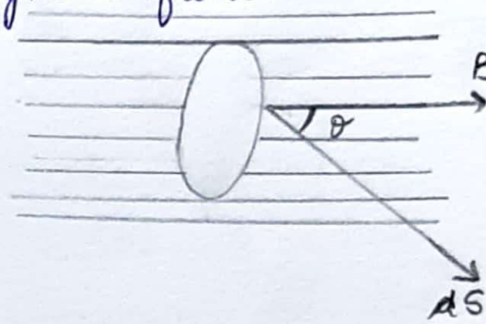
ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX -

It represents total magnetic lines of force passing normally through a given area placed in a magnetic field.

$$\boxed{\phi_B = B\cdot S = BS\cos\theta}$$



Unit - Weber (Wb) = Tm^2 → SI unit
Maxwell → CGS unit

ELECTROMAGNETIC INDUCTION -

The phenomenon to generate induced current or induced emf by changing the magnetic flux linked with a closed circuit is known as Electromagnetic Induction.

FARADAY'S LAWS

1. First Law - Whenever there is change in magnetic flux linked with the closed loop, an emf induces in the loop which lasts as long as the change in flux continues.

2. Second Law - The induced emf in a closed loop or circuit is directly proportional to the rate of change of magnetic flux linked with the closed loop or circuit.

i.e. $e \propto (-)\frac{d\phi}{dt}$

$$\boxed{e = -\left(\frac{d\phi}{dt}\right)}$$

* The negative sign is due to Lenz Law.

LENZ LAW -

Current induced in the loop due to changing magnetic flux is such that it tends to oppose the rate of change of magnetic flux.

- Lenz law is in accordance with law of conservation of energy.

INDUCED CURRENT -

If N is the number of turns and R is the resistance of a coil, the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, then induced current flowing through the coil is

$$e = -\frac{d\phi}{dt} \qquad I = \frac{|e|}{R} \Rightarrow I = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{— (1)}$$

$$\because I = \frac{dq}{dt}$$

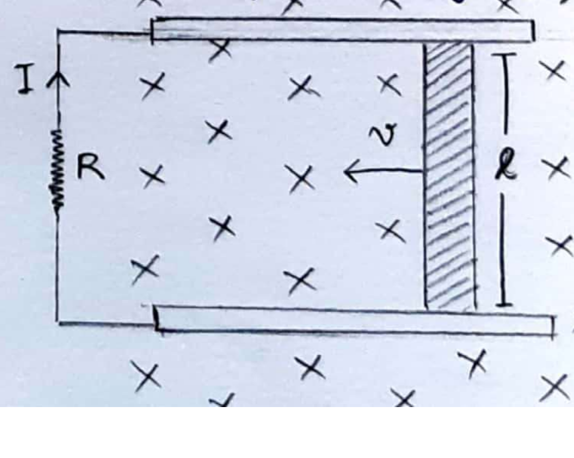
$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{using (1)}$$

$$\boxed{q = -\frac{1}{R}\int d\phi}$$

MOTIONAL EMF -

The potential difference induced in a conductor of length l moving with ~~vel~~ velocity v in a direction perpendicular to magnetic field B is given by

$$\mathcal{E} = \int (v \times B) \cdot dl = vBl$$



$B \rightarrow$ magnetic field
$l \rightarrow$ length of conducting wire
$v \rightarrow$ velocity

$$e = -\frac{d\phi}{dt}$$

$d\phi = BA$ $\qquad$ $e = -\frac{d}{dt}(BA)$

$A = lx$ $\qquad$ $e = -\frac{d}{dt}B(lx)$

$$\boxed{e = Blv}$$

• emf will not be induced if any two of Blv are parallel.

FORCE

$$I = \frac{Blv}{R}$$

$$F = BIl$$

$$\boxed{F = \frac{B^2l^2v}{R}}$$

POWER

$$P = F \times v$$

$$\boxed{P = \frac{B^2l^2v^2}{R}}$$

EDDY CURRENT

The current induced in bulk piece of conductor when magnetic flux linked with the conductor changes is known as eddy currents.

$$\boxed{i = \frac{e}{R}}$$

Applications -

1. Magnetic Braking
2. Induction furnace
3. Speedometer
4. Electromagnetic damping
5. Energy meter

Disadvantages -

1. Lot of heat energy is produced which damages the core of material.
2. Excessive heating may lead to fire.
3. Reduces the efficiency of the machine.

