# ELECTROSTATIC POTENTIAL & CAPACITANCE

(CHAPTER-2)

**ELECTROSTATIC POTENTIAL:-** It is defined as the amount of work done to bring unit +ve charge from ∞ to that point along any path without any acceleration.

$$\boxed{V = \frac{W}{q_0}}$$

→ It is a scalar quantity

→ SI unit- J/C or V

→ Dimension- $[M^1 L^2 T^{-3} A^{-1}]$

**POTENTIAL DIFFERENCE:-** It is defined as the amount of work done to bring unit +ve charge from one point to another point along any path without any acceleration.

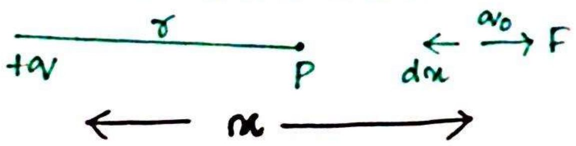
$$\Delta V = \frac{W}{q_0}$$

$$\Rightarrow W = q_0 \Delta V$$

if $q_0 = q$

$$\boxed{W = q \Delta V}$$

**POTENTIAL DUE TO A POINT CHARGE:-**




P is any point at a dist. r from +q charge, work done to bring $q_0$ from infinity to point P is equal to

$$W = \int \vec{F} \cdot \vec{dx}$$

$$= \int_{\infty}^{r} F\,dx \cos 180^\circ$$

$$= -\int_{\infty}^{r} K \frac{qq_0}{x^2}\, dx$$

$$= -Kqq_0 \left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$= Kqq_0 \left[\frac{1}{x}\right]_{\infty}^{r}$$

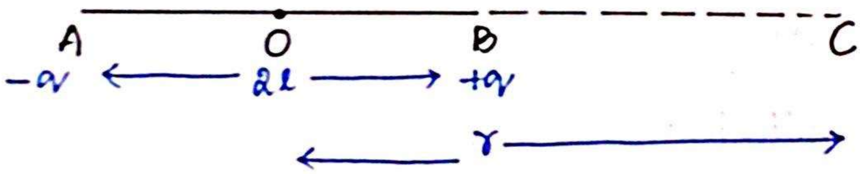
$$= Kqq_0 \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$W = \frac{Kqq_0}{r}$$

$$V = \frac{W}{q_0}$$

$$\Rightarrow \boxed{V = \frac{Kq}{r}}$$

## POTENTIAL DUE TO DIPOLE ON THE AXIAL LINE:-




Due to +q, charge potential,

$$V_1 = \frac{Kq}{(r-l)}$$

Due to -q, charge potential,

$$V_2 = \frac{-Kq}{(r+l)}$$

Net potential,

$$= \frac{Kq}{r-l} - \frac{Kq}{r+l}$$

$$= Kq\left(\frac{1}{r-l} - \frac{1}{r+l}\right)$$

$$= Kq\left[\frac{r+l-r+l}{(r-l)(r+l)}\right]$$

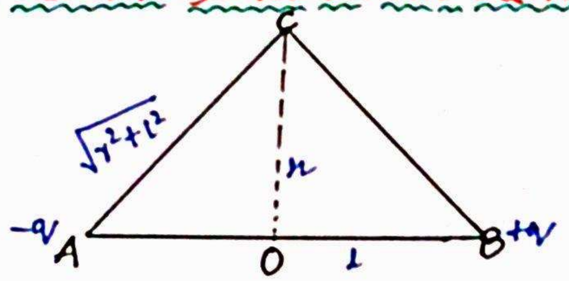
$$= \frac{Kq \cdot 2l}{r^2-l^2}$$

$$\boxed{V = \frac{KP}{r^2-l^2}}$$

For ideal dipole, $l <<<< r$, so $l^2$ can be neglected.

