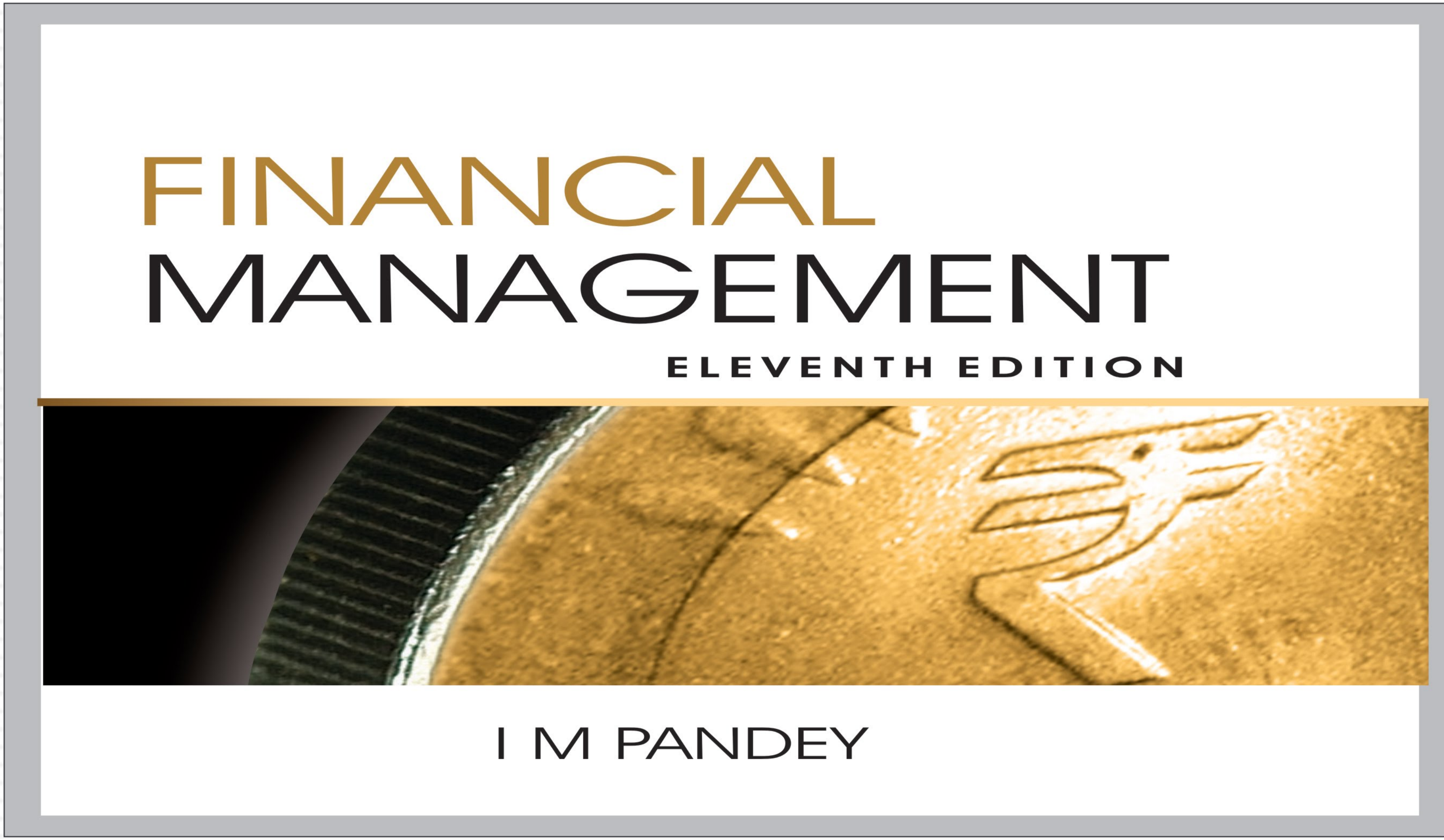

CHAPTER 2

VALUE AND RETURN

LEARNING OBJECTIVES

- Understand what gives money its time value.
- Explain the methods of calculating present and future values.
- Highlight the use of present value technique (discounting) in financial decisions.
- Introduce the concept of internal rate of return.

