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Basics of Neural Network Programming

Binary Classification

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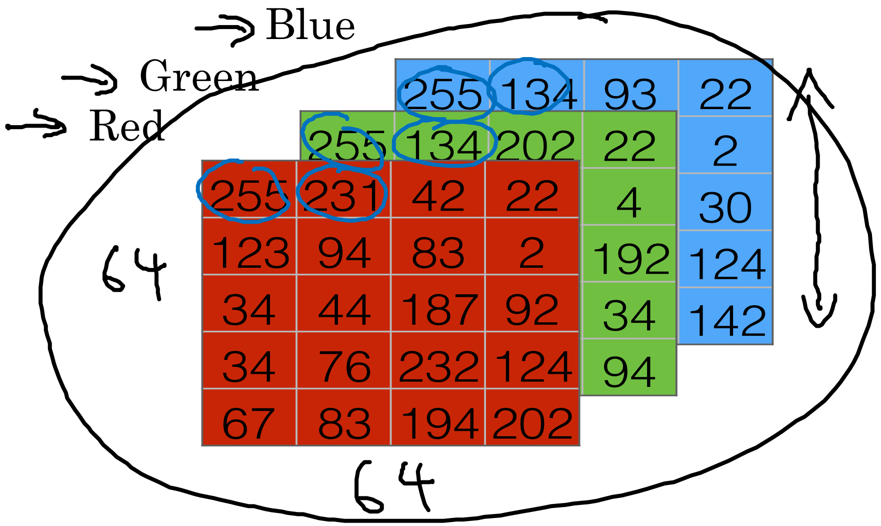
Binary Classification

64

64

$\longrightarrow$ 1 (cat) vs 0 (non cat)

y



$$x = \begin{bmatrix} 255 \\ 231 \\ \vdots \\ \vdots \\ 255 \\ 134 \\ \vdots \end{bmatrix}$$

$64 \times 64 \times 3 = 12288$

$n = n_x = 12288$

$x \longrightarrow y$

Notation

$(x, y) \qquad x \in \mathbb{R}^{n_x}, \; y \in \{0, 1\}$

m training examples: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots, (x^{(m)}, y^{(m)})\}$

$M = M_{train} \qquad M_{test} = \#\text{test examples.}$

$$X = \underbrace{\left.\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}\right\} n_x}_{m} \qquad \cancel{\begin{bmatrix} — & x^{(1)T} & — \\ & \vdots & \\ — & x^{(m)T} & — \end{bmatrix}}$$

$X \in \mathbb{R}^{n_x \times m} \qquad X.\text{shape} = (n_x, m)$

$Y = [y^{(1)} \; y^{(2)} \; \dots, y^{(m)}]$

$Y \in \mathbb{R}^{1 \times m}$

$Y.\text{shape} = (1, m)$

Basics of Neural Network Programming

Logistic Regression

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Logistic Regression

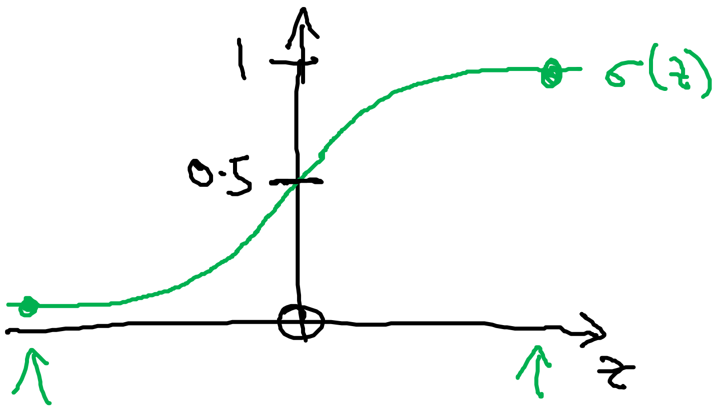
Given x, want $\underline{\hat{y}} = \underline{P(y=1 \mid x)}$

$\underline{0 \leq \hat{y} \leq 1}$

$x \in \mathbb{R}^{n_x}$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$X_0 = 1, \quad x \in \mathbb{R}^{n_x + 1}$

$\hat{y} = \sigma(\theta^T x)$

$$\underbrace{\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{matrix} \} b \leftarrow \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} w \leftarrow \end{matrix}}$$

$$\sigma(z) = \frac{1}{1 + \underline{e^{-z}}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1 + \underline{e^{-z}}} \approx \frac{1}{1 + \text{Bignum}} \approx 0$$

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Basics of Neural Network Programming

Logistic Regression cost function

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Logistic Regression cost function

$\rightarrow \hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$

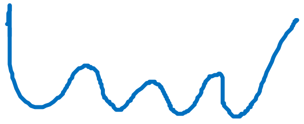
$z^{(i)} = w^T x^{(i)} + b$

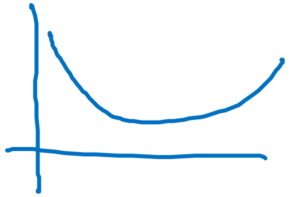
Given $\{(x^{(1)}, y^{(1)}),\dots,(x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$
$y^{(i)}$
$z^{(i)}$

i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y}-y)^2$





$$\boxed{\mathcal{L}(\hat{y},y)} = -(\boxed{y\log\hat{y}} + (1-y)\log(1-\hat{y})) \leftarrow$$

If $y=1$: $\mathcal{L}(\hat{y},y) = -\log\hat{y} \leftarrow$ Want $\log\hat{y}$ large, want $\hat{y}$ large.

