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IPCV H4

1] Transformations :-

→ Given

- f : 1-D discrete signal
- N : length of the signal
- g Transformed signal : $g = A f$ — ①

Transformation matrix (A), Original signal (f)

- Backtransformation : $f = B g$ — ②

Original signal (f), Backtransformation matrix (B), Transformed signal (g)

i] Discrete Fourier Transform [DFT]

→ We know the formula for DFT on a discrete signal as :

$$\boxed{X(K) = \frac{1}{\sqrt{M}} \sum_{m=n}^{M-1} x(n) \cdot e^{-\frac{i2\pi nK}{N}}}$$

Comparing eqn ① with the above eqn to get entry for transformation matrix A

- $g = A f$
- $x(K) = \frac{1}{\sqrt{M}} \sum_{M=n}^{M-1} x(n) \; e^{-\frac{i2\pi nK}{M}}$

$\underbrace{\frac{1}{\sqrt{M}} \; e^{-\frac{i2\pi nK}{M}}}_{A}$

Given:
$N = M = 8$

$$\therefore a_{p,m} = \frac{1}{\sqrt{M}} \cdot e^{-\frac{i2\pi nK}{m}}$$

$$= \boxed{\frac{1}{\sqrt{8}} \cdot e^{-\frac{i\pi pm}{4}}}$$

Repeating the same step by comparing eqn ② with formula of DFT to get

$$b_{m,p} = \frac{1}{\sqrt{m}} \cdot e^{\frac{i\pi p m 2}{m}}$$

$$b_{m,p} = \boxed{\frac{1}{\sqrt{8}} \cdot e^{\frac{i\pi p m}{4}}}$$

- Now we are good to go to find out all the elements of the matrices A and B.
- Before to that, we'd need this formula : $\boxed{\exp(\phi) = \cos(\phi) + i\sin(\phi)}$
- Since the original signal is of length $N = 8$, both the pointers 'p' & 'm' will be in the range of 0 to 7.
- Let us solve for matrix A

fig :

$$\begin{array}{c} \quad m \rightarrow \\ A = \downarrow p \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

- when $p = 0$ & $m = 0$

$$a_{0,0} = \frac{1}{\sqrt{8}} \cdot e^{\frac{-i\pi(0)\times(0)}{4}}$$

$$= \frac{1}{\sqrt{8}} \left(\cos\left(\frac{-i\pi(0)(0)}{4}\right) + i\sin\left(\frac{-i\pi(0)(0)}{4}\right) \right)$$

$$= \frac{1}{\sqrt{8}} \times (1 + 0)$$

$$= \frac{1}{\sqrt{8}}$$

- when $P=0$ & $m=1$

$$a_{0,1} = \frac{1}{\sqrt{8}}\left(\cos\left(\frac{-i\pi(0)(1)}{4}\right) + i\sin\left(\frac{-i\pi(0)(1)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}}(1)$$

$$= \frac{1}{\sqrt{8}}$$

- when $P=0$ & $m=2$

$$a_{0,2} = \frac{1}{\sqrt{8}}\left(\cos\left(\frac{-i\pi(0)(2)}{4}\right) + i\sin\left(\frac{-i\pi(0)(2)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}}(1)$$

$$= \frac{1}{\sqrt{8}}$$

IIIy,

$a_{0,3} = a_{3,0} = 1/\sqrt{8}$

$a_{0,4} = a_{4,0} = 1/\sqrt{8}$

$a_{0,5} = a_{5,0} = 1/\sqrt{8}$

$a_{0,6} = a_{6,0} = 1/\sqrt{8}$

$a_{0,7} = a_{7,0} = 1/\sqrt{8}$

$a_{1,0} = 1/\sqrt{8}$

$a_{2,0} = 1/\sqrt{8}$

- when $P=1$, & $m=1$.

$$a_{1,1} = \frac{1}{\sqrt{8}}\left(\cos\left(\frac{-i\pi(1)(1)}{4}\right) + i\sin\left(\frac{-i\pi(1)(1)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}}\left(\frac{\sqrt{2}}{2} + i\cdot\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \frac{1}{\sqrt{8}}\left(\frac{1}{\sqrt{2}} - i\cdot\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{8}} \cdot \left(\frac{1 - i}{\sqrt{2}}\right)$$

$\rightarrow$ This is a complex conjugate of a future term.

- when $p=1$ & $m=2$

$$a_{1,2} = \frac{1}{\sqrt{8}}\left(\cos\left(\frac{-i\pi(1)(2)}{4}\right) + i\sin\left(\frac{-i\pi(1)(2)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}}\left(0 + i(-1)\right)$$

$$= \frac{1}{\sqrt{8}}(-i)$$

- when $p=1$ & $m=3$

$$a_{1,3} = \frac{1}{\sqrt{8}}\left(\cos\left(\frac{-i\pi(1)(3)}{4}\right) + i\sin\left(\frac{-i\pi(1)(3)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}}\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$$

$$= \frac{1}{\sqrt{8}}\left(\frac{-1}{\sqrt{2}} - i \cdot \frac{1}{\sqrt{2}}\right)$$

(Taking '$-$' common)

$$= \frac{1}{\sqrt{8}} \cdot -\left(\frac{1+i}{\sqrt{2}}\right)$$

* [For better readability, let's rename this as K].

$$= \frac{1}{\sqrt{8}} \cdot -K$$

If we see closely, $a_{1,1}$ is a complex conjugate of $a_{1,3}$!!

