Whole material

Linear Programming Problems:

1. What is linear programming problem? What are its basic requirements and its basic structure?

The linear programming is a method of choosing best alternative from a set of feasible alternatives, in situation where objective function and constraints can be expressed as linear mathematical functions.

Following are the requirements for applying LPP.

a) Objective function should be clearly measurable in quantitative terms. It could be for example maximization of sales, or profit or minimization of cost etc.
b) The activities could be measurable in quantitative terms.
c) The resources should be in measurable terms. (i.e. availability of the Raw material, time etc.)
d) There should be series of feasible alternative solutions from which best solution can be obtained.

2. Explain with an example different components of the Linear Programming Problem.

Let us consider the following example:

A firm is engaged in producing two products, A and B. Each unit of product A requires 2kg of raw material and 4 labor hrs. for processing, whereas each unit of product B requires 3 kg of raw material and 3 hrs. of labor of the same type. Every week the firm has an availability of 60kg of raw material and 96 labor hrs. One unit of product A sold yields Rs 40 and one unit of product B sold gives Rs 35 as profit.

Following are the three components of Linear Programming Problem:

I. The Objective Function: This is nothing but the goal of Linear Programming Problem. It can be a maximization or minimization objective function. In case of profit or sales it is maximization whereas in case of cost or time it is a minimization problem.

In the above example as we are talking about profit per unit sale of product A or B, in this problem objective function is of maximization.

Max $Z = 40X1 + 35X2$, Where $X1$= no. of units of Product A and $X2$ = no. of units of Product B

II. The Constraints: Resources are always in limited supply and the limitation is always indicated by constraints. It is nothing but the mathematical relationship which indicates limited supply of the various resources.

In the above example the constraints are as follows:

$2X1 + 3X2 <= 60$ (Raw Material constraint)

$4X1 + 3X2 <= 96$ (Labor Hour constraint)

It is interested to note that both the constraints are linear in nature. Also the sign <= indicates that some amount of resource is still left after attaining the optimality point.

III. Non-negativity condition: as we are taking X1 & X2, no. of units produce of product A and B they can not take negative values as negative production is not possible.

In the above example it is $X1>= 0$ and $X2>= 0$.

3. What are the assumptions of Linear Programming Problem?

Linear programming is based on the following assumptions.

1. Proportionality: it means that whatever profit we are getting by selling one unit of product A or B is same as simple multiple of the profit that we are getting by selling 1000 units. This means Rs.30/unit sell of A then for 1000 units it is Rs.30000 so we do not consider the concepts like Economies of Scale etc. when we are solving LPP.

2. Additivity: Another assumption is that total profit obtained using objective function is a total contribution by all the products. Similarly in constraints total use of raw material is addition of use of raw material by all the activity. This means no where we consider the effect of interaction among the resources.

Interaction is possible when some product is by product of the other product. Such situation frequently occurs during the production of chemicals.

3. Continuity: Many times solution to the LPP gives fraction values like X1= 4.25 and X2= 7.75 this indicates production of product A should be 4.25 and production of product B should be 7.75. Production in fraction is not possible so if the process is a one time process then we round up this values to X1= 4 and X2= 8 and if it is a continuous process then we can consider the fraction amount as production in process for the next production cycle. Rounding up all the times may not give us feasible solution.

4. Certainty: One more assumption is that a LPP assumes that all the parameters like profit per unit, constraints coefficient, inequality/equality of the constraints all are know with certainty. So the LPP is deterministic in nature.

5. Finite Choices: Also LPP assumes that a limited number of choices are available to decision maker and the decision variables do not assume negative values. Output cannot be negative.

4. Describe the complete procedure of finding graphical solution of LPP. What do you understand by convex region? Discuss one special case in the graphical solution of such problem.

Using graphical approach of solution only two variables linear programming problem can be solved. Following are the steps to solve problem graphically.

a) Identify the problem: it means decision variables, the objective function and constraint restrictions
b) Draw graph that includes all the constraints and identify feasible region.
c) Obtain the point on the feasible region that optimizes the objective function
d) Interpret the result

Let us consider the following example:

Maximize Z = 40X1 + 35X2   Profit

Subject to

2X1 + 3X2 <= 60 (Raw material constraint)

4X1 + 3X2 <= 96 (Labor hours constraint)

X1, X2 >= 0

Graphing the restriction: First of all we will prepare graph by considering X1 and X2 axis and draw all the constraints on that graph. For plotting first constraint i.e. 2X1 + 3X2 <= 60 we will consider equality sign and put first X2 = 0 in the equation which will give us intersection point of line with X1 axis and then we will put X1 = 0 to get intersection point of the line with X2 axis. Joining these points we are able to plot straight line of first equation on the graph. Same process we will repeat for the second constraint.

(Note: plot graph to show the actual process)

Obtaining the optimal solution: Considering the inequalities of the constraints find out the common region for all the constraints which we will call a feasible region. There are many points in the feasible region but all are not giving the optimal solution so we will consider the best points which can give the optimal solution.

(Note: show the Optimality points in the graph)

Convex Sets: If we plot all the constraints of the above mentioned example then we can get diagram which is available on (page 27 figure 2.2) then convex set is represented by OPQR region where two sides are formulated by non negativity conditions and two sides are formulated by constraints. The concept of convex set in the context of a two variable problem can be understood as follows. If any two points are selected in the region and the line segment formed by joining these two points lies completely in this region, including on its boundary then this represents a convex set. If we select two points in the feasible region and draw line connecting these points and if some portion is outside the feasible region then it is not convex feasible region. Thus for the feasible region to be convex no part of any line obtainable by joining a pair of points in that region should lie outside it. (Refer Figure 2.3 Page 28).

Some special cases in graphical solution:

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1. Multiple Optimal Solutions: In this case we get more than one point on feasible region for which objective function is optimal. There are two conditions which should be satisfied in order that multiple optimal solutions exist.

a) The objective function should be parallel to a constraint that forms an edge or boundary on the feasible region, and

b) The constraint should form a boundary on the feasible region in the direction of optimal movement of the objective function. In other word the constraint must be a binding constraint.

Ex. Maximize $Z = 8X1 + 16X2$

Subject to $X1 + X2 <= 200$

$X2 <= 125$

$3X1 + 6X2 <= 900$

$X1, X2 >= 0$

(Note: Based on the marks of the question draw graph and show the example of multiple optimal solution.)

2. Infeasibility: If solution satisfies all the constraints then it is called feasible solution. So if solution does not satisfies all the constraints simultaneously then it is called infeasible solution.

Ex. Maximize $Z = 20X1 + 30X2$

Subject to $2X1 + X2 <= 40$

$4X1 – X2 <= 20$

$X1 >= 30$

$X1, X2 >= 0$ In this case we have one common region for the first two constraints and second region for the third constraint i.e. X1 >= 30 so we can conclude that there is no common region which is common to all the constraints. This is called infeasibility. **In case of infeasibility there is no optimal solution.**

3. Unbounded solution: For a maximization type of linear programming problem, unboundedness occurs when there is no constraint on the solution so that one or more of the decision variables can be increased indefinitely without violating any of the constraints. Practically if we find that solution is unbounded then we can conclude that problem is not correctly formulated.

Maximize $Z = 10X1 + 20X2$

Subject to $2X1 + 4X2 >= 16$

$X1 + 5X2 >= 15$

$X1, X2 >= 0$

Unbounded solution means that there is no optimal solution to the give problem.

