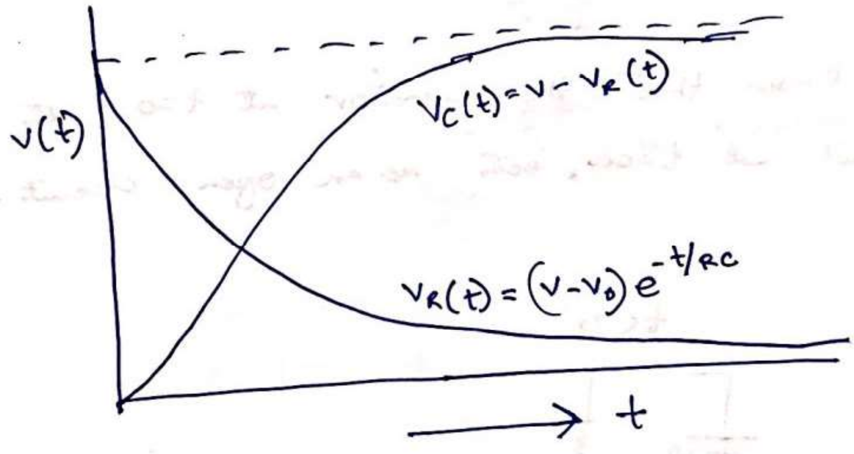
Using Inverse Laplace,

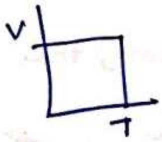
$$\therefore\ i(t) = \left(\frac{V - V_0}{R}\right) e^{-\frac{t}{RC}} \quad \therefore \text{ exponentially decreasing}$$



- At $\tau = RC$, the current response will be 36.7 % of its initial response.
- Steady state current = 0 .
- At time t=0, Capacitor acts as a short circuit.

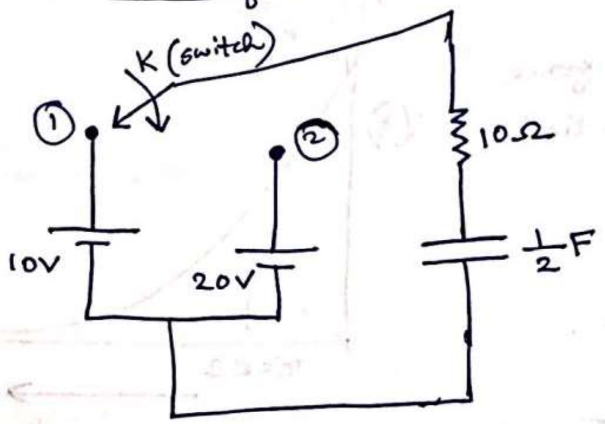
At $t = \infty$, Capacitor acts as an open circuit.



Now, if a gate function,  is applied.

i) input $v(t) = V\left[u(t) - u(t-T)\right]$

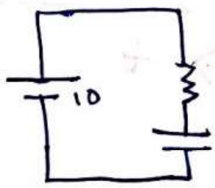
Switching Circuit



1. Suppose switch K is at position 1 for a long time. Now at $t=0$, switch is moved from position 1 to position 2. Now find out the transient current through the circuit.

We know that the capacitor at $t=0$ acts as a short circuit and $t=\infty$, acts as an open circuit.

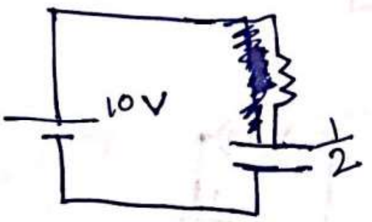
$t<0$



In this condition, the circuit remains for a long time.
∴ Steady state is reached.

2. At time t=0, the switch is at position 1. After 5 sec, the switch is moved from position 1 to position 2. Now find out the transient current through the circuit and the find out the transient voltage across the capacitor.

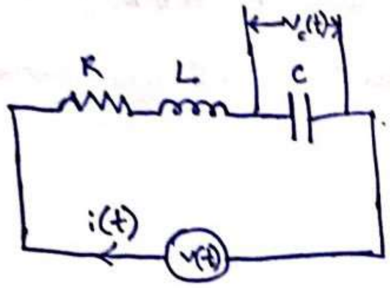
$t \to 0-5$sec



$t \to 5s - \infty$

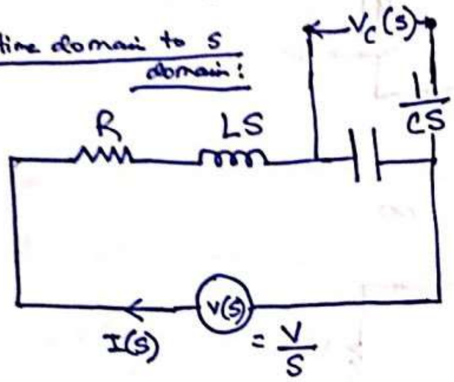
3.67 V → Initial Voltage

Transient Response of RLC Series Circuit



o/p $\rightarrow V_c(t)$

Converting from time domain to s domain:



