Project Proposal: Optimized Projections for Compressed Sensing

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April 29, 2016

1 Overview

Most of the applications (like MRI, CT), become time-consuming due to the large chunks of data that it has to collect and process. The research in past years has proved and observed that the required data can be recovered even if we have only a subset of linear combinations of the big chunk of measurements. Shannon's sampling theorem was one of such discoveries but it requires the signals to be band-limited. As band-limited signals are infinite time signals, most of the real life signals are not band-limited.

A signal is said to be sparse if it has a sparse representation (very less number of non-zero coefficients) in some basis space. Compressive sensing is a technology where a sparse signal can be recovered completely even when we know just a few samples which are linear combinations of the signal data (subject to a few conditions on the measurement matrix and recovery algorithm). Measurement matrix converts a big, sparse signal (signal to be measured) into a lower dimension signal.
Mathematically,
x: n length column vector - original signal to be measured
$\boldsymbol{D}$: $n \times k$ dimension basis matrix in which original signal is sparse
α : k length column vector - coefficients when x is represented wrt the basis $\boldsymbol{D}$ and follows, $||alpha||_0 \leq T \ll n$
$\boldsymbol{P}$: $p \times n$ dimension measurement matrix used to measure signal x, such that $T < p \ll n$
y: p length column vector - the measured low dimension column vector from which we are expected to retrieve the original signal x

$$x = \boldsymbol{D}\alpha$$

$$y = \boldsymbol{P}x$$

Thus,

$$y = \boldsymbol{PD}\alpha$$

And the α that we seek in order to reconstruct x from y is the solution to

$$\min_{\alpha} ||\alpha||_0 \text{ such that } y = \boldsymbol{PD}\alpha \tag{1}$$

Hence the problem of compressed sensing is divided in 2 parts

1. Find a good measurement matrix so that we can have the best recovery of x from y
2. Come up with an algorithm that tries to find the α closest to the solution to (1) ((1) is an NP-hard problem)

In our project, we will study the methods to find a better measurement matrix (in place of the standard random Gaussian one), i.e., the modification will be done on the part 1 of the above-mentioned problems and state-of-the-art algorithms will be used for part 2. The proposed algorithm is an iterative method for improving the projections based on the mutual-coherence (μ) of the overall new dictionary ($\boldsymbol{PD}$).

$$\mu(\boldsymbol{D}) = \max_{1 \leq i,j \leq k \text{ and } i \neq j} \frac{|d_i^T d_j|}{||d_i||.||d_j||}$$

2 Results

1. **Performance with synthesized data**
   Mainly 2 experiments were done:

   - Using random Gaussian matrix as a dictionary (setting completely as described in paper)
   - Using DCT matrix as dictionary (with a hope to characterize the performance with images sparse in DCT basis)

