Predictive Analytics Lecture 3

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Stat 422/722
at The Wharton School of the University of Pennsylvania

January 31 & February 1, 2017

The Coin Example from Last Class I

I want to explain the coin example from last class in the context of likelihood. Imagine you flip a coin three times and get heads, heads, tails; thus, $y_1 = 1, y_2 = 1, y_3 = 0$. There is a true probability of heads called θ. We don't know it.

What is the probability of the data? We employ the mass / density function:

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And now we can calculate the probability of seeing the data assuming θ. Assume $\theta = 0.5$ then,

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i.e. the most likely model for this data is a weighted coin with probability of heads of 2/3.

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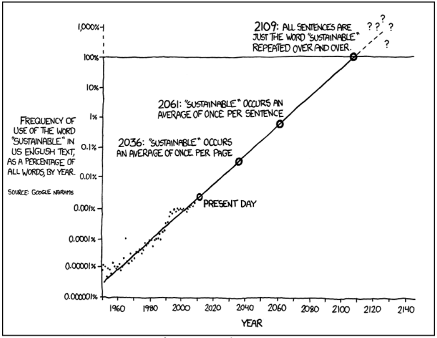

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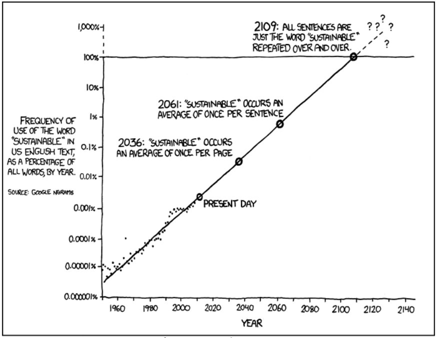

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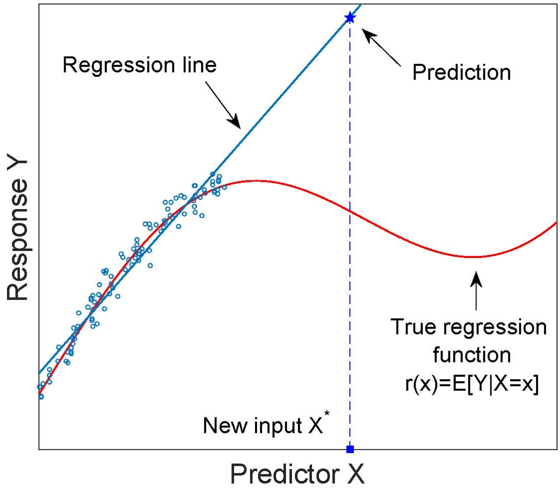
It is important to ask the question for a new observation $\boldsymbol{x}^*$ if it is within the space of $\boldsymbol{x}$'s in the historical data. (Hardly anyone does this when $p > 2$... but you should)!

Reconciliation of These Silly Cartoons



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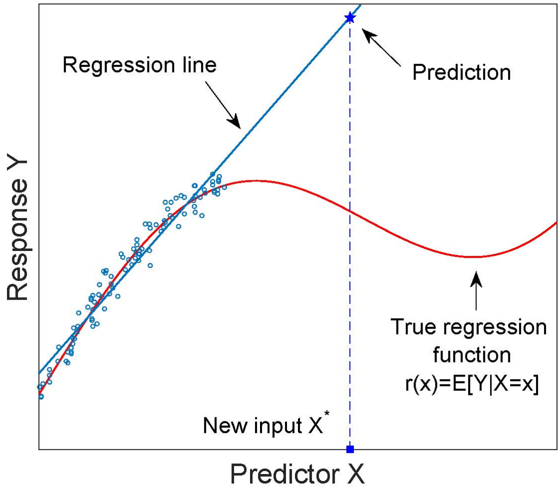

Be aware that extrapolation methods of different algorithms differ considerably! [R Demo]

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What to measure? ✓ Who to measure it on???