$$\boxed{V = \frac{KP}{r^2}}$$

## POTENTIAL DUE TO AN ELECTRIC DIPOLE ON THE EQUITORIAL LINE:-




Due to $+q$ charge, potential

$$V_1 = \frac{Kq}{\sqrt{r^2+l^2}}$$

Due to $-q$ charge, potential

$$V_2 = \frac{K(-q)}{\sqrt{r^2+l^2}}$$

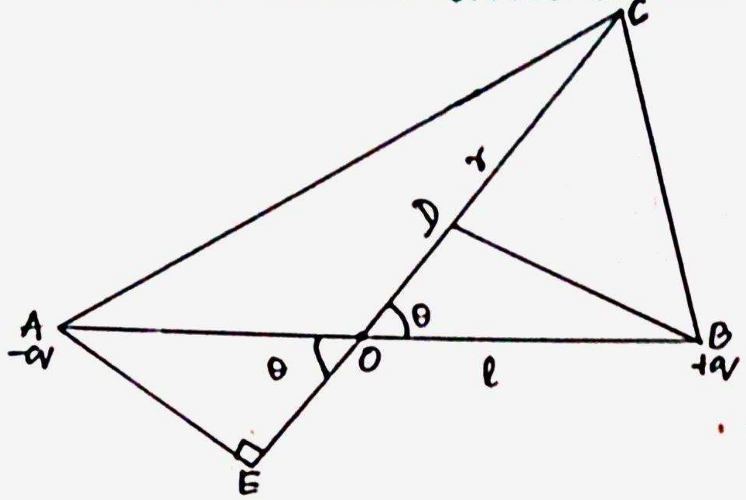
Net potential,

$$V = V_1 + V_2$$

$$= \frac{Kq}{\sqrt{r^2+l^2}} - \frac{Kq}{\sqrt{r^2+l^2}}$$

$$\boxed{V = 0}$$

# POTENTIAL DUE TO AN ELECTRIC DIPOLE AT ANY POINT:-




C is any point at a dist r from the centre of the dipole making an angle θ.

Due to +q,

$$V_1 = \frac{Kq}{BC}$$

$$= \frac{Kq}{CD} \quad (\because BC \sim CD)$$

$$= \frac{Kq}{OC - OD}$$

$$V_1 = \frac{Kq}{r - l\cos\theta}$$

Due to -q,

$$V_2 = \frac{K(-q)}{AC}$$

$$= \frac{K(-q)}{CE} \quad (\because AC \sim CE)$$

$$= \frac{K(-q)}{OC + OE}$$

$$V_2 = \frac{-Kq}{r + l\cos\theta}$$

Total potential,

$$V = V_1 + V_2$$

$$= \frac{Kq}{r-l\cos\theta} - \frac{Kq}{r+l\cos\theta}$$

$$= Kq\left[\frac{1}{r-l\cos\theta} - \frac{1}{r+l\cos\theta}\right]$$

$$= Kq\left[\frac{r+l\cos\theta - r + l\cos\theta}{(r-l\cos\theta)(r+l\cos\theta)}\right]$$

$$= \frac{Kq\,2l\cos\theta}{r^2-l^2\cos^2\theta} = \frac{KP\cos\theta}{r^2-l^2\cos^2\theta}$$

$$\boxed{V = \frac{KP\cos\theta}{r^2-l^2\cos^2\theta}}$$

for ideal dipole, $l <<< r$, $l^2$ can be neglected.

$$\boxed{V = \frac{KP\cos\theta}{r^2}}$$

## POTENTIAL DUE TO SYSTEM OF CHARGES:-

**POTENTIAL ENERGY:-** It is defined as the amount of work done to bring the charges from $\infty$ to their respective positions to constitute the systems.

(1) 2 particles system:-

$$q_1 \overset{r_{12}}{\text{———}} q_2$$

To bring $q_1$, work done $W_1 = 0$

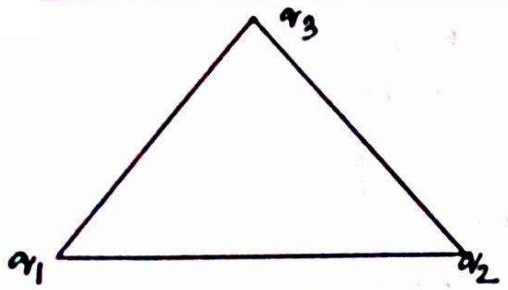
To bring $q_2$, work done $W_2 = \frac{Kq_1q_2}{r_{12}}$