IIIly writing for other terms of the matrix directly

i.e

$a_{1,4} = 1/\sqrt{8}\,(-1)$

$a_{1,5} = 1/\sqrt{8}\,\left(-\left(\frac{1-i}{\sqrt{2}}\right)\right)$

$a_{1,6} = 1/\sqrt{8}\,(i)$

$a_{1,7} = 1/\sqrt{8}\,\left(\frac{1+i}{\sqrt{2}}\right)$

$a_{2,0} = 1/\sqrt{8}\,(1)$

$a_{2,1} = 1/\sqrt{8}\,(-i)$

$a_{2,2} = 1/\sqrt{8}\,(-1)$

$a_{2,3} = 1/\sqrt{8}\,(i)$

$a_{2,4} = 1/\sqrt{8}\,(1)$

$a_{2,5} = 1/\sqrt{8}\,(-i)$

$a_{2,6} = 1/\sqrt{8}\,(-1)$

$a_{2,7} = 1/\sqrt{8}\,(i)$

$a_{3,0} = 1/\sqrt{8}\,(1)$

$a_{3,1} = 1/\sqrt{8}\,(-k)$

$a_{3,2} = 1/\sqrt{8}\,(i)$

$a_{3,3} = 1/\sqrt{8}\,(\bar{k})$

$a_{3,4} = 1/\sqrt{8}\,(-1)$

$a_{3,5} = 1/\sqrt{8}\,(k)$

$a_{3,6} = 1/\sqrt{8}\,(-i)$

$a_{3,7} = 1/\sqrt{8}\,(-\bar{k})$

$a_{4,0} = 1/\sqrt{8}\,(1)$

$a_{4,1} = 1/\sqrt{8}\,(-1)$

$a_{4,2} = 1/\sqrt{8}\,(1)$

$a_{4,3} = 1/\sqrt{8}\,(-1)$

$a_{4,4} = 1/\sqrt{8}\,(1)$

$a_{4,5} = 1/\sqrt{8}\,(-1)$

$a_{4,6} = 1/\sqrt{8}\,(1)$

$a_{4,7} = 1/\sqrt{8}\,(-1)$

$a_{5,0} = 1/\sqrt{8}\,(1)$

$a_{5,1} = 1/\sqrt{8}\,(-\bar{k})$

$a_{5,2} = 1/\sqrt{8}\,(-i)$

$a_{5,3} = 1/\sqrt{8}\,(k)$

$a_{5,4} = 1/\sqrt{8}\,(-1)$

$a_{5,5} = 1/\sqrt{8}\,(\bar{k})$

$a_{5,6} = 1/\sqrt{8}\,(i)$

$a_{5,7} = 1/\sqrt{8}\,(-k)$

$a_{6,0} = 1/\sqrt{8}\,(1)$

$a_{6,1} = 1/\sqrt{8}\,(i)$

$a_{6,2} = 1/\sqrt{8}\,(-1)$

$a_{6,3} = 1/\sqrt{8}\,(-i)$

$a_{6,4} = 1/\sqrt{8}\,(1)$

$a_{6,5} = 1/\sqrt{8}\,(i)$

$a_{6,6} = 1/\sqrt{8}\,(-1)$

$a_{6,7} = 1/\sqrt{8}\,(-i)$

$a_{7,0} = 1/\sqrt{8}\,(1)$

$a_{7,1} = 1/\sqrt{8}\,(k)$

$a_{7,2} = 1/\sqrt{8}\,(i)$

$a_{7,3} = 1/\sqrt{8}\,(-\bar{k})$

$a_{7,4} = 1/\sqrt{8}\,(-1)$

$a_{7,5} = 1/\sqrt{8}\,(-k)$

$a_{7,6} = 1/\sqrt{8}\,(-i)$

$a_{7,7} = 1/\sqrt{8}\,(\bar{k})$

If we notice $1/\sqrt{8}$ occurs in all the values, so taking this common
∴ The final transformation matrix A can be written as:

$$A = \frac{1}{\sqrt{8}} \times \begin{array}{c} \\ p \downarrow \end{array} \begin{array}{c|cccccccc} m \rightarrow & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & \bar{K} & -i & -K & -1 & -\bar{K} & \bar{i} & K \\ 2 & 1 & -i & -1 & i & 1 & -i & -1 & i \\ 3 & 1 & -K & i & \bar{K} & -1 & K & -i & -\bar{K} \\ 4 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 5 & 1 & -\bar{K} & -i & K & -1 & \bar{K} & i & -K \\ 6 & 1 & i & -1 & -i & 1 & i & -1 & -i \\ 7 & 1 & K & i & -\bar{K} & -1 & -K & -i & \bar{K} \end{array}$$

* Repeating the same process to find the back transformation matrix B.

- when $m=0$ & $p=0$

$$b_{0,0} = \frac{1}{\sqrt{8}} \cdot e^{\frac{i\pi(0)(0)}{4}}$$

$$= \frac{1}{\sqrt{8}} \left(\cos\left(\frac{i\pi(0)(0)}{4}\right) + i\sin\left(\frac{i\pi(0)(0)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}} (1)$$

- when $m=0$ & $p=1$

$$b_{0,1} = \frac{1}{\sqrt{8}} \left(\cos\left(\frac{i\pi(0)(1)}{4}\right) + i\sin\left(\frac{i\pi(0)(1)}{4}\right)\right)$$

$$= \frac{1}{\sqrt{8}} (1)$$

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* • I hope it's fine to skip all the redundant calculations again & directly print the backtransform matrix B ! *

• Again using the same notations as in $a_{p,m}$

i.e : $\left(k = \frac{1+i}{\sqrt{2}}\right)$ & $\left(\bar{k} = \frac{1-i}{\sqrt{2}}\right)$

$$\therefore B = \frac{1}{\sqrt{8}} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & k & i & -\bar{k} & -1 & -k & -i & \bar{k} \\ 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -\bar{k} & -i & k & -1 & \bar{k} & i & -k \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -k & i & \bar{k} & -1 & k & -i & -\bar{k} \\ 1 & -i & -1 & i & 1 & -i & -1 & i \\ 1 & \bar{k} & -i & -k & -1 & -\bar{k} & i & k \end{bmatrix}$$

ii) Discrete Cosine Transform :-

→ We already know the formula of DCT on a discrete signal (1D) as

$$\boxed{x(k) = \sum_{m=0}^{M-1} f_m \cdot C_p \cdot \cos\left(\frac{(2m+1)p\pi}{2M}\right)}$$

• Comparing eqn ① & ② again to generate expressions to tabulate A & B matrices

i.e $g = Af$

$$x(k) = \left(\sum_{m=0}^{M-1} f_m\right) C_p \cdot \cos\left(\frac{\pi(2m+1)p}{2M}\right)$$

Illy for $f = Bg$ too.

Finally, the expressions are:

$$a_{p,m} = C_p . \cos\left(\frac{\pi(2m+1)p}{2(8)}\right)$$

$$= C_p . \cos\left(\frac{(2m+1)p\pi}{16}\right)$$

& $$b_{m,p} = C_p . \cos\left(\frac{(2m+1)p\pi}{16}\right)$$

- For $N=8$, it is also known that the value of C_p follows this rule:
  - when $p > 0$:
    $C_p = \sqrt{1/4}$
  - when $p \leq 0$:
    $C_p = \sqrt{1/8}$

- The values of p & m ranges b/n 0 to 7
- Let us find the values of matrix A

- when $p=0$ & $m=0$

$$a_{0,0} = C_p . \cos\left(\frac{(2(0)+1)0\pi}{16}\right)$$

$$= \sqrt{\frac{1}{8}} . \cos(0)$$

$$= \sqrt{\frac{1}{8}}$$

- when $p=0$ & $m=1$

$$a_{0,1} = c_p \cdot \cos\left(\frac{(2(1)+1)\,0 \cdot \pi}{16}\right)$$

$$= \sqrt{\frac{1}{8}} \times 1$$

- when $p=0$ & $m=2$

$$a_{0,2} = \sqrt{\frac{1}{8}} \cdot \cos\left(\frac{(2(2)+1)\,0 \cdot \pi}{16}\right)$$

$$= \sqrt{\frac{1}{8}}$$

- Illy,

$$a_{0,3} = a_{0,4} = a_{0,5} = a_{0,6} = a_{0,7} = \sqrt{1/8}$$

- when $p=1$ & $m=0$

$$a_{1,0} = \sqrt{\frac{1}{4}} \cdot \cos\left(\frac{(2(0)+1)\,\pi(1)}{16}\right)$$

$$= \sqrt{\frac{1}{4}} \cdot \cos\left(\frac{\pi}{16}\right)$$

- when $p=1$ & $m=1$

$$a_{1,1} = \sqrt{\frac{1}{4}} \cdot \cos\left(\frac{(2(1)+1)\,\pi(1)}{16}\right)$$

$$= \sqrt{\frac{1}{4}} \cdot \cos\left(\frac{3\pi}{16}\right)$$

Illy.

