AIM: FIND THE MINIMUM SPANNING TREE(MST) USING
I) KRUSKAL'S ALGORITHM
II) PRIM'S ALGORITHM

THEORY:

A minimum spanning tree (MST) or minimum weight spanning tree is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight. That is, it is a spanning tree whose sum of edge weights is as small as possible. More generally, any edge-weighted undirected graph (not necessarily connected) has a minimum spanning forest, which is a union of the minimum spanning trees for its connected components.

There are many use cases for minimum spanning trees. One example is a telecommunications company trying to lay cable in a new neighborhood. If it is constrained to bury the cable only along certain paths (e.g. roads), then there would be a graph containing the points (e.g. houses) connected by those paths. Some of the paths might be more expensive, because they are longer, or require the cable to be buried deeper; these paths would be represented by edges with larger weights. Currency is an acceptable unit for edge weight – there is no requirement for edge lengths to obey normal rules of geometry such as the triangle inequality. A spanning tree for that graph would be a subset of those paths that has no cycles but still connects every house; there might be several spanning trees possible. A minimum spanning tree would be one with the lowest total cost, representing the least expensive path for laying the cable.

A) Kruskal's Algorithm

Kruskal's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which

- form a tree that includes every vertex
- has the minimum sum of weights among all the trees that can be formed from the graph

How Kruskal's algorithm works

It falls under a class of algorithms called greedy algorithms that find the local optimum in the hopes of finding a global optimum.

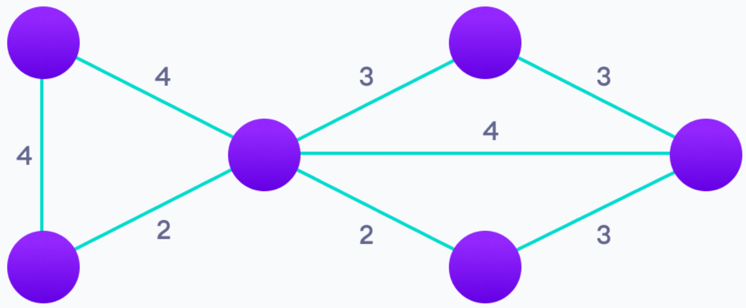
We start from the edges with the lowest weight and keep adding edges until we reach our goal.

The steps for implementing Kruskal's algorithm are as follows:

1. Sort all the edges from low weight to high

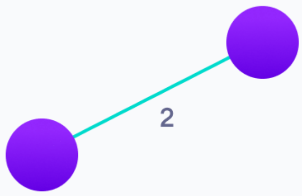
2. Take the edge with the lowest weight and add it to the spanning tree. If adding the edge created a cycle, then reject this edge.
3. Keep adding edges until we reach all vertices.