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3. Something else?

[R demo]

Optimal Design: Split Between Extremes

Recall the formula from Stat 102 / 613:

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General Optimal Design of Linear Models

We seek the best linear approximation of $f(x)$ which is $\beta_0 + \beta_1 x_1 + \ldots + \beta_p x_p$. We pick the $\boldsymbol{x}$'s to give us the best linear approximation. What criteria? JMP gives two ways:

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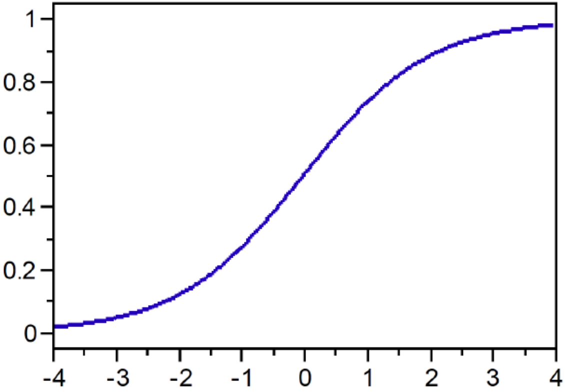
This is (I would say) the most interpretable link function situation we've got.

The Logistic Function (an "S" Shape)



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This does not have a simple, closed form solution. The computer iterates numerically using gradient methods. It usually uses the $\ln(\cdot)$ of above, since it's (1) numerically more stable and (2) the expression is easier to work with. When it "converges" on the values of the parameters that maximize the above, these are shipped to you as $\left\{\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_p\right\}$. This is called "running a logistic regression". The above looks complicated but it is instant on a modern computer for most real-world datasets.

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Recall that predictions in linear regression were easy:

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AKA the "most likely criterion". We will return to prediction and evaluation of predictive performance later but first... inference.

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So on top the computer iterates to find $\left\{\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_p\right\}$, plugs it in and computes the likelihood and on the bottom the computer independently iterates to find $\left\{\hat{\beta}_0\right\}$, plugs it in and computes the likelihood, then together, the LR.

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There is something special about a χ^2 r.v. with one degree of freedom. Note this cool fact from probability theory: $Q \sim \chi^2_1 \Rightarrow \sqrt{Q} \sim \mathcal{N}\left(0, 1\right)$ i.e. a "z-score". This is how JMP produces standard errors for logistic regression coefficients.

Telecom Company Churn Example

In marketing lingo, “churn” refers to a customer canceling their service. Studies suggest that it costs 5-10x more to acquire a new customer than to retain an old customer. Thus, predicting churn is of major interest!

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We begin just trying to model y: churn vs. x: tenure (the number of months customer is currently subscribed for). What do you think the relationship will be i.e. what is the sign of $\partial/\partial x[f(x)]$ generally speaking?

Results of Simple Logistic Regression

[JMP demo]

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$$\begin{aligned} Q &= 2\ln(LR) = 2\left(\ell\left(\hat{\theta};x\right) - \ell\left(\hat{\theta}_R;x\right)\right) \\ &= 2\left(-(3595.9341) - -(4075.0729)\right) \\ &= 2\left(479.1389\right) = 958.2778 \end{aligned}$$

and $\chi^2_{1,5\%} = \texttt{qchisq(.95, 1)} = 3.84$. So this passes the test (comfortably). We reject H_0 and conclude that the model is linearly useful.

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Also note that $LR = 1.22 \times 10^{208}$ but $e^{-3595.9341} = 0$ i.e. it's less than the smallest number a computer can represent (without special handling). Numbers are cool things... and logs are pretty powerful.

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Predict estimated expected probability of churn for someone who has 1 month of tenure

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How about 2 months of tenure?

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i.e. a difference in about 0.8% as measured on a probability scale.

Basic Predictions II

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How about 101 months of tenure?