Duality

Mathematical formulation of the primal is as follows:

Maximize $Z = cx$

Subject to $ax <= b$

$X >= 0$

c = row matrix containing the coefficient in the objective function,

x = column matrix containing the decision variables

a = matrix containing the coefficients in the constraints

b = column matrix containing the RHS values of the constraints

The dual of the above primal problem:

Minimize $G = b'y$

Subject to $a'y >= c'$

$y >= 0$

b' = transpose of the b matrix of the primal problem
a' = transpose of the coefficient matrix of the primal problem
c' = transpose of the matrix of the objective function coefficients of the primal problem
y = column matrix of the dual variables.

1. What is Duality?

Linear programming problems exist in 'pairs' so that corresponding to every given linear programming problem there is another LPP.
Some important features of the primal and dual are as follows:

Primal	Dual
Maximization Problem	Minimization Problem
No. of variables in Primal	No. of constraints in dual
No. of constraints in the primal	No. of variables in the dual
<= type constraints	>= type constraints
<= type constraints	Non – negative variables
= type constraint in primal	Unrestricted variables in dual
Unrestricted variables in primal	= type constraints in dual
Objective function coefficient i.e. Ci's	RHS constant for ith constraint
RHS constant for ith constraint	Objective function coefficient for ith variable
Coefficient matrix A for primal constraints	Coefficient matrix A' for the dual constraints

- Objective function value for both the primal and dual is same.
- If one problem has unbounded solution then its dual has no feasible solution.
- Optimal solution to the dual can be read from the optimal solution of primal and vice versa. So both need not be solve.
- If primal problem is having 3 variables and 7 constraints then it involves many artificial variables which is difficult to solve, in such situation one can solve dual instead of primal where dual is of 7 variable and 3 constraints.

Economic Interpretation of Duality:

From the point of view of owner of a company **(Primal)**

A firm is engaged in producing two products, A and B. each unit of product A requires 2 kg of raw material and 4 labor hours for processing, whereas each unit of product B requires 3 kg of raw material and 3 hours of labor, of the same type. Every week the firm has an availability of 60 kg of raw material and 96 labor hours. One unit of product A sold yields Rs. 40 and one unit of product B sold gives Rs. 35 as profit.

Maximize $Z = 40X1 + 35X2$ (profit)
Subject to $2X1 + 3X2 <= 60$ (Raw material constraint)
$4X1 + 3X2 <= 96$ (Labor hours constraint)
$X1, X2 >= 0$ (Non negativity restriction)

In the above mentioned problem owner wants to increase profit as much as possible by considering the given constraints. The optimal solution to this problem indicates that producing 18 units of A and 8 units of B per week would yield the maximum profit equal to Rs. 1000.

From the point of view of the hirer of a company **(Dual)**

Now suppose that the firm is approached by an individual who would like to rent the facilities of the firm for one week. The firm has its assets in the form of 60 kg of raw material and 96 labor hours. If we let y1 represent the rental rate per kg of raw material and y2 the rental rate per labor hour, the firm would receive a total rental equal to $60y1 + 96y2$. We shall compute the minimum value of the rental so that the firm will know as to what minimum offer shall be economically acceptable to it. The constraints can be set up by

keeping in mind that the alternative of renting must be at least as favorable as the other ones. The rental rates of the resources should be at least as attractive as producing products A and B. We know that production of one unit of A requires 2 kg of raw material and 4 labor hours. Thus the total rental for these amounts of resources should be greater than or equal to the profit obtainable from one unit of the product i.e. Rs. 40 Thus we should have 2y1 + 4y2>= 40 (Product A) in similar way we can write 3y1 + 3y2 >= 35
Thus the dual problem will be,
Minimize G = 60y1 + 96y2
Subject to 2y1 + 4y2>= 40
3y1 + 3y2 >= 35 y1, y2 >= 0

Transportation Problem

1. Prove that Transportation Problem is special case of Linear Programming Problem. OR Give the mathematical formulation of a transportation problem.

For proving this let us consider the following transportation problem.

Source	**Destination**				**Supply**
	1	**2**	**3**	**4**	
1	15	18	22	16	30
2	15	19	20	14	40
3	13	16	23	17	30
Demand	20	20	25	35	100

If we want to solve the same problem using linear programming method then we have to check that it is possible to write LPP formulation or not. If possible then we can conclude that Transportation Problem is a special case of Linear Programming Problem. Mostly it will be a minimization problem as we want to reduce transportation cost.

Minimize Z = 15X11 + 18X12 + 22X13 + 16X14 + 15X21 + 19X22 + 20X23 + 14X24 + 13X31 + 16X32 + 23X33 + 17X34 (Objective function)

Subject to
X11 + X12 + X13 + X14 = 30
X21 + X22 + X23 + X24 = 40
X31 + X32 + X33 + X34 = 30 (Supply and Demand Constraints)
X11 + X21 + X31 = 20
X12 + X22 + X32 = 20
X13 + X23 + X33 = 25
X14 + X24 + X34 = 35
Xij >= 0 i = 1 2 3 4, j = 1 2 3 (Non negativity condition)

Above formulation contains Objective function, Subject to constraints and Non negativity constraint. So we can conclude that all the transportation problem can be solved using linear programming method.

2. What is Degeneracy in Transportation problem solution? Explain the whole process of removing degeneracy from the solution.

We know that basic feasible solution of transportation problem contains m+n-1 occupied cells. Which mean that numbers of occupied cells are one less than the total number of rows and columns. Sometimes it may happen that no. of occupied cells are less than m+n-1 and such solution is called degenerate solution.

- Degeneracy may occur at the first instance when we obtain initial basic feasible solution or
- When we are trying to obtain optimal solution to the given problem.
- If degeneracy has occurred then it is not possible to find out all ui and vj values and thus not possible to find out optimal solution.

- § To overcome degeneracy we can add an infinitesimally small amount close to zero to one empty cell and treat the cell as an occupied cell. This is called epsilon denoted by e.
- § Epsilon is very small in amount but enough to cause a change in the total cost and the other non zero amount.
- §
- § k + e = k e + e = e
  k – e = k e – e = 0
  0 + e = e k * e = 0
- § It is important to remember that an epsilon cannot be placed in any randomly selected unoccupied cell.
- § The cell chosen for inserting epsilon must be an independent cell, originating from which a closed loop cannot be traced.
- § When more than such unoccupied independent cells are available then the cell with lowest value may be selected.

3. What is the process of tracing a closed loop for finding optimal solution of Transportation Problem?

Once the basic feasible solution is obtained attempt is to be made to find out optimal solution. For finding optimal solution one may consider few empty cells.

- § Once empty cell is selected then moving clockwise draw an arrow from this cell to an occupied cell in the same row or column as the case may be.
- § After that move vertically or horizontally to another occupied cell and draw an arrow.
- § Again move horizontally or vertically to another occupied cell and continue the procedure until reaching the original empty cell.
- § In the process of moving from one occupied cell to another a) move only horizontally or vertically, but never diagonally, and b) step over empty and if the need be over occupied cells without changing them. Thus loop always have right angled turns with corners only on the occupied cells.
- § After drawing a close loop place plus and minus signs alternately in the cells on each turn of the loop beginning with plus (+) sign in the empty cell.

The following points may also be considered while drawing a close loop.