Example of Kruskal's algorithm



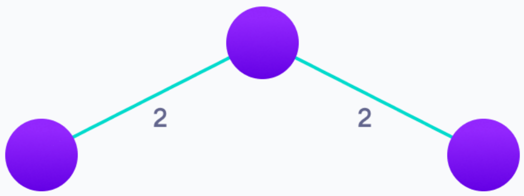
Step: 1

Start with a weighted graph



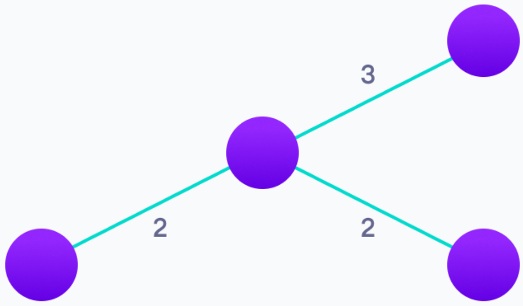
Step: 2

Choose the edge with the least weight, if there are more than 1, choose anyone



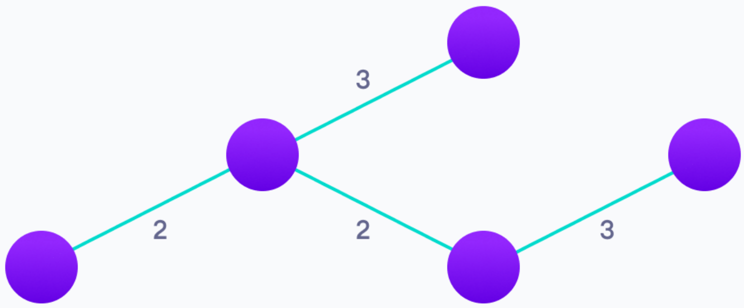
Step: 3

Choose the next shortest edge and add it



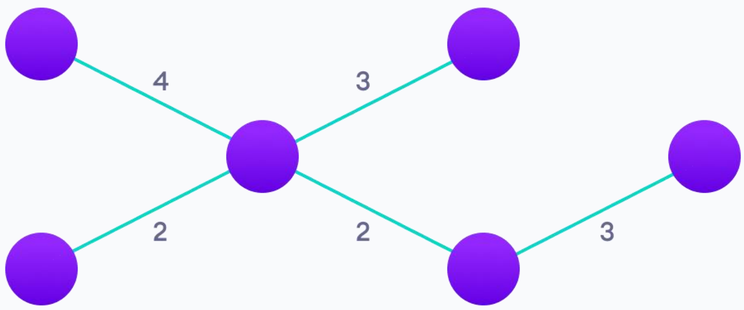
Step: 4

Choose the next shortest edge that doesn't create a cycle and add it



Step: 5

Choose the next shortest edge that doesn't create a cycle and add it



Step: 6

Repeat until you have a spanning tree

Kruskal Algorithm Pseudocode

Any minimum spanning tree algorithm revolves around checking if adding an edge creates a loop or not.

The most common way to find this out is an algorithm called Union FInd. The Union-Find algorithm divides the vertices into clusters and allows us to check if two vertices belong to the same cluster or not and hence decide whether adding an edge creates a cycle.

```
KRUSKAL(G):
A = ∅
For each vertex v ∈ G.V:
   MAKE-SET(v)
For each edge (u, v) ∈ G.E ordered by increasing order by weight(u, v):
   if FIND-SET(u) ≠ FIND-SET(v):
   A = A ∪ {(u, v)}
   UNION(u, v)
return A
```

Kruskal's vs Prim's Algorithm

Prim's algorithm is another popular minimum spanning tree algorithm that uses a different logic to find the MST of a graph. Instead of starting from an edge, Prim's algorithm starts from a vertex and keeps adding lowest-weight edges which aren't in the tree, until all vertices have been covered.

Kruskal's Algorithm Complexity

The time complexity Of Kruskal's Algorithm is: O(E log E).

Kruskal's Algorithm Applications

- In order to layout electrical wiring
- In computer network (LAN connection)

B) Prim's Algorithm

In this tutorial, you will learn how Prim's Algorithm works. Also, you will find working examples of Prim's Algorithm in C, C++, Java and Python.

Prim's algorithm is a minimum spanning tree algorithm that takes a graph as input and finds the subset of the edges of that graph which

- form a tree that includes every vertex
- has the minimum sum of weights among all the trees that can be formed from the graph

How Prim's algorithm works

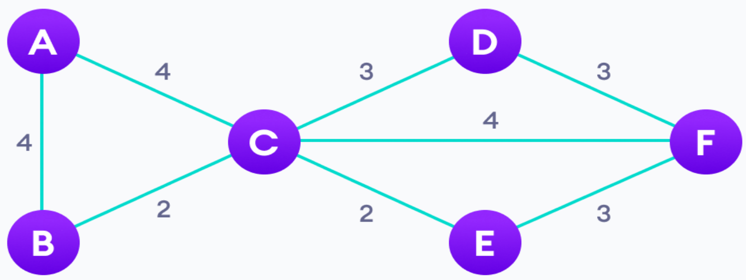
It falls under a class of algorithms called greedy algorithms that find the local optimum in the hopes of finding a global optimum.

We start from one vertex and keep adding edges with the lowest weight until we reach our goal.

The steps for implementing Prim's algorithm are as follows:

1. Initialize the minimum spanning tree with a vertex chosen at random.
2. Find all the edges that connect the tree to new vertices, find the minimum and add it to the tree
3. Keep repeating step 2 until we get a minimum spanning tree