Time Preference for Money

- **Time preference for money** is an individual's preference for possession of a given amount of money *now*, rather than the same amount at some future time.
- Three reasons may be attributed to the individual's time preference for money:
  - risk
  - preference for consumption
  - investment opportunities

Required Rate of Return

- The time preference for money is generally expressed by an interest rate. This rate will be positive even in the absence of any risk. It may be therefore called the **risk-free rate.**
- An investor requires compensation for assuming risk, which is called **risk premium**.
- The investor's **required rate of return** is:

  Risk-free rate + Risk premium.

Required Rate of Return

- Would an investor want Rs. 100 today or after one year?
- Cash flows occurring in different time periods are not comparable.
- It is necessary to adjust cash flows for their differences in timing and risk.
- Example : If preference rate =10 percent
  - An investor can invest if Rs. 100 if he is offered Rs 110 after one year.
  - Rs 110 is the future value of Rs 100 today at 10% interest rate.
  - Also, Rs 100 today is the present value of Rs 110 after a year at 10% interest rate.
  - If the investor gets less than Rs. 110 then he will not invest. Anything above Rs. 110 is favourable.

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Time Value Adjustment

- Two most common methods of adjusting cash flows for time value of money:
  - **Compounding**—the process of calculating **future values** of cash flows and
  - **Discounting**—the process of calculating **present values** of cash flows.

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Future Value

- **Compounding** is the process of finding the future values of cash flows by applying the concept of compound interest.
- **Compound interest** is the interest that is received on the original amount (principal) as well as on any interest earned but not withdrawn during earlier periods.
- **Simple interest** is the interest that is calculated only on the original amount (principal), and thus, no compounding of interest takes place.

Future Value

- The general form of equation for calculating the future value of a lump sum after n periods may, therefore, be written as follows:

$$F_n = P(1+i)^n$$

- The term $(1 + i)^n$ is the **compound value factor** (*CVF*) of a lump sum of Re 1, and it always has a value greater than 1 for positive i, indicating that *CVF* increases as i and n increase.

$$F_n = P \times \mathrm{CVF}_{n,i}$$

Future Value

- In Microsoft Excel: Use FV function.

FV(rate,nper,pmt,pv,type)

Where: rate= interest rate. nper= n periods, pmt= annuity value, pv= present value, type= 1 for beginning of the period and 0 for end for end of period.

Future Value: Example

- If you deposited Rs 55,650 in a bank, which was paying a 15 per cent rate of interest on a ten-year time deposit, how much would the deposit grow at the end of ten years?

- First find out the compound value factor at 15 per cent for 10 years which is 4.046.
- Multiple 4.046 by Rs 55,650 to get the compound value:

$$FV = 55{,}650 \times CVF_{10,\,0.12} = 55{,}650 \times 4.046 = \text{Rs } 225{,}159.90$$

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Future Value of an Annuity

- **Annuity** is a fixed payment (or receipt) each year for a *specified* number of years.

$$F_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

- The term within brackets is the **compound value factor for an annuity** of Re 1, which we shall refer as *CVFA*.

$$F_n = A \times \mathrm{CVFA}_{n,i}$$

Future Value of an Annuity: Example

- Suppose that a firm deposits Rs 5,000 at the end of each year for four years at 6 per cent rate of interest. How much would this annuity accumulate at the end of the fourth year?
- First find *CVFA* which is 4.3746.
- Multiple 4.375 by Rs 5,000 to obtain a compound value of Rs 21,875:

$$F_4 = 5{,}000(\text{CVFA}_{4,\,0.06}) = 5{,}000 \times 4.3746 = \text{Rs }\ 21{,}873$$

Sinking Fund

- **Sinking fund** is a fund, which is created out of fixed payments each period to accumulate to a future sum after a specified period. For example, companies generally create sinking funds to retire bonds (debentures) on maturity.