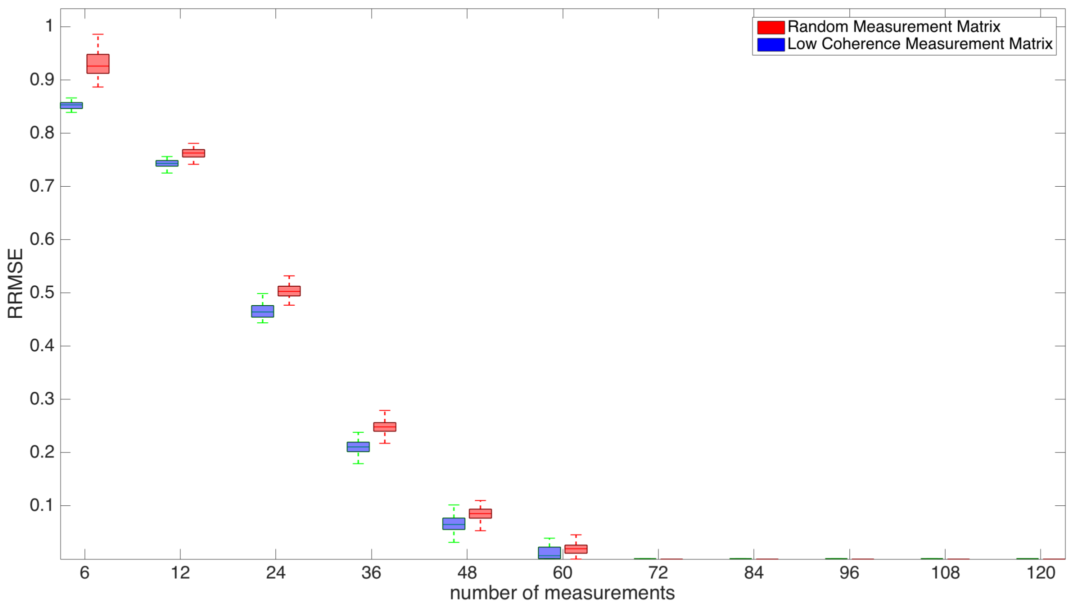

Figure 1: Boxplot for Gaussian random matrix as dictionary

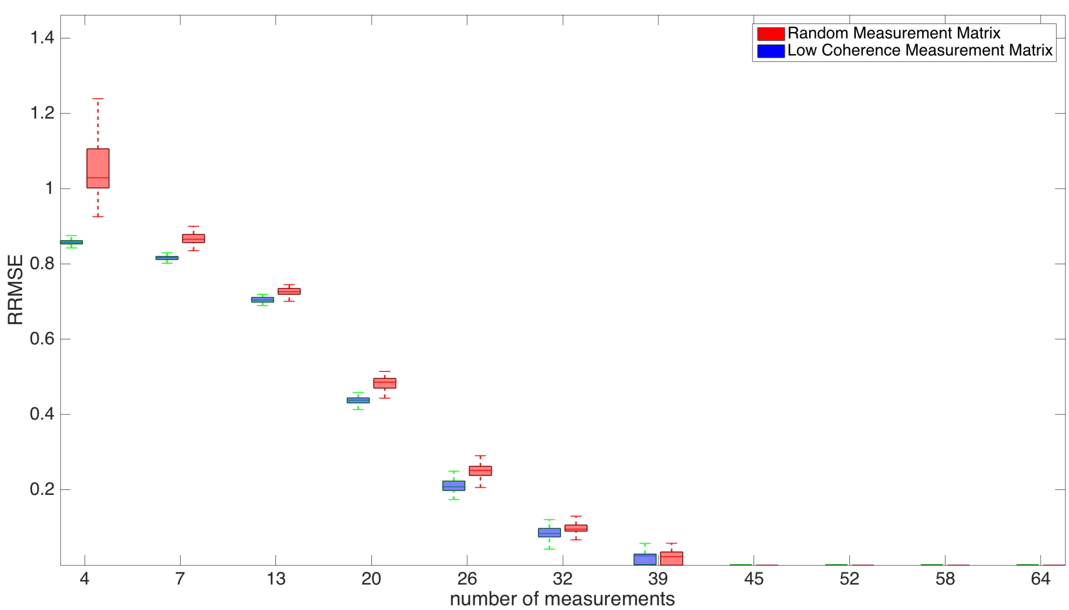

Figure 2: Boxplot for 2-D DCT dictionary

2. Performance on images

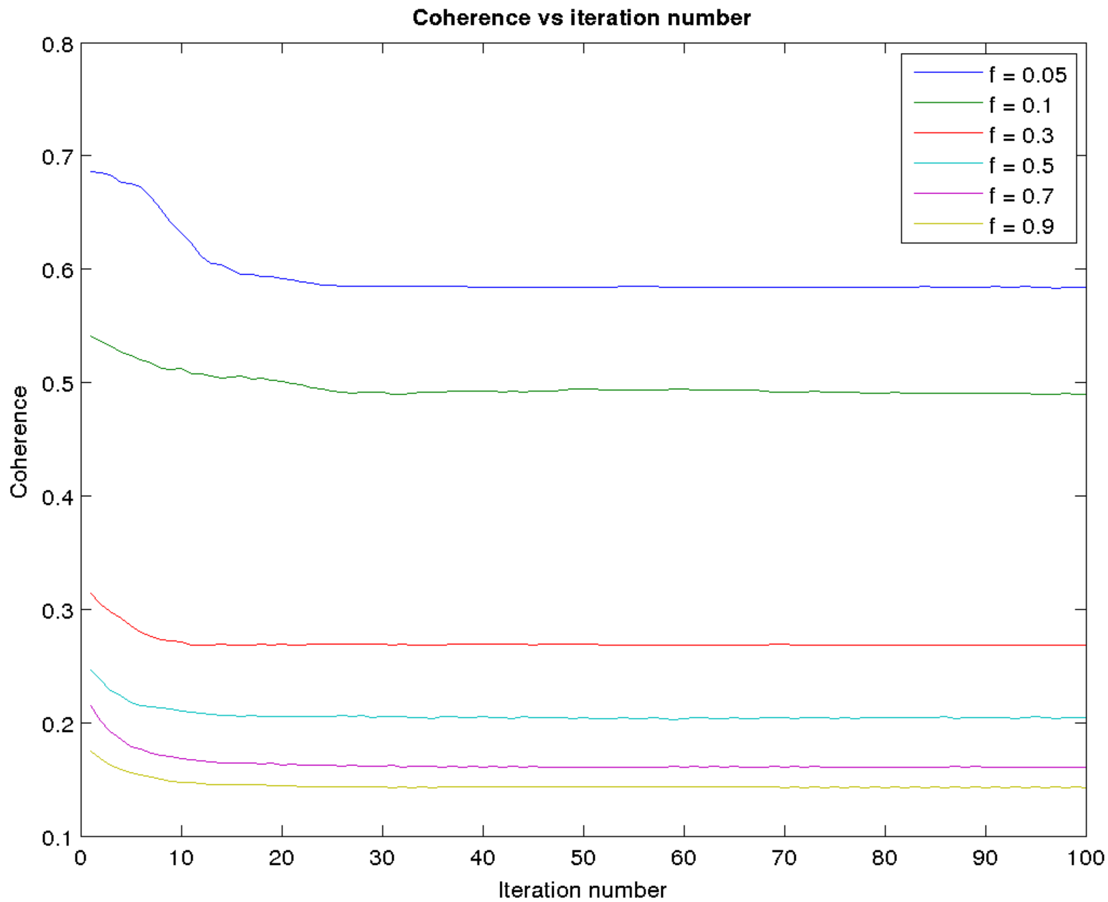

Figure 3: Change in coherence

3 Observations and Speculations

- μ starts at higher values for smaller no of measurements. This is because at higher dimensions, 2 random vectors are almost orthogonal with a high probability.
- μ decreases by a less amount for higher no of measurements. This is probably because the value of μ is already lower for higher number of measurements and so the further amount by which our algorithm reduces μ is reduced.
- As a consequence of the above reason the convergence rate is also high. As the iteration starts with a low value, it reaches its steady state value in fewer number of iterations.
- The relative root mean square error reduces with no of measurements for both the sensing matrices (random gaussian matrix and low coherence matrix) but the error is smaller when we use our low coherence sensing matrix instead of the random matrix.
- We see that the value of coherence is not decreasing strictly with the number of iterations. This is because after each iteration we are taking a lower rank appoimation of the Gram matrix and then for taking the square root we are forcing the negative eigenvalues to be positive. These drastic steps leads us to a matrix which may not necessarily have a lower coherence. However, it turns out that eventually the algorithm converges to a low coherence value.

4 References

- Michael Elad
  Optimized Projections for Compressed Sensing
  IEEE Transactions on Signal Processing, volume: 55, issue: 12, 5695 - 5702, Dec 2007
- Slides on Compressive Sensing by Prof. Ajit Rajwade