Example of Prim's algorithm

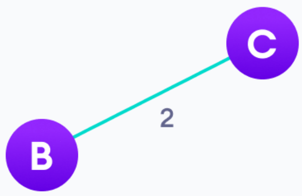


Step: 1

Start with a weighted graph

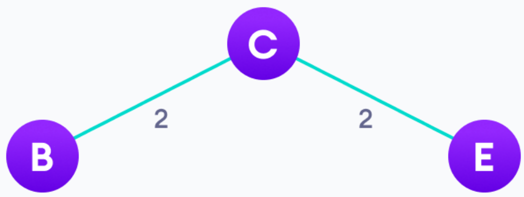
Step: 2

Choose a vertex



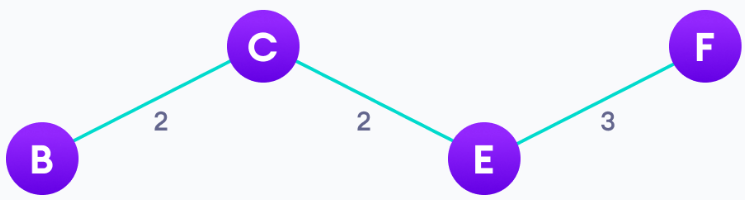
Step: 3

Choose the shortest edge from this vertex and add it



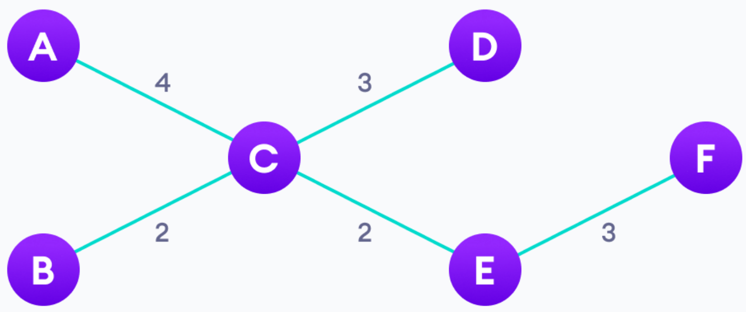
Step: 4

Choose the nearest vertex not yet in the solution



Step: 5

Choose the nearest edge not yet in the solution, if there are multiple choices, choose one at random



Step: 6

Repeat until you have a spanning tree

Prim's Algorithm pseudocode

The pseudocode for prim's algorithm shows how we create two sets of vertices U and V-U. U contains the list of vertices that have been visited and V-U the list of vertices that haven't. One by one, we move vertices from set V-U to set U by connecting the least weight edge.

```
T = ∅;
U = { 1 };
while (U ≠ V)
   let (u, v) be the lowest cost edge such that u ∈ U and v ∈ V - U;
   T = T ∪ {(u, v)}
   U = U ∪ {v}
```

Prim's vs Kruskal's Algorithm

Kruskal's algorithm is another popular minimum spanning tree algorithm that uses a different logic to find the MST of a graph. Instead of starting from a vertex, Kruskal's algorithm sorts all the edges from low weight to high and keeps adding the lowest edges, ignoring those edges that create a cycle.

Prim's Algorithm Complexity

The time complexity of Prim's algorithm is $O(E \log V)$.

Prim's Algorithm Application

- Laying cables of electrical wiring
- In network designed
- To make protocols in network cycles

A) KRUSKAL'S ALGORITHM

SOURCE CODE:

```
#include<iostream>
#include <algorithm>
#include<vector>
using namespace std;

class edge
{

public:
   int s;
   int d;
   int w;

   edge()
   {

   }
   edge(int src,int des,int wei)
   {
      s=src;
      d=des;
      w=wei;
   }
};


bool compare(edge e1,edge e2)
{
   return e1.w<e2.w;
}


int findparent(int i,int* parent )
{
   if(parent[i]==i)
   {
    return i;
   }
   return findparent(parent[i],parent);
}


class graph
{

public:
```

```
int e,n;
edge* v;

graph(int n,int e)
{

    this->n=n;
    this->e=e;
    v=new edge[e];
    for(int i=0; i<e; i++)
    {
        int x,y,w;
        cout<<"ENTER VERTICES AND WEIGHT OF EDGE "<<i+1<<" :          ";
        cin>>x>>y>>w;
        edge e(x,y,w);
        v[i]=e;
    }
}

edge* unionfind()
{
    int* parent=new int[n];
    for(int i=0; i<n; i++)
    {
        parent[i]=i;

    }

    sort(v,v+e,compare);

    edge* output;
    output=new edge[n-1];
    int count=0,i=0;
    while(count!=n-1)
    {
        edge c=v[i];
        int sourceparent=findparent(v[i].s,parent);
        int desparent=findparent(v[i].d,parent);

        if(sourceparent!=desparent)
        {
            output[count]=c;
            parent[sourceparent]=desparent;
            count++;
        }
        i++;

    }

    int sum=0;
    cout<<endl<<"-------MST-------\n";
```

```
        for(int i=0; i<n-1; i++)
        {
            cout<<output[i].s<<"        "<<output[i].d<<"        "<<output[i].w<<endl;
            sum+=output[i].w;
        }
        cout<<"\nWEIGHT OF MST IS "<<sum;
        return output;
    }
};


int main()
{
    int n,e;
    cout<<"KRUSKAL'S ALGORITHM\nENTER NUMBER OF VERTICES :        ";
    cin>>n;
    cout<<"ENTER NUMBER OF EDGEES :  ";
    cin>>e;
    graph g(n,e);
    edge* mst=g.unionfind();
}
```