Potential Energy,

$U = W = W_1 + W_2$

$$\rightarrow \boxed{U = \frac{Kq_1q_2}{r_{12}}}$$

(2) 3 particles system:-




to bring $q_1$, work done $W_1 = 0$

to bring $q_2$, work done $W_2 = \frac{Kq_1q_2}{r_{12}}$

to bring $q_3$, work done $W_3 = \frac{Kq_2q_3}{r_{23}} + \frac{Kq_1q_2}{r_{12}}$

Total potential energy,

$$U = \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_1q_2}{r_{12}}$$

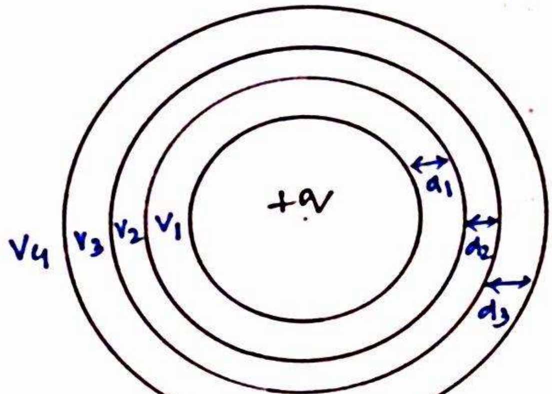
$$\boxed{U = K\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_2}{r_{12}}\right)}$$

(3) 4 particle system

$$\boxed{U = \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_1q_4}{r_{14}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_2q_4}{r_{24}} + \frac{Kq_3q_4}{r_{34}}}$$

## EQUIPOTENTIAL SURFACES:

Any surface which has same electrostatic potential at every point on it is called an equipotential surfaces.




$d_1 < d_2 < d_3$

$V_1 > V_2 > V_3 > V_4$

## PROPERTIES:-

1. Along the electric field, potential decreases
2. Work done to move a charge on equipotential surface

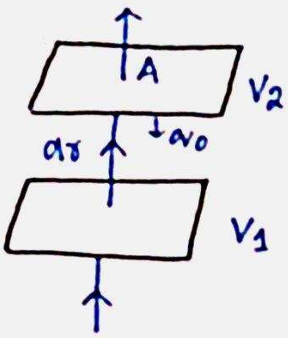
$$W = q\Delta V = q \cdot 0 = 0$$

3. Two equipotential surface never cross each other.

   **Reason**:- If they cross at every point on the line of intersection two potential appear which is not possible.

4. Electric field is always $\perp^r$ to the equipotential surface.

## RELATION BETWEEN ELECTRIC FIELD INTENSITY AND POTENTIAL:-




Work done to move $q_0$ from A to B.

$$dw = \vec{F} \cdot \vec{dr}$$
$$= q_0 \vec{E} \cdot \vec{dr}$$
$$= q_0 E dr \cos 180°$$
$$= -q_0 E dr$$

Again. $dw = q_0 dv$

Now, $-q_0 E dr = q_0 dv$

$\Rightarrow -E dr = dv$

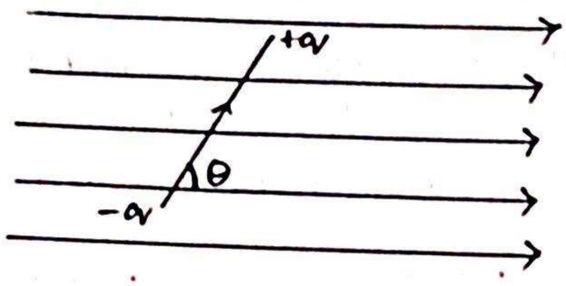
$\Rightarrow \boxed{E = -\frac{dv}{dr}}$

Electric field intensity is negative of potential gradient.

