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Learning Tetris Using the Noisy Cross-Entropy Method

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The cross-entropy method is an efficient and general optimization algorithm. However, its applicability in reinforcement learning (RL) seems to be limited because it often converges to suboptimal policies. We apply noise for preventing early convergence of the cross-entropy method, using Tetris, a computer game, for demonstration. The resulting policy outperforms previous RL algorithms by almost two orders of magnitude.

1 Introduction

Tetris is one of the most popular computer games (see, e.g., Fahey, 2003). Despite its simple rules, playing the game well requires a complex strategy and lots of practice. Furthermore, Demaine, Hohenberger, and Liben-Nowell (2003) have shown that Tetris is hard in a mathematical sense as well. Finding the optimal strategy is NP-hard even if the sequence of tetrominoes is known in advance. These properties make Tetris an appealing benchmark problem for testing reinforcement learning (and other machine learning) algorithms.

Reinforcement learning (RL) algorithms are quite effective in solving a variety of complex sequential decision problems. Despite this, RL approaches tried to date on Tetris show surprisingly poor performance. The aim of this note is to show how to improve RL for this hard combinatorial problem. In this article, we put forth a modified version of the cross-entropy (CE) method (de Boer, Kroese, Mannor, & Rubinstein, 2004).

2 Applying the Cross-Entropy Method to Tetris

2.1 Value Function and Action Selection. Following the approach in Bertsekas and Tsitsiklis (1996), we shall learn state-value functions that are linear combination of several basis functions. We use 22 such basis functions: maximal column height, individual column heights, differences of column heights, and the number of holes. More formally, if s denotes a Tetris state

and $\phi_i(s)$ is the value of basis function i in this state, then according to weight vector w, the value of state s is

$$V_w(s) := \sum_{i=1}^{22} w_i \phi_i(s). \tag{2.1}$$

During a game, the actual tetromino is test-placed in every legal position, and after erasing full rows (if any), the value of the resulting state is calculated according to V_w. Finally, we choose the column and direction with the highest value.

2.2 The Cross-Entropy Method. The cross-entropy (CE) method is a general algorithm for (approximately) solving global optimization tasks of the form

$$w^* = \arg\max_w S(w), \tag{2.2}$$

where S is a general real-valued objective function, with an optimum value $\gamma^* = S(w^*)$. The main idea of CE is to maintain a distribution of possible solutions and update this distribution at each step (de Boer et al., 2004). Here a very brief overview is provided.

The CE method starts with a parametric family of probability distributions $\mathcal{F}$ and an initial distribution $f_0 \in \mathcal{F}$. Under this distribution, the probability of drawing a high-valued sample (having value near γ^*) is presumably very low; therefore, finding such samples by naive sampling is intractable. For any $\gamma \in \mathbb{R}$, let $g_{\geq\gamma}$ be the uniform distribution over the set $\{w : S(w) \geq \gamma\}$. If one finds the distribution $f_1 \in \mathcal{F}$ closest to $g_{\geq\gamma}$ with regard to the cross-entropy measure, then f_0 can be replaced by f_1 and γ-valued samples will have larger probabilities. For many distribution families, the parameters of f_1 can be estimated from samples of f_0. This estimation is tractable if the probability of the γ-level set is not very low with regard to f_0. Instead of the direct computation of the $\mathcal{F}$-distribution closest to $g_{\geq\gamma^*}$, we can proceed iteratively. We select a γ_0 appropriate for f_0, update the distribution parameters to obtain f_1, select γ_1, and so on, until we reach a sufficiently large γ_k. Below we sketch the special case when w is sampled from a member of the gaussian distribution family.

Let the distribution of the parameter vector at iteration t be $f_t \sim N(\mu_t, \sigma_t^2)$. After drawing n sample vectors $w_1, \ldots, w_n$ and obtaining their value $S(w_1), \ldots, S(w_n)$, we select the best $\lfloor \rho \cdot n \rfloor$ samples, where $0 < \rho < 1$ is the selection ratio. This is equivalent to setting $\gamma_t = S(w_{\lfloor \rho \cdot n \rfloor})$. Denoting the set of indices of the selected samples by $I \subseteq \{1, 2, \ldots, n\}$, the mean and

the deviation of the distribution is updated using

$$\mu_{t+1} := \frac{\sum_{i \in I} w_i}{|I|} \tag{2.3}$$

and

$$\sigma^2_{t+1} := \frac{\sum_{i \in I} (w_i - \mu_{t+1})^T (w_i - \mu_{t+1})}{|I|}. \tag{2.4}$$

2.3 The Cross-Entropy Method and Reinforcement Learning. Applications of the CE method to RL include the parameter tuning of radial basis functions (Menache, Mannor, & Shimkin, 2005) and adaptation of a parameterized policy (Mannor, Rubinstein, & Gat, 2003). We apply CE to learn the weights of the basis functions, drawing each weight from an independent gaussian distribution.

