Vortex Identification in the solar photosphere

INTRODUCTION

The cross sectional area of vortex motions has a direct or indirect impact on the on the heat dissipation in the solar atmosphere towards the downflow region. The horizontal flow in the solar atmosphere observe a flow from the granule interiors to the downflow regions in the inter-granular lanes, forming vortex flows in these lanes. These vortical flows exhibit a complex 3D vector flow field. Spatial sampling of these individual granules and inter-granular layers is performed to confirm the observation of vortex formation. This is performed by identifying the edges and core of the vortices found in a frame of Solar surface.

FOURIER LOCAL CORRELATION TRACKING (FLCT)

Multi-Order Solar EUV Spectrograph (MOSES) are utilized to capture high-resolution cotemporal spectral and spatial information of solar features over a large two-dimensional field of view.
A pair of 2D maps $I_1(x, y, t_1)$ and $I_2(x, y, t_2)$ are taken using MOSES and a 2D vector field is applied to I_1 and I_2 such that they resemble. The purpose of FLCT is to maximize the cross correlation function between I_1 and I_2 and it does so by using two techniques namely- Fast Fourier Transform (FFT) to compute correlation and second order accurate Taylor expansion to calculate the local peak (to sub pixel accuracy).
A 2D velocity field is constructed for I_1 and I_2, starting at same location followed by computing their velocity vectors. Parts of these are then emphasized and de-emphasized using a windowing function. Localization of these images are done by multiplying them with Gaussian function of width σ and centered at (x_i, y_j). The sub-images obtained are S_1 and S_2 with following expressions:

$$S_1^{i,j}(x, y) = I_1(x, y)e^{-(x-x_i)^2+(y-y_j)^2/\sigma^2} \tag{1}$$

$$S_2^{i,j}(x, y) = I_2(x, y)e^{-(x-x_i)^2+(y-y_j)^2/\sigma^2} \tag{2}$$

S_1 and S_2 are the equations of localization for emphasis.For (i, j)th pixel, the cross-correlation function of S_1 and S_2 is defined by

$$C^{i,j}(\delta x, \delta y) = \int\int dxdyS_1^{i,j*}(-x, -y)S_2^{i,j}(\delta x - x, \delta y - y) \tag{3}$$

The aim is to maximize $C(\delta x, \delta y)$ by finding (x_i, y_j) for each pair of S_1 and S_2. Using the amplitude between these shifts and δt, velocity can be determined as: $v_x = \delta x/\delta t$ and $v_y = \delta y/\delta t$. $C(\delta x, \delta y)$ can be computed using convolution theorem and Fourier transforms. Using this Eq.(3) can be re-written as:

$$C^{i,j}(\delta x, \delta y) = F^{-1}(s_1 * s_2) \tag{4}$$

where F^{-1} is inverse Fourier transform. Eq.(4) represents the FLCT and its peak is given by $|C^{i,j}(\delta x, \delta y)|$.

VORTICES AND NAVIER-STOKES EQUATION

The Navier-Stokes equation is a differential equation of the type:

$$\frac{\partial u}{\partial t} = F(u) \tag{5}$$

$$u(x, y, t) = (u_1(x, y, t), u_2(x, y, t)) \tag{6}$$

where function $u(x, y, t)$ is the velocity of the fluid at a given point in space, (x, y), and time, t. Also, using Newton's Law which says that Force is a product of mass and acceleration embedded with Eq.(5), we can write,

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - u_1\frac{\partial u_1}{\partial x} - u_2\frac{\partial u_1}{\partial y} - \frac{\partial p}{\partial x} \tag{7}$$

$$\frac{\partial u_2}{\partial t} = \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} - u_1\frac{\partial u_2}{\partial x} - u_2\frac{\partial u_2}{\partial y} - \frac{\partial p}{\partial y} \tag{8}$$

Since the vortices are not stationary, all the calculations are carried around vorticity, w, essentially the shell of vortex instead of velocity at equilibrium. Therefore, vorticity is given by

$$w = \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \tag{9}$$

$w > 0$: Fluid spins clockwise
$w < 0$: Fluid spins counterclockwise
The Navier-Stokes equation in Eq.(1) can therefore be rewritten as follows:

$$\frac{\partial u}{\partial t} = G(w) \tag{10}$$

Therefore, equilibrium vortex solution $G(w) = 0$;
Equilibrium vortex solution is $dG(w)/dw < 0$

The characteristic equation for δu is given as follows:

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0 \tag{11}$$

where P, Q and R are the three invariants of the velocity gradient tensor.
Lugt [9] proposed the following definition for a vortex, "A vortex is the rotating motion of a multitude of material particles around a common center." The problem with this definition is that it is too vague. Although it is consistent with visual observations, it does not lend itself readily to implementation in a detection algorithm. The existence of a vortex requires a priori knowledge of the orientation and motion of its core. Even so, $\lambda 2$ method along with many other methods have been able to adequately identify vortices in most computational datasets.