Ways to minimize -

1. By laminating the core.
2. By making slots on the conductor surface.

INDUCTANCE -

The flux linkage of a closely wound coil is directly proportional to the current I i.e. $\phi_B \propto I$. The flux linked with the coil having 'N' turns will be
$N\phi_B \propto I$. The constant of proportionality in this relation is called inductance.

SELF INDUCTANCE

The phenomenon of production of induced emf in a coil, when a current pass through it, undergoes a change.

$\because$ Total flux linked with coil, $N\phi \propto I$

$$\boxed{N\phi = LI}$$

where, ϕ = flux linked with each turn and
L = coefficient of self-induction or self inductance

Also, induced emf, $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

SI unit - Henry (H)

where, $L = \frac{\varepsilon}{\ldots}$

Self Inductance of Long Solenoid -

The magnetic field B at any point inside such a solenoid is constant;

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

where,

μ_0 = magnetic permeability

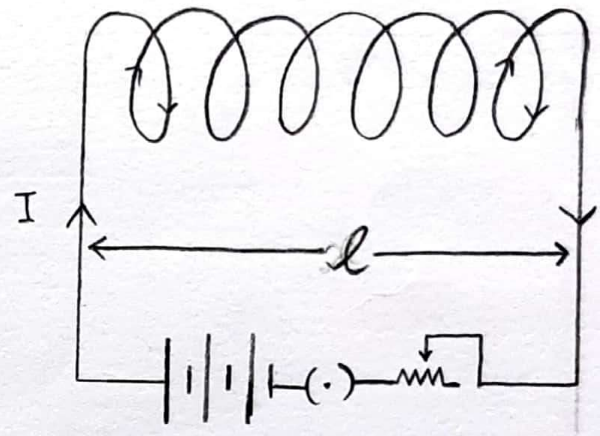
N = total no. of turns

l = length of the solenoid

n = no. of turns per unit length

ϕ = B × area of each turn

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A$$



$$N\phi = LI$$

where $L = \frac{\mu_0 N^2 A}{l}$

$$\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

MUTUAL INDUCTANCE -

The phenomenon according to which an opposing emf is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called inductance.

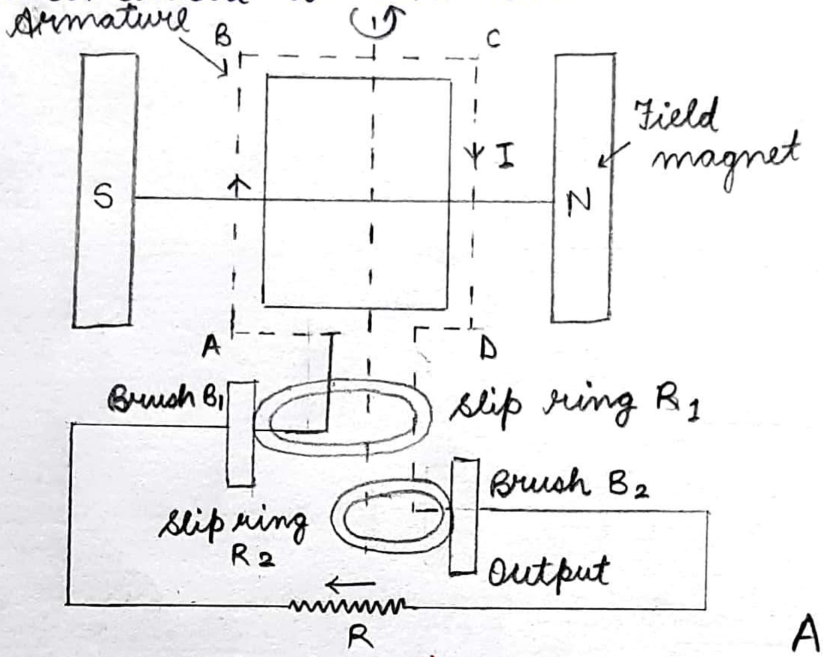
$$\phi \propto I$$

$$\phi = MI$$

$$\boxed{e = -M \frac{dI}{dt}}$$

AC GENERATOR

An AC generator is based on the phenomenon of electromagnetic induction, which states that a coil is rotated in uniform magnetic field, the magnetic flux linked with a conductor changes and an emf is induced in the coil.