$a_{1,2} = \sqrt{1/4} \cdot \cos\left(\frac{5\pi}{16}\right)$

$a_{1,3} = \sqrt{1/4} \cdot \cos(7\pi/16)$

$a_{1,4} = \sqrt{1/4} \cdot -(\cos(7\pi/16))$

$a_{1,5} = \sqrt{1/4} \cdot -(\cos(5\pi/16))$

$a_{1,6} = \sqrt{1/4} \cdot -(\cos(3\pi/16))$

$a_{1,7} = \sqrt{1/7} \cdot -(\cos(\pi/16))$

- writing all the further elements b/n $p = 2$ to 7 & $m = 0$ to 7 directly into the matrix to save some space from redundancy

i.e

$$A = \begin{bmatrix} \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} & \sqrt{1/8} \\ \sqrt{1/4}\cos(\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{1/4}\cos(2\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{1/4}\cos(3\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{1/4}\cdot\cos(5\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{1/4}\cdot\cos(6\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sqrt{1/4}\cdot\cos(7\pi/16) & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

$\Downarrow$ simplifying further by taking $\sqrt{1/8}$ common as we did in task 1.

- Also its better if we paraphase these $\cos(n\pi/16)$ fractions into $x, y, z, \bar{x}, \bar{y}, \bar{z}$ → makes matrix readable

(P.T.O)

• After taking $\sqrt{1/8}$ common, the paraphrased fractions would be:

$$x = \sqrt{2}\cos(\pi/16)$$
$$y = \sqrt{2}\cos(3\pi/16)$$
$$z = \sqrt{2}\cos(5\pi/16)$$
$$\bar{x} = \sqrt{2}\cos(2\pi/16)$$
$$\bar{y} = \sqrt{2}\cos(6\pi/16)$$
$$\bar{z} = \sqrt{2}\cos(7\pi/16)$$

$$\therefore A = \sqrt{\frac{1}{8}} \begin{array}{c} \begin{array}{cccccccc} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ x & y & z & \bar{z} & -\bar{z} & -z & -y & -x \\ \bar{x} & \bar{y} & -\bar{y} & -\bar{x} & -\bar{x} & -\bar{y} & \bar{y} & \bar{x} \\ y & -\bar{z} & -x & -z & z & x & \bar{z} & -y \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ z & -x & \bar{z} & y & -y & -\bar{z} & x & -z \\ \bar{y} & -\bar{x} & \bar{x} & -\bar{y} & -\bar{y} & \bar{x} & -\bar{x} & \bar{y} \\ \bar{z} & -z & y & -x & x & -y & z & -\bar{z} \end{bmatrix} \end{array}$$

* Repeating the same procedure to find matrix B.

** Since both the formula for A & B are same, the calculations also remain the same. Only difference would be that B matrix would be a transpose of A, since the index in $a_{p,m} / b_{m,p}$ changes!! **

Thus, writing the backtransform matrix $\underline{B}$ finally!

- Again taking $\sqrt{1/8}$ common across all the elements.

$$
B = \sqrt{\frac{1}{8}} \times
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
1 & x & \bar{x} & y & 1 & z & \bar{y} & \bar{z} \\
1 & y & \bar{y} & -\bar{z} & -1 & -x & -\bar{x} & y \\
1 & z & -\bar{y} & -x & -1 & \bar{z} & \bar{x} & y \\
1 & \bar{z} & -\bar{x} & -z & 1 & y & -\bar{y} & -x \\
1 & -\bar{z} & -\bar{x} & z & 1 & -y & -\bar{y} & x \\
1 & -z & -\bar{y} & x & -1 & -\bar{z} & \bar{x} & -y \\
1 & -y & \bar{y} & \bar{z} & -1 & x & -\bar{x} & z \\
1 & -x & \bar{x} & -y & 1 & -z & \bar{y} & -\bar{z}
\end{bmatrix}
\end{array}
$$

iii) Discrete Wavelet Transform :

→ [method referred from Preparatory Assignment 4]

- The $\underline{\text{1-D signal}}$ is represented using basis vectors

i.e ⇒ $\phi_{3,0}$, $\psi_{3,0}$, $\psi_{2,0}$, $\psi_{2,1}$, $\psi_{1,0}$, $\psi_{1,1}$, $\psi_{1,2}$, $\psi_{1,3}$

- Like we did before, we again have to compare eqn ① & ② i.e $g = Af$ & $f = Bg$ to retrieve expressions which could help calculate the matrices A & B.

(P.T.O)

- Continuous setting : $(\phi_{3,0})$

$$C_{3,0} = \sum_{m=0}^{m-1} f_m \cdot (\phi_{3,0})_m$$

$$g = A \cdot f$$

Illy for Discrete setting : $(\psi_{m,n})$

- $d_{3,0} = \sum_{m=0}^{M-1} f_m \cdot (\psi_{3,0})_m$

- $d_{2,0} = \sum_{m=0}^{m-1} f_m \cdot (\psi_{2,0})_m$ & so on....

* In Q1 in preparatory assn 4 it is given that, for discrete setting we sample at N equidistant grid points $\{1/2, 3/2, \ldots N - 1/2\}$

$\therefore$ For $\phi_{3,0}(x) = \frac{1}{\sqrt{8}} \underbrace{(1,1,1,1,1,1,1,1)^T}_{N \text{ times}}$

(Basis vectors)

For $N=8$,

** writing vectors for other co-efficients directly from the preparatory assn solution

i.e $\psi_{3,0} = 1/\sqrt{8}\,(1,1,1,1,-1,-1,-1,-1)^T$

$\psi_{2,0} = 1/2\,(1,1,-1,-1,0,0,0,0)^T$

$\psi_{2,1} = 1/2\,(0,0,0,0,1,1,-1,-1)^T$

$\psi_{1,0} = 1/\sqrt{2}\,(1,-1,0,0,0,0,0,0)^T$

$\psi_{1,1} = 1/\sqrt{2}\,(0,0,1,-1,0,0,0,0)^T$

$\psi_{1,2} = 1/\sqrt{2}\,(0,0,0,0,1,-1,0,0)^T$

$\psi_{1,3} = 1/\sqrt{2}\,(0,0,0,0,0,0,1,-1)^T$

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∴ The final transformation matrix A would be: (after taking $\sqrt{1}/\sqrt{8}$ common, like we did in previous task)

$$A = \sqrt{\frac{1}{8}} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

* Repeat the same procedure to find the backtransformation matrix.

- Again considering the fact that these basis vectors are orthogonal & orthonormal to each other, so we can cleverly use the backtransformation eqn as:

$$f = Bg$$

$$f = B(Af)$$

$$f = A^T(Af) \rightarrow \text{orthonormal}$$

$$\therefore B = A^T = \sqrt{1/8} \begin{bmatrix} 1 & 1 & \sqrt{2} & 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & \sqrt{2} & 0 & -2 & 0 & 0 & 0 \\ 1 & 1 & -\sqrt{2} & 0 & 0 & 2 & 0 & 0 \\ 1 & 1 & -\sqrt{2} & 0 & 0 & -2 & 0 & 0 \\ 1 & -1 & 0 & \sqrt{2} & 0 & 0 & 2 & 0 \\ 1 & -1 & 0 & \sqrt{2} & 0 & 0 & -2 & 0 \\ 1 & -1 & 0 & -\sqrt{2} & 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & -\sqrt{2} & 0 & 0 & 0 & -2 \end{bmatrix}$$

(b) Apart from being inverse to each other. It also appears that with Discrete wavelet transform, backtransformation matrix B equals to the transpose of A

i.e $B = A^T$,

Also, the basis vectors which we used to construct these matrices appear to be orthonormal!

