CURRENT ELECTRICITY

(CHAPTER-3)

ELECTRIC CURRENT (I)

It is defined as rate of flow of electric charge through any cross section of a conductor.

$$\boxed{\begin{aligned} I &= \frac{\text{total charge}}{\text{time taken}} \\ I &= \frac{q}{t} = \frac{ne}{t} \\ I &= \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \end{aligned}}$$

→ Scalar quantity

SI unit → A

CGS unit → st A

CURRENT DENSITY (J):-

It is the ratio of the current at that point in the conductor to the area of the cross section of the conductor at that point.

$$J = \frac{I}{A}$$

$$I = JA$$

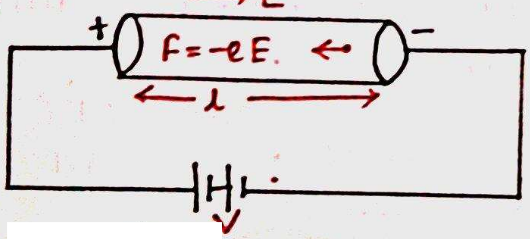
$$\rightarrow \boxed{I = \vec{J} \cdot \vec{A}}$$

→ Vector quantity.

DRIFT VELOCITY (v_d)

It is defined as avg. velocity gained by the free e^-s of a conductor in the opposite direction of the externally applied electric field.

DRIFT OF ELECTRONS:-



If $\vec{u}_1, \vec{u}_2, \ldots\ldots\ldots\ldots \vec{u}_N$ be the velocities of N no of free electrons,

Then, avg velocities of electrons $= \dfrac{\vec{u}_1 + \vec{u}_2 + \ldots\ldots + \vec{u}_N}{N} = 0$

Thus, there is no net flow of charge in any direction. In the presence of electric field, each e^- experiences a force, $\vec{F} = -e\vec{E}$

The negative sign indicate e^- are moving in the opp. direction of $\vec{E}$.

$$\vec{F} = -e\vec{E}$$

$$\Rightarrow m\vec{a} = -e\vec{E}$$

$$\Rightarrow \vec{a} = \frac{-e\vec{E}}{m}, \quad m = \text{mass of the electron.}$$

If n, no. of e^- gain velocity component

$$\vec{v}_1, \vec{v}_2, \ldots\ldots\ldots \vec{v}_n$$

$$\vec{v}_1 = \vec{u}_1 + \vec{a}t_1$$

$$\vec{v}_2 = \vec{u}_2 + \vec{a}t_2$$

$$\vdots$$

$$\vec{v}_n = \vec{u}_n + \vec{a}t_n$$

$$\vec{v}_1 + \vec{v}_2 + \ldots\ldots + \vec{v}_n = \vec{u}_1 + \vec{u}_2 + \ldots\ldots + \vec{u}_n + \vec{a}(t_1 + t_2 + \ldots + t_n)$$

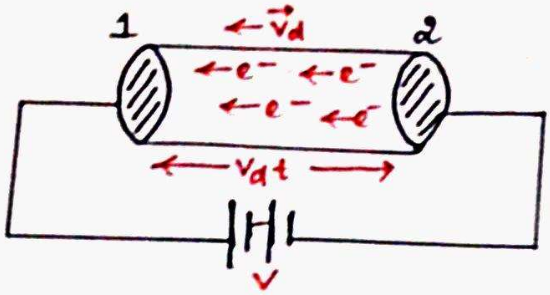
$$\Rightarrow \frac{\vec{v}_1 + \vec{v}_2 + \ldots + \vec{v}_n}{n} = \cancel{\frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_n}{n}}^{\,0} + \frac{\vec{a}(t_1 + t_2 + \ldots + t_n)}{n}$$

$$\Rightarrow \vec{v}_d = \vec{a}\tau, \quad v_d = \text{drift velocity}, \quad \tau = \text{relaxation time.}$$

τ is the avg. time elapsed between 2 successive collision of the electron.

$$\boxed{\vec{v}_d = \frac{-e\vec{E}\tau}{m}}$$

RELATION BETWEEN ELECTRIC CURRENT AND DRIFT VELOCITY :-



A = area of the cross-section
n = free electron density
t = time taken by electron to move from cross-section 1 to 2.