AC GENERATOR

Theory and Working

As the armature of coil is rotated in uniform magnetic field, angle θ changes continuously. Therefore, magnetic flux changes and an emf is induced. If e is the emf induced in the coil, then

$$e = -\frac{N d\phi}{dt} \quad \text{or} \quad e = -\frac{d}{dt}(NBA\cos\omega t)$$

$$\varepsilon = NBA\omega \sin\omega t$$

N → no. of turns in the coil

B → strength of magnetic field

A → area of each turn of coil

ω → angular velocity of rotation of the coil

and $I = \frac{e}{R} = \frac{NBA\omega}{R}\sin\omega t$, R → resistance of the coil

IMPORTANT QUESTIONS

1. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside, normal to the axis of solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? (NCERT)

Sol. Here, no. of turns per unit length,

$$n = \frac{N}{l} = 15 \text{ turns/cm} = 1500 \text{ turns/m}$$

$$A = \frac{N}{l} = \cancel{15\ tu}\ 2.0\ cm^2 = 2 \times 10^{-4}\ m^2$$

$$\frac{dI}{dt} = \frac{4-2}{0.1} \quad \text{or} \quad \frac{dI}{dt} = 20\ A\,s^{-1}$$

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA)$$

$$|e| = \frac{A\,d}{dt}\left(\mu_0 \frac{NI}{l}\right) = A\mu_0\left(\frac{N}{l}\right)\frac{dI}{dt}.$$

$$|e| = (2 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 1500 \times 20\ V$$

$$|e| = 7.5 \times 10^{-6}\ V$$

2. A 1 m long conducting rod rotates with an angular frequency of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exist everywhere. Calculate the emf developed between the centre and the ring. (NCERT)

Sol. Length of rod, $l = 1$ m, $\omega = 400\ rad\,s^{-1}$, $B = 0.5\ T$, $e = ?$

Average linear velocity,

$$v = \frac{0 + l\omega}{2} = \frac{l\omega}{2}, \quad e = Blv$$

$$e = Bl\frac{l\omega}{2} = \frac{Bl^2\omega}{2} = \frac{0.5 \times (1)^2 \times 400}{2} = 100V$$

3. A line charge λ per ~~p~~ unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis as shown in the figure.

Sol. Given, linear charge density, $\lambda = \frac{\text{Total charge}}{\text{length}} = \frac{Q}{2\pi R}$

where, radius of rim = R and mass of rim = M

Magnetic field extends over a circular region,

$$B = -B_0\hat{k}\ (r \leq a,\ a < R) = 0$$

Let the angular velocity of the wheel be ω, then the induced emf, $e = -\frac{d\phi}{dt}$

$$e = -\int E.\,dl = -\frac{d\phi}{dt}$$

$$E\int dl = -\frac{d}{dt}(\pi a^2 B)$$

$$E \times 2\pi a = -\pi a^2 \frac{dB}{dt}\ ;\ E = -\frac{a}{2}\cdot\frac{dB}{dt}$$

Force on charge, $F = QE = -\pi a^2 \lambda \frac{dB}{dt}$

$$F = \frac{dp}{dt} = M\cdot\frac{dv}{dt}$$

$$M\frac{dv}{dt} = -\pi a^2\lambda\frac{dB}{dt} \Rightarrow MR\left(\frac{d\omega}{dt}\right) = -\pi a^2\lambda\frac{dB}{dt}$$

$$d\omega = -\frac{\pi a^2\lambda}{MR}\,dB$$

$$\omega = -\frac{\pi a^2\lambda B}{MR} \Rightarrow \boxed{\omega = -\frac{\lambda a^2\pi}{MR}B\hat{k}}$$