$$V(s) = I(s)\left(R + LS + \frac{1}{CS}\right)$$

$$\Rightarrow I(s) = \frac{V(s)}{R + LS + \frac{1}{CS}}$$

$$\Rightarrow V_c(s) = I(s)\frac{1}{CS} = \frac{V(s)\,C\cancel{S}}{\cancel{S}\left(RCS + LCS^2 + 1\right)} \times \frac{1}{C\cancel{S}}$$

$$\Rightarrow \frac{V_c(s)}{V(s)} = \frac{1}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$\Rightarrow \frac{V_c(s)}{V(s)} = \frac{\left(\frac{1}{\sqrt{LC}}\right)^2}{s^2 + \frac{R}{L}s + \left(\frac{1}{\sqrt{LC}}\right)^2}$$ System Transfer Function

This equation is called a ~~se~~ second order system.

General ~~Transform~~ Transfer Function of any 2nd order system

$$\frac{\omega_n^2}{s^2 + 2\xi_\varphi \omega_n s + \omega_n^2}$$

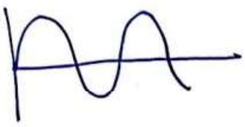
$\omega_n \rightarrow$ natural frequency

$\xi_\varphi \rightarrow$ damping contant

1) $\xi_\varphi = 0$

Inverse Laplace Transform of $\frac{\omega_n^2}{s^2+\omega_n^2}$

$\omega_n \sin \omega_n t$



(Undamped)

2) $0 < \xi_\varphi < 1$

$$\frac{\omega_n^2}{s^2 + 2\xi_\varphi \omega_n s + \omega_n^2} \Rightarrow \frac{a}{(s+b)^2 + a^2}$$

$$\Rightarrow \frac{\omega_n^2}{(s+\xi_\varphi \omega_n)^2 - \xi_\varphi \omega_n^2 + \omega_n^2}$$

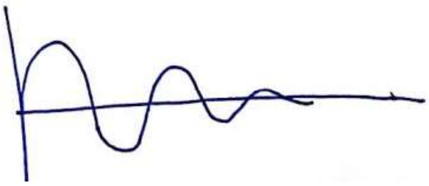
$$\Rightarrow \frac{\omega_n^2}{(s+\xi_\varphi \omega_n)^2 + \left(\omega_n\sqrt{1-\xi_\varphi^2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{1-\xi_\varphi^2}} \frac{\omega_n \, \omega_n\sqrt{1-\xi_\varphi^2}}{(s+\xi_\varphi \omega_n)^2 + \left(\omega_n\sqrt{1-\xi_\varphi^2}\right)^2}$$

Taking Inverse Laplace Transform,

$$\frac{\omega_n}{\sqrt{1-\xi_\varphi^2}} e^{-\xi_\varphi \omega_n t} \sin\left[\left(\omega_n\sqrt{1-\xi_\varphi^2}\right)t\right]$$

(Underdamped System)



3) $\xi_p = 1$

Taking Inverse Laplace Transform,

$$te^{-\omega_n t}$$

At $t=0$, 0

At $t=\infty$, 0

(Critically Damped)

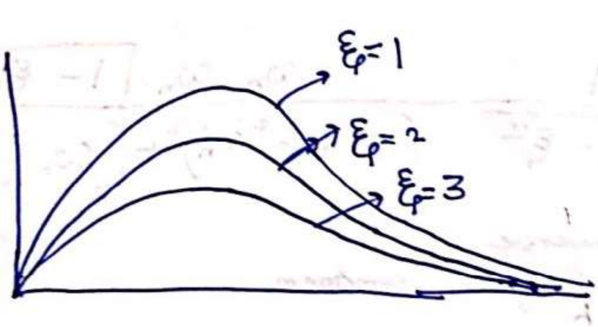
4) $\xi_p > 1$

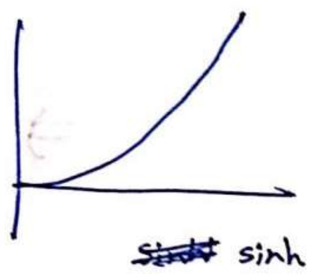
Like ②,

$$\frac{1}{\sqrt{\xi_p^2-1}} \cdot \frac{\omega_n \, \omega_n\sqrt{\xi_p^2-1}}{\left(s+\xi_p\omega_n\right)^2-\left(\omega_n\sqrt{\xi_p^2-1}\right)^2}$$

Taking ~~Inverse~~ Laplace Transform,

$$\frac{\omega_n}{\sqrt{\xi_p^2-1}} e^{-\xi_p \omega_n t} \sinh\left[\omega_n\sqrt{(\xi_p^2-1)}\, t\right]$$





~~Sinh~~ sinh

(Overdamped)

Now, $\omega_n = \sqrt{\frac{1}{LC}}$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$