- At even no. of at least four cells must be participate in the close loop.
- All cells having either + or – sign must be occupied cells.
- Closed loop may or may not be square or rectangular in shape. In large transportation tables loop may cross over.
- Movement on the path may be clockwise or anti clockwise, in any case result will be the same.

4. Which are the methods of finding IBFC (Initial Basic Feasible Solution) to the transportation problems?

[Note:

- To write answer of this question explains all the three methods in detail with the help of example. Methods are 1) North West Corner rule 2) Least Cost Method 3) Vogal's approximation method.
- Also explain that least cost method is better than North West corner rule as importance is given to the least cost cell in least cost method where as in North West corner rule cell is selected without any logic. Also write that vogel's approximation is better than least cost method as we are considering the difference of the least cost and this method is more structured than other two methods.
-

5. What do you mean by the following terms?

1. Unbalanced transportation problem: In this problem total demand is not equal to total supply. If problem is unbalanced then it is not possible to find out solution, so first we have to make it balanced by adding either dummy row or column i.e. source or destination as the case may be. (give one example to show the situation)

2. Restricted route: there are some routes which are prohibited because of very high cost of transportation or due to any other reason. To avoid this route we have to keep 'M' in the cell. Where M is very large number. By doing this we can avoid this route from the calculation. (give one example to show the situation).

Travelling Salesmen Problem:

1. What is travelling salesman problem?

Travelling salesman problem is like, A salesman is assigned n cities to visit. He is given distance/ time/ cost between all pairs of cities and instructed to visit each of the cities once in a continuous trip and return to the original, using the shortest route. In this context, we define the visit to the cities in a sequence, ending in the same cite where it begins, including a visit to each cities only once and this will form a tour. He can start from any of the city.

<u>The method used for the travelling salesman problem is call 'Branch and Bound technique'</u>. For the understanding of the entire process consider the following hypothetical example

From Item	To			
	A	**B**	**C**	**D**
A	-	4	7	3
B	4	-	6	3
C	7	6	-	7
D	3	3	7	-

Step 1: In travelling salesman problem the very first step is to replace ' - ' by 'M' where M is very big number, this is because salesman does not travel from territory A to territory A. After doing this the more important is to find out Upper Bound of the problem. Upper Bound indicates that in any case solution will not exceed the given limit.

Upper Bound can be obtain using the sequence in its natural order. i.e. A-B-C-D-A = 4+6+7+3 = 20

Step 2: once upper bound is obtained efforts can be made to find out lower bound using Hungarian algorithm. During this process following out comes are possible.

I. Continuous path may be obtained in the first attempt. For eg. B-A-D-C-B which may have length equal to 20 (i.e. UB) or less then 20 in any of the case we may conclude that optimal solution has obtained and stop procedure
II. Second possibility is that we may obtain discontinuous path like A-D-A & B-C-B. In such situation go to next step.

Step 3: If we obtain two sub path as mentioned in the previous step then find out that which path is having minimum length. i.e. A-D-A or B-C-B. Suppose A-D-A is having length 6 and B-C-B is having length 12. Total of this two is 18 which will be called Lower Bound (LB) of the solution. This implies that now whatever solution we will obtain in the subsequent steps will fall between 20 and 18.

As A-D-A is having length 6 we will restrict first A-D path by putting M and solve entire matrix using Hungarian method to find out optimal solution, if we get continuous path then tentative optimal solution is obtained other wise repeat same process again.

In the next step we may block D-A by putting M and solve entire matrix using Hungarian method to find out optimal solution. If continuous path is obtained then the desire solution is obtained otherwise repeat same process again.

Compare length of continuous paths which we have obtained in the last step. Consider the path having minimum length and call it final optimal solution to the given problem. For eg. final path may be like A-C-D-B-A whose length is 19 which is between UB (20) and LB (18).

Assignment Problem:

1. What is an Assignment Problem? Is it true to say that it is a special case of the transportation problem? Explain.

Assignment problem is a special type of linear programming problem. There are many situations where the assignment of people, machine and so on to be done on one to one basis. Assignment of workers to machines, Clerks to various checkout counters, salesmen to different sales areas, service crews to different districts are the typical examples. Assignment is a problem because different people are having different ability to finish different set of work. Few people can take more time and few may take less time to finish work. Thus in the assignment problem efforts are made to find out solution to the problem in such a way that cost of assignment is minimum. i.e. if time is given then minimum time is required, if cost is given then minimum cost is involved. [Give one example of assignment problem.]

It is true to say that it is a special case of the transportation problem because one job is assigned to only one worker and one worker does not have more than one job in any case. So total no. of jobs are same as total no. of worker so total supply and total demand is same. Consider the following assignment problem.

Clerks	**Jobs (Time taken in hrs.)**			
	A	**B**	**C**	**D**
I	4	7	5	6
II	6	8	7	4
III	3	5	5	3
IV	6	6	4	2

Here we have four jobs and four clerks. We have to find solution in such a manner that each clerk should get only one job and total time taken should be minimum. By making some modification same problem can be converted to the transportation problem as follows,

Clerks	**Jobs (Time taken in hrs.)**				
	A	**B**	**C**	**D**	**Supply**
I	4	7	5	6	1
II	6	8	7	4	1
III	3	5	5	3	1
IV	6	6	4	2	1
Demand	1	1	1	1	**4**

The above form is called transportation problem where demand is same as supply. This problem can be solved using any of the method which can be applied to the transportation problem.

2. How can you formulate an assignment problem as a standard linear programming problem? Illustrate.

Linear Programming Problem contains three main parts, 1. Objective Function, 2. Subject to constraints and 3. Non negativity condition.

So if we are able to write assignment problem such that it contains all the three parts of linear programming problem then it can be conclude that assignment problem is a special case of linear programming problem.

Consider the following example,

Clerks	Jobs (Time taken in hrs.)			
	A	**B**	**C**	**D**
I	4	7	5	6
II	6	8	7	4
III	3	5	5	3
IV	6	6	4	2

X_{ij} = ith clerks has assigned jth job.

Objective Function:

Minimize $Z = 4X_{11}+7X_{12}+5X_{13}+6X_{14}+6X_{21}+8X_{22}+7X_{23}+4X_{24}+3X_{31}+5X_{32}+5X_{33}+3X_{34}+6X_{41}+6X_{42}+4X_{43}+2X_{44}$

Subject to constraints,

$X_{11}+X_{12}+X_{13}+X_{14} = 1$ $\quad$ $X_{11}+X_{21}+X_{31}+X_{41} = 1$

$X_{21}+X_{22}+X_{23}+X_{24} = 1$ $\quad$ $X_{12}+X_{22}+X_{32}+X_{42} = 1$

$X_{31}+X_{32}+X_{33}+X_{34} = 1$ $\quad$ $X_{13}+X_{23}+X_{33}+X_{43} = 1$

$X_{41}+X_{42}+X_{43}+X_{44} = 1$ $\quad$ $X_{14}+X_{24}+X_{34}+X_{44} = 1$

Non negativity condition

$X_{ij} = 1$ if jth job is assign to ith worker

$X_{ij} = 0$ if jth job is not assign to ith worker

Considering the above steps we can conclude that assignment problem is a special case of linear programming problem.

3. What are the different methods of solving Assignment problem? Which one is best? Why?

There are four different methods using which one can solve assignment problem.