- The factor used to calculate the annuity for a given future sum is called the *sinking fund factor* (*SFF*).

$$A = F_n\left[\frac{i}{(1+i)^n - 1}\right]$$

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Example

- Suppose a 20-year-old student wants to start saving for retirement. She plans to save Rs.3 a day. Every day, she puts Rs.3 in her drawer. At the end of the year, she invests the accumulated savings (Rs.1,095) in an online stock account. The stock account has an expected annual return of 12%.
  - When she is 65 years old, she will get Rs. **1,487,261.89**

$$A = F_n \left[\frac{i}{(1+i)^n - 1} \right]$$

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Present Value

- **Present value** of a future cash flow (inflow or outflow) is the amount of current cash that is of equivalent value to the decision-maker.
- **Discounting** is the process of determining present value of a series of future cash flows.
- The *interest rate* used for discounting cash flows is also called the *discount rate*.

Present Value of a Single Cash Flow

- Formula to calculate the present value of a lump sum to be received after some future periods:

$$P = \frac{F_n}{(1+i)^n} = F_n\left[(1+i)^{-n}\right]$$

- The term in parentheses is the **discount factor** or **present value factor** (*PVF*), and it is always less than 1.0 for positive *i*, indicating that a future amount has a smaller present value.

$$PV = F_n \times \mathrm{PVF}_{n,i}$$

- In MS Excel use pv function

PV(rate,nper,pmt,fv,type)

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Example

- Suppose that an investor wants to find out the present value of Rs 50,000 to be received after 15 years. Her interest rate is 9 per cent. First, we will find out the present value factor, which is 0.275. Multiplying 0.275 by Rs 50,000, we obtain Rs 13,750 as the present value:

$$PV = 50{,}000 \times PVF_{15,\ 0.09} = 50{,}000 \times 0.275 = \text{Rs } 13{,}750$$

Present Value of an Annuity

- The computation of the present value of an annuity can be written in the following general form:

$$P = A\left[\frac{1}{i} - \frac{1}{i(1+i)^n}\right]$$

- The term within parentheses is the **present value factor of an annuity** of Re 1, which we would call *PVFA*, and it is a sum of single-payment present value factors.

$$P = A \times \text{PVAF}_{n,i}$$

- In MS Excel use pmt function.

PMT(rate,nper,pv,fv,type)

Example

- Suppose you have borrowed a 3-year loan of Rs 10,000 at 9 per cent from your employer to buy a motorcycle. If your employer requires three equal end-of-year repayments, then the annual instalment will be

$$10{,}000 = A \times \text{PVFA}_{3,0.09}$$

$$10{,}000 = A \times 2.531$$

$$A = \frac{10{,}000}{2.531} = \text{Rs } 3{,}951$$

Capital Recovery and Loan Amortisation

- **Capital recovery** is the annuity of an investment made today for a specified period of time at a given rate of interest. Capital recovery factor helps in the preparation of a **loan amortisation (loan repayment) schedule**.

$$A = P\left[\frac{1}{\mathrm{PVAF}_{n,i}}\right]$$

$$A = P \times \mathrm{CRF}_{n,i}$$

- The reciprocal of the present value annuity factor is called the ***capital recovery factor (CRF)***.

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Loan Amortisation Schedule

End of Year	Payment	Interest	Principal Repayment	Outstanding Balance
0				10,000
1	3,951	900	3,051	6,949
2	3,951	625	3,326	3,623
3	3,951	326	3,625*	0

Present Value of an Uneven Periodic Sum

- In most instances the firm receives a stream of uneven cash flows. Thus the present value factors for an annuity cannot be used.
- The procedure is to calculate the present value of each cash flow and aggregate all present values.