In compenent form,

$$E_x = \frac{-\partial v}{\partial x} \quad , \quad E_y = \frac{-\partial v}{\partial y} \quad , \quad E_z = \frac{-\partial v}{\partial z}$$

## POTENTIAL ENERGY OF DIPOLE IN UNIFORM ELECTRIC FIELD:-




θ is the angle between dipole moment and electric field intensity.
Work done to rotate the dipole from $\theta_1$ angle to $\theta_2$ angle.

$$W = \int_{\theta_1}^{\theta_2} \tau \cdot d\theta$$

$$= \int_{\theta_1}^{\theta_2} PE\sin\theta \cdot d\theta$$

$$= PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= PE \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= PE \left[-\cos\theta_2 + \cos\theta_1\right]$$

$$\boxed{W = PE(\cos\theta_1 - \cos\theta_2)}$$

When $\theta_1 = 90^\circ, \theta_2 = \theta$

$$W = PE(\cos 90^\circ - \cos\theta) = -PE\cos\theta$$

$$\therefore \boxed{U = -PE\cos\theta}$$

**CASE-1**
When $\theta = 0$
$\boxed{U = -PE}$
(minimum)
It is in stable equilibrium.

**CASE-2**
When $\theta = 180^\circ$
$\boxed{U = PE}$
(maximum)
It is in unstable equilibrium.

**CASE-3**
When $\theta = 90^\circ$
$\boxed{U = 0}$

## POTENTIAL ENERGY OF 2 PARTICLE SYSTEM : IN EXTERNAL ELECTRIC FIELD :-


Work done to bring $q_1$,

$$W = q_1(V_1 - V_\infty)$$

$$W = q_1 V_1$$

Work done to bring $q_2$,

$$W = q_2 V_2 + \frac{K q_1 q_2}{r_{12}}$$

Potential Energy,

$$\boxed{U = q_1 V_1 + q_2 V_2 + \frac{K q_1 q_2}{r_{12}}}$$

## BEHAVIOUR OF CONDUCTOR IN ELECTROSTATICS :-

① Net electrostatic field is zero in the interior of a conductor.

② Just outside the surface of a charged conductor, electric field is normal to the surface.

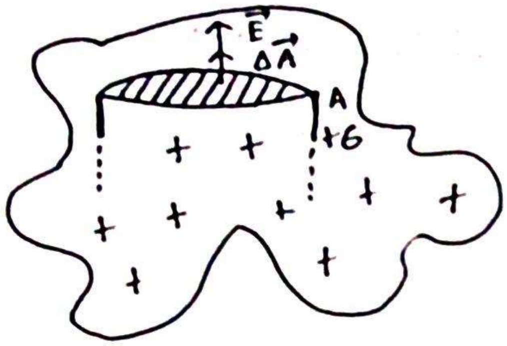
③ The net charge in the interior of the conductor is 0.

④ Potential is constant within and on the surface of the conductor :

$$E = \frac{-dV}{dr} \Rightarrow \frac{-dV}{dr} = 0 \Rightarrow V = \text{constant}$$

(Q) Prove that electric field at the surface of the charged conductor is directly proportional to the surface charge density.

In order to calculate electric field inside the conductor, let us assume pill box shaped gaussian surface.




According to gauss law,

$$\oint \vec{E} \cdot \vec{dA}$$
$$= \oint E dA \cos 0$$
$$= EA$$

According to gauss's law,

$$\phi = \frac{q_{en}}{\epsilon_0} = EA = \frac{6A}{\epsilon_0}$$

$$\Rightarrow E = \frac{6}{\epsilon_0}$$

$$\Rightarrow E \propto 6$$

$$\boxed{\vec{E} = \frac{6}{\epsilon_0}\hat{n}}$$

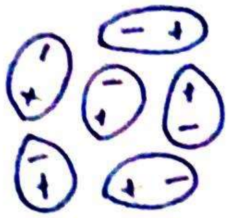
**ELECTROSTATICS SHIELDING:-** The phenomenon of making a region free from any $e^-$ field is called electrostatics shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