1. Complete Enumeration Method
2. Transportation Method
3. Linear Programming Method
4. Hungarian Algorithm

1. Complete Enumeration Method:

In this method we consider all possible assignments and select the one which is having either minimum cost or maximum profit. For eg. let us consider a problem having two jobs J1 & J2 and two workers W1 & W2 then two assignments are possible, J1-W1, J2-W2 & J1-W2, J2-W1. Over all we can conclude that if it is a 3 jobs 3 worker problem then we will have 3! Combinations which are equal to 6 combinations and for n jobs and n workers problem it will be n! combinations.

This method gives perfect result but as no. of jobs are increases it becomes very difficult to find all the assignments and thus the optimal solution.

2. **Transportation Method:** [This part is already explained in question 1 one can write that part here.]
3. **Simplex Method:** [This part is already explained in question 2 so one can take help from that part.]
4. **Hungarian Method**: From the efforts point of view Hungarian algorithm is better as compare to above three methods as they involve many iterations. Using Hungarian algorithm one can find comparatively better result but not as accurate as Complete Enumeration Method.

   Hungarian Algorithm is available in text book as well as class note book so one may use that material to answer this question.

4. What do you mean by following terms?

a) Unbalanced Assignment Problem: When no. of jobs and no. of workers are not equal then that problem is called unbalanced assignment problem. To make such problem we have to add either dummy row or column whatever is required. Dummy row or column can be added by taking all the values equal to zero. Then usual process of Hungarian Algorithm can be applied.

Workers	Jobs (time in minutes)		
	J1	J2	J3
W1	20	18	16
W2	25	14	11

Above problem is unbalanced as there are 3 jobs and 2 workers so we have to add one dummy worker row.

b) Restricted Assignment Problem: When some task cannot be performed by some worker then that assignment problem is called restricted assignment problem. To restrict assignment in Hungarian method we can put 'M' at specific place where 'M' means very large number. Obviously Hungarian algorithm deals with minimization problem so assignment having 'M' will not be considered for the calculation.

Workers	Jobs (time in minutes)		
	J1	J2	J3
W1	20	18	M
W2	M	14	11
W3	15	17	18

In the above assignment problem job 1 cannot be performed by worker 2 and job 3 cannot be performed by worker 1 so it is a type of restricted assignment for which we have kept 'M' at specific places.

c) More then one optimal solution: Sometimes it is possible to have more than one optimal solution to the assignment problem.

Sales Representatives	Sales Territories			
	I	II	III	IV
A	200	150	170	220
B	160	120	150	140
C	190	195	190	200
D	180	175	160	190

The above problem is having more than one optimal solution.

Sensitivity Analysis

1. Explain the concept of sensitivity analysis. What are different factors affecting the given solutions and how do we resolve them? Give brief comment on each of them.

Sensitivity Analysis: Optimal solution is always found under deterministic assumptions. This means that we assume complete certainty in the data and relationship of variables- for eg. Prices are fixed, resources are known, and time needed to produce is known. But in real world available resource and prices are changes frequently and thus this approach is less useful.

Sensitivity analysis is useful to check how sensitive is the optimal solution to changes in profits, resources, or other input parameters? An important function of sensitivity analysis is to allow managers to experiment with values of the input parameters.

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Sensitivity analysis is used to examine the effects of changes in three factors:
1. Contribution rates for each variables (profit or cost coefficient in the objective function)
2. Technological coefficients (numbers-multipliers in the constraints, left hand side)
3. Available resources (the right hand side quantities in each constraint)

Sensitivity analysis is alternatively called post optimality analysis or parametric programming.
Sensitivity analysis involves what-if? type of questions. What if the profit on one product increases, what if available labor hours increases etc.

There are mainly two approaches to determining, how sensitive an optimal solution is to changes. The first one is simply trial and error approach. This approach usually involves resolving the entire problem preferably by computer, each time when parameter is changed. It takes long time.

The approach we prefer is the analytic post optimality method. After an LP problem has been solved, we attempt to determine a range of changes in problem parameters that will not affect the optimal solution or change the variables in the solution. This is done without resolving the whole problem.

To understand the different factors affecting the given solution we consider following example:
The Abc Ltd. company manufactures quality compact disc (CD) players and stereo receivers. Each of these products requires a certain amount of skilled artisanship, of which there is a limited weekly supply. The firm formulates the following LP problem in order to determine the best production mix of CD players(X1) and receivers(X2).

Maximize profit = Rs. 50X1 + Rs. 120X2

Subject to 2X1 + 4X2 <= 80 (hours of electricians' time available)

3X1 + 1X2 <= 60 (hours of audio technicians' time available)

X1, X2 >= 0

If this problem is graphical solved then the solution to the problem is X1 = 0 and X2 = 20 this means produce 20 units of receivers and zero unit of CD players. Maximum profit is 120*20 = Rs. 2400.

1. Change in the Objective Function Coefficient:

In real life problems, contribution rates (Usually profit or cost) in the objective functions fluctuate periodically, as do most of a firm's expenses. We can see that in graphical solution feasible solution region remains exactly the same but value of the objective function will change as multipliers are changing.

For eg. if profit contribution of receiver changes from Rs.120 to Rs.150 then objective function value changes to Z = 150*20 = Rs. 3000

Here there is no need to find out new solution. Whatever optimality points we have got in the original solution, same points can be used to find out new solution. If change in price is small then may be optimality point will remain same.

2. Change in the technological coefficients:

By technological changes we mean change in coefficients of the constraints on the left hand side of the constraints. Sometimes one or more technological coefficients change simultaneously.

In the given example CD players required 3 hours for audio technicians' time, if it reduces to 2 hours then definitely solution will change. Slope of the second constraint will change and because of that feasible region will get changed. By drawing new line for the changed constraint we may have new feasible region and thus optimal solution.

3. Changes in the resources or right hand side values:

Changes in the resources are nothing but the changes in right hand side of the constraints. Right hand side of the constraint gives the total availability of the resources.

In the above example 80 hours of electricians'' time is available. If this time increase to 100 hours and extra 20 hours are allowed at higher cost (over time scale to the employees) then entire solution may get changed.

In such situation we have to draw entire constraint again as it will get changed, and as a result of that optimal solution will be changed.

Quantitative Techniques in general

1. "All quantitative techniques have hardly any real life applications." Do you agree with the statement? Discuss.

I disagree with this statement as we can see from the following discussion that all the quantitative techniques has real life applications, may be with some limitations sometime but we can not find any quantitative technique which has no real life application. To support this argument consider following points along with examples.

1. Probability: Concept of probability is widely used in almost all the areas now days. For eg. Weather forecasting, predicting market share of any brand, predicting arrival rate of the customers etc.
2. Linear programming problem: This is widely used when constraints are given and we suppose to maximize profit or minimize cost or utilization of the resources. (Give some example)
3. Goal programming problem: When we have more than one goal then we can use goal programming problem. (Give some example)

4. Transportation problem: Here the ultimate objective is to reduce transportation cost. It is widely used in transporting material from source to destination. (Give some example)
5. Assignment problem: It is a problem where the objective is to minimize total cost of doing work or to maximize total profit by making optimal assignment. (Give some example)
6. Travelling salesmen problem: Using this concept many problem can be solved in marketing area where scheduling of salesmen is very much required and that to also at minimum cost. (Give some example)
7. Queuing Theory: Where ever there is queue knowledge of this technique is useful. We frequently come across a situation where we are supposed to stand in a queue or we are supposed to manage queue. (Give some example)
8. Simulation Models: Instead of performing experiment on real setup it is always desirable to perform experiment on models and use result of these experiments for the implementation purpose. (Give some example)
9. Markov Chains: Using this concept one can predict future market share of a particular product and one can have knowledge about the probability of brand switching in future time points. (Give some example)

From the above discussion it revealed that quantitative techniques always have real life applications.