distance betⁿ two cross-section = $v_d t$

volume bounded by two cross-section = $Al = Av_d t$

no. of electrons in that volume = $nAv_d t$

no. of electron passes through the cross-section 1 in time t = $nAv_d t$

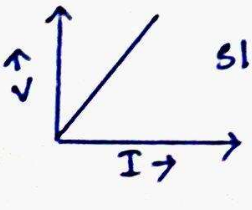
$$I = \frac{q}{t} = \frac{nAv_d t \cdot e}{t} = neAv_d$$

$$\boxed{I = neAv_d}$$

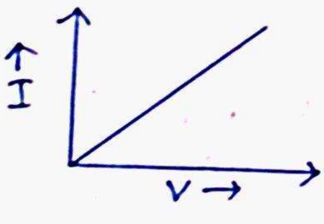
OHM'S LAW:- The potential difference between two ends of a conductor is directly proportional to current passing through it at constant temperature.

$$V \propto I$$

$$\Rightarrow V = IR$$



Slope = $\frac{V}{I} = R$



Slope = $\frac{I}{V} = \frac{1}{R}$.

DEDUCTION OF OHM'S LAW:-

$$I = neAv_d$$

$$= neA\left(\frac{eV\tau}{ml}\right) = \left(\frac{ne^2A\tau}{ml}\right)V$$

$$V = \left(\frac{ml}{nAe^2\tau}\right) I$$

$$\Rightarrow V = RI \quad , \quad R = \frac{ml}{nAe^2\tau}$$, constant for a particular conductor at constant temp.

$$\Rightarrow V \propto I$$

LIMITATIONS OF OHM'S LAW :-

1. only valid at constant temp.
2. some substances do not obey ohm's law.

VECTOR FORM OF OHM'S LAW :-

$$J = \frac{I}{A}$$

$$= \frac{neAv_d}{A}$$

$$= n \cdot \frac{eE\tau}{m} \cdot e$$

$$= \left(\frac{ne^2\tau}{m}\right) E$$

$$J = \sigma E$$

In vector form.

$$\boxed{\vec{J} = \sigma \vec{E}}$$

RESISTANCE (R) :- It is defined as the opposition offered to the flow of current

SI unit → Ω

CGS unit → St Ω / ab Ω

$$\boxed{R = \frac{ml}{nAe^2\tau}}$$

Resistance depends on :-

1. Geometry of conductor
2. Nature of material
3. Temperature.

CONDUCTANCE (G) :- It is defined as the reciprocal of resistance.

$$G = \frac{1}{R} = \boxed{\frac{nAe^2\tau}{mL}}$$

SI unit :- Ω^{-1}

RESISTIVITY (ρ) :-

$$R = \frac{ml}{nAe^2\tau}$$

$$= \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}$$

$\Rightarrow R = \rho \frac{l}{A}$ where $\boxed{\rho = \frac{m}{ne^2\tau}}$, which is constant for a particular material at constant temp.

DEFINATION OF ρ :-

$\rho = \frac{RA}{L}$, $A = 1m^2$, $l = 1m$, $\rho = R$

It is defined as resistance of a rod of that material of length 1 m and area of cross section 1 m^2.

SI unit $\rightarrow$ Ωm

$R = \frac{\rho l}{A}$, $R \propto l$ (A is constant)

$R \propto \frac{1}{A}$ (l is constant)

SPECIAL CASE :-

CASE-I

When A is not constant

$$R = \rho \frac{l}{A} \times \frac{l}{l}$$

$$= \frac{\rho l^2}{Vol}$$

$$\Rightarrow \boxed{R \propto l^2}$$

CASE-II

when l is not constant

$$R = \rho \frac{l}{A} \times \frac{A}{A} = \frac{\rho l A}{A^2}$$

$$= \frac{\rho \times Vol}{A^2}$$

$$\Rightarrow \boxed{R \propto \frac{1}{A^2}}$$

CONDUCTIVITY (σ) :-

It is defined as reciprocal of resistivity

$$\boxed{\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}}$$

SI unit :- $\Omega^{-1} m^{-1}$

MOBILITY (μ)