If $y=0$: $\mathcal{L}(\hat{y},y) = -\log(1-\hat{y}) \leftarrow$ Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

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Basics of Neural Network Programming

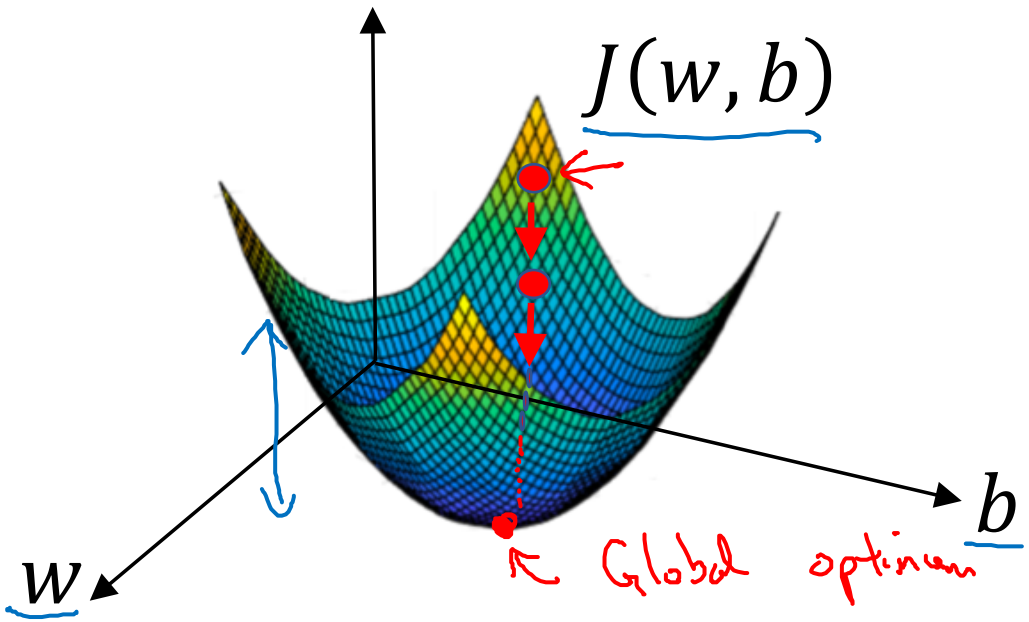
Gradient Descent

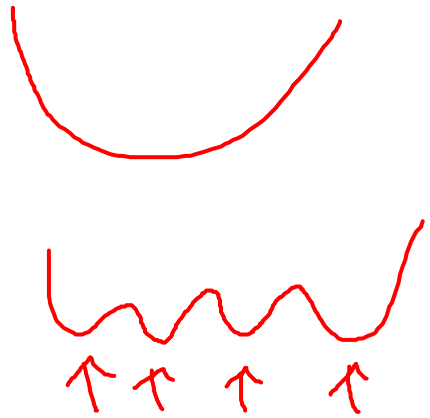
Gradient Descent

Recap: $\hat{y} = \sigma(w^T x + b)$, $\sigma(z) = \frac{1}{1+e^{-z}}$

$$\underline{J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\boxed{\hat{y}^{(i)}}, \boxed{y^{(i)}}) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})$$

Want to find w, b that minimize $J(w,b)$





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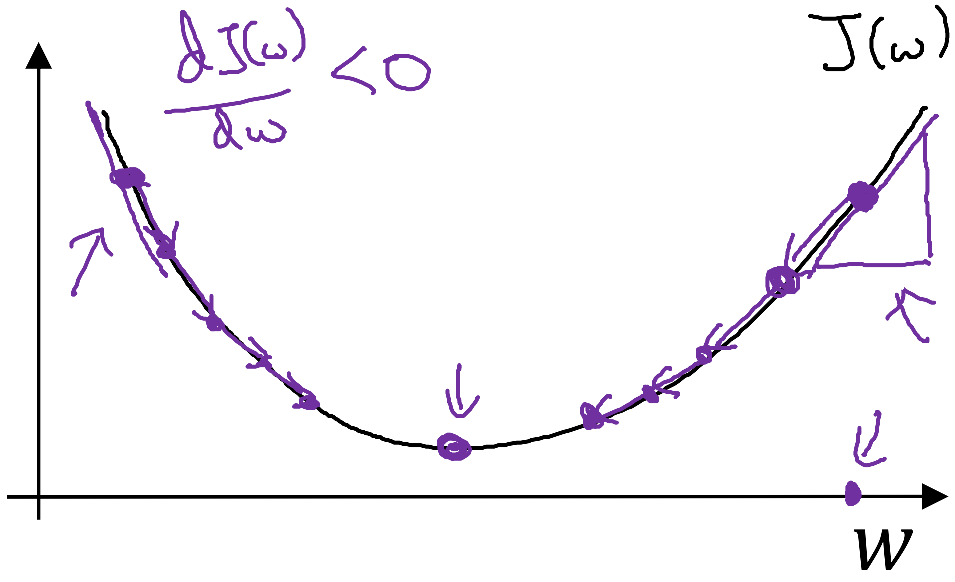
Gradient Descent