[Note:

1. It is not necessary to give whole example of assignment or transportation problem instead of that you may discuss the situation where it is applicable otherwise answer will be too long compare to its mark.

2. Such type of open ended questions may be asked, in which you are suppose to give your opinion based on your knowledge so don't leave it.]

2. Discuss the role and scope of quantitative methods for scientific decision making in business management.

(Note: Points are discussed here, more material and examples you can add on your own.)

Whole material

Quantitative methods for scientific decision making in business management is mostly based on the formulation of the mathematical models.

Mathematical models are having following advantages:

1. Models can accurately represent reality. If properly formulated a model can be extremely accurate. A valid model is one that is accurate and correctly represents the problem.
2. Models can help a decision maker formulate problems.
3. Models can give us insight and information. For eg. Using profit model we can see what impact changes in revenues and expenses will have on profits.
4. Models can save time and money in decision making and problem solving. It usually takes less time, efforts, and expense to analyze a model.
5. A model may be the only way to solve some large or complex problems in a timely fashion. A large company for example, may produce literally thousands of size of nuts, bolts, and fasteners. The company may want to make the highest profits possible given its manufacturing constraints. A mathematical model may be the only way to determine the highest profits the company can achieve under these circumstances.
6. Model can be used to communicate problems and solutions to others.

Quantitative techniques mainly concerned with the techniques of applying scientific knowledge besides the development of science. It provides an understanding which gives the expert/manager new insights and capabilities to determine better solutions in his decision making problems with great speed, competence and confidence.

Finance, Budgeting and Investment

I. Cash flow analysis, long range capital requirement, dividend policies
II. Credit policies, credit risks
III. Claim and complaint procedure.

Marketing

I. Product selection, timing, competitive actions.
II. Advertising media with respect to cost and time
III. Number of salesmen, frequency of calling of account
IV. Effectiveness of market research

Physical Distribution

I. Location and size of warehouses, distribution centers
II. Distribution policy.

Purchasing, Procurement and Exploration

I. Rules of buying
II. Determining the quantity and timing of purchases
III. Equipment replacement policies

Human Resource

I. Forecasting the manpower requirement, recruitment policies and job assignments
II. Selection of suitable personnel

Production

I. Scheduling and sequencing the production run by proper allocation of machines.
II. Calculating the optimum product mix.
III. Selection, location and design of the sites for the production plant.

Research and Development

I. Reliability and evaluation of alternative design
II. Control of developed projects
III. Co ordination of multiple research projects.
IV. Determination of time and cost requirements.

Above mentioned are the applications of quantitative techniques in various areas of business.

Integer Programming Problem

1. What is Integer Programming Problem? What are the various types of Integer Programming Problem?

Integer Programming Problem is a special case of linear programming problem where all or some variables are restricted to take non negative integer value. In real life situation we come across a problem where non integer solution is impractical. For eg. allocation of goods to trucks wagons or aircrafts, in capital budgeting selection of project etc. are the situations where integer programming problem is applicable. Formulation part of the integer programming problem is same as linear programming problem only difference is that along with non negativity condition we have to add integer variable condition.

Integer programming problem are of three types:

1. Pure Integer Programming Problem: In such integer programming problem all the variables are restricted to take integer variables.

For eg.

Maximise Z = 20X1 + 8X2

Subject to 5X1 + 7X2 <= 63

3X1 + 5X2 <= 42

X1, X2 >= 0 and Integer

This is pure integer programming problem.

2. Mixed Integer Programming Problem: in such problem few variables are allow to take any positive integer value but few are allow to take only positive integer value.

For eg.

Three machines are available to make 5000 units which are required for the internal use of a company as components to another product. If the production is to be done by any one of these machines, a set up cost is incurred apart from the cost of making each unit on different machines. The cost data are as follows:

Machine	Set Up Cost (Rs.)	Cost per Unit (Rs.)	Maximum Production
1	8000	5	4000
2	5000	4	3000
3	4000	8	1000

Minimise Z = 8000d1 + 5000d2 + 4000d3 + 5X1 + 4X2 + 8X3

Subject to X1 + X2 +X3 >= 5000

X1 <= 4000d1

X2 <= 3000d2

X3 <= 1000d3

X1, X2, X3 >= 0 d1, d2, d3 = 0 or 1 (i.e. Integer variables)

3. Zero – One Integer programming problem/ Binary Integer programming problem

In such integer programming problem variables are restricted to take either of the two value i.e. 0 or 1. Any assignment problem is an example of the Zero-one or Binary integer programming problem.

For eg.

Clerks	**Jobs (Time taken in hrs.)**			
	A	**B**	**C**	**D**
I	4	7	5	6
II	6	8	7	4
III	3	5	5	3
IV	6	6	4	2

X_{ij} = ith clerks has assigned jth job.

Objective Function:

Minimize Z = 4X11+7X12+5X13+6X14+6X21+8X22+7X23+4X24+3X31+5X32+ 5X33+3X34+6X41+6X42+4X43+2X44

Subject to constraints,

X11+X12+X13+X14 = 1	X11+X21+X31+X41 = 1
X21+X22+X23+X24 = 1	X12+X22+X32+X42 = 1
X31+X32+X33+X34 = 1	X13+X23+X33+X43 = 1
X41+X42+X43+X44 = 1	X14+X24+X34+X44 = 1

Non negativity condition

X_{ij} = 1 if jth job is assign to ith worker

X_{ij} = 0 if jth job is not assign to ith worker

As all variables can take either of the two values i.e. 0 or 1.

Goal Programming Problem

1. What is Goal Programming Problem? Explain its distinct features compare to Linear Programming Problem.

In traditional linear programming problem models include only one goal at a time i.e. maximization of profit or minimization of cost or other resources like time, raw material etc. but often there are situations where we are suppose to achieve more than one goal at a time. For example profit maximization is one goal but along with this goal we may have a goal of employees' retention and minimization of environmental pollution. Linear programming problem cannot help us in such situation because we have multiple goals, some are maximization type and some are minimization type.

Above mentioned situation can be handled by Goal programming problem. Features of the Goal programming problem are as follows:

- Goal programming allows handling multi-objective situations at a time.
- It uses the concept of penalties in the format of linear programming problem.
- For the application of Goal programming problem a target value (goal) is set for each of the objectives/goals.
- Since all goals are unlikely to be satisfied simultaneously a penalty is assigned for each unit of deviation from the target value in each direction.
- Thus revised linear programming problem using deviational variables is formulated whose optimal solution comes as close as possible to achieving the stated goals in the sense that it minimizes the sum of penalties incurred for under achieving or over achieving the various goals.
- Thus in goal programming problem objective function is minimization/maximization type where we suppose to minimize/maximize deviations from the goal. Sometimes deviations are negative and sometimes they are positive.
- Goal programming problem may be with single goal or multiple goals.
- Goal programming problem can be categorize in two ways, Preemptive and non preemptive goal programming.
- In non preemptive goal programming weight is not assigned to the goals, thus we treat all the goals equally.
- In preemptive goal programming weight is assigned (i.e. P1, P2, P3) as per the importance of the goal. Thus more weight is given to the goal which is more important and less weight to the goal which is less important.

For eg.