Mobility of a charge is defined as drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

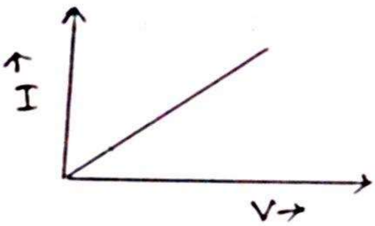
$$\mu = \frac{eE\tau}{m} \times \frac{1}{E}$$

$$\boxed{\mu = \frac{e\tau}{m}} \text{ (for electron)}$$

$$\boxed{\mu = \frac{q\tau}{m}} \text{ (general)}$$

* For a particular charge, $\mu \propto \tau \propto \frac{1}{\text{temp}}$.
* At constant temperature, $\mu \propto \frac{q}{m}$.

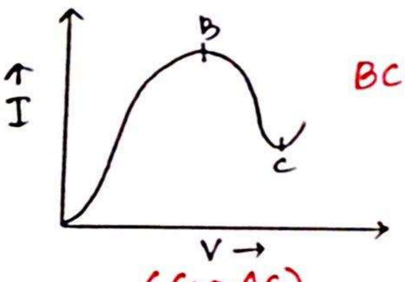
OHMIC SUBSTANCE :- Substance which obeys ohm's law.



eg:- all metals carrying low current.

NON-OHMIC SUBSTANCE :- Substance which doesn't obey ohm's law.

eg:- dil H_2SO_4, water voltameter, vacuum diode, GaAs



(GaAs)

BC = –ve region

TEMPERATURE DEPENDANCE OF RESISTIVITY :-

$\rho_0 \rightarrow$ initial velocity at temp T_0

$\rho \rightarrow$ final velocity at temp T

$\rho - \rho_0 \rightarrow$ change in resistivity.

$\Rightarrow \rho - \rho_0 \propto (T - T_0)$

$\Rightarrow \rho - \rho_0 \propto \rho_0$

$\Rightarrow \rho - \rho_0 \propto \rho_0 (T - T_0)$

$\Rightarrow \rho - \rho_0 = \alpha \rho_0 (T - T_0)$, α = temp. coeffiuent of resistivity.

$$\Rightarrow \boxed{\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)}}$$

It is defined as the ratio between change in resistivity per original resistivity for degree rise of temp.

SI unit → K^{-1}

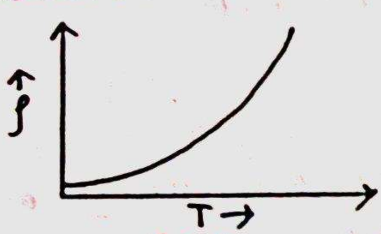
CONDUCTOR:- for conductor, α = +ve i.e ρ increases with ↑ in temp.
Cause:- When temp ↑, K.E of free e^- ↑ so no of collision per sec ↑. Hence, resistivity ↑.

SEMICONDUCTOR:- α = −ve, with ↑ in temp, ρ ↓
Cause:- When temp ↑, charge carrier density ↑ses which dominate the effect of τ.

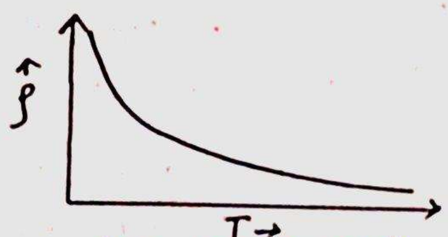
As $\rho = \frac{m}{ne^2\tau}$, so ρ decreases.

INSULATOR:- α = −ve, ρ ↓ses with temperature.

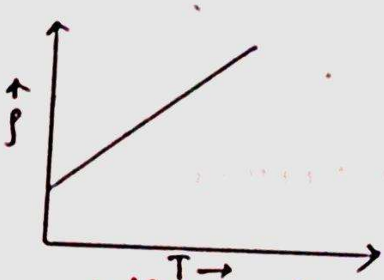
$\rho \sim T$ GRAPHS :-



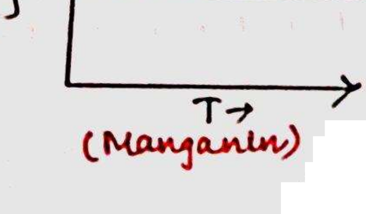
(Cu)



(Semiconductor)



(Nichrome)



(Manganin)

USE OF ALLOY IN MAKING RESISTOR :-

1. ρ of alloy is very high
2. They have very small temp. of coefficient.
3. least affected by atmospheric conditions such as air, moisture, pressure.