PV of Uneven Cash Flows: Example

- Consider that an investor has an opportunity of receiving Rs 1,000,Rs 1,500, Rs 800, Rs 1,100 and Rs 400 respectively at the end of one through five years. Find out the present value of this stream of uneven cash flows. The investor's required interest rate is 8 per cent? The present value is calculated as follows:

$$\text{Present value} = \frac{1000}{(1.08)} + \frac{1{,}500}{(1.08)^2} + \frac{800}{(1.08)^3} + \frac{1{,}100}{(1.08)^4} + \frac{400}{(1.08)^5} = Rs3{,}927.60$$

$$\begin{aligned} PV &= 1{,}000 \times \text{PVF}_{1,.08} + 1{,}500 \times \text{PVF}_{2,.08} + 800 \times \\ &= \text{PVF}_{3,.08} + 1{,}100 \times \text{PVF}_{4,.08} + 400 \times \text{PVF}_{5,.08} \\ &= 1{,}000 \times .926 + 1{,}500 \times .857 + 800 \times .794 + 1{,}100 \\ &\quad \times .735 + 400 \times .681 = \text{Rs } 3{,}927.60 \end{aligned}$$

Present Value of Perpetuity

- **Perpetuity** is an annuity that occurs *indefinitely*. Perpetuities are not very common in financial decision-making.
- The PV of an annuity is:

$$\text{Present value of a perpetuity} = \frac{\text{Perpetuity}}{\text{Interest rate}}$$

Present Value of a Perpetuity: Example

- Let us assume that an investor expects a perpetual sum of Rs 500 annually from his investment. What is the present value of this perpetuity if his interest rate is 10 per cent?

$$P = \frac{500}{0.10} = \text{Rs } 5,000$$

Present Value of Growing Annuities

- The present value of a constantly growing annuity is given below:

$$P = \frac{A}{i-g}\left[1-\left(\frac{1+g}{1+i}\right)^{n}\right]$$

- Present value of a constantly growing perpetuity is given by a simple formula as follows:

$$P = \frac{A}{i-g}$$

Example

- Assume that you earn an annual salary of Rs 1,000 with the provision that you will get annual increment at the rate of 10 per cent. It means that you shall get the following amounts from year 1 through year 5.

End of Year	*Amount of Salary (Rs)*	
1	$1{,}000 = 1{,}000 \times 1.10^0$	1,000
2	$1{,}000 \times 1.10 = 1{,}000 \times 1.10^1$	1,100
3	$1{,}100 \times 1.10 = 1{,}000 \times 1.10^2$	1,210
4	$1{,}210 \times 1.10 = 1{,}000 \times 1.10^3$	1,331
5	$1{,}331 \times 1.10 = 1{,}000 \times 1.10^4$	1,464

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Example

Present value of Salary at required rate of return of 12% is

$$P = \frac{1{,}000(1.10)^0}{(1.12)^1} + \frac{1{,}000(1.10)^1}{(1.12)^2} + \frac{1{,}000(1.10)^2}{(1.12)^3} + \frac{1{,}000(1.10)^3}{(1.12)^4} + \frac{1{,}000(1.10)^4}{(1.12)^5}$$

$$= 1{,}000 \times \frac{1}{(1.12)^1} + 1{,}100 \times \frac{1}{(1.12)^2} + 1{,}210 \times \frac{1}{(1.12)^3} + 1{,}331 \times \frac{1}{(1.12)^4} + 1{,}464 \times \frac{1}{(1.12)^5}$$

Year End	*Amount of Salary (Rs)*	*PVF @ 12%*	*PV of Salary (Rs)*
1	1,000	0.893	893
2	1,100	0.797	877
3	1,210	0.712	862
4	1,331	0.636	847
5	1,464	0.567	830
	6,105		4,309