Whole material

Refer example given on page no. 395 & 398 (N D Vohra) example no. 7.20, 7.22. Important as they are different type of examples.

Queuing Theory

1. What is Queuing problem? Explain the general structure of queuing system.

Queuing theory is also known an waiting line theory, and it was developed by A K Erlang's to analyze telephone traffic congestion with a view to satisfying the randomly arising demand for the services of Copenhagen automatic telephone system in year 1909. In real life there are many places where we can find customers and server. At many places we can see the formation of queue where customers are waiting for their turn to get service.

The waiting lines develop because the service to the customer may not be given immediately as the customer reaches the service facility. Thus lake of adequate service facility would cause waiting lines of customers to be formed.

Few examples of queue are as follows:

- Sales of theatre tickets
- Banking transaction
- Transfer of electronic messages
- Flow of automobile traffic through a road network
- Calls at police control room

General structure of queuing system is as follows:

1. **Arrival process**: the arrivals from the input population may be classified on different bases as follows:

a) According to sources: There are two types of sources of arriving population, one is Infinite source (Population of Baroda city arriving at railway ticket window) and second is finite source (no. of machines in a company which are moving to the maintenance department)

b) According to numbers: customers may arrive for service individually (at library of a college) or in a group (at restaurant or movie)

c) According to time: customers may arrive in the system at known times or they may arrive at random way.

2. **Service System:**

a) Structure of the service system

i) a single service facility: where there is only one server and one queue (in bank there may be only one window for depositing money)

ii) Multiple, Parallel Facilities with Single Queue:

Here there are many server but only one queue so person can decide about the server (in service station of the automobile such situation can be observed).

iii) Multiple, Parallel Facilities with Multiple Queues:

In this model each server has different queue. For example different cash counters for paying electricity bills.

b) Speed of Service:

Speed of the service can be mentioned two ways as follows,

1^{st} approach is - Server is able to provide service to the 6 customers in an hour.

2^{nd} approach is – Server takes 10 minites to provide service to each customer.

Both the statements carries same meaning but the only difference is that, 1^{st} statement gives rate in terms of number of customers but the second statement gives time (rate) of providing service to one customer.

3. Queue Structure:

a) First Come First Served:

In this structure service is provided on the basis of the arrival order of the customer. If customer has arrived first then he will get first chance for the service. This structure can be seen at most of the places.

b) Last Come First served:

Sometimes service is provided in reverse order of the arrival. i.e. the one who has come last will get service first and vice versa. This generally happens in case of files arrive at some department of office for the clearing purpose.

c) Service in random order:

Here the customer is chosen on random basis. So no such order is followed.

d) Priority service:

Here the service is provided on the basis of priority. For example in hospital if there is a queue and some serious patient has arrived then service is provided to that patient first as it is a priority.

2. What are the operating characteristics of queuing system? Also explain in detail deterministic and probabilistic queuing model that you have studied. / Explain (M/M/1) Model of queuing theory.

Analysis of queuing theory is based on the various characteristics of queuing system which are as follows:

1. Queue length- the average number of customers in the queue waiting for getting service is called queue length. Large queue indicates poor service capacity.
2. System length- summation of the average number of customers in the queue and the customers who are getting service is called system length.
3. Waiting time in the queue- the average time that customer spend in a queue for waiting for his/her turn is called waiting time in the queue.
4. Total time in the system- it is the summation of average time that customer spends in a queue and for getting service.
5. Server idle time- it is the time for which server has no customer in the system to provide service. Less server idle time indicates better utilization of the capacity.

Queuing Models can be divided in two parts: 1. Deterministic Queuing Model, 2. Probabilistic Queuing Model.

Deterministic Queuing Model: In deterministic queuing model Arrival rate i.e. lambda and Service rate i.e. Mu is known with certainty. In reality there are very few situations where these two factors are known with certainty. Because of this reason use of this model is very limited.

There are two possibilities,

Lambda > Mu : this implies that arrival rate is greater than service rate and waiting line will increase indefinitely and server will remain busy every time.

Lambda <= Mu: in this case there will be no waiting time and for proportion of the time service facility would be idle.

Probabilistic Queuing Model: These are as follows,

a) Poisson-exponential, Single server model- infinite population,
b) Poisson-exponential, single server model-finite population and,
c) Poisson-exponential, multiple server model-infinite population.

Out of this three our focus is mainly on first two models.

Probabilistic models are called Poisson-exponential because arrival of the customers to the server follows Poisson distribution where as service time for the customer follows exponential distribution. Poisson distribution deals with discrete random variable where as exponential distribution deals with continuous random variables.

A) Poisson-exponential Single server model- Infinite population: Assumptions of this model are as follows:

I. The arrivals follow Poisson distribution with a mean arrival rate of say lambda.

II. The service time has exponential distribution. And the average service rate is Mu.
III. Arrivals are from infinite population.
IV. The customers are served on a first-come-first-served basis.
V. There is only a single service station.

Example of this model is Railway ticket booking window. In reality population is not infinite in this case but for a single server population of city like Vadodara can be considered as infinite population.

Write all the formulas with explanation. i.e. Po, Pn, Ls, Lq, Ws, Wq etc.

B) Poisson-exponential Single server model- Finite population: Assumptions of this model are same as the assumption mentioned above; the only difference is that instead of infinite population now we have finite population.

Example of this model is maintenance department of a company. Let say there are only five machines in the company and if any machine goes out of order then it will go to maintenance department for repair. Here population is finite and it is only of five machines.

Write all the formulas with explanation. i.e. Po, Pn, Ls, Lq, Ws, Wq etc.

3. Explain in detail the cost involved in the queuing problem. How one can find out tradeoff between these costs?

Queuing theory is ultimately useful for providing hassle free service to the customers by utilizing available resources optimally.

Costs involved in the queuing problem are mainly of two types: 1. Cost of serving, 2. Cost of waiting. Both the costs are inversely related to each other. This means if cost of serving increases then cost of waiting reduces and vice versa. So the important task is to find out tradeoff between these two such that total cost is minimum.

1. Cost of serving: this involves cost like setting up physical infrastructure as well as salary to the person who is providing service. If to increase the capacity of the system one more server is set then such costs add to the total cost.

2. Cost of waiting: this involves cost of loss of goodwill; if customer changes the service provider instead of waiting in the queue then it is called cost of waiting. If machines are waiting for a long to get repair then ultimately we are loosing resource in terms of machine capacity. Similarly if employee is waiting in a queue in purchase department for getting necessary raw material or tools then we are losing capacity in terms of employee out put.

If we try to reduce cost of waiting then obviously we have to add more servers which will increase the cost of serving and vice versa.

Draw diagram available in Figure 10.3 (Determination of Optimal Service Level) available at page no. 534. (This is must.)

Simulation Modeling

1. What is Simulation? Describe process of simulation and mention advantages and disadvantages of Simulation.

Simulation has wide utility that we know, for eg if we talk about production unit then 'Boing Aircraft Manufacturing Company' prepare model before preparing actual aircraft. They are testing it under different condition and draw conclusions about the design of the air craft.

Simulation does not provide solution to the management problem but it can give picture which is very close to the actual situation. This is because we make use of random nos. in simulation process. So ultimately to study some probabilistic situations we can use simulation concept.

