

# Connectivity in Commercial Internet

Ankita Pasad  
Parthasarathi Panda  
Pranav Jain

Internet Economics: Course Project

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# Overview

- 1 Introduction and Basics
  - Structure of the Internet
  - Market
- 2 Duopoly
  - Single Homing: Problem Formulation
- 3 Oligopoly
  - Preliminaries
  - No Merger
  - Merger
- 4 Multihoming
  - Pre-existing Multihoming
  - Multihoming in response to degradation
- 5 References

# Internet

- Largest example of deregulated communications network
- Very strong network externalities
- Hierarchical structure
  - Reduces complexity of routing table
  - Transparent interconnection agreements
  - Flow of money is from bottom to top

# Hierarchy of Internet

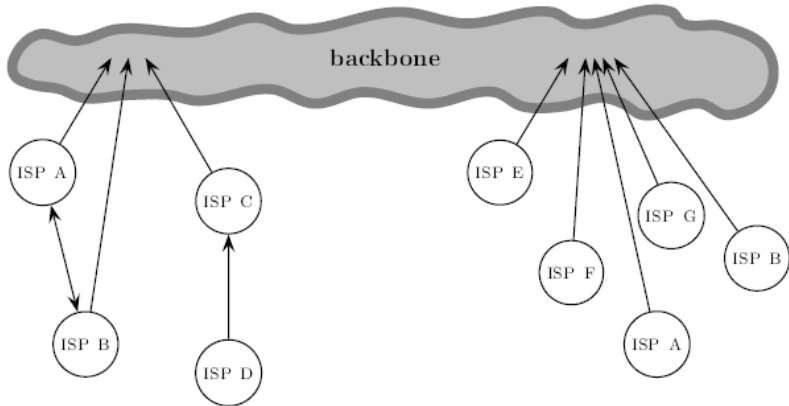


Figure: 1

# Internet Backbone Providers (IBP)

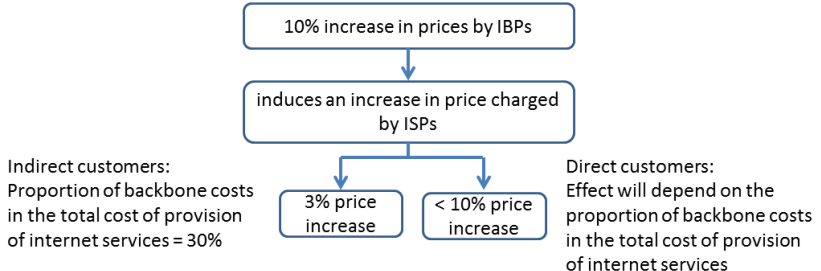
- ISP: Provides service to end-users (consumers and businessmen)
- IBP: High bandwidth long-distance transmission, routing and interconnection to ISP
- Compete for new unattached/attached customers
- Interconnect with other IBPs, but main source of income is via customers and not peering
- Incentives to interconnect

## Effect of increase in backbone prices

How do users try to combat this increase?

- Retail-level substitution
- Wholesale-level substitution

## Retail-level substitution



## Wholesale-level substitution

### Various strategies by ISPs

Purchase transit from other ISP

No increase in purchase of transit due to an increase in price of the backbone service

Secondary peering

Limited impact:  
Local traffic is usually a small fraction of the total traffic

Co-operation for an alternate backbone OR  
Buy/tie-up with an existing backbone

Difficult to organize in short term  
Expensive as a permanent resort

- Co-ordination and management difficulties
- Long-term contracts and switching costs
- Discriminatory offers by hypothetical monopolist
- Inefficient routing and increased delays

Entry of a new small backbone

- Costly to build a large network so fast
- Above issues remain
- Convincing the customers to shift