Assignment #4

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Problem 2 (Discrete wavelet transform)

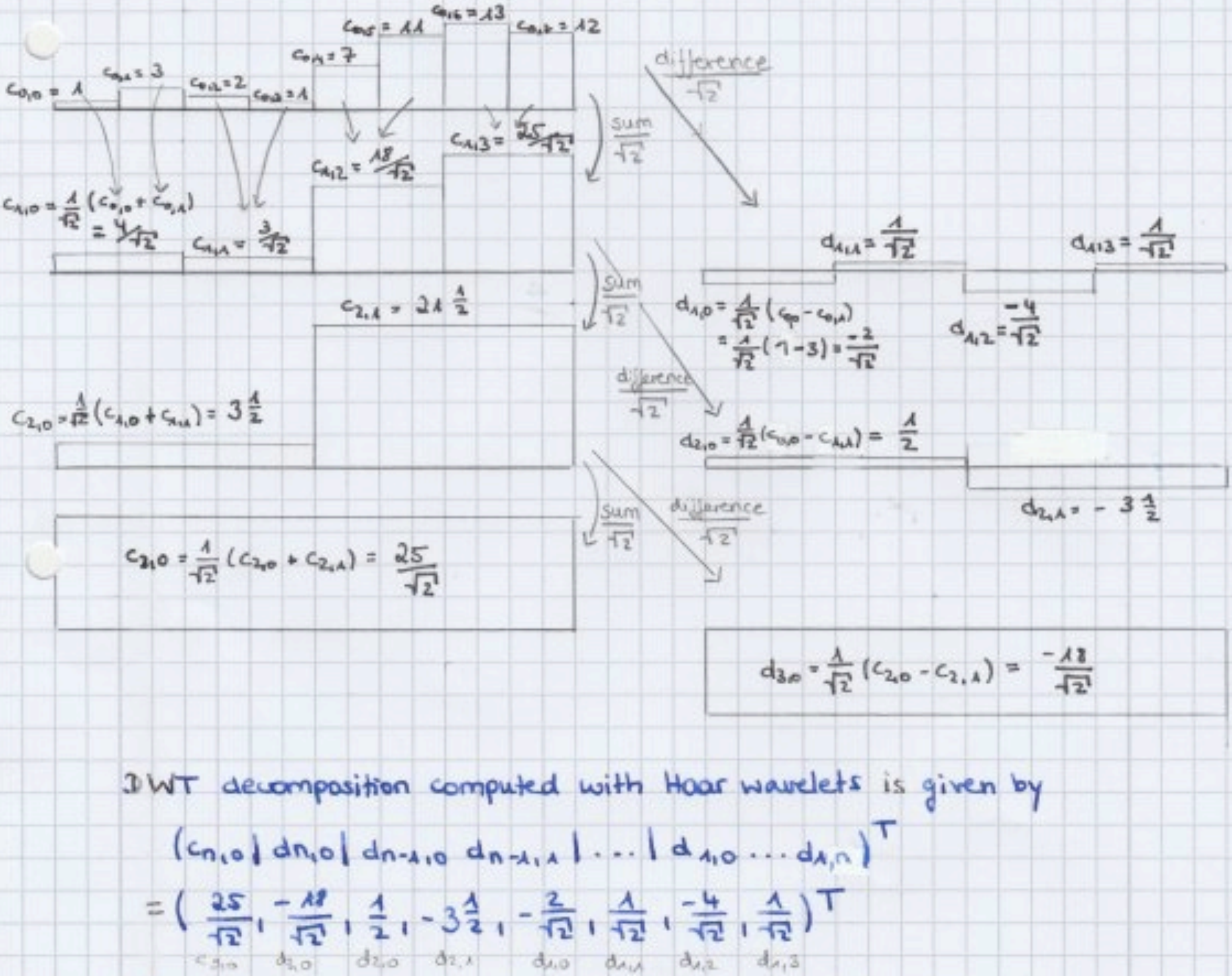
(a) $c_{j,k} = \frac{1}{\sqrt{2}} \overbrace{(c_{j-1,2k} + c_{j-1,2k+1})}^{\text{sum}}$ $\qquad c_{0,k} = f_k$

$d_{j,k} = \frac{1}{\sqrt{2}} \overbrace{(c_{j-1,2k} - c_{j-1,2k+1})}^{\text{difference}}$

$k = 0, \dots, 2^n - 1 \qquad j = 0, \dots, n$

In this example: $k = 0, \dots, 7$ $\qquad f := (1,3,2,1,7,11,13,12)^T$

$n = 3 \quad j = 0, \dots, 3$



DWT decomposition computed with Haar wavelets is given by

$(c_{n,0} \mid d_{n,0} \mid d_{n-1,0}\, d_{n-1,1} \mid \dots \mid d_{1,0} \dots d_{1,n})^T$

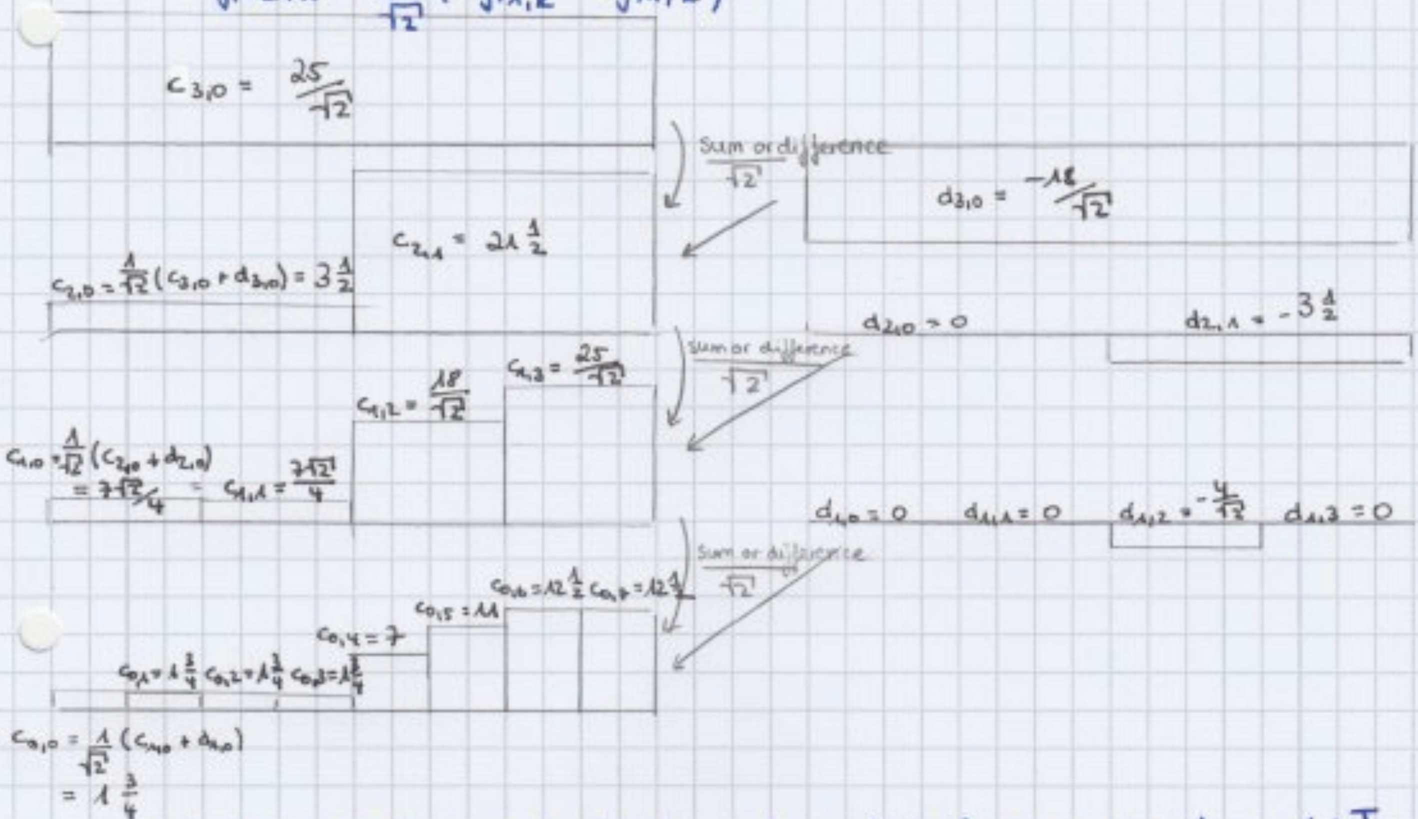
$= \Big(\underbrace{\frac{25}{\sqrt{2}}}_{c_{3,0}}, \underbrace{-\frac{18}{\sqrt{2}}}_{d_{3,0}}, \underbrace{\frac{1}{2}}_{d_{2,0}}, \underbrace{-3\frac{1}{2}}_{d_{2,1}}, \underbrace{-\frac{2}{\sqrt{2}}}_{d_{1,0}}, \underbrace{\frac{1}{\sqrt{2}}}_{d_{1,1}}, \underbrace{\frac{-4}{\sqrt{2}}}_{d_{1,2}}, \underbrace{\frac{1}{\sqrt{2}}}_{d_{1,3}} \Big)^T$