"The idea behind simulation is to duplicate (carbon copy) a real world situation with a mathematical model that does not affect operations." For eg. When we start new counter for the selling of ticket we don't now

the pattern of arrival but with the help of some historical data and simulation technique we may duplicate the actual situation and can avoid some difficulties in near future.
The seven steps for the simulation are as follows:

1. Define Problem (i.e. problem is of inventory management or queuing theory)
2. Introduce important variables (i.e. Variables under study like no. of people arrival in an hour etc.)
3. Construct simulation model (For eg. One may construct simulation model using Montecarlo simulation technique)
4. Specifies values of variables to be tested (Decide about the intervals of Random Variables)
5. Conduct the simulation (Create probability interval and select random nos., on the basis of selected random nos. try to simulate the whole phenomenon)
6. Examine the results (Examine out comes of the entire process and do necessary calculation to further crystallize the situation.
7. Select best course of action (On the basis of simulated situation one may take the important decision.)

Advantages and Disadvantages of simulation:
There are eight advantages of simulation make it one of the most widely used quantitative analysis tool.

Advantages:

1. It is relatively straightforward and flexible. It is easy to calculate and du to this reason it is possible to compare many scenarios side by side.
2. Because of the advance software not it has become very easy to simulate the given complex situations.
3. It can be used to analyze large complex real world problems for which mathematical model cannot be build, like effect of economic, social, environmental and political factors on some new business. Simulation can be successfully used for the modeling of hospitals, educational systems, national and state economies and food systems.
4. It allows what-if? Types of question, in which managers are mostly interested as they want to know the future situation in advanced.
5. Using simulation experiments can be done with the model and not with the real system. So in case any damage is made then it is always to the model only. For eg. to implement new system in the hospital it is not necessary to implement it practically but it can be implemented on paper with the help of simulation and can have some idea about the out comes.
6. It allows to study interactive effect of the variables.
7. Time compression is possible. Effect of demand for the products for many months or for many years can be obtained in short time with the help of computer.
8. Real world complications can be avoided using simulation. For eg. Queuing model requires Poisson arrivals and exponential service time but simulation has no such limitations as it can work with user define probability distributions.

Disadvantages:

1. Simulation models for the complex situation are very expensive and time consuming. It may sometimes require months or years.
2. Simulation does not generate optimal solutions to the problem like LPP, PERT, CPM etc. Simulation is a trial-and-error approach that can produce different solutions in repeated runs.
3. Each simulation model is unique and its solution can not be transfer to the other condition directly.
4. Managers must generate all of the conditions and constraints for solutions that they want to examine as answer is not produce by itself as in case of LPP.

2. What is Monet Carlo Simulation? Explain with an example all the steps of Monte Carlo Simulation.

When a system contains elements which contains chance factor in their behavior, the Monte Carlo method of simulation can be applied.

Monte Carlo Simulation: the basic idea in this is to generate values for the variables which are forming model. There are many variables in the system which are probabilistic in nature and which can be simulated and study using Monte Carlo Simulation model. Few examples are as follows:

1. Inventory demand on a daily or weekly basis
2. Lead time for inventory orders to arrive
3. Times between machine breakdowns
4. Service times
5. Times to complete project activities

In the above examples some variables are discrete like demand of the product where as some are continuous like time between machine breakdowns. For discrete variables we have to generate discrete random variables where as for continuous variables we have to generate continuous variables.

Five steps of Monte Carlo Simulation Model are as follows:

1. Establishing probability distributions for important input variables
2. Write cumulative probability distribution for each variable in step 1
3. Establishing an interval of random numbers for each variable
4. Generating random numbers
5. Simulating a series of trials

Let us understand all the five steps with the help of simple example:

A bakery sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using random number table generate the demand for next 10 weeks.

Step 1: Establishing probability distributions for important input variables

First of all we have to generate probability distribution on the basis of the given data. Frequency is available to us using which we can obtain probability for respective demand/week as follows.

Demand/week	**Frequency**	**Probability**
0	2	2/50 = 0.04
5	11	11/50 = 0.22
10	8	8/50 = 0.16
15	21	21/50 = 0.42
20	5	5/50 = 0.1
25	3	3/50 = 0.06
Total	50	

Step 2: Write cumulative probability distribution for each variable in step 1.

Using last column of the above table we can find cumulative probability distribution for demand variable as follows:

Whole material

Demand/week	Frequency	Probability	Cumulative Probability
0	2	2/50 = 0.04	0.04
5	11	11/50 = 0.22	0.26
10	8	8/50 = 0.16	0.42
15	21	21/50 = 0.42	0.84
20	5	5/50 = 0.1	0.94
25	3	3/50 = 0.06	1
Total	50		

Step 3: Establishing an interval of random numbers for each variable
Considering cumulative probability column we can find out random number interval as follows:

Demand/week	Frequency	Probability	Cumulative Probability	Random no. Intervals
0	2	2/50 = 0.04	0.04	00-03
5	11	11/50 = 0.22	0.26	04-25
10	8	8/50 = 0.16	0.42	26-41
15	21	21/50 = 0.42	0.84	42-83
20	5	5/50 = 0.1	0.94	84-93
25	3	3/50 = 0.06	1	94-100
Total	50			

Step 4: Generating random numbers:
Using random no. table we can generate random numbers. (table is available on page no. 1024,1025) Let say we have generated following random nos. 35 52 90 13 23 73 34 57 35 83. We have generated 10 random nos. because we want to generate demand for next 10 weeks.

Step 5: Simulating a series of trials
For doing this step we have to take each selected random variable one by one and then to check the interval in which it falls. Selected interval will give us corresponding demand nos. So at the end we can say that we have generated values of the variable called Demand/week.

Random No.	Random Interval	Demand /week
35	26-41	10
52	42-83	15
90	84-93	20
13	04-25	5
23	04-25	5
73	42-83	15
34	26-41	10
57	42-83	15
35	26-41	10
83	42-83	15

Third column gives the simulated demand/week for next 10 weeks based on the previous data.

3. Give some applications of Simulation in business environment. / How Simulation is useful to the managers?

Following are the managerial applications of simulation.

1. Simulation of an inventory system:

Inventory management is a vital problem for the production manager as we know demand and lead time both are fluctuating. Whatever formulas we have to find out economic order quantity are not dealing with the fluctuating demand and lead time. Concept of simulation can help us in such situation. To understand this let us consider the following example.

Consider the case of dealer of a certain product for which the probability distribution of daily demand and the probability distribution of the lead time, both developed on the basis of the historical data and observations.

Probability distribution of daily demand

Units demanded	3	4	5	6	7	8
Probability	0.09	0.12	0.25	0.28	0.15	0.11

Probability distribution of lead time

Lead time (days)	2	3	4	5
Probability	0.20	0.30	0.35	0.15

The ordering cost is known to be Rs 80 per order; the holding cost per unit per day is estimated at Rs.2 while the unit shortage cost, representing the loss in profits is Rs 20 per unit per day.

Production manager is eager to know what would be the specific re order levels and re order quantities, what would be the inventory costs.

To answer such questions simulation can be used.

2. Simulation of Queuing System:

As we have studied in the queuing problems, arrival of the customers follows Poisson distribution and Service time of the customers follows exponential distribution. These assumptions are not true in many real life situations and in such cases it is difficult to apply mathematical model. Simulation provides the answer in such cases. To understand this let us consider following example.