COLOUR CODE OF CARBON RESISTOR :-

- B → Black → 0
- B → Brown → 1
- R → Red → 2
- O → Orange → 3
- Y → Yellow → 4
- G → Green → 5
- B → Blue → 6
- V → Violet → 7

G → Grey → 8
W → White → 9

G → Gold → 5%
S → Silver → 10%
N → No colour → 20%

$\}$ Tolerance

TRICK TO REMEMBER :- B B ROY of Great Britain had a Very Good Wife Wearing Gold & Silver Necklace.

INTERNAL RESISTANCE (r) :- The opposition offered by electrolyte due to flow of electric current is called internal resistance.

CAUSE :- Due to collision of ions.

It depends on (1) nature of electrolyte and electrode.

(2) area of electrode dipped in electrolyte.

(more is the area, less is the internal resistance)

(3) distance between the two electrodes

(more is the separation, more is the internal resistance)

(4) temperature

(when temp ↑es, internal resistance ↓ because viscosity decreases)

RELATION BETWEEN EMF AND POTENTIAL DIFFERENCE :-

(A) DISCHARGING CONDITION

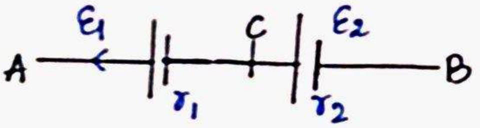
$$\boxed{\mathcal{E} = V + Ir}$$

(B) CHARGING CONDITION

$$\boxed{V = \mathcal{E} + Ir}$$

COMBINATION OF CELL :-

(A) SERIES



for cell 1, $V_{AC} = \mathcal{E}_1 - Ir_1$

$$\Rightarrow V_A - V_C = \mathcal{E}_1 - Ir_1 \quad \text{—— (1)}$$

for cell 2, $V_{CB} = \mathcal{E}_2 - Ir_2$

$$\Rightarrow V_C - V_B = \mathcal{E}_2 - Ir_2 \quad \text{—— (2)}$$

Adding (1) & (2),

$$V_A - V_B = \mathcal{E}_1 + \mathcal{E}_2 - I(r_1 + r_2) \quad \text{—— (3)}$$

for the combination,

$$V_{AB} = \mathcal{E} - Ir$$

$$\Rightarrow V_A - V_B = \mathcal{E} - Ir \quad \text{—— (4)}$$

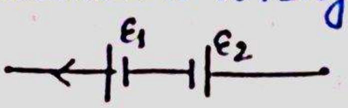
From eqn (3) & (4),

$$\mathcal{E}_1 + \mathcal{E}_2 - I(r_1 + r_2) = \mathcal{E} - Ir$$

$$\Rightarrow \boxed{\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2} \qquad \boxed{r = r_1 + r_2}$$

SPECIAL CASE :-

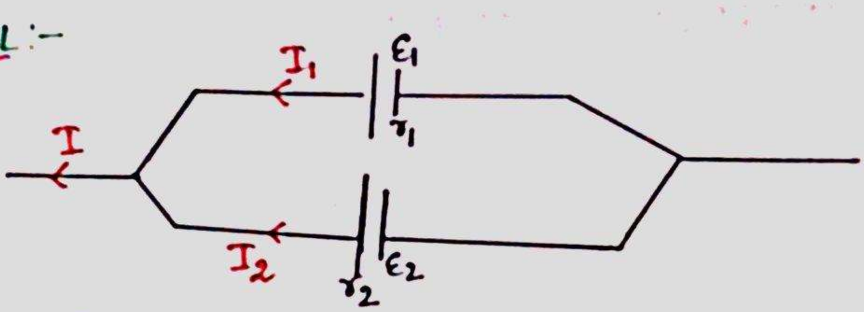
If the connection is wrong



$$\boxed{\varepsilon = \varepsilon_1 - \varepsilon_2}$$ (If $\varepsilon_1 > \varepsilon_2$)

