Vivekanand Education Society's Institute of Technology

NAME: NARENDER KESWANI

ROLL NO: 24

DIVISION: B

DEPARTMENT: MASTER OF COMPUTER APPLICATION (M.C.A)

SUBJECT: MCA11 - Mathematical Foundation for Computer Science 1 (MFCS)

EXAM: CONTINOUS ASSESTMENT (CA)

PROFESSOR: RUCHI RAUTELA (RR)

DATE: 04/03/2022

Vivekanand Education Society's Institute of Technology, Chembur, Mumbai Department Of MCA Year:2021-22 (Odd Sem) Assignment 1

Class : FIRST YEAR MCA	Division: A & B
Semester: I	Subject: Mathematical Foundation for Computer Science 1
Assignment: 1	Topic: Statistics & Probability
Each question carries 5 marks.	

Q1	Calculate Karl Pearson's coefficient of skewness for the following data: *Class interval* / *Frequency* / *Class interval* / *Frequency* 130 – 134 / 3 / 150 – 154 / 19 135 – 139 / 12 / 155 – 159 / 12 140 – 144 / 21 / 160 – 164 / 5 145 – 149 / 28	CO1
Q2	A sample of 12 fathers and their eldest sons gave the following data about their height in inches : Father: 65 63 67 64 68 62 70 66 68 67 69 71 Son : 68 66 68 65 69 66 68 65 71 67 68 70 Calculate coefficient of rank correlation.	CO2
Q3	The probability that an electric component will fail in less than 1200 hrs of continuous use is 0.24. Using normal distribution, find probability that among 200 such components fewer than 45 will fail in less than 1200 hrs of continuous use.	CO4
Q4	The level of calcium in the blood of healthy, young adults varies with a mean of 9.5 mg per deciliter and a SD of 0.4. A clinic measures the blood calcium level of 180 healthy women and finds x = 9.57mg. Is this an indication that the mean calcium level in this population differs from 9.5mg?($Z\alpha$ = 1.96)	CO5

NARENDER KESWANI

Q.1. Calculate Karl Pearson's coefficient of skewness for the following data

Class Interval	Frequency
130 - 134	3
135 - 139	12
140 - 144	21
145 - 149	28
150 - 154	19
154 - 159	12
160 - 164	5

Sol.

Class Interval	Frequency (f_i)	Mid (x_i)	f_ix_i	$f_ix_i^2$	c.f.
129.5 - 134.5	3	132	396	52272	3
134.5 - 139.5	12	137	1644	225228	15
139.5 - 144.5	21	142	2982	423444	36
144.5 - 149.5	28	147	4116	605052	64
149.5 - 154.5	19	152	2888	438976	83
154.5 - 159.5	12	157	1884	295788	95
159.5 - 164.5	5	162	810	131220	100
	Σf_i		Σf_ix_i	$\Sigma f_ix_i^2$	
	=100		=14720	=2171980	

Formula Used:

Karl Pearson's coefficient of skewness = $\frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$

For Mean,

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{Mean} = \frac{14720}{100}$$

$$\boxed{\text{Mean} = 147.20}$$

For Mode,

where $l = 144.5$ [Lower Limit]
f_1 = frequency of mode class = 28
f_0 = frequency of preceding class = 21
f_2 = frequency of succedding class = 19
h = class length of mode class = 5

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 144.5 + \left(\frac{28 - 21}{2 \times 28 - 21 - 19}\right) \times 5$$

$$\text{Mode} = 144.5 + \left(\frac{7}{16}\right) \times 5$$

$$\text{Mode} = 144.5 + 2.1875$$

$$\boxed{\text{Mode} = 146.6875}$$

For Standard Deviation (S.D)

$$\sigma = \sqrt{\frac{\Sigma fx^2}{n} - \frac{(\Sigma fx)^2}{n}}$$

$$\sigma = \sqrt{\frac{2171980}{100} - \frac{(14720)^2}{100}}$$

$$\sigma = \sqrt{\frac{2171980 - 2166784}{100}}$$

$$\sigma = \sqrt{\frac{5196}{100}}$$

$$\sigma = \sqrt{51.96}$$

$$\boxed{\sigma = 7.2083}$$

Karl Pearson's coefficient of skewness

$$= \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}$$

$$= \frac{147.20 - 146.6875}{7.2083}$$

$$= \underline{0.0710}$$

∴ Hence, the Karl Pearson's coefficient of skewness is 0.0710.

Q.2. A sample of 12 fathers and their eldest sons gave the following data about their height in inches.

Father : 65, 63, 67, 64, 68, 62, 70, 66, 68, 67, 69, 71
Son : 68, 66, 68, 65, 69, 66, 68, 65, 71, 67, 68, 70

Calculate coefficient of rank correlation.

Sol.