**DI-ELECTRICS AND POLARISATION:-**

| POLAR-DIELECTRIC | NON-POLAR DI-ELECTRIC |
|---|---|
| ① The di-electric in which centre of +ve charge doesn't coincide with the centre of −ve charge is called polar-di-electric. | The dielectric in which centre of +ve charge exactly coincide with the centre of −ve charge is called non-polar dielectric |
| For eg :- $H_2O$, $HCl$, $CH_3OH$, $CH_3COOH$ etc. | For eg:- $CO_2$, $N_2$, $O_2$, $H_2$ etc. |
| ② unsymmetrical shape. | symmetrical shape |
| ③ It possess a permanent dipole moment of the order $10^{-30}$ cm. | There is no permanent dipole moment. |

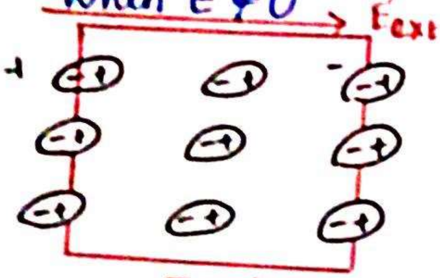
## POLARISATION IN EXTERNAL FIELD:-

When $\vec{E}=0$



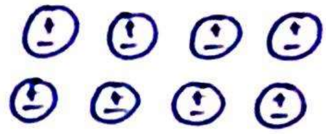
$\Sigma P=0$ because of random orientation of the individual atom.

When $\vec{E}\neq 0$



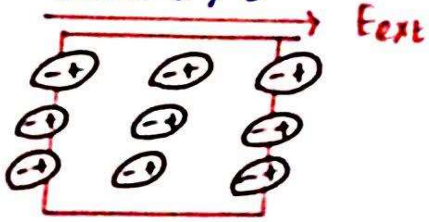

$\Sigma P\neq 0$

$\vec{E}=0$



$\Sigma P=0$ as the centre coincide.

When $\vec{E}\neq 0$




$\Sigma P\neq 0$

## CAPACITANCE:-

The device that can store charge or energy is called capacitor. The capacity of a capacitor is called capacitance.

Representation:- 

$$Q\propto V$$

$$\Rightarrow Q=CV$$

$$\Rightarrow \boxed{C=\frac{Q}{V}}\ ,\quad C \text{ is capacitance.}$$

It is defined as the ratio between amount of charge stored in the plates to the potential maintained across its plate.

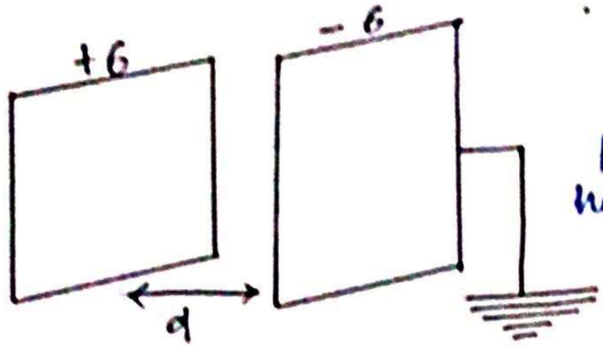
SI unit $\longrightarrow$ F

CGS unit $\longrightarrow$ st F, abF

Dimension $\rightarrow$ $[M^{-1}L^{-2}T^{4}A^{2}]$

# PARALLEL PLATE CAPACITOR :-






It consists of 2 parallel metal plates separated by some distance having some insulating medium between them. 1 plate is given +ve charge and other plate is connected to earth.

