CSBE Class 12 Mathematics

Chapter-3

Matrices

- A **matrix** is an ordered rectangular array of numbers, real or complex or functions.
- A matrix having m rows and n columns is called a matrix of **order** m × n.
- **Column matrix**: A matrix with one column is denoted by $[a_{ij}]_{m\times 1}$.
- **Row matrix**: A matrix with one row is denoted by $[a_{ij}]_{1\times n}$.
- **Square matrix**: An m × n matrix is a square matrix if m = n.
- **Diagonal matrix**: A = $A = [a_{ij}]_{m\times n}$ is a diagonal matrix if $a_{ij} = 0$, when $i \neq j$
- **Scalar matrix**: $A = \left[a_{ji}\right]_{n\times n}$ is a scalar matrix if $a_{ij} = 0$ when $i \neq j$, $a_{ij} = k$ (k is *some constant), when I=j.*
- **Identity matrix**: $A = \left[a_{ij}\right]_{n\times n}$ is an identity matrix, if

  $a_{ij} = 1,\ when\ i = j,\ a_{ij} = 0,\ when\ i \neq j.$
- **Zero matrix**: A zero matrix has all its elements as zero.
- **Equality of two matrices**: $A = \left[a_{ij}\right] = \left[b_{ij}\right] = B$ if (i) A and B are of same order, (ii) for all possible values of $i\ and\ j.$
- **Scalar multiplication**: $kA = k\left[a_{ij}\right]_{m\times n} = \left[k\left(a_{ij}\right)\right]_{m\times n}$

  Also $-A = (-1)A$
- $A - B = A + (-1)B$

  $A + B = B + A$

  $(A + B) + C = A + (B + C)$, where $A = [a_{ij}]$, $B = [b_{ij}]$ and $C = [c_{ij}]$ are of same order.
- $k(A + B) = kA + kB$, where A and B are of same order, k is constant.
- $(k + l)A = kA + lA$, where k and l are constant.
- $If\ A = [a_{ij}]_{m\times n}\ and\ B = [b_{jk}]_{n\times p},\ then\ AB = C = [c_{ik}]_{m\times p},$ where

$$C_{t1} = \sum_{j=i}^{n} a_{ij} b_{jk}$$

- (i) A(BC) = (AB)C,

(ii) A(B + C) = AB + AC,

(iii) (A + B)C = AC + BC

- $If\ A = \left[a_{ij}\right]_{m\times n},\ then\ A'\ or\ A^{T} = \left[a_{ji}\right]_{n\times m}$
- (i) (A')' = A, · (ii) (kA)' = kA', · (iii) (A + B)' = A' + B', · (iv) (AB)' = B'A'
- **Symmetric matrix**: A is a symmetric matrix if A' = A.
- **Skew-aymmetric matrix**: A is a skew symmetric matrix if A' = –A.
- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix. In fact, A = $\frac{1}{2}$ (A + A') + $\frac{1}{2}$ (A - A'), where $\frac{1}{2}$ (A + A') is a symmetric matrix and $\frac{1}{2}$ (A - A') is a skew-symmetric matrix.
- **Equivalent matrices**: Two matrices A and B are equivalent that is, A - B is A is obtained from the other by a sequence of elementary operations. Elementary operations of a matrix are as follows:

(i) $\mathrm{R}_i \leftrightarrow \mathrm{R}_j$ or $\mathrm{C}_i \to \mathrm{C}_j$ (interchange rows or columns)

(ii) $\mathrm{R}_i \to k\mathrm{R}_j$ or $\mathrm{C}_i \to k\mathrm{C}_j$

(iii) $\mathrm{R}_i \to \mathrm{R}_i + k\mathrm{R}_j$ or $\mathrm{C}_i \to \mathrm{C}_i + k\mathrm{C}_j$

- If A and B are two square matrices such that AB = BA = I, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.
- Inverse of a square matrix, if it exists, is unique.