Vortex Identification in the solar photosphere

INTRODUCTION

The cross sectional area of vortex motions has a direct or indirect impact on the on the heat dissipation in the solar atmosphere towards the downflow region. The horizontal flow in the solar atmosphere observe a flow from the granule interiors to the downflow regions in the intergranular lanes, forming vortex flows in these lanes. These vortical flows exhibit a complex 3D vector flow field. To confirm the observation of vortex formation, spatial sampling of these individual granules and inter-granular layers is required.

EQUATION OF A FREE VORTEX FLOW

The Equation of vortex motion can be explained as follows:

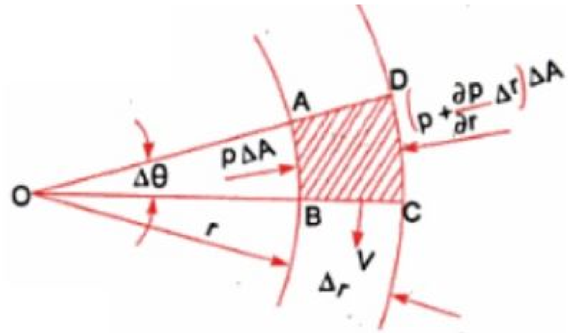

Figure 1: *Equation for motion of vortex*

1. The pressure force $P\Delta A$ on face AB
2. $(p + \frac{\partial Q}{\partial t}\Delta r)\Delta A$ force on face CD
3. Centrifugal force $(mV^2)/r$ acting in a direction away from the center 'O'.

Mass of element = Mass density x Volume, i.e.,$m = \rho\Delta r\Delta A$
Therefore, the centrifugal force equals $\rho\Delta r\Delta AV^2/r$
Equating the forces in radial directions we get

$$(p + \frac{\partial p}{\partial r}\Delta r)\Delta A - p\Delta A = \rho\Delta r\Delta A\frac{V^2}{r} \Rightarrow \frac{\partial p}{\partial r}\Delta r = \rho\Delta r\Delta A\frac{V^2}{r} \Rightarrow \frac{\partial p}{\partial r} = \rho\frac{V^2}{r} \quad (1)$$

The above equation (1) gives the pressure variation along the radical direction for a free vortex flow in a horizontal plane.The pressure variation in vertical plane is measured upwards in z direction using the hydro-static law :

$$(\frac{\partial p}{\partial z}) = \rho g \Rightarrow dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz \Rightarrow \ \Rightarrow dp = \rho\frac{V^2}{r}dr - \rho g dz \quad (2)$$

Equation (2) represents the equation of motion for vortex flow. The torque of a free vortex flow is zero, therefore $d(mVr)/dt = 0$; where m is the mass of the fluid particle and r is the radial distance and V is the velocity. Integrating this equation, we get $V * r = C$ where C is a constant. Substituting this in equation (2) we get,

$$\frac{\rho C^2}{r^2 r}dr - \rho g dz = \frac{\rho C^2}{r^3 r}dr - \rho g dz \quad (3)$$

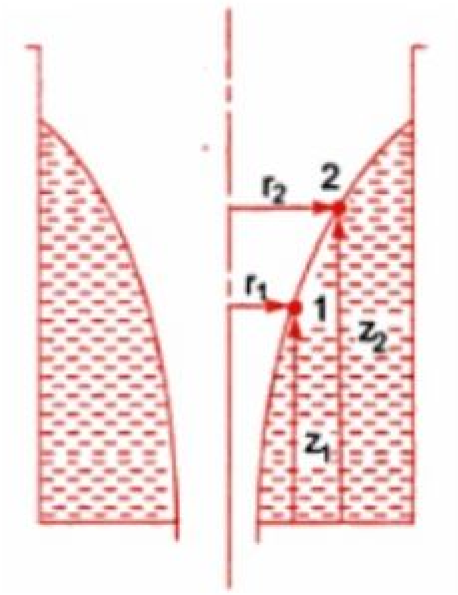

Figure 2: *Equation for motion of vortex*

Points 1 and 2 in Figure 2 are at a radial distance of r1 and r2 from the center axis of the cylinder as shown. The height of 1 and 2 from the bottom of the cylinder is z1 and z2 respectively. Integrating Equation 3for points 1 and 2, we get,

$$\int_1^2 dp = \int_1^2 \frac{\rho C^2}{r^3 r} dr - \int_1^2 \rho g dz \tag{4}$$

Solving Eq.4 we get

$$p_2 - p_1 = -\frac{\rho C^2}{2}[\frac{1}{r_2^2} - \frac{1}{r_1^2}] - \rho g[z_2 - z_1] \tag{5}$$

$$z_1 + \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \tag{6}$$

Equation 6 represents the equation of a free vortex motion.
The steady state error is the difference between the input and the reference signals in a limit for which the time goes to infinity i.e.,
Steady state error for Ramp input R(s) = $1/s^2$: $e_(ss)$ = $1/K_v$ = $\frac{1}{\lim_{s\to\infty} sG(s)}$
where K_v is the velocity error constant for type I and type II systems.
For the compensated system obtained in PART A the steady state percentage can be calculated as

$$e_(ss) = \frac{1}{\lim_{s\to\infty} sG(s)G_c(s)G_l(s)}$$
$$\Rightarrow e_(ss) = 2.86 percent$$

Following code (Figure 4) is used in MATLAB to achieve the response of control system to a unit ramp and evaluation of percentage steady state. **References**

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