Lesson 9.1

Decision Theory with Unknown State Probabilities

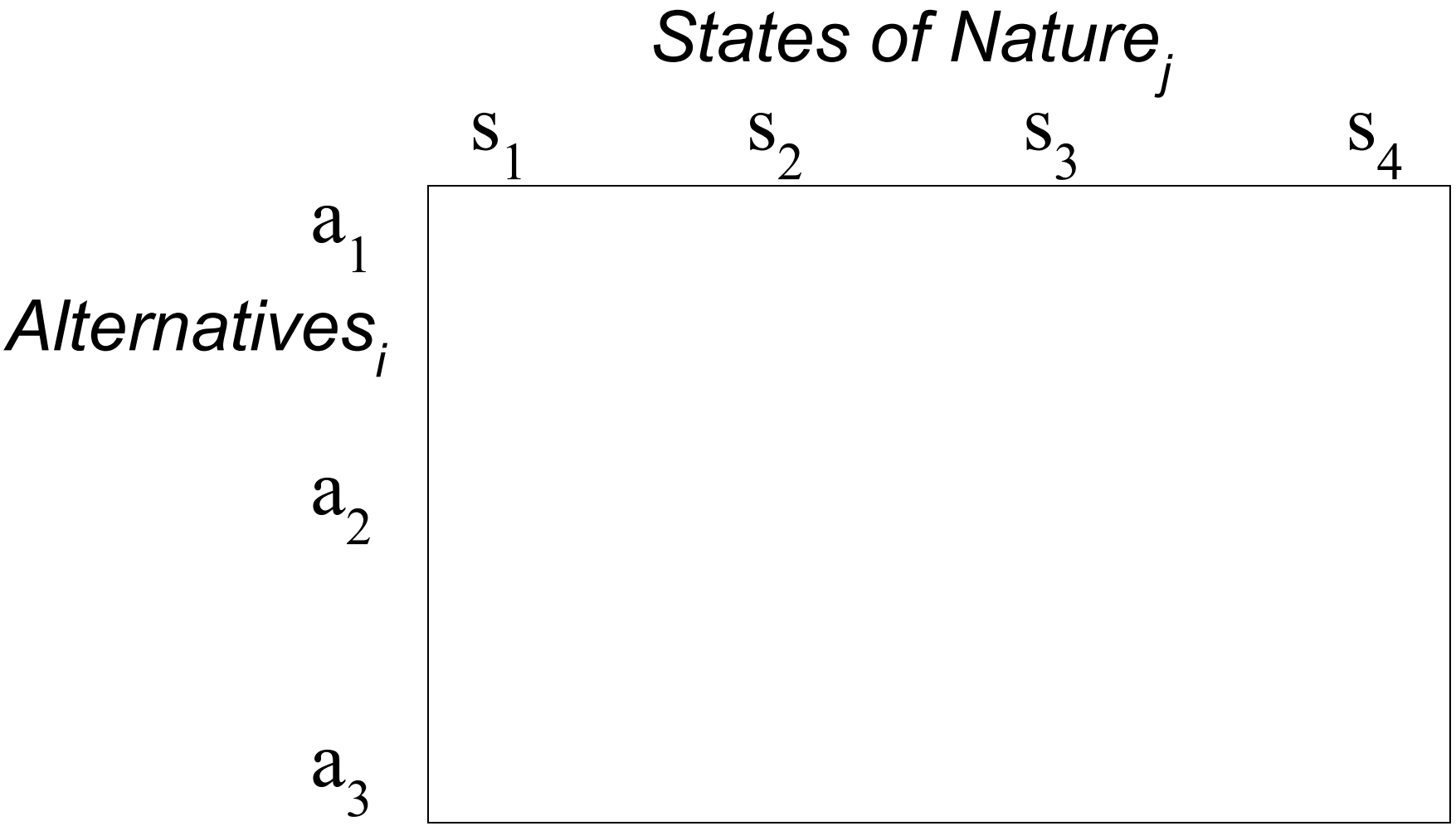
Decision Theory

- Most management decisions are made in an environment of uncertainty.
- Decision theory provides a orderly way of choosing among several alternative strategies when decisions are made under uncertainty or risk.