Repeat {

learning rate

$$\left(w := w - \alpha \boxed{\frac{dJ(w)}{dw}} \right)$$

}

"dw"

$$w := w - \alpha \, dw$$

$$\frac{dJ(w)}{dw} = ?$$

$J(w,b)$

$$w := w - \alpha \boxed{\frac{dJ(w,b)}{dw}} \qquad \boxed{\frac{\partial J(w,b)}{\partial w}} \rightarrow dw$$

$$b := b - \alpha \boxed{\frac{dJ(w,b)}{db}} \qquad \boxed{\frac{\partial J(w,b)}{\partial b}} \rightarrow db$$

∂ "partial derivative"

d

J

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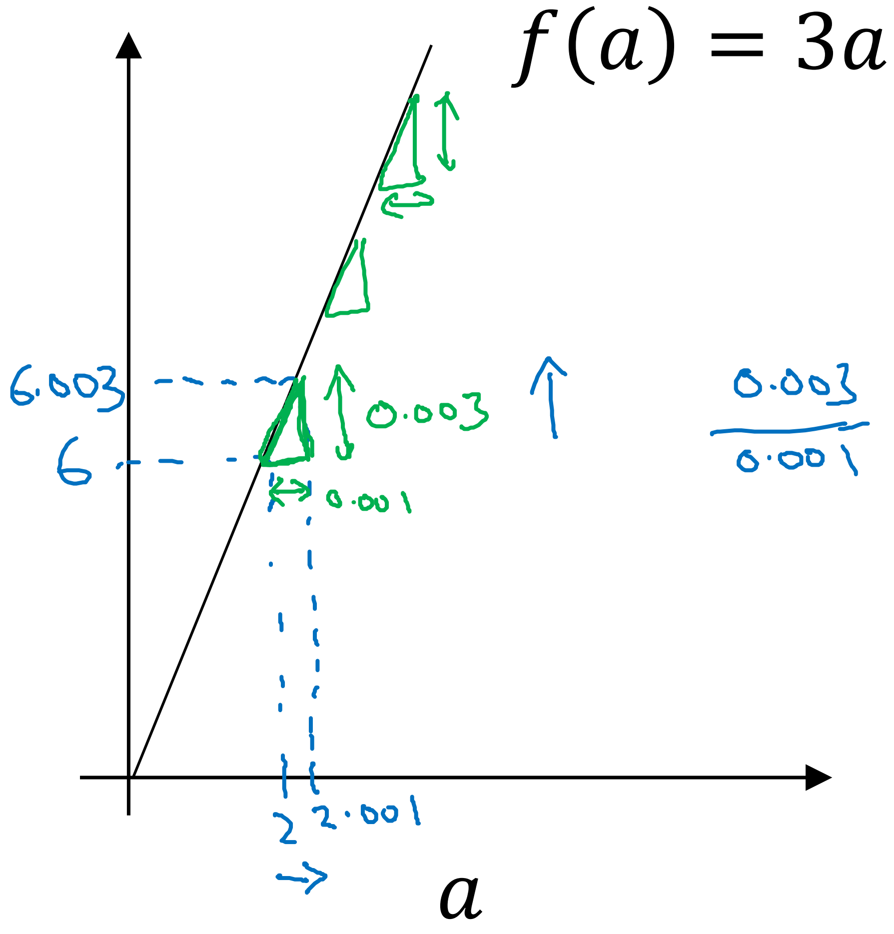
Basics of Neural Network Programming

Derivatives

Intuition about derivatives



$\frac{0.003}{0.001}$ $\frac{\text{height}}{\text{width}}$

$\rightarrow$ $a = 2$ $f(a) = 6$

$a = 2.001$ $f(a) = 6.003$

slope (derivative) of $f(a)$ at $a=2$ is 3

$\rightarrow$ $a = 5$ $f(a) = 15$

$a = 5.001$ $f(a) = 15.003$

slope at $a=5$ is also 3

$$\frac{d f(a)}{da} = 3 = \frac{d}{da} f(a)$$

0.001

0.00000001

0.0000000001

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Cost function should be as small as possible

For the derivatives, the value is very small, like 0.00000000001. For the sake of visualization, value considered is 0.001.

For straight line, derivative is constant and remain same at different points.
For x^2, derivative is $2x$ and doubles for value of x.