A = common area
d = separation betⁿ 2 plates
σ = surface charge density = q/A

$\vec{E}$ = electric field betⁿ 2 plates

$\vec{E} = \vec{E_1} + \vec{E_2}$

$= \frac{\sigma}{2\epsilon_0}\hat{i} + \frac{\sigma}{2\epsilon_0}\hat{i}$

$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{i}$

We know, $E = \frac{-dv}{dr}$ $\Rightarrow -dv = \vec{E}\cdot\vec{dr}$

$\Rightarrow -dv = E\,dr\cos 180^\circ$

$\Rightarrow dv = E\,dr$

$\Rightarrow \int_0^v dv = \int_0^d E\,dr$

$\Rightarrow V = E[r]_0^d$

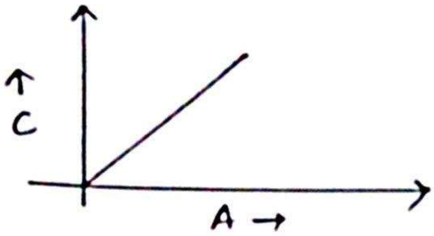
$\Rightarrow V = Ed$

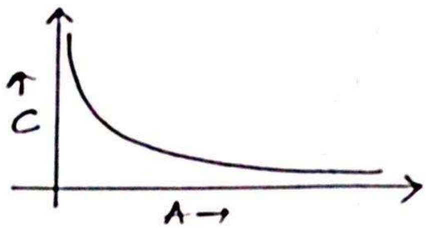
$= \frac{\sigma}{\epsilon_0}d$

$\Rightarrow \boxed{V = \frac{\sigma}{\epsilon_0}d = \frac{q}{A\epsilon_0}d}$

$\boxed{C = \frac{q}{V} = \frac{A\epsilon_0}{d}}$

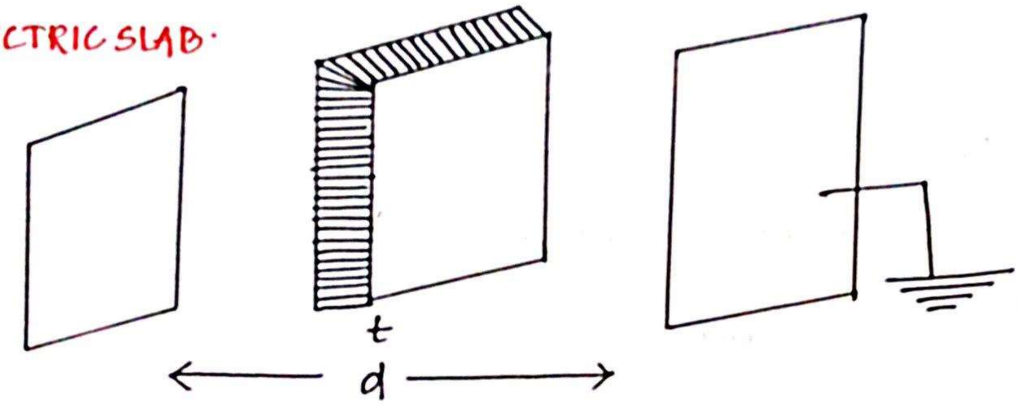
$C \propto A \quad , \quad C \propto \frac{1}{d}$




## CASE-1

DI-ELECTRIC SLAB·




t = thickness of di-electric slab

electric field,

$$E_{air} = \frac{\sigma}{\epsilon_0} \quad , \quad E_{die} = \frac{\sigma}{\epsilon_0 K}$$

$$V = E \cdot d$$

$$= E_{air}(d-t) + E_{die}\, t$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{\epsilon_0 K} t$$

$$= \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$$= \frac{q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$$C = \frac{q}{V} = \frac{q}{\frac{q\left(d-t+\frac{t}{K}\right)}{A\epsilon_0}} = \boxed{\frac{A\epsilon_0}{d - t + \frac{t}{K}}}$$

If the whole space is di-electric.

$$t = d$$

$$C = \frac{A\epsilon_0}{d/K}$$

$$C_{dil} = \frac{K\epsilon_0 A}{d}$$

$$\boxed{C_{dil} = K C_{air}}$$

## CASE-II :-

## CONDUCTING SLAB :-

$t$ = thickness of conducting slab

$$E_{air} = \frac{\sigma}{\epsilon_0} \quad , \quad E_{cond} = 0$$

$$V = E \cdot d$$

$$= E_{air}(d-t) + E_{cond} \cdot t$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + 0$$

$$V = \frac{\sigma}{\epsilon_0}(d-t)$$

$$V = \frac{q}{A\epsilon_0}(d-t)$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{A\epsilon_0}(d-t)} = \boxed{\frac{\epsilon_0 A}{d-t}}$$