2.4 Preventing Early Convergence. Preliminary investigations showed that applicability of CE to RL problems is restricted severely by the phenomenon that the distribution concentrates to a single point too fast. To prevent this, we adapt a trick frequently used in particle filtering: at each iteration, we add some extra noise to the distribution: instead of equation 2.4, we use

$$\sigma^2_{t+1} := \frac{\sum_{i \in I} (w_i - \mu_{t+1})^T (w_i - \mu_{t+1})}{|I|} + Z_{t+1}, \tag{2.5}$$

where Z_{t+1} is a constant vector depending only on t.

3 Experiments

In the experiments we used the standard Tetris game described in Bertsekas and Tsitsiklis (1996), scoring 1 point for each cleared row. Each parameter had an initial distribution of $N(0, 100)$. We set $n = 100$ and $\rho = 0.1$. Each drawn sample was evaluated by playing a single game using the corresponding value function. After each iteration, we updated distribution parameters using equations 2.3 and 2.5 and evaluated the mean performance of the learned parameters. This was accomplished by playing 30 games using V_{μ_t}, and averaging the results. (The large number of evaluation games was necessary because Tetris strategies have large performance deviations; Fahey, 2003.)

In experiment 1 we tested the original CE method (corresponding to $Z_t = 0$). As expected, deviations converge to 0 too fast, so the mean performance settles at about 20,000 points. In experiment 2 we used a constant noise rate

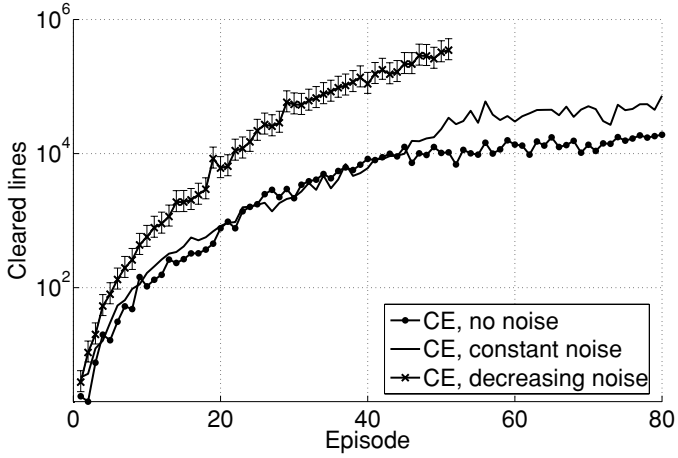

Figure 1: Tetris scores on logarithmic scale. Mean performance of 30 evaluations versus iteration number. Error bars denoting 95% confidence intervals are shown on one of the curves.

of $Z_t = 4$, raising mean performance to a 70,000 point level. Our analysis showed that further improvement was counteracted by the amount of noise, which was too high, and thus prevented convergence of the distributions. Therefore, in experiment 3, we applied a decreasing amount of noise, $Z_t = \max(5 - \frac{t}{10}, 0)$. With this setting, average score exceeded 300,000 points by the end of episode 50, and the best score exceeded 800,000 points. In experiments 2 and 3, noise parameters were selected in an ad hoc manner, and no optimization was carried out. Results are summarized in Figure 1 and in Table 1.

Assuming an exponential score distribution (Fahey, 2003), we calculated the 95% confidence intervals of the mean. For better visibility, confidence intervals have been plotted only for experiment 3.

Learning took more than one month of CPU time on a 1 GHz machine using Matlab. The main reason for the long learning course is the fact that for Tetris, evaluation time of the value function scales linearly with the score, and the score is very noisy.[1] Related to this, the computational overhead of the CE method is negligibly small.

Because of the large running times, experiments consisted of only a single training run. However, preliminary results on simplified Tetris problems show that over multiple trials, the method consistently converges to the same region.

[1] It is conjectured that the length of a game can be approximated from its starting sequence (Fahey, 2003), which could reduce evaluation time considerably.

Table 1: Average Tetris Scores of Various Algorithms.

Method	Mean Score	Reference
Nonreinforcement learning		
Hand-coded	631,167	Dellacherie (Fahey, 2003)
Genetic algorithm	586,103	(Böhm et al., 2004)
Reinforcement learning		
Relational reinforcement learning+kernel-based regression	≈50	Ramon and Driessens (2004)
Policy iteration	3183	Bertsekas and Tsitsiklis (1996)
Least squares policy iteration	<3000	Lagoudakis, Parr, and Littman (2002)
Linear programming + Bootstrap	4274	Farias and van Roy (2006)
Natural policy gradient	≈6800	Kakade (2001)
CE+RL	21,252	
CE+RL, constant noise	72,705	
CE+RL, decreasing noise	348,895	

3.1 Comparison to Previous Work. Tetris has been chosen as a benchmark problem by many researchers to test their RL algorithms. Table 1 summarizes results known to us, comparing them to our algorithm and to two state-of-the-art non-RL algorithms as well.

The comparison shows that our method improves on the performance of the best RL algorithm by almost two orders of magnitude and gets close to the best non-RL algorithms (Fahey, 2003; Böhm, Kókai, & Mandl, 2004).

We think that by applying the performance enhancement techniques in Böhm et al. (2004)—more basis functions, exponential value function—further significant improvement is possible.

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