$$\boxed{r = r_1 + r_2}$$

(B) PARALLEL :-



for cell 1, $V = \varepsilon_1 - I r_1 \Rightarrow I_1 = \dfrac{\varepsilon_1 - V}{r_1}$ — ①

for cell 2, $I_2 = \dfrac{\varepsilon_2 - V}{r_2}$ — ②

Similarly for the combination,

$$I = \frac{\varepsilon - V}{r} \quad \text{— ③}$$

$$I = I_1 + I_2$$

$$\Rightarrow \frac{\varepsilon - V}{r} = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$\Rightarrow \frac{\varepsilon}{r} - \frac{V}{r} = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2} \Rightarrow \frac{\varepsilon}{r} - \frac{V}{r} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Comparing both sides,

$$\frac{\varepsilon}{r} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \& \quad \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} \Rightarrow r = \frac{r_1 r_2}{r_1 + r_2}$$

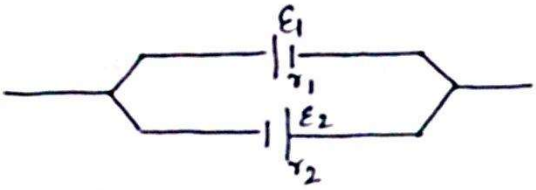
$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 \cdot r_2} \cdot r \quad = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \cdot \frac{r_1 r_2}{r_1 + r_2}$$

$$\boxed{\varepsilon = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}$$

SPECIAL CASE :-

CASE-I

If connection is wrong



$$\boxed{\mathcal{E} = \frac{\mathcal{E}_1 r_2 - \mathcal{E}_2 r_1}{r_1 + r_2}}$$

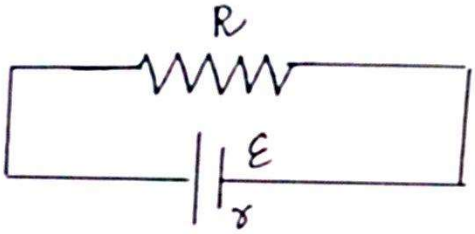
CASE-II :-

If $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$

$r_1 = r_2 = r$

$$\boxed{\mathcal{E}_{net} = \mathcal{E}}$$

EXPRESSION OF CURRENT :-



$$\mathcal{E} = V + Ir$$

$$\Rightarrow \mathcal{E} = IR + Ir$$

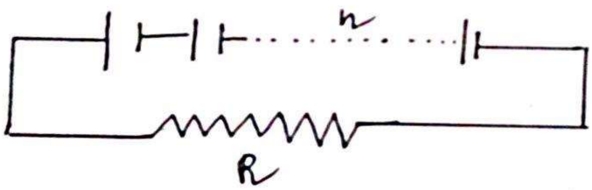
$$\Rightarrow \mathcal{E} = I(R+r)$$

$$\Rightarrow I = \frac{\mathcal{E}}{R+r}$$

$$\boxed{I = \frac{\text{net emf}}{\text{net resistance}}}$$

COMBINATION OF IDENTICAL CELL :-

(A) SERIES COMBINATION :-



n = no. of cells connected in series

net emf $= n\mathcal{E}$

$$I = \frac{n\mathcal{E}}{nr+R} = \frac{n\mathcal{E}}{nr+R} = \frac{n\mathcal{E}}{nr+R}$$

CASE-I

If $R >>> nr$

$$\boxed{I = \frac{n\varepsilon}{R}},$$

Current depends on no. of cells.

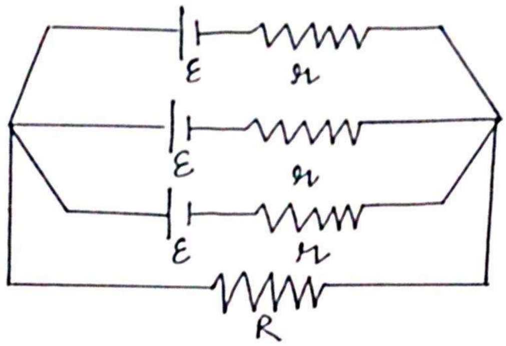
CASE-II

If $R <<< nr$

$$\boxed{I = \frac{\varepsilon}{r}}$$

Series connection is useful when external resistance is very large.