Two takeaways-
1. Derivative of a function means slope. Slope can be different at different points.
2. Derivative can be calculated using formula

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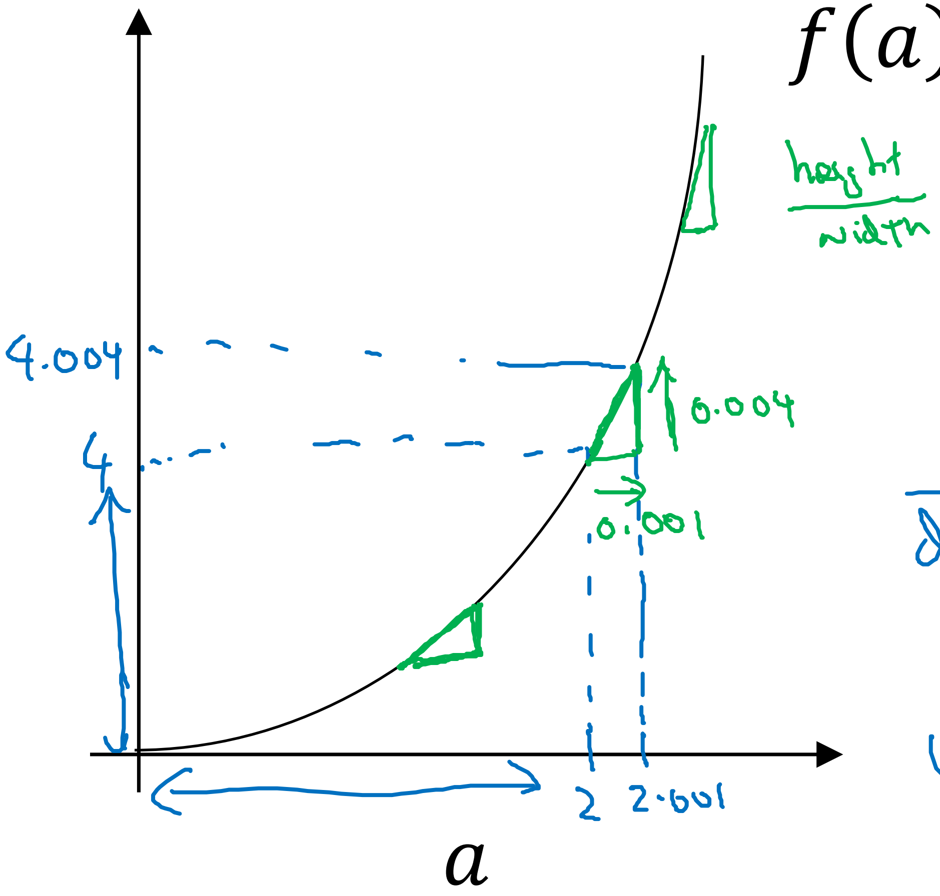
More derivatives examples

Intuition about derivatives

$$f(a) = a^2$$



$$\frac{d}{da} a^2 = 2a$$

0.001

$(2a) \times 0.001$

0.001 ←

0.00000.....01 ←

$a = 2$ $f(a) = 4$

$a = 2.001$ $f(a) \approx 4.004$

$(4.004\boxed{001})$

slope (derivative) of $f(a)$ at $a = 2$ is 4.

$\boxed{\frac{d}{da} f(a) = 4}$ when $\boxed{a = 2}$.

$a = 5$ $f(a) = 25$

$a = 5.001$ $f(a) \approx 25.010$

$\boxed{\frac{d}{da} f(a) = 10}$ when $\boxed{a = 5}$

$\frac{d}{da} f(a) = \underline{\frac{d}{da} a^2} = \boxed{2a}$

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More derivative examples

$f(a) = a^2$ $\qquad \frac{d}{da} f(a) = \underbrace{2a}_{4}$ $\qquad a = 2 \quad f(a) = 4$

$a = 2.001 \quad f(a) \approx 4.004$

$f(a) = a^3$ $\qquad \frac{d}{da} f(a) = \underbrace{3a^2}_{3 \times 2^2 = 12}$ $\qquad a = 2 \quad f(a) = 8$

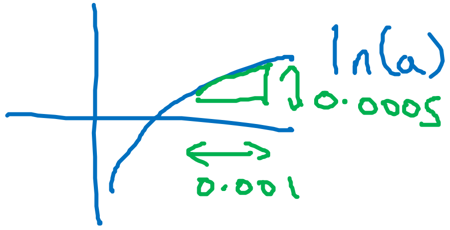
$a = \underline{2.001} \quad f(a) \approx 8.\underline{012}$

$f(a) = \log_e(a)$

$\ln(a)$

$\frac{d}{da} f(a) = \frac{1}{a}$

$\frac{d}{da} f(a) = \boxed{\frac{1}{2}}$



$a = 2 \quad f(a) \approx 0.69315$

$a = 2.\underline{001} \quad \underline{f(a)} \approx 0.69365$

0.0005

$\underline{0.0005}$

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Basics of Neural Network Programming

