CBSE Class 12 Mathematics
Chapter-11
Three Dimensional Geometry

- **Direction cosines of a line** : Direction cosines of a line are the cosines of the angles made by the line with the positive direct ions of the coordinate axes.
- *If* l, m, n are the direct ion cosines of a line, then $l^2 + m^2 + n^2 = 1$
- Direct ion cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\frac{x_2 - x_1}{PQ}, \ \frac{y_2 - y_1}{PQ}, \ \frac{z_2 - z_2}{PQ}$$

· where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Direction ratios of a line are the numbers which are proportional to the direct ion cosines of a line.
- If l, m, n are the direct ion cosines and a, b, c are the direct ion ratios of a line

Then, $l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

- **Skew lines**: Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.
- **Angle between two skew lines**: Angle between skew lines is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.
- If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then,

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_{12}}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

- Vector equation of a line that passes through the given point whose position vector is $\bar{a}$ and parallel to a given vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$

- Equation of a line through a point (x_1, y_1, z_1) and having direct ion cosines l, m, n is $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$
- The vector equation of a line which passes through two points whose position vectors are $\bar{a}$ and $\bar{b}$ is $\bar{r}=\bar{a}+\lambda(\bar{b}-\bar{a})$
- Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$
- If θ is the acute angle between $\bar{r}=\bar{a}_1+\lambda\bar{b}_1$ and $\bar{r}=\bar{a}_2+\lambda\bar{b}_2$ then,

  $\cos\theta=\left|\frac{\vec{b}_1\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$
- If $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are the equations of two lines, then the acute angle between the two lines is given by

  $\cos\theta=|l_1l_2+m_1m_2+n_1n_2|$
- Shortest distance between two skew lines is the line segment perpendicular to both the lines.
- Shortest distance between $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$ is $\left|\frac{\left(\vec{b_1}\times\vec{b_2}\right).\left(\vec{a_2}-\vec{a_1}\right)}{|b_1\times b_2|}\right|$
- Shortest distance between the lines: $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_1}=\frac{y-y_2}{b_1}=\frac{z-z_2}{c_1}$ is

$$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

- Distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is $\left|\frac{\vec{b} \times \left(\vec{a_2} - \vec{a_1}\right)}{|b|}\right|$
- In the vector form, equation of a plane which is at a distance d from the origin, and $\hat{n}$ is the unit vector normal to the plane through the origin is $\bar{r}.\hat{n} = d$
- Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is $lx + my + nz = d$.
- The equation of a plane through a point whose position vector is a and perpendicular to the vector $\vec{N}$ is $\left(\vec{r} - \vec{a}\right).\vec{N} = 0$.
- Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x_1, y_1, z_1) is $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$
- Equation of a plane passing through three non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$
- Vector equation of a plane that contains three non collinear points having position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is $\left(\vec{r} - \vec{a}\right).\left[\left(\vec{b} - \vec{a}\right) \times \left(\vec{c} - \vec{a}\right)\right] = 0$.
- Equation of a plane that cuts the coordinates axes at $(a, 0, 0)$, $(0, b, 0)$ *and* $(0, 0, c)$ is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
- Vector equation of a plane that passes through the intersection of planes $\vec{r}.\vec{n_1} = d_1$ and $\vec{r}.\vec{n_2} = d_2$ is $\vec{r}\left(\vec{n_1} - \lambda\vec{n_2}\right) = d_1 + \lambda d_2$, where λ is any non-zero constant.
- Cartesian equation of a plane that passes that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is $(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2 = 0$
- Two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are coplanar if $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 0$.
- Two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are

coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$

- In the vector form, if θ is the angle between the two planes, $\vec{r}.\vec{n_1}=d_1$ and $\vec{r}.\vec{n_2}=d_2$, then $\theta=\cos^{-1}\left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$
- The angle ϕ between the line $\vec{r}=\vec{a}+\lambda\vec{b}$ and the plane $\vec{r}.\hat{n}=d$ is $\sin\phi=\frac{\vec{b}.\hat{n}}{|\vec{b}||\hat{n}|}$
- The angle θ between the planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ is given by $\cos\theta=\left|\frac{A_1A_2++B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$
- The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r}.\hat{n}=d$ is $\left|d-\vec{a}.\hat{n}\right|$.
- The distance from a point (x_1,y_1,z_1) to the plane Ax + By + Cz + D = 0 is $\left|\frac{Ax_1+By_1+Cz_1+D}{\sqrt{A^2+B^2+C^2}}\right|$
- Equation of any plane that is parallel to a plane that is parallel to a plane Ax + By + Cz + D = 0 is Ax + By + Cz + k = 0, where k is a different constant other than D.