Father (x)	Son (y)	Rank (Rf) of Father	Rank (Rs) of Son	d_i = x − y	d_i^2
65	68	9	5.5	3.5	12.25
63	66	11	9.5	1.5	2.25
67	68	6.5	5.5	1	1
64	65	10	11.5	−1.5	2.25
68	69	4.5	3	1.5	2.25
62	66	12	9.5	2.5	6.25
70	68	2	5.5	−3.5	12.25
66	65	8	11.5	3.5	12.25
68	71	4.5	1	−3.5	12.25
67	67	6.5	8	−1.5	2.25
69	68	3	5.5	−2.5	6.25
71	70	1	2	−1	1
					Σd_i = 72.5

In x, 68 is repeated 2 times corresponding to ranks 4, 5. Therefore, (4+5)/2 = 4.5 is assigned for rank 4, 5 with $m_1 = 2$.

In x, 67 is repeated 2 times corresponding to ranks 6, 7. Therefore, $\frac{6+7}{2} = 6.5$ is assigned for rank 6, 7 with $m_2 = 2$.

In y, 68 is repeated 4 times corresponding to ranks 4, 5, 6, 7. Therefore, $(4+5+6+7)/4 = 5.5$ is assigned for rank 4, 5, 6, 7 with $m_3 = 4$.

In y, 66 is repeated 2 times corresponding to ranks 9, 10. Therefore $(9+10)/2 = 9.5$ is assigned for rank 9, 10 with $m_4 = 2$.

In y, 65 is repeated 2 times corresponding to ranks 11, 12. Therefore, $(11+12)/2 = 11.5$ is assigned for rank 11, 12 with $m_5 = 2$.

Formula Used:

$$\rho = 1 - \frac{6 \times \left(\Sigma d^2 + \Sigma \frac{m(m^2-1)}{12}\right)}{n(n^2-1)}$$

$$\rho = 1 - 6 \times \frac{\left(72.5 + \frac{2\times(2^2-1)}{12} + \frac{2\times(2^2-1)}{12} + \frac{4\times(4^2-1)}{12} + \frac{2\times(2^2-1)}{12}\right)}{12\times(12^2-1)}$$

$$\rho = 1 - 6 \times \frac{(72.5 + 0.5 + 0.5 + 5 + 0.5 + 0.5)}{12\times(144-1)}$$

$$\rho = 1 - \frac{477}{1716} = 1 - 0.278$$

$$\boxed{\rho = 0.722}$$

∴ The Rank correlation coefficient is 0.722.

Q3. The probability that an electric component will fail in less than 1200 hrs of continous use is 0.24. Using normal distribution, find probability that among 200 such components fewer than 45 will fail in less than 1200 hrs of continous use.

Sol.

$n = 200$

$p = 0.24$

$q = 1 - p = 1 - 0.24 = 0.76$

$\text{Mean} = np$

$\text{Mean} = 200 \times 0.24$

$\boxed{\text{Mean} = 48}$

$\sigma = \sqrt{npq}$

$\sigma = \sqrt{200 \times 0.24 \times 0.76}$

$\boxed{\sigma = 6.03}$

Formula Used:

$$z = \frac{x - np}{\sqrt{npq}}$$

$$z = \frac{x - 48}{6.03}$$

Since, $x = 44.5$

$$z = \frac{44.5 - 48}{6.03}$$

$z = -0.5804$

$\therefore \boxed{z \cong -0.6}$

For $Z = 0.6$, $p = 0.2257$ [From Table]

$P(X \leq 44.5) = P(X \leq 0.6)$

$P(X \leq 44.5) = 0.5 - 0.2257 \longrightarrow (\frac{1}{2} = 0.5)$

$\boxed{P(X \leq 44.5) = 0.2743}$

$\therefore$ Hence, probability that among 200 such components fewer than 45 will fail in less than 1250 hrs of continous use is 0.2743.

Q.4. The level of calcium in the blood of healthy, young adults varies with a mean of 9.5 mg per decilitter and a SD of 0.4. A clinic measures the blood calcium level of 180 healthy women and find $\bar{x} = 9.57$ mg. Is this an indication that the mean calcium level in this population differs from 9.5 mg? ($Z_\alpha = 1.96$)

Sol. $\bar{x} = 9.57$

$\sigma = 0.4$

$n = 180$

Step 1: $H_0 : \mu = 9.5$

Step 2: $H_1 : \mu \neq 9.5$

Step 3: $$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{9.57 - 9.5}{\frac{0.4}{\sqrt{180}}}$$

$$z = \frac{0.07}{0.03}$$

$$\boxed{z = 2.33}$$

Step 4: With alpha = 0.05 & since this is two tailed test (i.e. the $\neq$ & =), the critical region will consist of Z score beyond $\pm$ 1.96 (in proportion of tail, 0.0250 was closet to Z = 1.96, so 0.0250 x 2 = 0.05) Therefore, look for (1 - 0.0250 = 0.9750) (in the table)

Step 5: Since, 2.33 lies outside the critical interval $\pm$ 1.96, hence it lies in the rejection region, we reject the null hypothesis at the level of significance 0.05 and conclude that there is enough evidence to show the mean is different from 9.5. Hence, we have shown that the average level of calcium in the blood of healthy women is different from 9.5. (which is of other healthy young adults).