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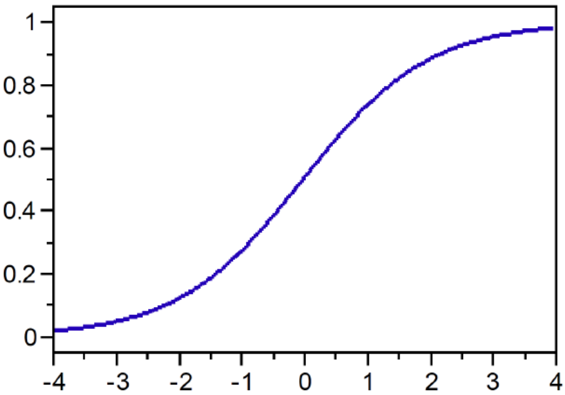
i.e. a difference in about 0.2% as measured on a probability scale (i.e. a 4x difference from before). But isn't the model supposedly to be linear??

The Logistic Function



A move of one unit in x when $x \approx 0$ is a much bigger move than one unit in x when $x \approx 3$

Parameter Standard Error

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$$\underbrace{Q = z^2}_{\substack{\text{fact from}\\\text{probability}\\\text{theory}}} = \left(\frac{\hat{\beta}_1}{s_{\hat{\beta}_1}}\right)^2 \Rightarrow s_{\hat{\beta}_1} = \frac{\left|\hat{\beta}_1\right|}{\sqrt{Q}}$$

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are expectedly about the same

$$s_{\hat{\beta}_1} = \frac{|-0.0387682|}{\sqrt{761.00}} = 0.0014 \quad \text{(via the Wald test)}$$

$$s_{\hat{\beta}_1} = \frac{|-0.0387682|}{\sqrt{958.28}} = 0.0013 \quad \text{(via the LR test)}$$

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Now let's use all variables. Questions:

- Which variable(s) should we leave out?

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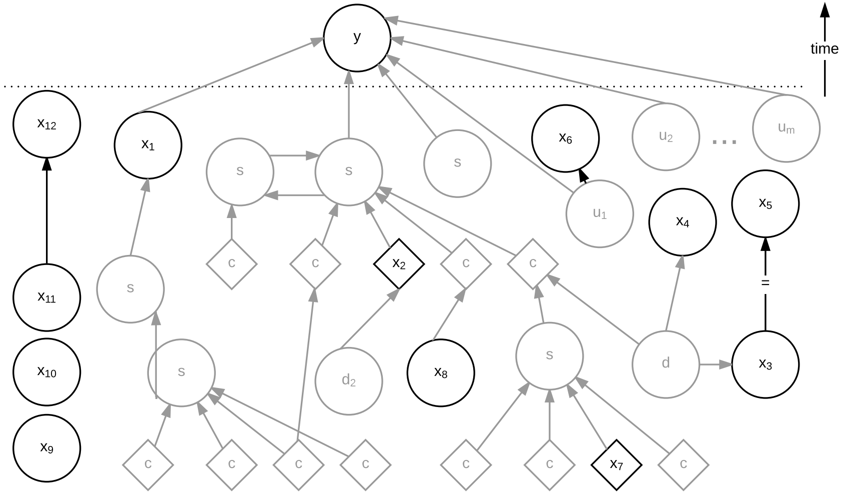
- Which variable(s) should we leave out? `customerID` — no causal mechanism.
- Why are we getting biased estimates and zeroes? Perfect multicollinearity. Solution? Kill columns until you don't get the error anymore. Turns out 6 have to be removed.

Realistic Predictors Illustration (updated)



Multivariate Logistic Regr. Interp. II

Now let's use all variables. Questions:

- Which variable(s) should we leave out? `customerID` — no causal mechanism.
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- Are all these variables "significant"?