(B) PARALLEL COMBINATION :-



Total emf = ε

Net. internal resistance = r/n

Net. resistance of entire network = $R + \frac{r}{n}$

$$I = \frac{\varepsilon}{R + \frac{r}{n}} = \frac{n\varepsilon}{nR + r}$$

CASE-I

If $R >>>> r$, r can be neglected

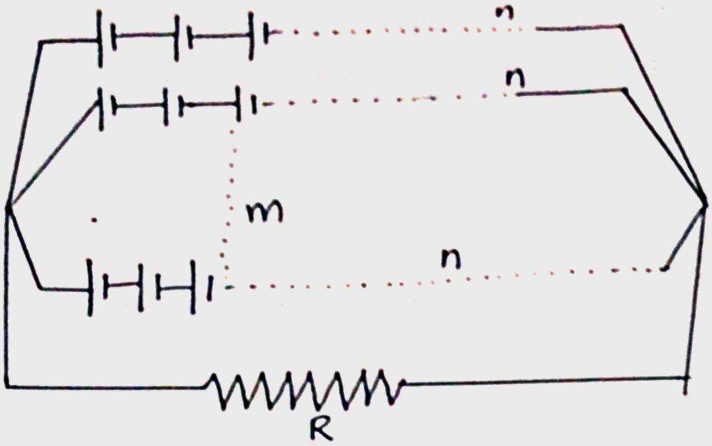
$$I = \frac{n\varepsilon}{nR} = \boxed{\frac{\varepsilon}{R}}$$

CASE-II

If $R <<< r$, R can be neglected

$$I = \frac{n\varepsilon}{r} = \boxed{n\left(\frac{\varepsilon}{R}\right)}$$

MIXED CONNECTION :-



n = no. of cells in each row
m = no. of such rows

Net emf = nE

Net internal resistance = $\frac{1}{R'} = \frac{1}{nr} + \frac{1}{nr} + \ldots\ldots m = \frac{m}{nr}$

$$R' = \frac{nr}{m}$$

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{Rm + nr}$$

$$mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2 = (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr}$$

Current will be max. when $\sqrt{mR} = \sqrt{nr}$

$$\Rightarrow mR = nr$$

$$\Rightarrow \boxed{R = \frac{nr}{m}}$$

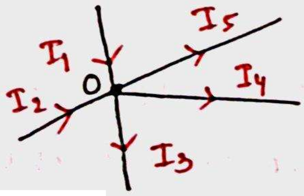
⇒ Total external resistance = Total internal resistance.

KIRCHHOFF'S LAWS :-

(a) KIRCHHOFF CURRENT LAW / JUNCTION LAW :-

It states that algebraic sum of currents meeting at a junction is zero.

$$\boxed{\Sigma I = 0}$$



The current coming towards the junction is taken as +ve.
The current going away from the junction is taken as −ve.

$\Rightarrow I_1 + I_2 - I_3 - I_4 - I_5 = 0$

$\Rightarrow \boxed{I_1 + I_2 = I_3 + I_4 + I_5}$

So, net current coming towards the junction = net current going out of the junction.

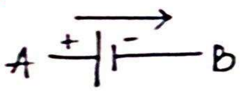
(b) KVL / loop law :-

It states that the algebraic sum of potential differences across cells and resistors in a close loop is 0.

$$\boxed{\Sigma \Delta V = 0}$$

SIGN CONVENTION :-

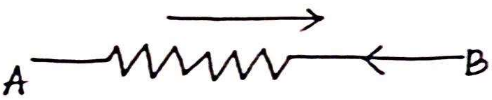
① If one moves from +ve to −ve of a cell, then emf is −ve



$\Delta V = V_B - V_A$

$\boxed{\mathcal{E} = -ve}$

② If one move opposite to direction of current then the product of current and resistance (IR) is taken as +ve.