If the whole space is conductor,

$$d = t$$

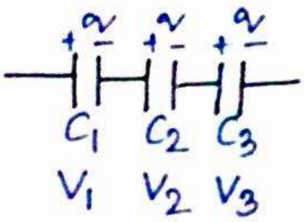
$$C = \frac{\epsilon_0 A}{t-t}$$

$$\boxed{C = \infty}$$

# COMBINATION OF CAPACITORS:-

## (a) SERIES

In this connection, negative part of one capacitor is connected to positive plate of other




In this connection, charge remains same but potential gets divided.

$$V = V_1 + V_2 + V_3$$

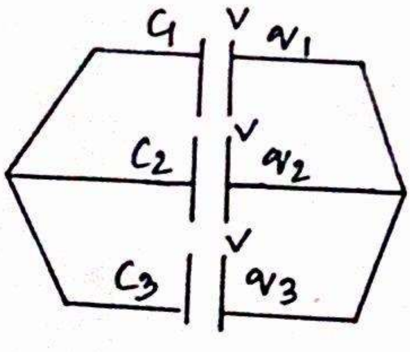
$$\Rightarrow \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

## (b) PARALLEL

In this connection, +ve terminals of all capacitors are connected at one point and -ve terminal are connected at one point

In this connection, potential diff remains constant but charge divides.




$$q = q_1 + q_2 + q_3$$

$$\Rightarrow CV = C_1V + C_2V + C_3V$$

$$\Rightarrow \boxed{C = C_1 + C_2 + C_3}$$

# ENERGY STORED IN A CAPACITOR :-

The amount of work done to add charge to a capacitor is stored in the form of electric potential energy in the space between the two plates.

Let $q$ is the charge given to a capacitor and $V$ is the potential difference at any instant.

$$C = \frac{q}{V}$$

Add $dq$ amount of charge, work done

$$dW = dq\,V$$

Integrating both sides,

$$\int_0^W dW = \int_0^Q dq \cdot V$$

$$\Rightarrow W = \int_0^Q \frac{q}{C}\,dq$$

$$\Rightarrow W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2C}\left[Q^2 - 0^2\right]$$

$$\Rightarrow \boxed{W = \frac{Q^2}{2C}}$$

Potential Energy, $\boxed{U = \frac{Q^2}{2C}}$

$$U = \frac{1}{2}(CV)^2/C = \frac{1}{2}\frac{C^2V^2}{C}$$

$$\boxed{U = \frac{1}{2}CV^2}$$

$$U = \frac{1}{2}\frac{Q}{V}\cdot V^2$$

$$\boxed{U = \frac{1}{2}QV}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$

$$= \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2}{Ad}$$

$$= \frac{\epsilon_0 A}{2d} \times E^2 d^2 \times \frac{1}{Ad}$$

$$= \frac{E^2 \epsilon_0}{2}$$

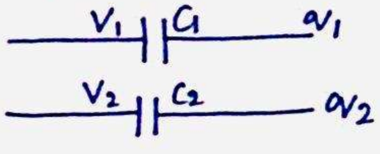
$$u = \frac{1}{2}\epsilon_0 E^2$$

## ENERGY STORED IN COMBINATION :-

$$\boxed{U = U_1 + U_2 + U_3 + \cdots}$$

## COMMON POTENTIAL :-

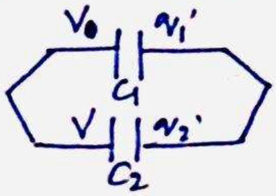
Before connection




$q_1 = C_1 V_1$

$q_2 = C_2 V_2$

After connection




$q_1' = C_1 V$

$q_2' = C_2 V$

According to conservation of charge.

$$q_1 + q_2 = q_1' + q_2'$$

$$\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow \boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

## ENERGY LOSS:-

Before combination,

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

After combination,

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}(C_1 + C_2)V^2 \quad = \frac{1}{2}(C_1 + C_2)\left\{\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right\}^2.$$

$$= \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$$

$$\Delta u = U_i - U_f$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left\{\frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}\right\}$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2 > 0$$

So, $U_i > U_f$.

So energy is lost. The lost energy appears in the form of heat in connecting wire