(b) smallest absolute values: $d_{2,0} = \frac{1}{2}$, $d_{1,0} = \frac{-2}{\sqrt{2}}$, $d_{1,1} = \frac{1}{\sqrt{2}}$, $d_{1,3} = \frac{1}{\sqrt{2}}$

setting these values to zero yields: $\left(\frac{25}{\sqrt{2}}, -\frac{18}{\sqrt{2}}, 0, -3\frac{1}{2}, 0, 0, -\frac{4}{\sqrt{2}}, 0 \right)^T$

(c) $c_{j,2k} = \frac{1}{\sqrt{2}} \left(c_{j+1,k} + d_{j+1,k} \right)$

$c_{j,2k+1} = \frac{1}{\sqrt{2}} \left(c_{j+1,k} - d_{j+1,k} \right)$



Reconstructed signal: $\left(1\frac{3}{4}, 1\frac{3}{4}, 1\frac{3}{4}\ 1\frac{3}{4}, 7, 11, 12\frac{1}{2}, 12\frac{1}{2}\right)^T$

(d)

Problem 3 (Huffman Coding)

(a) "EYJAFJALLAJÖKULL" → 16 symbols

E: |	1x	$p(E) = \frac{1}{16}$
Y: |	1x	$p(Y) = \frac{1}{16}$
J: |||	3x	$p(J) = \frac{3}{16}$
A: |||	3x	$p(A) = \frac{3}{16}$
F: |	1x	$p(F) = \frac{1}{16}$
L: ||||	4x	$p(L) = \frac{4}{16} = \frac{1}{4}$
Ö: |	1x	$p(Ö) = \frac{1}{16}$
K: |	1x	$p(K) = \frac{1}{16}$
U: |	1x	$p(U) = \frac{1}{16}$

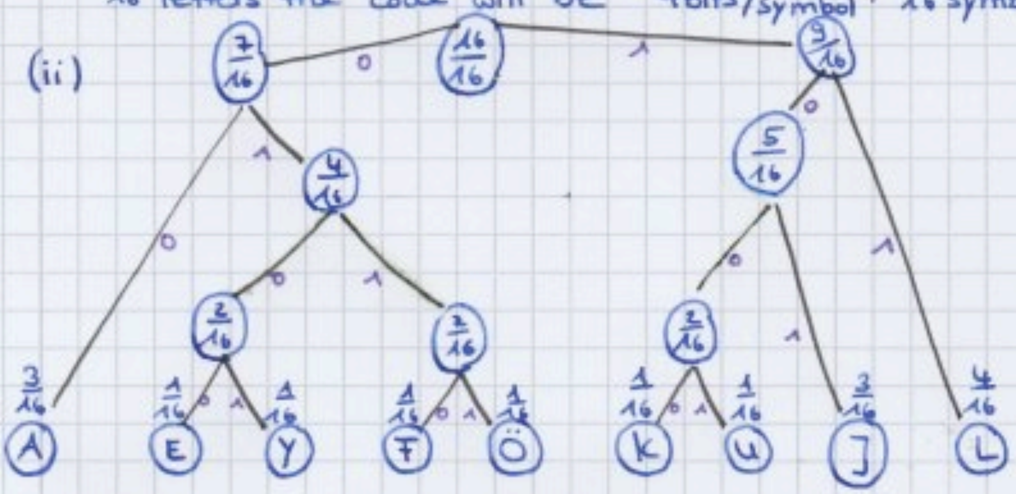
2^3 2^2 2^1 2^0
8 4 2 1

Order by increasing probability

E – Y – F – Ö – K – U – J – A – L

(i) Storing 9 unique letters will need 4 bits/symbol when there should be an equal number of bits per symbol. Since we have 16 letters the code will be 4 bits/symbol · 16 symbol = 64 bits long

(ii)



00 0100 0101 0110 0111 1000 1001 101 11 Code words

The Huffman code for the source word: "EYJAFJALLAJÖKULL" is:

"0100 0101 101 00 0110 101 00 11 11 00 101 0111 1000 1001 11 11"

4 4 3 2 4 3 2 2 2 2 3 4 4 4 2 2 bits

8 11 13 17 20 22 24 26 28 31 35 39 43 45 47

Is the obtained code unique? No it is not. Combining the two smallest probabilities can be done in many orders + you could also change the orde 0 (left branch) and 1 (right branch) is assigned to 1 (left branch) 0 (right branch).

(iii) 4 + 4 + 3 + 2 + 4 + 3 + 2 + 2 + 2 + 2 + 3 + 4 + 4 + 4 + 2 + 2 = 47 bits

(iv) The entropy assigns a lower bound for the average code word length (ideal code word length):

$$H = -\sum_k p_k \operatorname{ld} p_k$$

$$H = -\big(p_E \log_2 p_E + p_Y \log_2 p_Y + p_F \log_2 p_F + p_{Ö} \log_2 p_{Ö}$$

$$+ p_K \log_2 p_K + p_U \log_2 p_U + p_J \log_2 p_J + p_A \log_2 p_A + p_L \log_2 p_L\big)$$