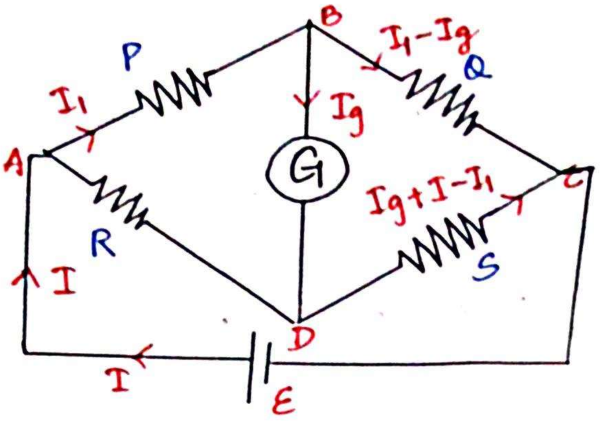
$\Delta V = V_B - V_A$

$\Rightarrow \boxed{IR = +ve}$

WHEATSTONE BRIDGE :-

P, Q, R, S are the 4 resistors connected in wheatstone bridge.

G → resistance of galvanometer.



Using KVL,

ABDA

$$-PI_1 - GI_g + R(I - I_1) = 0$$

$$-I_1P - I_gG + (I - I_1)R = 0 \quad \text{—— (I)}$$

BCDB

$$-Q(I_1 - I_g) + S(I - I_1 + I_g) + GI_g = 0 \quad \text{—— (II)}$$

The bridge is said to be balanced when no current passes through galvanometer

i.e $I_g = 0$

eqn (I) & (II) becomes,

$$-I_1P + (I - I_1)R = 0$$

$$\boxed{(I - I_1)R = I_1P} \quad \text{—— (III)}$$

$$-QI_1 + SI - SI_1 = 0$$

$$\boxed{I_1Q = (I - I_1)S} \quad \text{—— (IV)}$$

Dividing (III) & (IV),

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

It is the balanced condition of wheatstone bridge.

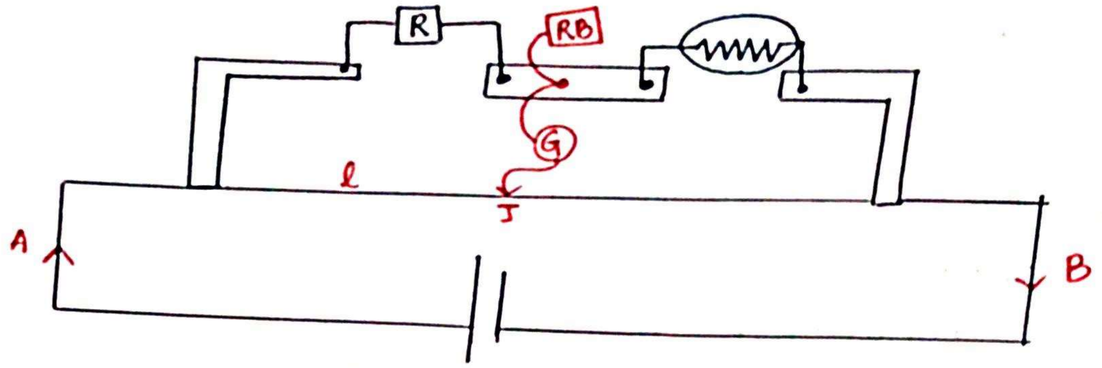
Q):- What happens to the balanced condition if cell & galvanometer are interchanged?

No change.

METER BRIDGE :-

It is an electrical device used to measure unknown resistance.

PRINCIPLE :- It works on the balanced condition of wheatstone bridge.



R = known resistance from resistance box
S = unknown resistance
J = null point such that AJ = l

According to balanced condition of wheatstone bridge.

$$\frac{R}{S} = \frac{R_{AJ}}{S_{BJ}}$$

$$\Rightarrow \frac{R}{S} = \frac{l}{100-l}$$

$$\Rightarrow \boxed{S = \frac{R(100-l)}{l}}$$

(Q) :- When is metre bridge most sensitive?
If It is obtained at the middle of the wire

(Q) :- Why thick copper strips are used?
Because of negligible resistance

(Q) :- What happens to balancing length if resistance R increases?
Increases.