Computation Graph

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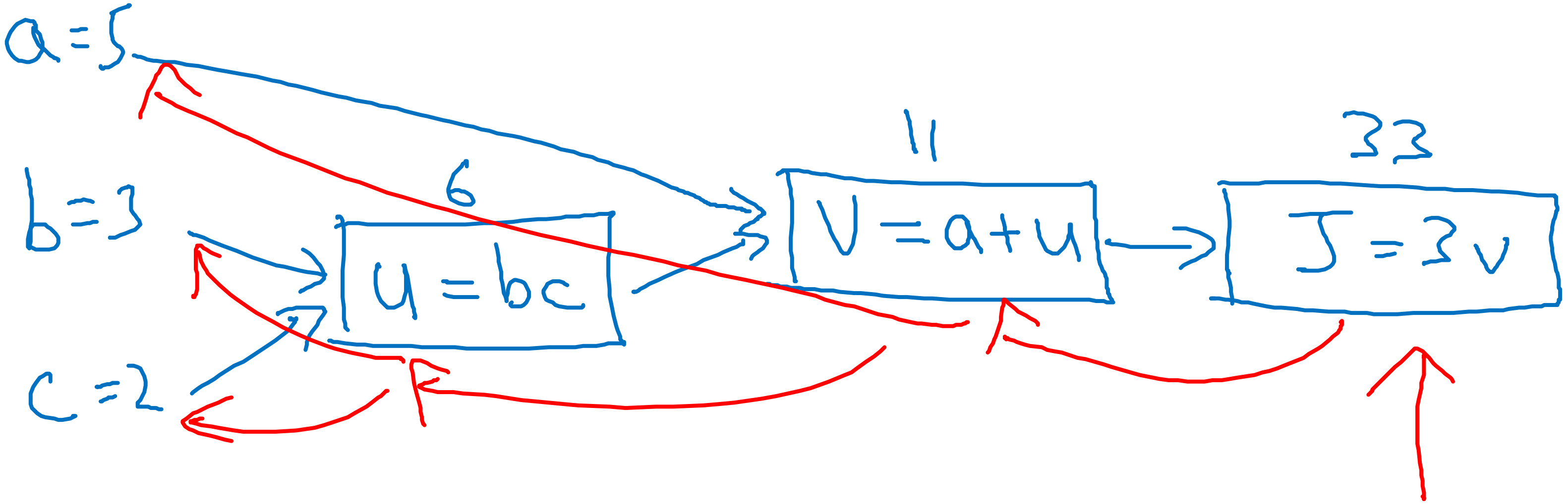
Computation Graph

$$J(a,b,c) = 3(\underbrace{\underbrace{a + \underbrace{bc}_{u}}_{v}}_{J}) = 3(5 + 3 \times 2) = 33$$

$u = bc$

$v = a + u$

$J = 3v$



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Derivatives with a Computation Graph

Computing derivatives

$a = 5$ $\quad \frac{dJ}{da}$ "da" $= 3$

$b = 3$

$c = 2$

6: $u = bc$

11: $v = a + u$

33: $J = 3v$

$\frac{dJ}{dv}$ "dv" $= 3$

$\frac{dJ}{dv} = ? = 3$

$a \to v \to J$

$J = 3v$
$v = 11 \to 11.001$
$J = 33 \to 33.003$

$f(a) = 3a$

$\frac{df(a)}{da} = \frac{df}{da} = 3$

$J = 3v$

$\frac{dJ}{dv} = 3$

$\frac{dJ}{da} = 3 = \frac{dJ}{dv} \frac{dv}{da}$

3×1

$\frac{dv}{da} = 1$

$a = 5 \to 5.001$
$\to v = 11 \to 11.001$
$J = 33 \to 33.003$

$\frac{d\,\text{FinalOutputVar}}{d\,\text{var}}$

dJdvar

"dvar"

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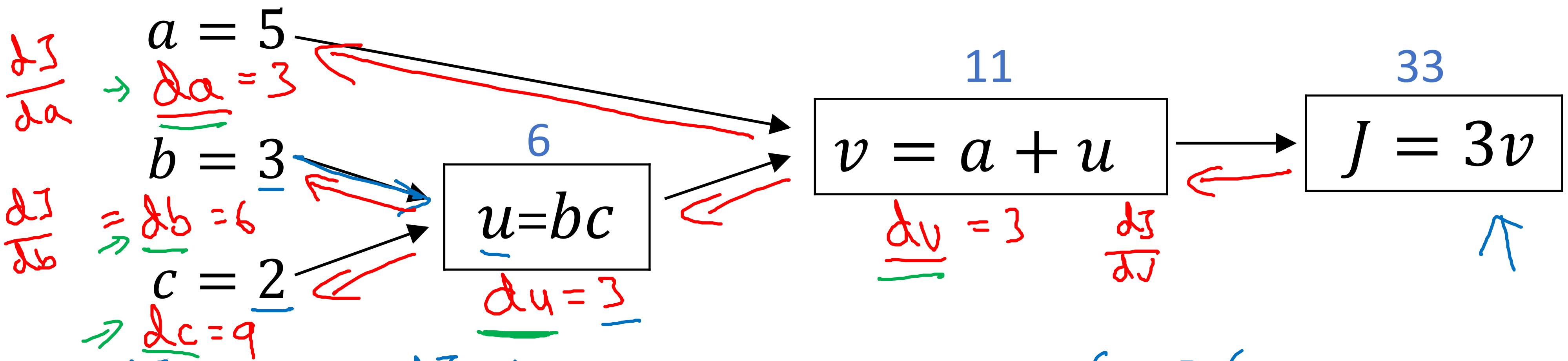
One step of a back propagation on a computation graphyields derivative of final output variable.