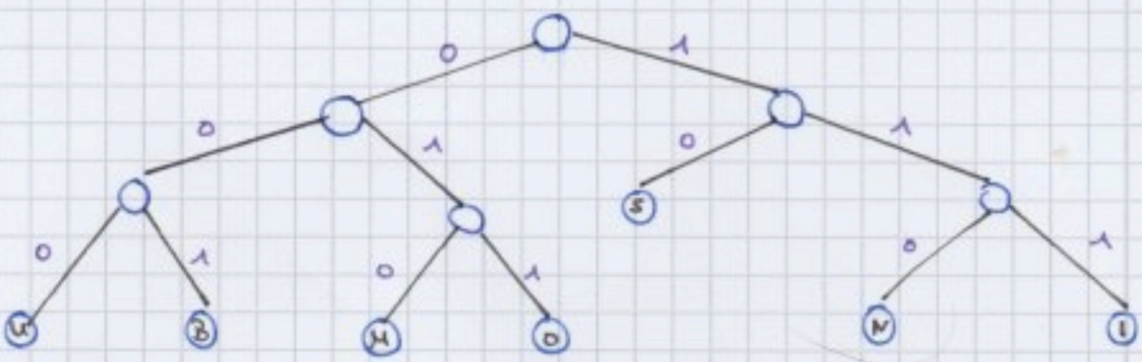
$$= -\Big(\underset{E}{\tfrac{1}{16} \log_2 \tfrac{1}{16}} + \underset{Y}{\tfrac{1}{16} \log_2 \tfrac{1}{16}} + \underset{F}{\tfrac{1}{16} \log_2 \tfrac{1}{16}} + \underset{Ö}{\tfrac{1}{16} \log_2 \tfrac{1}{16}} + \underset{K}{\tfrac{1}{16} \log_2 \tfrac{1}{16}}$$

$$+ \underset{U}{\tfrac{1}{16} \log_2 \tfrac{1}{16}} + \underset{J}{\tfrac{3}{16} \log_2 \tfrac{3}{16}} + \underset{A}{\tfrac{3}{16} \log_2 \tfrac{3}{16}} + \underset{L}{\tfrac{4}{16} \log_2 \tfrac{4}{16}}\Big)$$

$$= -(-2{,}9056) = 2{,}9056 \text{ bits/symbol}$$

$$\lceil 2.9056 \text{ bits/symbol} \cdot 16 \text{ symbols} \rceil = \lceil 46{,}49 \rceil = \underline{\underline{47 \text{ bits}}}$$

(b) (i) U = 000 B = 001 M = 010 O = 011 S = 10 N = 110 I = 111



(ii) 10 | 000 | 001 | 010 | 111 | 10 | 10 | 111 | 011 | 110

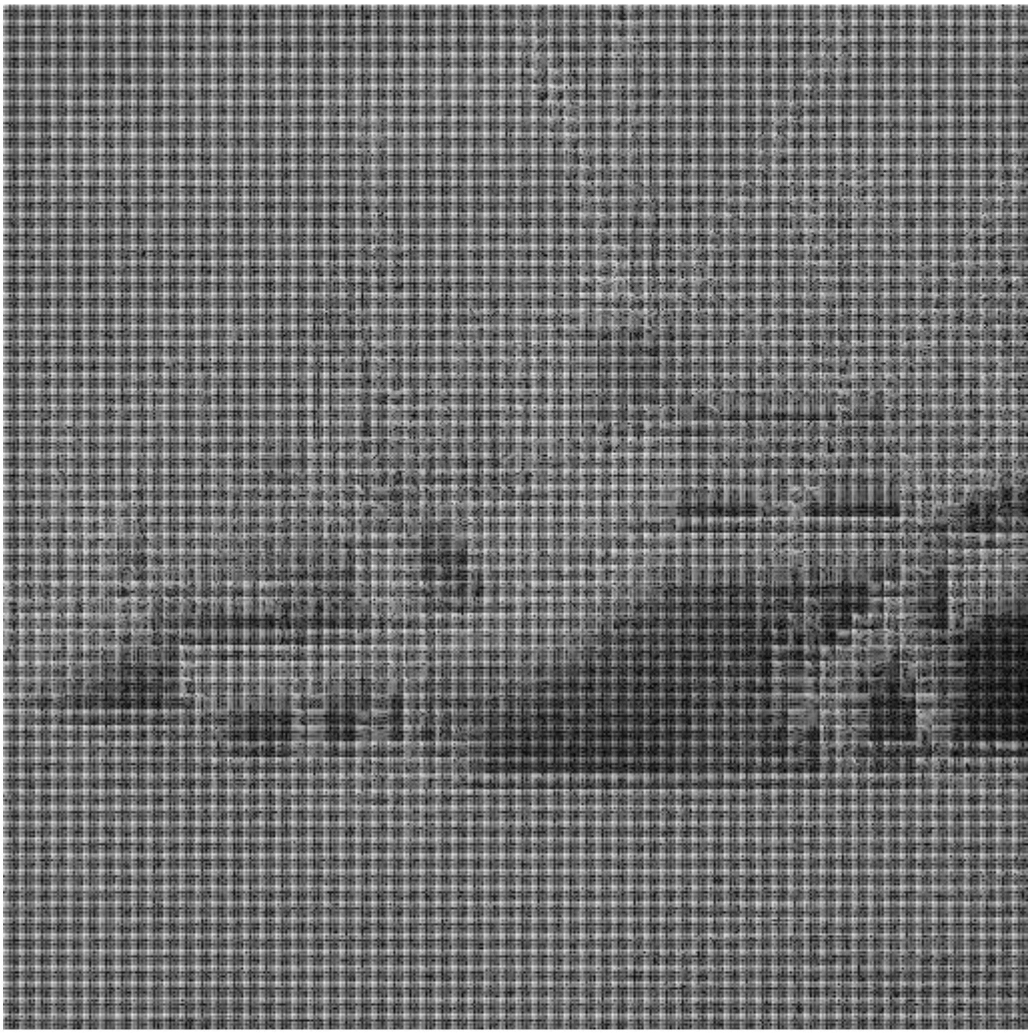
S U B M I S S I O N

Source word: <u>SUBMISSION</u>

4.b)

Menu option 1: DCT of whole image:

Menu option 2 spectra: DCT 8*8:

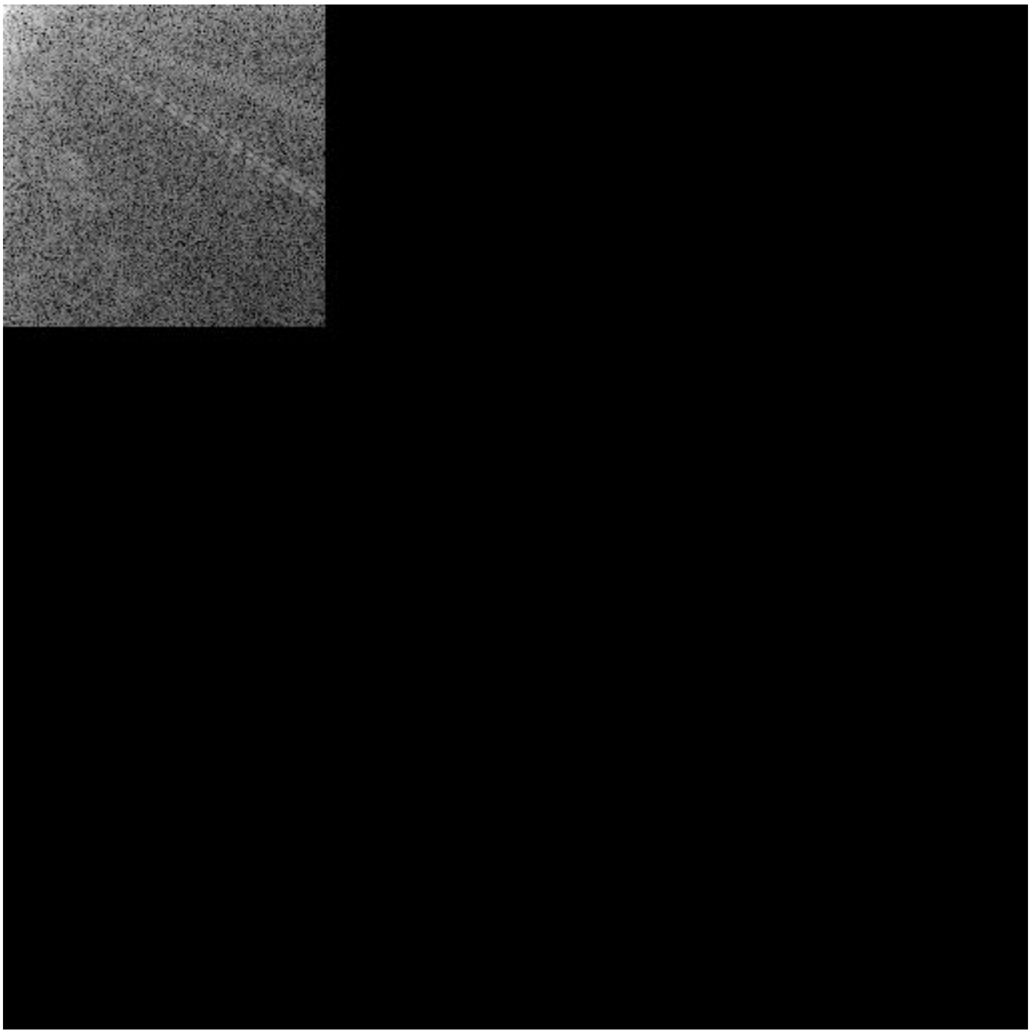


For option 1 it took around a minute to compute output but for option 1 it was within seconds(2 sec).

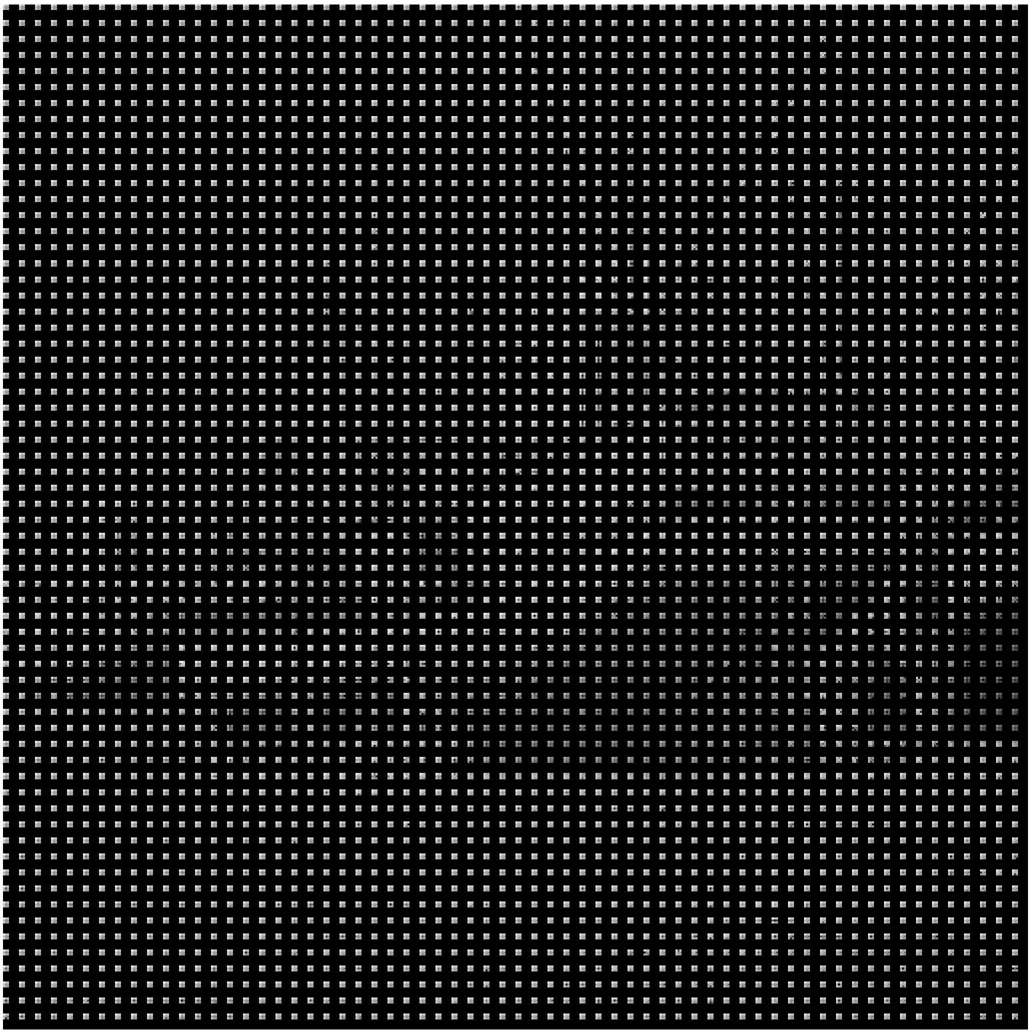
In option 02 spectra, we can see it has been divided into blocks of 8*8 rather than whole image.

4c)

Option 3 Spectrum:



Option 4 Spectrum:

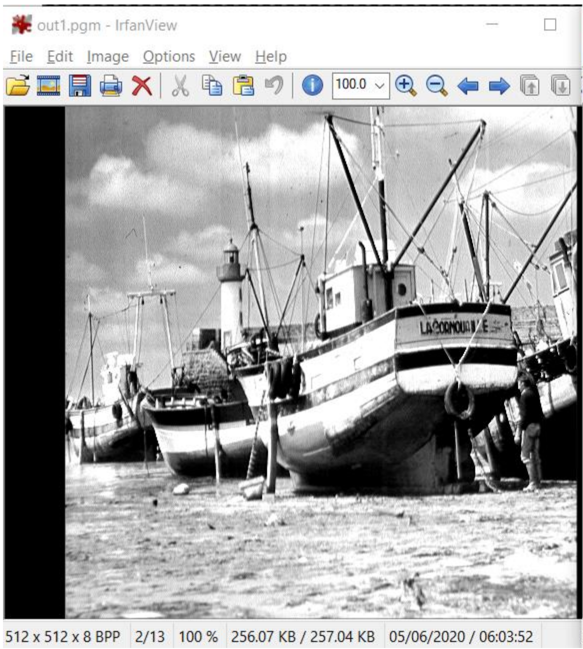


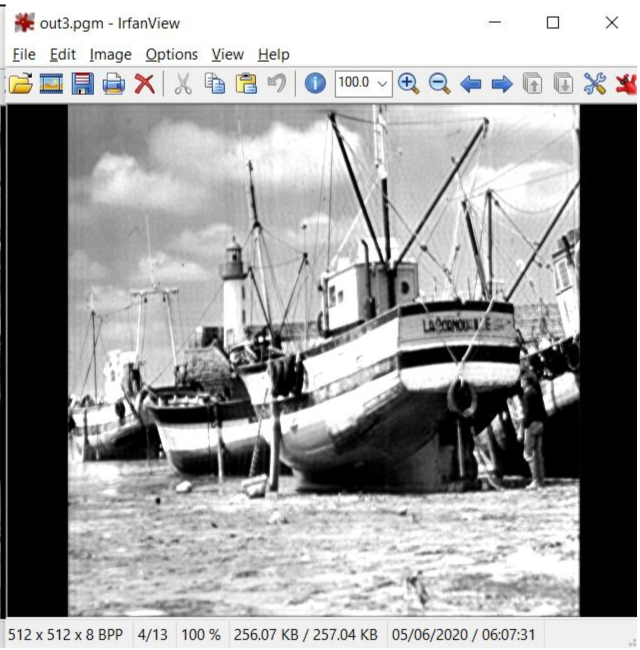
Yes the high and low frequency have been removed as we nullify the coefficients of high and low value.