In a large workshop undertaking servicing jobs, the mechanics obtain their tools and requirements from a central store. The manager, worried about the waiting time of mechanics, is in the process of determining whether more attendants be hired for the store for raising the level of service. The idle time cost for the mechanics and the wages required to be paid to the attendants being known, he wishes to ascertain how many attendants may be employed to minimize the total cost involved. For helping the manager solve his problem using simulation, we proceed as follows. The following data on the times between successive arrivals and the service times for the mechanics have been obtained from the past 200 observations made on the system presently in operation

Distribution of inter-arrival time:

Time (Minutes)	**Frequency**	**Probability**
0	12	0.06
3	18	0.09
6	50	0.25
9	74	0.37
12	32	0.16
15	14	0.07

Distribution of service time:

Time (Minutes)	Frequency	Probability
4	8	0.04
6	20	0.10
8	36	0.18
10	88	0.44
12	48	0.24

Generating set of random nos. we can have future data of inter arrival time and service time.

[Note: length of the example can be decided as per the marks allotted to the question. Also it is not necessary to stick to the same example; you may give your own simple examples of inventory management and queuing theory.]

Markov Chains

1. What do you understand by Markov process? In what areas of management can they be applied successfully?

Markov Analysis: it is a technique that deals with the probabilities of future occurrences by analyzing presently known probabilities. This analysis allows us to predict the future by using the state probabilities and the matrix of transition probabilities. The technique has numerous applications in business.

Markov analysis makes the assumption that the system starts in an initial state or condition. For example, two competing manufacturers might have 40% and 60% of the market sales, respectively as initial states. Perhaps in two months the market shares for the two companies will change to 45% and 55% of the market respectively. To predict such future probabilities Markov analysis is useful. For this initial probabilities are arranged in matrix form which is called **Transition Probability Matrix,** it is denoted by 'P'. To find out future probabilities transition probability matrix is useful, for eg. if we want to find out market shares of two products after two periods then we have to find out P*P and if we want to find out market shares of two products after three periods then we have to find out P*P*P. This process is called **Markov Process.**

Assumptions of Markov Analysis:

1. There are a limited or finite number of possible states.
2. The probability of changing states remains the same over time.
3. We can predict any future state from previous state and the matrix of transition probabilities.
4. The size and total no. of manufacturers and customers do not change during the analysis.
5. All the states are collectively exhaustive and mutually exclusive.

In the following areas Markov Process can be applied.

1. In marketing if one wants to study the market share after some periods or wants to know the probability of brand switching then Markov analysis is useful.
2. In production area if one wants to have knowledge about the smooth functioning of machines then Markov analysis is useful.
3. In management college if three specializations i.e. HR, MKT and Finance are offered to the students and one wants to study the preference pattern of the students over a period of time then this concept is important.
4. [You may discuss example of three detergent powder brands, in detail, which is available in the book]

2. What do you mean by States and State Probabilities? Explain the concepts of Mutually Exclusive and Collectively exhaustive States in Markov analysis.

States: They are used to identify all the possible conditions of a system.
For eg. (1) if we are studying machine problem with two machine and our target is to know the probability that machine will run properly in the next month given that it is running/not running properly in the current month. Then in this case machine is having two states, one is **proper functioning** of machine and second is **not proper functioning** of machine.
(2) Suppose there are only three brands of the detergent powder in the whole market and one wants to study the brand switching probability of the three brands. In this case each brand will be one state so three brands will form three different states.
In Markov analysis there are two more assumptions which are called mutually exclusive states and collectively exhaustive states.
Mutually exclusive states: suppose there are three brands (i.e. A, B, & C) of detergent powder are available in the market and it is known that person who is buying brand A is not buying brand B & C similarly one who is buying brand B is not buying brand A & C and one who is buying brand C is not buying brand A & B then we can say that all the states are mutually exclusive.
Collectively exhaustive: If it is assumed that there are only three brands of detergent powder i.e. A, B & C and whole market is covered by these three brands only then summation of the market shares of all the three brands is equal to 1. In this case all the three brands are called collectively exhaustive.
State probabilities: It is a probability of moving from one state to another state. For eg. probability that person who is using brand A detergent powder will use brand B detergent power after one month is 0.15 i.e. 15% then it is called state probability of moving from state A to B.

3. What is Transition Probability Matrix? How it is useful to make future predictions.

Transition probability matrix: the concept that allows us to get from a current state, such as market shares, to a future state is the matrix of transition probabilities. This is a matrix of conditional probabilities of being in a future state given a current state. The following definition is helpful:
Let Pij = conditional probability of being in state j in the future given the current state of i.
For eg. P12 is the probability of being in state 2 in the future given the event was in state 1 in the period before:
Let P = matrix of transition probabilities

$$P = \begin{pmatrix} P11\ P12 \ldots\ldots\ldots\ P1n \\ P21\ P22 \ldots\ldots\ldots\ Pn \\ \\ Pm1\ Pm2 \ldots\ldots\ldots\ Pmn \end{pmatrix}$$

Consider the example of three brands of detergent powder. Probability of brand shifting is as follows.

Current	Next month
D1	(0.60) D1
	(0.30) D2
	(0.10) D3
D2	(0.20) D1
	(0.50) D2
	(0.30) D3
D3	(0.15) D1
	(0.05) D2
	(0.80) D3

Above table gives probabilities of switching from D1 to D1, D2 and D3. Similarly for D2 & D3. If we write the same thing in the matrix form as follows, then it is called transition probability matrix.

$$P = \begin{pmatrix} 0.60 & 0.30 & 0.10 \\ 0.10 & 0.50 & 0.30 \\ 0.15 & 0.05 & 0.80 \end{pmatrix}$$

This matrix gives probability of brand switching from detergent D1 to D1, D2 & D3 after one month. If we want to find probability of brand switching after two months then we can find P*P by multiplying matrix P with the same matrix P and probability of brand switching after three months can be obtained using P*P*P. Thus Probability transition matrix can be used for finding future probabilities.

Second use of probability transition matrix is to find out market share after one month, two month and so on. Formula for finding market share after one month is

$Q(1) = Q(0) * P$

Market share after two months is

$Q(2) = Q(0) * P*P$

Market share after three months is

$Q(3) = Q(0) * P*P*P$

4. What is an absorbing state? Give some examples of absorbing states.

Absorbing States: Generally it is possible for the system to go from one state to any other state between any two periods. In some cases however if we move to some state then it is not possible to move to other states, this means we are absorbed in a particular state so further movement is not possible such states are called absorbing states.

If we are in absorbing state, then we cannot move to another state in future. Also a person is in an absorbing state now; the probability of being in an absorbing state in the future is 100%.

Example 1: An example of Accounts receivable application.

Consider the following four states,

State 1: Paid all bills

State 2: bad debt, overdue more than three months

State 3: overdue less than one month

State 4: overdue between one and three months

Consider the following matrix,

This Month	**Next Month**			
	Paid	**Bad Debt**	**<1 Month**	**1 to 3 Months**
Paid	1	0	0	0
Bad debt	0	1	0	0
Less than 1 month	0.6	0	0.2	0.2
1 to 3 months	0.4	0.1	0.3	0.2

Here Paid and Bad debt are the absorbing states.

Example 2: Suppose a person is having Rs. 20 and he is about to play a game. In each trial of the game he may either gain Rs.10 or loss Rs.10 with probability p and 1-p respectively. He has decided that if he doubles the money then he will stop or if he loss the entire amount then he will stop.

In such experiment the state where he has double the money (i.e. Rs. 40) and the state where he losses entire amount (i.e. Rs. 0) are called absorbing states as he can not move to any other states once he reach to any of these states.

Whole material