Forward propagation to derive cost function
Backward propagation to compute derivative

Computing derivatives



$$\frac{dJ}{du} = 3 = \underbrace{\frac{dJ}{dv}}_{3} \cdot \underbrace{\frac{dv}{du}}_{1}$$

$$\frac{dJ}{db} = \underbrace{\boxed{\frac{dJ}{du}}}_{\to 3} \cdot \underbrace{\frac{du}{db}}_{=2} = 6$$

$$\frac{dJ}{da} = \boxed{\frac{dJ}{du}} \cdot \frac{du}{da} = 9$$

$$3 \times 3$$

$u = 6 \to 6.001$

$v = 11 \to 11.001$

$J = 33 \to 33.003$

$b = 3 \to 3.001$

$u = b \cdot c = 6 \to 6.002$ $c = 2$

$J = 33.006$ $.006$

$v = 11.002$

$J = 3v$

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Basics of Neural Network Programming

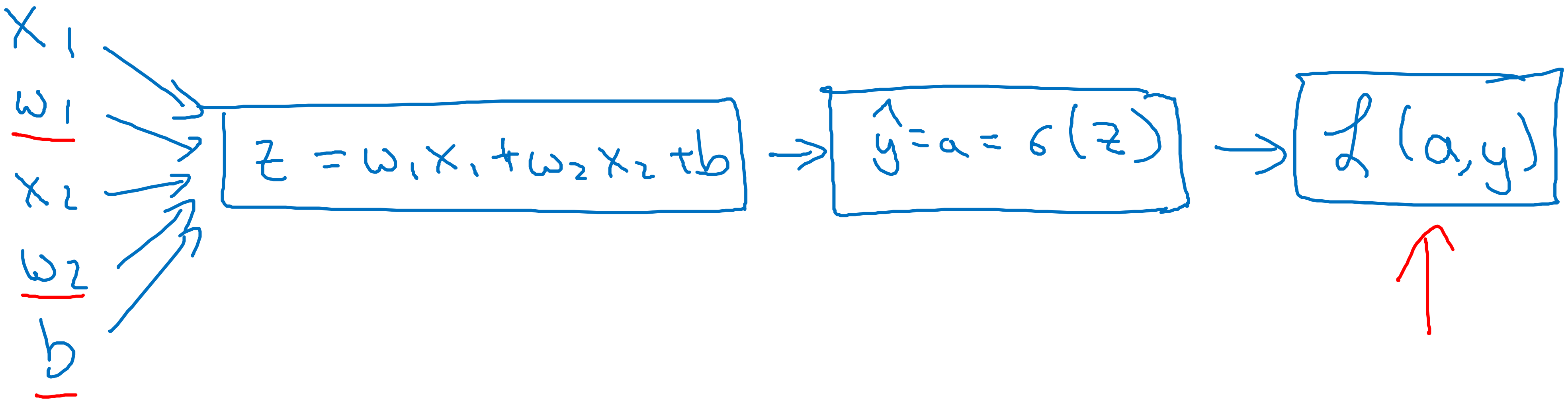
Logistic Regression Gradient descent

Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y) \log(1 - a))$$



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Logistic regression derivatives

x_1, w_1, x_2, w_2, b → $z = w_1 x_1 + w_2 x_2 + b$ → $a = \sigma(z)$ → $\mathcal{L}(a, y)$

$$\text{"}dz\text{"} = \frac{dL}{dz} = \frac{dL(a,y)}{dz} = a - y = \frac{dL}{da} \cdot \frac{da}{dz}$$

$$\frac{da}{dz} = a(1-a)$$

$$\text{"}da\text{"} = \frac{dL(a,y)}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$\frac{\partial L}{\partial w_1} = \text{"}dw_1\text{"} = x_1 \cdot dz. \qquad dw_2 = x_2 \cdot dz. \qquad db = dz.$$

$$\boxed{\begin{aligned} w_1 &:= w_1 - \alpha\, dw_1 \\ w_2 &:= w_2 - \alpha\, dw_2 \\ b &:= b - \alpha\, db. \end{aligned}}$$

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Basics of Neural Network Programming

Gradient descent on m examples

Logistic regression on m examples

$$\underline{J(w,b)} = \underbrace{\frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(a^{(i)}, y^{(i)})}$$

$$(x^{(i)}, y^{(i)})$$

$$\rightarrow a^{(i)} = \hat{y}^{(i)} = \sigma(z^{(i)}) = \sigma(w^T x^{(i)} + b)$$

$$\underline{dw_1^{(i)}}, \underline{dw_2^{(i)}}, \underline{db^{(i)}}$$

$$\underline{\frac{\partial}{\partial w_1} J(w,b)} = \frac{1}{m}\sum_{i=1}^{m} \underbrace{\frac{\partial}{\partial w_1} \mathcal{L}(a^{(i)}, y^{(i)})}_{\underline{dw_1^{(i)}}} \quad - (x^{(i)}, y^{(i)})$$

Logistic regression on m examples

$J=0;\ dw_1=0;\ dw_2=0;\ db=0$

→ For $i=1$ to m

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J += -[y^{(i)} \log a^{(i)} + (1-y^{(i)}) \log(1-a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

$n=2$

dw_3
$\vdots$
dw_n

$J /= m$ ←

$dw_1 /= m;\ dw_2 /= m;\ db /= m.$ ←

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$

Vectorization

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Vectorization

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What is vectorization?