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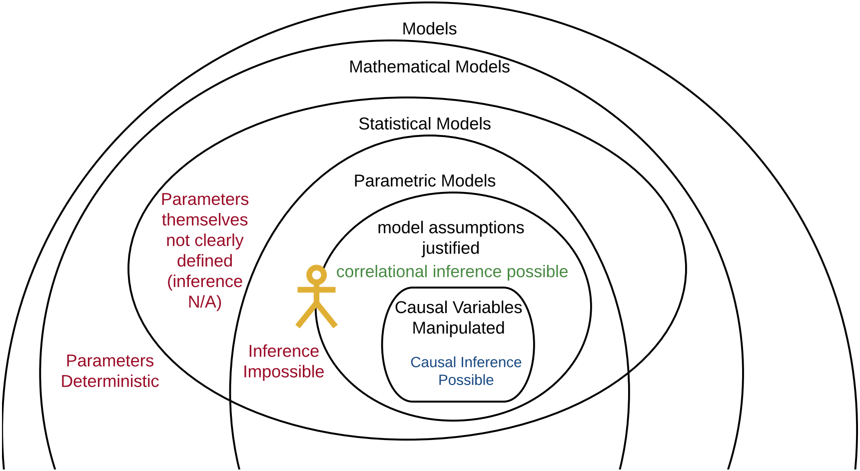
- Which of the "driving" variables are *not* able to be manipulated? Senior citizen, total charges (?)
- Should you (the manager) try to incentivize electronic checks? Paperless billing? Dropping multiple phone lines? You can try these things if you have nothing to lose, but remember, they are not guaranteed to be causal! And they may backfire!! (Can you think of

Remember Where you At!!



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We now cover evaluating classification models in general (not only in the context of logistic regression models specifically).

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		$\hat{y}$ (decision) 1	$\hat{y}$ (decision) 0
y (truth)	1	true positive (TP)	false negative (FN)
	0	false positive (FP)	true negative (TN)

Why do "correlations rock" here??

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Why do "correlations rock" here?? We are purely evaluating predictive performance... no inferential claims!

Confusion Matrix for Churn Model

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JMP gives us the matrix [JMP], but they don't annotate it well. Here are some numbers I like to see:

		$\hat{y}$ 1	$\hat{y}$ 0	Totals	Model Errors
y	1	$TP = 1012$	$FN = 857$	$P = 1869$	$FNR = 45.9\%$
	0	$FP = 531$	$TN = 4632$	$N = 5163$	$FPR = 10.2\%$
	Totals	$\hat{P} = 1543$	$\hat{N} = 5489$	$n = 7032$	
	Use errors	$FDR = 34.3\%$	$FOR = 15.6\%$		$\boxed{ME = 19.7\%}$

There are other metrics commonly reported

- Sensitivity = Recall = $\frac{TP}{TP+FN} = \frac{TP}{P}$, the proportion of positives successfully recovered (large value = good model), 54.9% above
- Specificity = $\frac{TN}{TN+FP} = \frac{TN}{N}$, the proportion of negatives successfully recovered (large value = good model), 89.8% above

Misclassification Error

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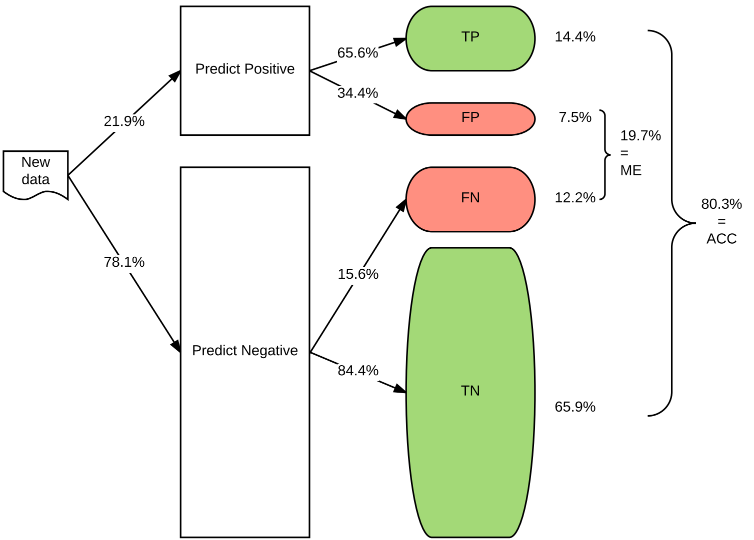
Misclassification Error