OUTPUT:

"C:\Users\NARENDER KESWANI\Documents\FYMCA\DSA\kruskal.exe"

```
KRUSKAL'S ALGORITHM
ENTER NUMBER OF VERTICES :       8
ENTER NUMBER OF EDGEES :         12
ENTER VERTICES AND WEIGHT OF EDGE 1 :    0 1 2
ENTER VERTICES AND WEIGHT OF EDGE 2 :    1 5 5
ENTER VERTICES AND WEIGHT OF EDGE 3 :    1 7 6
ENTER VERTICES AND WEIGHT OF EDGE 4 :    2 7 6
ENTER VERTICES AND WEIGHT OF EDGE 5 :    7 5 6
ENTER VERTICES AND WEIGHT OF EDGE 6 :    1 3 5
ENTER VERTICES AND WEIGHT OF EDGE 7 :    4 5 3
ENTER VERTICES AND WEIGHT OF EDGE 8 :    3 4 10
ENTER VERTICES AND WEIGHT OF EDGE 9 :    4 5 9
ENTER VERTICES AND WEIGHT OF EDGE 10 :   1 6 5
ENTER VERTICES AND WEIGHT OF EDGE 11 :   2 7 6
ENTER VERTICES AND WEIGHT OF EDGE 12 :   1 3 7

-------MST-------
0       1       2
4       5       3
1       5       5
1       3       5
1       6       5
1       7       6
2       7       6

WEIGHT OF MST IS 32
Process returned 0 (0x0)   execution time : 89.553 s
Press any key to continue.
```

B) PRIM'S ALGORITHM:

SOURCE CODE:

```
#include <bits/stdc++.h>
using namespace std;

#define V 5

int minKey(int key[], bool mstSet[])
{
   int min = INT_MAX, min_index;

   for (int v = 0; v < V; v++)
   {

      if (mstSet[v] == false && key[v] < min)
      {
         min = key[v], min_index = v;
      }
   }

   return min_index;
}

void printMST(int parent[], int graph[V][V])
{
   int a = 0;
   cout<<"Edge \tWeight\n";
   for (int i = 1; i < V; i++)
   {
      cout<<parent[i]<<" - "<<i<<" \t"<<graph[i][parent[i]]<<" \n";
      a = a +  graph[i][parent[i]];
   }
   cout<<"The weight of graph is: "<<a<<endl;
}

void primMST(int graph[V][V])
{
   int parent[V];

   int key[V];

   bool mstSet[V];

   for (int i = 0; i < V; i++)
   {

      key[i] = INT_MAX, mstSet[i] = false;
   }
```

```
    key[0] = 0;
    parent[0] = -1;
    for (int count = 0; count < V - 1; count++)
    {
        int u = minKey(key, mstSet);

        mstSet[u] = true;


        for (int v = 0; v < V; v++)
        {


            if (graph[u][v] && mstSet[v] == false && graph[u][v] < key[v])
            {
                parent[v] = u, key[v] = graph[u][v];
            }
        }
    }

    printMST(parent, graph);
}

int main()
{
    /* Let us create the following graph
            2 3
    (0)--(1)--(2)
    | / \ |
    6| 8/ \5 |7
    | / \ |
    (3)-------(4)
                    9           */
    int graph[V][V] =
    { { 0, 2, 0, 6, 0 },
        { 2, 0, 3, 8, 5 },
        { 0, 3, 0, 0, 7 },
        { 6, 8, 0, 0, 9 },
        { 0, 5, 7, 9, 0 }
    };

    primMST(graph);

    return 0;
}
```

OUTPUT:

"C:\Users\NARENDER KESWANI\Documents\FYMCA\DSA\prims.exe"

```
Edge    Weight
0 - 1   2
1 - 2   3
0 - 3   6
1 - 4   5
The weight of graph is: 16

Process returned 0 (0x0)   execution time : 0.053 s
Press any key to continue.
```

CONCLUSION:

From this practical, I have learned how to implement minimum spanning tree using Kruskal & prims algorithms.