Decision Theory

- **Uncertainty** exists when the decision maker is unable to ascertain or subjectively estimate the probabilities of the various states of nature.
- **Risk** exists when the decision maker does not know with certainty the state of nature, but the probabilities of various outcomes is known.

Payoff Matrix



Payoff Matrix

Alternatives$_i$	*States of Nature*$_j$ s_1	s_2	s_3	s_4
a_1	c_{11}	c_{12}	c_{13}	c_{14}
a_2	c_{21}	c_{22}	c_{23}	c_{24}
a_3	c_{31}	c_{32}	c_{33}	c_{34}

Payoff Matrix

Alternatives$_i$	*States of Nature*$_j$ s_1	s_2	s_3	s_4
a_1	c_{11}	c_{12}	c_{13}	c_{14}
a_2	c_{21}	c_{22}	c_{23}	c_{24}
a_3	c_{31}	c_{32}	c_{33}	c_{34}

C_{ij} is the consequence of state I under alternative j

Home Health Example

Suppose a home health agency is considering adding physical therapy (PT) services for its clients. There are three ways to do this:

Option A: contract with an independent practitioner at $60 per visit.

Option B: hire a staff PT at a monthly salary of $4000 plus $400/mo. for a leased car plus $7/visit for supplies and travel.

Option C: independent practitioner at $35/visit but pay for fringe benefits at $200/mo. and cover the car and expenses as in Option B.

Source: Austin, CJ and Boxerman, SB, Quantitative Analysis for Health Services Administration, AUPHA/Health Administration Press, Ann Arbor, Michigan, 1995

Payoff Matrix: Home Health Example

	States of Nature $_j$			
	s_1	s_2	s_3	s_4
Demand of Patient Services: Visits/ mo.	30	90	140	150

Assumption: Probabilities of States of Nature are unknown.

Payoff Matrix: Home Health Example

$Alternatives_i$	
a_1	•Contract with independent Contractor at $60/visit. Net Profit = (75 - 60) * D = 15*D
a_2	
a_3	

Assumption: Charge $75 per visit.

Payoff Matrix: Home Health Example

$Alternatives_i$	
a_1	•Contract with independent Contractor at $60/visit. Net Profit = (75 - 60) * D = 15*D
a_2	•Pay monthly salary of $4,000 •Car allowance $400 •Expenses @$7 a visit Net Profit = - 4,000 - 400 + (75 - 7) * D = -4,400 + 68*D
a_3	

Assumption: Charge $75 per visit.

Payoff Matrix: Home Health Example

$Alternatives_i$	
a_1	•Contract with independent Contractor at $60/visit. Net Profit = (75 - 60) * D = 15*D
a_2	•Pay monthly salary of $4,000 •Car allowance $400 •Expenses @$7 a visit Net Profit = - 4,000 - 400 + (75 - 7) * D = -4,400 + 68*D
a_3	•Contract @ $35 per visit •Car allowance $400 •Fringe benefits of $200 •Expenses @$7 a visit Net Profit = -400 -200+ (75 - 35 -7) * D = -600 + 33*D

Assumption: Charge $75 per visit.

Payoff Matrix

Total Profit (Alt 1) = 15*D

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	450	1350	2100	2250
a_2				
a_3				

Payoff Matrix

Total Profit (Alt 2) = -4,400 + 68D

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	450	1350	2100	2250
a_2	-2360	1720	5120	5800
a_3				

Payoff Matrix

Total Profit (Alt 3) = -600 + 33D

	s_1 = 30	s_2 = 90	s_3 = 140	s_4 = 150
a_1	450	1350	2100	2250
a_2	-2360	1720	5120	5800
a_3	390	2370	4020	4350

Payoff Matrix

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	450	1350	2100	2250
a_2	-2360	1720	5120	5800
a_3	390	2370	4020	4350

Payoff Matrix

No alternative dominates any other alternative

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	450	1350	2100	2250
a_2	-2360	1720	5120	5800
a_3	390	2370	4020	4350

Criteria for Decision Making

Maximin Criterion- criterion that maximizes the minimum payoff for each alternative.