$z = \underbrace{w^T x}_{} + b$

$w = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$ $x = \begin{bmatrix} \vdots \\ \vdots \end{bmatrix}$

$w \in \mathbb{R}^{n_x}$

$x \in \mathbb{R}^{n_x}$

Non-vectorized:

```
z = 0
for i in range(n-x):
    z += w[i] * x[i]
z += b
```

Vectorized

$z = \underbrace{\text{np.}\underline{\text{dot}}(w, x)}_{w^T x} + b$

→ GPU
→ CPU

SIMD – single instruction multiple data.

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More vectorization examples

Neural network programming guideline

Whenever possible, avoid explicit for-loops.

$u = Av$

$u_i = \sum_i \sum_j A_{ij} v_j$

```
u = np.zeros((n,1))
for i ...
    for j ....
        u[i] += A[i][j]*v[j]
```

```
u = np.dot(A,v)
```

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Vectors and matrix valued functions

Say you need to apply the exponential operation on every element of a matrix/vector.

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \rightarrow u = \begin{bmatrix} e^{v_1} \\ e^{v_2} \\ \vdots \\ e^{v_n} \end{bmatrix}$$

```
u = np.zeros((n,1))
for i in range(n):
    u[i]=math.exp(v[i])
```

```
import numpy as np
u = np.exp(v)

np.log(v)
np.abs(v)
np.maximum(v, 0)
v**2          1/v
```

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Logistic regression derivatives

$J = 0,\ \boxed{\cancel{dw1 = 0,\ dw2 = 0}},\ db = 0$ $\quad dw = np.zeros((n\_x, 1))$

for i = 1 to n:

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)})]$

$dz^{(i)} = a^{(i)}(1 - a^{(i)})$

for $j = 1 \ldots n_x$

$dw_j \mathrel{+}= \ldots$

$\boxed{\cancel{dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}}}$ $\quad n_x = 2 \quad dw \mathrel{+}= x^{(i)} dz^{(i)}$

$\boxed{\cancel{dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}}}$

$db \mathrel{+}= dz^{(i)}$

$J = J/m,\ \boxed{\cancel{dw_1 = dw_1/m,\ dw_2 = dw_2/m}},\ db = db/m$

$dw \mathrel{/}= m.$

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Basics of Neural Network Programming

Vectorizing Logistic Regression

Vectorizing Logistic Regression

$z^{(1)} = w^T x^{(1)} + b$ $\qquad$ $z^{(2)} = w^T x^{(2)} + b$ $\qquad$ $z^{(3)} = w^T x^{(3)} + b$

$a^{(1)} = \sigma(z^{(1)})$ $\qquad$ $a^{(2)} = \sigma(z^{(2)})$ $\qquad$ $a^{(3)} = \sigma(z^{(3)})$

$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$ $\quad (n_x, m)$ $\quad \mathbb{R}^{n_x \times m}$

$[\;\text{———}\;] \; w^T \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$

$Z = [z^{(1)} \; z^{(2)} \cdots z^{(m)}] = w^T X + \underbrace{[b \; b \cdots b]}_{1 \times m} = \underbrace{[w^T x^{(1)} + b \quad w^T x^{(2)} + b \quad \cdots \quad w^T x^{(m)} + b]}_{1 \times m}$

$Z = \text{np.dot}(w.T, X) + b$ $\quad (1,1) \quad \mathbb{R}$

"Broadcasting"

$A = [a^{(1)} \; a^{(2)} \cdots a^{(m)}] = \sigma(Z)$

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Basics of Neural Network Programming

Vectorizing Logistic Regression’s Gradient Computation

Vectorizing Logistic Regression

$dz^{(1)} = a^{(1)} - y^{(1)}$ $\qquad dz^{(2)} = a^{(2)} - y^{(2)}$ $\qquad \ldots$

$$\boxed{dZ = [dz^{(1)} \; dz^{(2)} \; \ldots \; dz^{(m)}]} \leftarrow$$

$1 \times m$

$A = [a^{(1)} \ldots a^{(m)}]$ $\qquad Y = [y^{(1)} \ldots y^{(m)}]$

$\rightarrow dZ = A - Y = [a^{(1)} - y^{(1)} \quad a^{(2)} - y^{(2)} \quad \ldots]$

$\rightarrow dw = 0$

$dw \mathrel{+}= x^{(1)} dz^{(1)}$

$dw \mathrel{+}= x^{(2)} dz^{(2)}$

$\vdots$

$dw \mathrel{/}= m$

$\cancel{dw_1}$

$\cancel{dw_2}$

$\vdots$

$db = 0$

$db \mathrel{+}= dz^{(1)}$

$db \mathrel{+}= dz^{(2)}$

$\vdots$

$db \mathrel{+}= dz^{(m)}$

$db \mathrel{/}= m.$

$$db = \frac{1}{m} \sum_{i=1}^{m} dz^{(i)}$$

$$= \frac{1}{m} \, np.sum(dZ)$$

$$\boxed{dw = \frac{1}{m} X \, dZ^T}$$

$$= \frac{1}{m} \begin{bmatrix} | & & | \\ x^{(1)} & \ldots & x^{(m)} \\ | & & | \end{bmatrix} \begin{bmatrix} dz^{(1)} \\ \vdots \\ dz^{(m)} \end{bmatrix}$$

$$= \frac{1}{m} \left[x^{(1)} dz^{(1)} + \ldots + x^{(m)} dz^{(m)} \right]$$

$n \times 1$

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Implementing Logistic Regression