POTENTIOMETER

It is an electrical device which is used to measure emf of a cell.

PRINCIPLE OF POTENTIOMETER :-

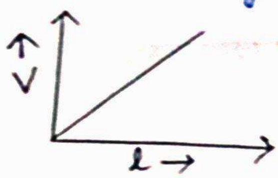
$$V = IR$$

$$\Rightarrow V = I\rho l/A$$

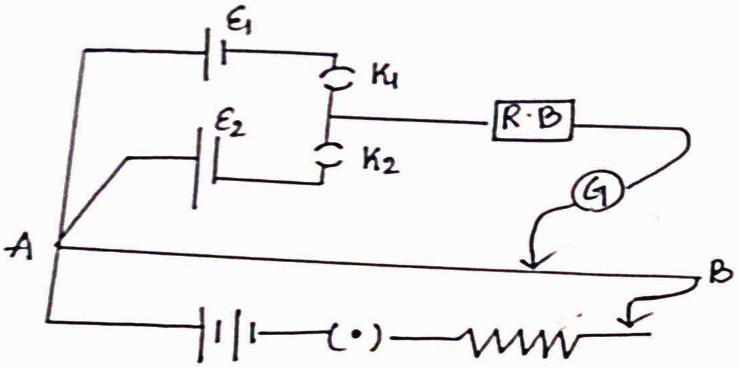
$$\Rightarrow V = \left(\frac{I\rho}{A}\right) l$$

$$\Rightarrow \boxed{V = Kl} \quad , \quad K = \rho I/A$$

The principle is that when a constant current flows through a wire of uniform cross-section and composition, the potential drop across any length of the wire is directly proportional to that length.



① Comparison of emf :-



E_1 & E_2 → are two primary cells
K_1 & K_2 → Two way key
RB → Resistance box
E → Driving cell
K → Key of Auxillary / primary circuit
R → Rheostat
AB → potentiometer wire

Close K_1, K_2 is open

$$E_1 \propto l_1$$

$$\Rightarrow E_1 = K l_1 \quad \text{—(I)}$$

Close K_2, K_1 is open.

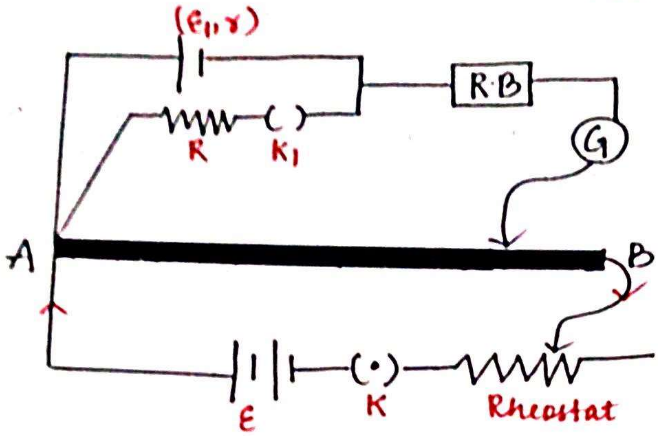
$$E_2 \propto l_2$$

$$\Rightarrow E_2 = K l_2 \quad \text{—(II)}$$

Eqⁿ (I) by Eqⁿ (II),

$$\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$$

(2) DETERMINATION OF INTERNAL RESISTANCE OF GIVEN PRIMARY CELL:-



CASE-I

K_1 is open

$E_1 \propto l_1 \Rightarrow \boxed{E_1 = K l_1}$ —— (I)

CASE-II

V_1 is closed.

$V \propto l_2 \Rightarrow \boxed{V = K l_2}$ —— (II)

$$\frac{\text{eq}^n \text{(I)}}{\text{eq}^n \text{(II)}} = \frac{E_1}{V} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{I(R+r)}{IR} = \frac{l_1}{l_2}$$

$$\Rightarrow 1 + \frac{r}{R} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{r}{R} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \boxed{r = \left(\frac{l_1 - l_2}{l_2}\right) R}$$

l_1 = balancing length when only E_1 is connected

l_2 = balancing length when R is connected