Already... what is one broad, general metric to evaluate the model? Misclassification error cost function (or Accuracy):

$$
\begin{aligned}
ME &:= \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{y_i \neq \hat{y}_i} \\
ACC &:= 1 - ME = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{y_i = \hat{y}_i}
\end{aligned}
$$

This essentially treats both types of errors (the FN's and the FP's) equally (more on this later).

Production Classifier Flowchart



There's a Ton of Metrics...

From wikipedia...

		Predicted condition			
	Total population	Predicted Condition positive	Predicted Condition negative	Prevalence = $\frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	
True condition	condition positive	**True positive**	**False Negative** (Type II error)	True positive rate (TPR), Sensitivity, Recall, probability of detection = $\frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False negative rate (FNR), Miss rate = $\frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$
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		False discovery rate (FDR) = $\frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$	Negative predictive value (NPV) = $\frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$	Negative likelihood ratio (LR−) = $\frac{\text{FNR}}{\text{TNR}}$	

Others (from above) commonly used:

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So now it's simple, vary p_0 and pick the best model according to your cost / error / loss function (the ME at the moment). Let's just do every p_0!

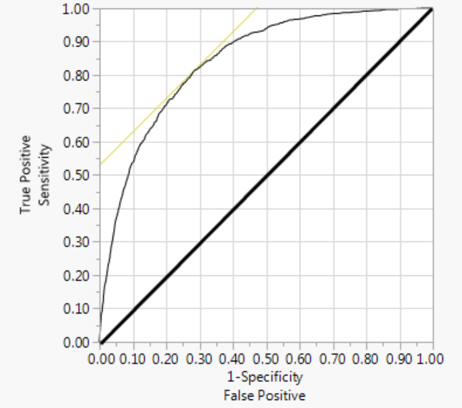
Receiver-Operator Characteristic Table

ROC Table

Prob	1-Specificity	Sensitivity	Sens-(1-Spec)	True Pos	True Neg	False Pos	False Neg
.	0.0000	0.0000	0.0000	0	5163	0	1869
0.8117	0.0000	0.0005	0.0005	1	5163	0	1868
0.8104	0.0000	0.0011	0.0011	2	5163	0	1867
0.8093	0.0000	0.0016	0.0016	3	5163	0	1866
0.8092	0.0000	0.0021	0.0021	4	5163	0	1865
0.8090	0.0000	0.0027	0.0027	5	5163	0	1864
0.8085	0.0000	0.0032	0.0032	6	5163	0	1863
0.8083	0.0000	0.0037	0.0037	7	5163	0	1862
0.8082	0.0000	0.0043	0.0043	8	5163	0	1861
0.8079	0.0000	0.0048	0.0048	9	5163	0	1860
0.8079	0.0000	0.0054	0.0054	10	5163	0	1859
0.8077	0.0000	0.0059	0.0059	11	5163	0	1858
0.8076	0.0002	0.0059	0.0057	11	5162	1	1858
0.8072	0.0002	0.0064	0.0062	12	5162	1	1857
0.8065	0.0002	0.0070	0.0068	13	5162	1	1856
0.8064	0.0002	0.0075	0.0073	14	5162	1	1855
0.8061	0.0002	0.0080	0.0078	15	5162	1	1854

Here, `Prob` is what we denoted p_0. "Best model" is not defined here by highest ACC (lowest ME), it's determined by highest **specificity** + **sensitivity** or equivalently, the highest **sensitivity** - **(1 - specificity)**.JMP indicates that row with a $\star$. This is an arbitrary metric, but is a good default.