J = 0, dw_1 = 0, dw_2 = 0, db = 0

$\boxed{\texttt{for i = 1 to m:}}$

$z^{(i)} = w^T x^{(i)} + b$

$a^{(i)} = \sigma(z^{(i)})$

$J \mathrel{+}= -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$

$dz^{(i)} = a^{(i)} - y^{(i)}$

$dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$

$dw \mathrel{+}= x^{(i)} * dz^{(i)}$

$db \mathrel{+}= dz^{(i)}$

J = J/m, $dw_1 = dw_1/m$, $dw_2 = dw_2/m$

db = db/m

for iter in range(1000):

$Z = w^T X + b$

$= np.dot(w.T, X) + b$

$A = \sigma(Z)$

$dZ = A - Y$

$dw = \frac{1}{m} X dZ^T$

$db = \frac{1}{m}$ np.sum(dZ)

$w := w - \alpha dw$

$b := b - \alpha db$

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Basics of Neural Network Programming

Broadcasting in Python

Broadcasting example

Calories from Carbs, Proteins, Fats in 100g of different foods:

$$
\begin{array}{r c}
 & \begin{array}{cccc} \text{Apples} & \text{Beef} & \text{Eggs} & \text{Potatoes} \end{array} \\
\begin{array}{r} \text{Carb} \\ \text{Protein} \\ \text{Fat} \end{array} &
\left[\begin{array}{cccc} 56.0 & 0.0 & 4.4 & 68.0 \\ 1.2 & 104.0 & 52.0 & 8.0 \\ 1.8 & 135.0 & 99.0 & 0.9 \end{array}\right]
\end{array}
= \underset{(3,4)}{A}
$$

59 cal

$\frac{56}{59} \approx 94.9\%$

0

1

Calculate % of calories from Carb, Protein, Fat. Can you do this without explicit for-loop?

```
cal = A.sum(axis = 0)
percentage = 100*A/(cal.reshape(1,4))
```

(3,4) / (1,4)

Broadcasting example

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \begin{bmatrix}100\\100\\100\\100\end{bmatrix} \cancel{100}$$

$$\underset{(m,n)\quad(2,3)}{\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}} + \underset{(1,n) \rightsquigarrow (m,n) \qquad (2,3)}{\begin{bmatrix}100 & 200 & 300\\100 & 200 & 300\end{bmatrix}}$$

$$\underset{(m,n)}{\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}} + \underset{(m,1) \to (m,n)}{\begin{bmatrix}100 & 100 & 100\\200 & 200 & 200\end{bmatrix}} =$$

General Principle

(m, n) matrix $\quad \begin{matrix} + \\ - \\ * \\ / \end{matrix} \quad \begin{matrix} (1, n) \rightsquigarrow (m, n) \\ (m, 1) \rightsquigarrow (m, n) \end{matrix}$

$(m, 1) \quad + \quad \mathbb{R}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 \\ 102 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + 100 = \begin{bmatrix} 101 & 102 & 103 \end{bmatrix}$$

Matlab/Octave: bsxfun

deeplearning.ai

Basics of Neural Network Programming

Explanation of logistic regression cost function (Optional)

Logistic regression cost function

$\hat{y} = \sigma(w^T x + b)$ where $\sigma(z) = \frac{1}{1+e^{-z}}$

Interpret $\hat{y} = P(y=1 \mid x)$

If $y=1$: $P(y \mid x) = \hat{y}$

If $y=0$: $P(\underline{y} \mid x) = \underline{1 - \hat{y}}$

Logistic regression cost function

$$\boxed{\text{If} \quad y = 1: \qquad p(y|x) = \hat{y}}$$

$$\boxed{\text{If} \quad y = 0: \qquad p(y|x) = 1 - \hat{y}}$$

$p(y|x)$

$$\boxed{p(y|x) = \hat{y}^{y} (1-\hat{y})^{(1-y)}}$$

If $y=1$: $p(y|x) = \hat{y} \quad \underbrace{(1-\hat{y})^{0}}_{=1}$

If $y=0$: $p(y|x) = \underbrace{\hat{y}^{0}}_{=1} \quad (1-\hat{y})^{(1-y)} = 1 \times (1-\hat{y}) = 1-\hat{y}$

$$\uparrow \log p(y|x) = \log \hat{y}^{y} (1-\hat{y})^{(1-y)} = y \log \hat{y} + (1-y) \log(1-\hat{y})$$

$$= -\mathcal{L}(\hat{y}, y) \downarrow$$

Andrew Ng

Cost on m examples

$$\log p(\underline{\text{labels in training set}}) = \log \prod_{i=1}^{m} p(y^{(i)} | x^{(i)})$$

$$\underline{\log p(\text{.....})} = \sum_{i=1}^{m} \underbrace{\log p(y^{(i)} | x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)}, y^{(i)})}$$

Maximum likelihood estimation

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Cost: $\underline{J(w,b)} = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$

(minimize)