The DCT for whole image with frequency reduction gives better image then 8*8.

The output of image 1 is better than that of 3 as we can see below. Image 1 is sharp and clear whereas in image 3 we reduce the frequency for compression which results in distorted image.

See image below for output option 1 and 3.



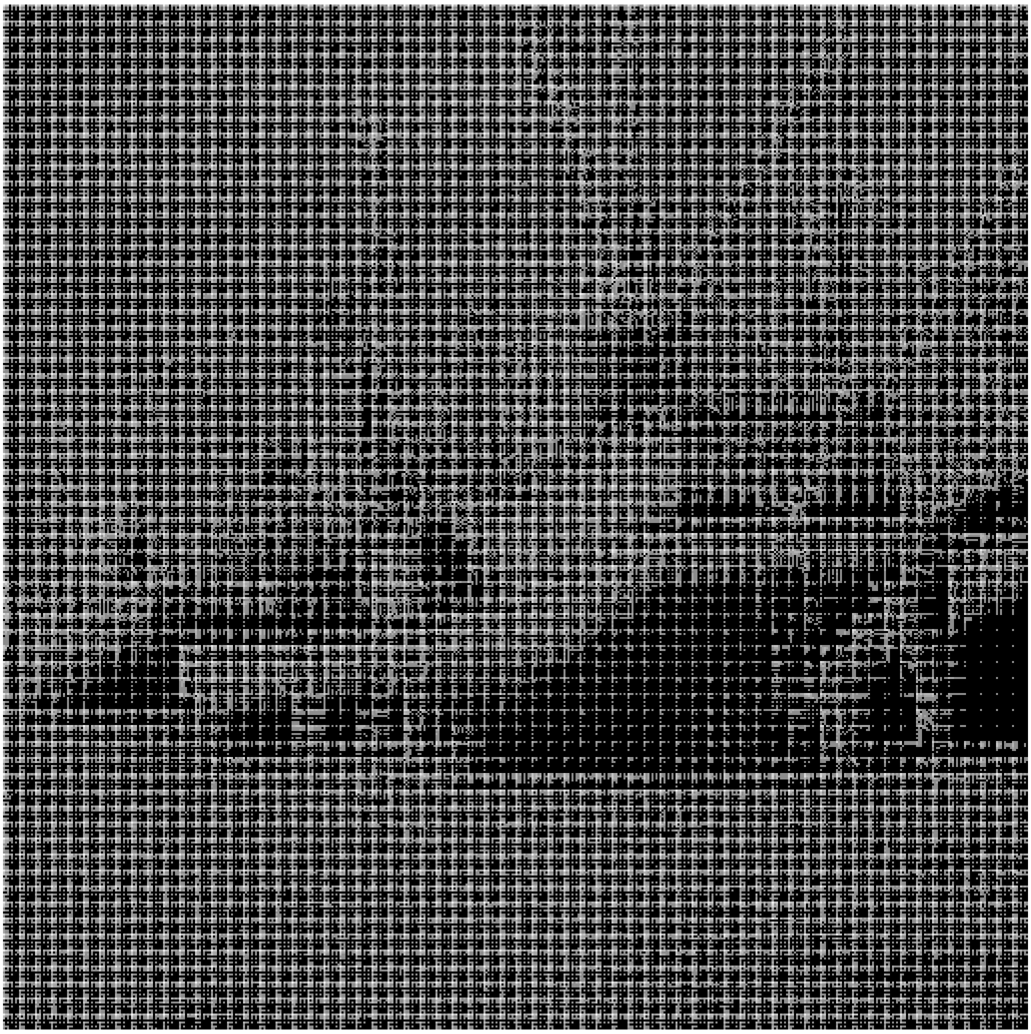


Output option 3:

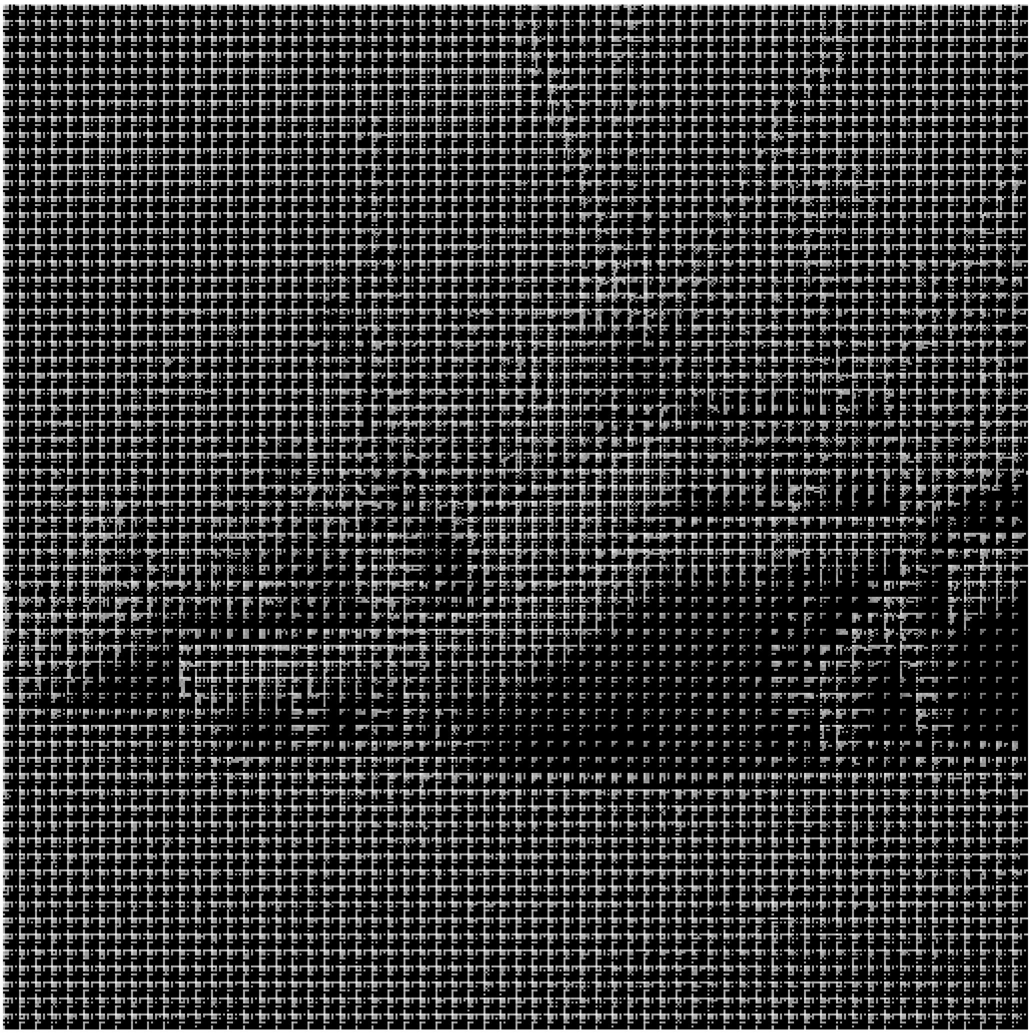
Output option 1:

4d)

Option 5 Spectra:



Option 6 Spectra:



The visual quality for second strategy of weighted matrix yields better results. It may be because it treats the high and low frequency separately rather than treating all frequency same as in first quantisation approach.