Steps:

1) Identify the minimum payoff for each alternative.
2) Pick the largest minimum payoff.

Maximin Decision Criterion

	s_1	s_2	s_3	s_4	Maximin
	30	90	140	150	
a_1	*450*	1350	2100	2250	***450***
a_2	*-2360*	1720	5120	5800	-2360
a_3	*390*	2370	4020	4350	390

Maximin Decision Criterion

The maximin criterion is a very conservative or risk adverse criterion. It is a pessimistic criterion. It assumes nature will vote against you.

Minimax Decision Criterion

If the values in the payoff matrix were costs, the equivalent conservative or risk adverse criterion would be the minimax criterion. It is a pessimistic criterion.

Criteria for Decision Making

Maximax Criterion- criterion that maximizes the maximum payoff for each alternative.

Steps:

1) Identify the maximum payoff for each alternative.
2) Pick the largest maximum payoff.

Maximax Decision Criterion

	s_1	s_2	s_3	s_4	Maximax
	30	90	140	150	
a_1	450	1350	2100	*2250*	2250
a_2	-2360	1720	5120	*5800*	***5800***
a_3	390	2370	4020	*4350*	4350

Maximax Decision Criterion

The maximax criterion is a very optimistic or risk seeking criterion. It is not a criterion which preserves capital in the long run.

Minimin Decision Criterion

If the values in the payoff matrix were costs, the equivalent optimistic criterion is minimin. It assumes nature will vote for you.

Criteria for Decision Making

Minimax Regret Criterion- criterion that minimizes the loss incurred by not selecting the optimal alternative.

Steps:

1) Identify the largest element in the first column.

2) Subtract each element in the column from the largest element to compute the opportunity loss and repeat for each column.

3) Identify the maximum regret for each alternative and then choose that alternative with the smallest maximum regret.

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	***450***	1350	2100	2250
a_2	-2360	1720	5120	5800
a_3	390	2370	4020	4350

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	***450*** **450 - 450** **0**	1350	2100	2250
a_2	-2360 **450 - (-2360)** **2810**	1720	5120	5800
a_3	390 **450 - 390** **60**	2370	4020	4350

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	***450***	1350	2100	2250
	450 - 450			
	0			
a_2	-2360	1720	5120	5800
	450 - (-2360)			
	2810			
a_3	390	***2370***	4020	4350
	450 - 390			
	60			

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	***450***	1350	2100	2250
	450 - 450	**2370 - 1350**		
	0	**1020**		
a_2	-2360	1720	5120	5800
	450 - (-2360)	**2370 - 1720**		
	2810	**650**		
a_3	390	***2370***	4020	4350
	450 - 390	**2370 - 2370**		
	60	**0**		

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1	s_2	s_3	s_4
	30	90	140	150
a_1	***450***	1350	2100	2250
	450 - 450	**2370 - 1350**	**5120 - 2100**	**5800 - 2250**
	0	**1020**	**3020**	**3550**
a_2	-2360	1720	***5120***	***5800***
	450 - (-2360)	**2370 - 1720**	**5120 - 5120**	**5800 - 5800**
	2810	**650**	**0**	**0**
a_3	390	***2370***	4020	4350
	450 - 390	**2370 - 2370**	**5120 - 4020**	**5800 - 4350**
	60	**0**	**1100**	**1450**

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1 30	s_2 90	s_3 140	s_4 150	Max Regret
a_1	0	1020	3020	3550	3550
a_2	2810	650	0	0	2810
a_3	60	0	1100	1450	1450

Minimax Regret: $\text{Regret}_j = \text{Max}\,[c_{ij}] - c_{ij}$

	s_1 30	s_2 90	s_3 140	s_4 150	Max Regret
a_1	0	1020	3020	3550	3550
a_2	2810	650	0	0	2810
a_3	60	0	1100	1450	*1450*

Minimax Regret Decision Criterion

The minimax regret criterion is also a conservative criterion. It is not as pessimistic as the maximin criterion.