VORTEX IDENTIFIATION - Lambda2 Method

The pressure minimum method for vortex identification is not a sufficient method as proposed by Jeong and Hussain [10].Pressure minimums can be created in absense of a vortex and viscous effects due to unsteady irrotational straining. To remove these effects produced by irrotaional straining the velocity gradient tensor, $\bar{D}$ can be written as $D_{ij} = \frac{\partial u_i}{\partial x_j}$. Since $\bar{D}$ is a second order tensor, it can be decomposed into symmetric and a skew-symmetric part $D_{ij} = S_{ij} + \Omega_{ij}$ where

$$S_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}); \Omega_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \tag{12}$$

Using these symmetric and skew-symmetric parts stated here, Eq.(11) can be expressed as

$$P = -tr(\bar{D}); Q = \frac{1}{2}(tr(\bar{D})^2 - tr(\bar{D^2})); R = -det(\bar{D}) \tag{13}$$

S_{ij} is the strain-rate tensor and Ω_{ij} is the spin tensor. They define a vortex as a connected region where $S^2 + \Omega^2$ has real eigenvalues such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$. A point belongs to the vortex core only if $\lambda_2 < 0$. λ_2 criterion is an Eulerian method to find the core of the vortex in this way.

VORTICITY IDENTIFICATION - Direct Lyapunov Exponent (DLE)

The Lyapunov exponent is a Lagrangian Method for identification of vortice stretching. It does so by providing a mesaure of separation of neighbouring particle trajectories from a given particle. The expansion particle can be calculated using Δ and using largest singular value $\lambda_{max}\Delta$.

Ridges (maximizing curves in DLE field) can be generated using largest DLE providing regions of maximum material streteching. To make sure that ridge produced is a Hyperbolic Lagrangian Coherent Structure (LCS), forward and backward integrations of the DLE is performed. The forward integration provides the repelling material lines and vice-versa.

$$\sigma_T = \frac{1}{|T|} ln\sqrt{\lambda_{max}(\Delta)} \tag{14}$$

Eq.(14) represents one of the definitions for largest direct Lyapunov exponent. This way the edges of a vortex can be identified using the idea that the particles in a coherent structure will move close and the others will move far apart.

The Eulerian method (λ_2 criterion) along with the DLE method can be used together to find the edges while also locating the core of the vortex. This will not only make the identification process easier but also reduce the computational costs since the DLE method has to run only once to calibrate λ_2. Although this also means that a range of λ_2 values will be attained along the edges of DLE field making it impossible to find a single λ_2 that would result in matching edges.

References

[1] Vivianne Holmen (2012). Methods for Vortex Identication.

[2] George H. Fisher (2008). FLCT: A Fast, Efficient Method for Performing Local Correlation Tracking: University of California, Berkeley

[3] I. Giagkiozis, V. Fedun, E. Scullion, and G. Verth (2017). Vortex Flows in the Solar Atmosphere: Automated Identification and Statistical Analysis

[4] Scullion, Eamon. The Multi-Scale Nature of Vorticity in the Solar Atmosphere: Northumbria University, UK

[5] Vaclav Kolar. On the relationship between a vortex and vorticity : Institute of Hydrodynamics, Academy of Sciences of the Czech Republic, 166 12 Prague 6, Czech Republic

[6] Chakraborty, P., Balachandar, S., Adrian, R.J. (2005), J. Fluid Mech. 535, 189214.

[7] Sven Wedemeyer-Bohm1,2, Eamon Scullion1, Oskar Steiner3, Luc Rouppe van der Voort1, Jaime de la Cruz Rodriguez4, Viktor Fedun5 Robert Erdelyi5 (2012). Magnetic tornadoes as energy channels into the solar corona

[8] Mei, C.C. (2002). Lecture Notes for 1.63 Advanced Fluid Dynamics of the Environment. Lecture 1.2: Kinematics of Fluid Motion the Eulerian Picture.

[9] H. J. Lugt. Vortex Flow in Nature and Technology. Wiley, 1972.

[10] J. Jeong and F. Hussain. On the Identification of a Vortex. J. Fluid Mechanics, 285:6994, 1995.

[11] Hans T. Courrier* and Charles C. Kankelborg on Using local correlation tracking to recover solar spectral information from a slitless spectrograph.