Receiver-Operator Characteristic Curve



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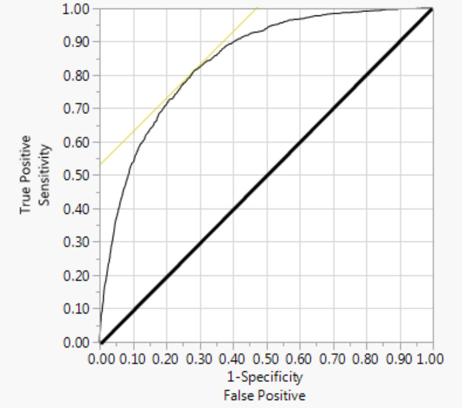

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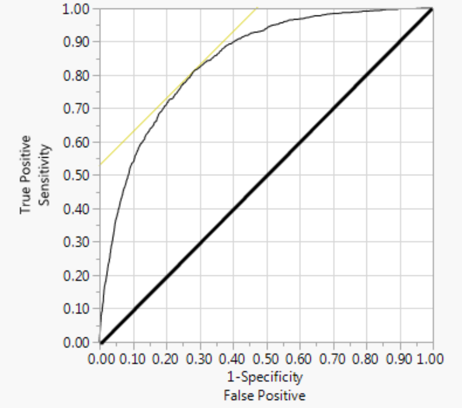

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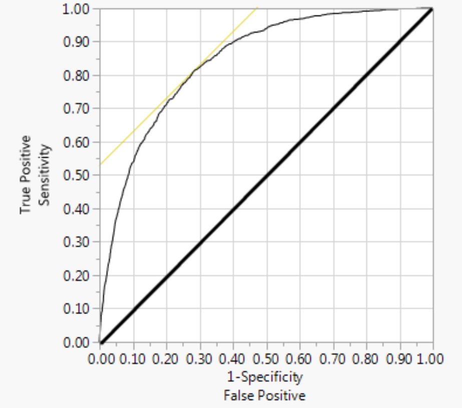

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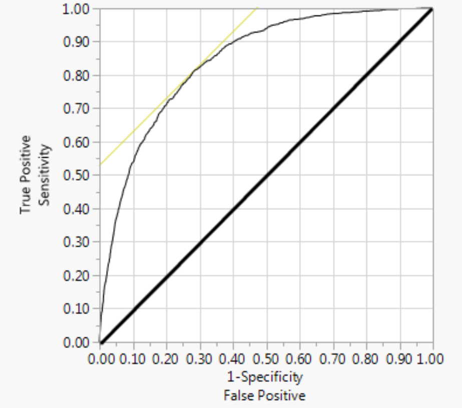

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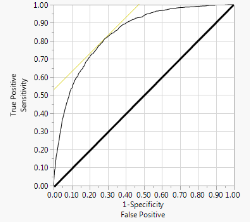
Area Under the Curve (AUC) Metric



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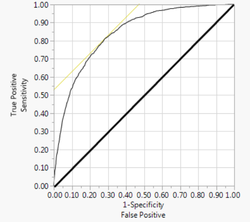

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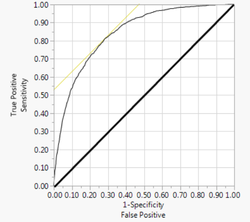
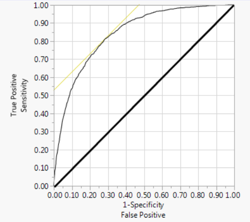


If you built a model by chance the "area under the curve" (or to the right of the curve) on the graph would be ... 0.5 since the graph is a unit square. Under the ROC curve itself (or to its right) is an area ... greater than 0.5. Here, it's 0.844.