Value of an Annuity Due

- **Annuity due** is a series of fixed receipts or payments *starting at the beginning of each period* for a specified number of periods.
- **Future Value of an Annuity Due**

$$F_n = A \times \text{CVFA}_{n,i} \times (1+i)$$

- **Present Value of an Annuity Due**

$$P = A \times \text{PVFA}_{n,i} \times (1+i)$$

Future Value of An Annuity: Example

- When you deposit Re 1 at the end of each year, the compound value at the end of 4 years is Rs 4.375. However, Re 1 deposited in the beginning of each of year 1 through year 4 will earn interest respectively for 4 years, 3 years, 2 years and 1 year:

$$F = 1 \times 1.06^4 + 1 \times 1.06^3 + 1 \times 1.06^2 + 1 \times 1.06^1$$
$$= 1.262 + 1.191 + 1.124 + 1.06 = \text{Rs}\, 4.637$$

Example

- The present value of Re 1 paid at the beginning of each year for 4 years is

$$1 \times 3.170 \times 1.10 = \text{Rs } 3.487$$

Multi-Period Compounding

- If compounding is done more than once a year, the actual annualised rate of interest would be higher than the nominal interest rate and it is called the **effective interest rate**.

$$\text{EIR} = \left[1 + \frac{i}{m}\right]^{n \times m} - 1$$

Effective Interest Rate: Example

- Suppose you invest Rs 100 now in a bank, interest rate being 10 per cent a year, and that the bank will compound interest semi-annually (i.e., twice a year). **What will your effective interest rate ?**
- Applying:

$$\text{EIR} = \left[1 + \frac{i}{m}\right]^{n \times m} - 1$$

$$\text{EIR} = \left[1 + \frac{i}{2}\right]^{1 \times 2} - 1 = \left[1 + \frac{0.10}{2}\right]^{2} - 1$$

$$= 1.1025 - 1 = 0.1025 \text{ or } 10.25\%$$

Continuous Compounding

- The **continuous compounding** function takes the form of the following formula:

$$F_n = P \times e^{i \times n} = P \times e^x$$

- Present value under continuous compounding:

$$P = \frac{F_n}{e^{in}} = F_n \times e^{-i \times n}$$

Net Present Value

- **Net present value** *(NPV) of a financial decision is the difference between the present value of cash inflows and the present value of cash outflows.*

$$\text{NPV} = \sum_{t=1}^{n} \frac{C_t}{(1+k)^t} - C_0$$

Present Value and Rate of Return

- A bond that pays some specified amount in future (without periodic interest) in exchange for the current price today is called a **zero-interest bond** or **zero-coupon bond**.
- In such situations, one would be interested to know what rate of interest the advertiser is offering. One can use the concept of present value to find out the **rate of return** or **yield** of these offers.
- The rate of return of an investment is called **internal rate of return** since it depends exclusively on the cash flows of the investment.

Internal Rate of Return

- The formula for Internal Rate of Return:

$$\text{NPV} = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} - C_0 = 0$$

- In MS Excel use rate function

RATE(nper,pmt,pv,fv,type,guess)

IRR Calculation: Example of Trial-Error Method

- Suppose your friend wants to borrow from you Rs 1,600 today and would return to you Rs 700, Rs 600 and Rs 500 in year 1 through year 3 as principal plus interest. What will be your rate of return?

Year	Cash Flow (Rs)	PVF at 8%	PV of Cash Flow (Rs)
1	700	0.926	648.20
2	600	0.857	514.20
3	500	0.794	397.00
			1,559.40

Year	Cash Flow (Rs)	PVF at 6%	PV of Cash Flow (Rs)
1	700	0.943	660.00
2	600	0.890	534.00
3	500	0.840	420.00
			1,614.00

Interpolating :

$$= 6\% + (7\% - 6\%) \times \frac{(1,614 - 1,600)}{(1,614 - 1,586)}$$

$$= 6\% + 1\% \times \frac{14}{28} = 6\% + 0.5\% = 6.5\%$$