Area Under the Curve (AUC) Metric



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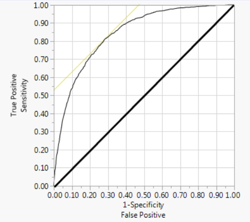

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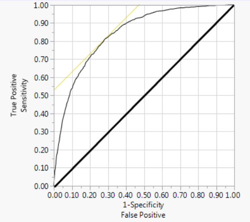

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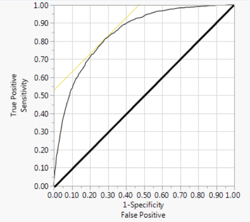

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Churn Example Where $p_0 = 0.10$

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y	1	$TP = 1012$	$FN = 857$	$P = 1869$	$FNR = 45.9\%$
	0	$FP = 531$	$TN = 4632$	$N = 5163$	$FPR = 10.2\%$
	Totals	$\hat{P} = 1543$	$\hat{N} = 5489$	$n = 7032$	
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Which numbers did not change? n, P and N. Why?

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Not necessarily... It depends on what your goal is!

Asymmetric Costs in a Classifier

These are always two types of errors but the costs are not always the same.

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Weighted Misclassification Error

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We now define two costs: (1) the cost of the *FP* denoted c_{FP} and (2) the cost of the *FN* denoted c_{FN}. We then define the weighted misclassification error evaluation metric:

$$ME_w := \frac{1}{n}\sum_{i=1}^{n} c_{FP}\mathbb{1}_{y_i=0\&\hat{y}=1} + c_{FN}\mathbb{1}_{y_i=1\&\hat{y}_i=0}$$

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We now vary p_0 to locate the model that optimizes this error to be minimum.

Minimum Weighted Misclassification Error

Let's assume that $c_{FN} = \$1000$ and $c_{FP} = \$100$ just for the example's sake. Note: this is a **cost ratio** of 10:1.

	Prob	TP	TN	FP	FN	COST
1	0.8117	1	5163	0	1868	1868000
2	0.8104	2	5163	0	1867	1867000
3	0.8093	3	5163	0	1866	1866000
4	0.8092	4	5163	0	1865	1865000
5	0.8090	5	5163	0	1864	1864000
6	0.8085	6	5163	0	1863	1863000
7	0.8083	7	5163	0	1862	1862000
8	0.8082	8	5163	0	1861	1861000
9	0.8079	9	5163	0	1860	1860000

We now calculate the cost and find the minimum model (i.e. the p_0 to ship). [JMP]

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We now calculate the cost and find the minimum model (i.e. the p_0 to ship). [JMP] Or alternatively, we can select the model with the closest $FN/FP \approx 10:1$ to match the stakeholder preference of the desired cost ratio. Why would this be good?

Expected Value Calculation

You can also imagine assignment of both costs *and* benefits:

$p_0 = 0.1$		$\hat{y}$	
		1	0
y	1	b_{TP}	c_{FN}
	0	c_{FP}	b_{TN}

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and then use the confusion matrix to estimate probabilities:

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The expected value would be?

$$\begin{aligned} \mathbb{E}[T] &= p_{TP} \times b_{TP} + p_{TN} \times b_{TN} + p_{FP} \times c_{FP} + p_{FN} \times c_{FN} \\ &\approx \hat{p}_{TP} \times b_{TP} + \hat{p}_{TN} \times b_{TN} + \hat{p}_{FP} \times c_{FP} + \hat{p}_{FN} \times c_{FN} \end{aligned}$$

Highest expected value model is shipped (ex. from Provost & Fawcett, 2013).

$\hat{p}$'s as Ordinal Values

One final point... If we were on a mission to find the top m churners. What would we do?

$\hat{p}$'s as Ordinal Values

One final point... If we were on a mission to find the top m churners. What would we do? Sort the $\hat{p}$'s and return the top m.