## Purchase of transit from other ISP

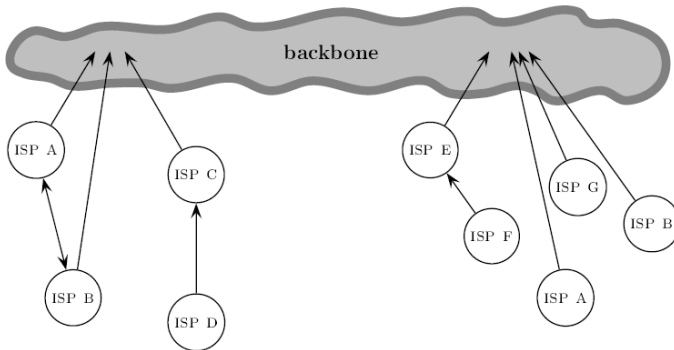


Figure: 2

Source: [1]

## Secondary Peering

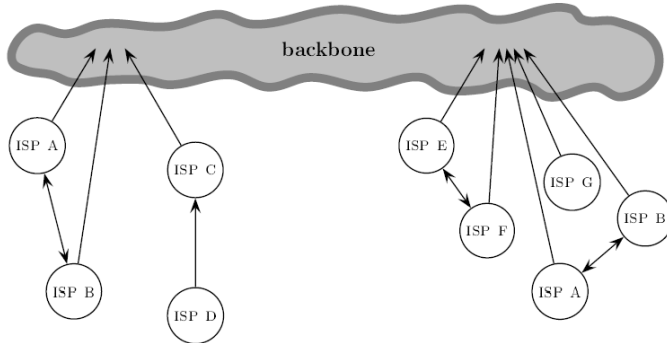


Figure: 3

Source: [1]

## Preliminaries

- Two backbones,  $i = 1, 2$
- $\beta_i$ : Installed base of customers ( $\beta_1 > \beta_2$ )
- Unattached customers of type  $\tau \in [0, 1]$
- $p_i$ : price of connecting to backbone  $i$
- $s_i$ : quality of service of backbone  $i$
- $q_i$ : equilibrium number of unattached customers taken by backbone  $i$
- $\nu$ : importance of connectivity (valuation to the consumers)
- $\theta$ : Quality of interconnection  $\in [0, 1]$

- Net surplus for an unattached customer of type  $\tau$

$$\tau + s_i - p_i \quad (1)$$

- Quality of service of backbone  $i$

$$s_i = \nu[(\beta_i + q_i) + \theta(\beta_j + q_j)] \quad (2)$$

- All unattached customer look at both backbones as substitutes, i.e.

$$p_1 - s_1 = p_2 - s_2 \quad (3)$$

- An indifferent customer, has valuation  $\tau = \hat{p}$  such that  $\tau + s_1 - p_1 = \tau + s_2 - p_2 = 0$ .

So, we have

$$\hat{p} = p_1 - s_1 = p_2 - s_2 \quad (4)$$

- Considering the net fraction of unattached customers as 1, we have,  $q_i + q_j + \hat{p} = 1$
- We thus get an equation for  $p_i$  in terms of  $\beta_i, \beta_j, q_i, q_j, \nu, \theta$  (using 2,3,4)

$$\begin{aligned} p_i &= 1 - (q_i + q_j) + s_i \\ &= 1 + \nu(\beta_i + \theta\beta_j) - (1 - \nu)q_i - (1 - \theta\nu)q_j \end{aligned} \quad (5)$$

- Nash Bargain

Choose  $\theta$  and payment  $t$  from backbone 2 to backbone 1

$$\begin{aligned} &\{[\pi_1(\theta) - F(\theta) + t] - [\pi_1(\min\theta_1^*, \theta_2^*) - F(\min\theta_1^*, \theta_2^*)]\}^\epsilon \\ &\times \{[\pi_2(\theta) - F(\theta) - t] - [\pi_2(\min\theta_1^*, \theta_2^*)] - F(\min\theta_1^*, \theta_2^*)\}^{1-\epsilon} \end{aligned} \quad (6)$$

$\epsilon$ : Relative bargaining power of backbone  $i$

# Equilibrium

Profit associated with the installed base is considered to be constant.

- Gross profit of backbone  $i = (p_i - c) * q_i$
- On differentiating, we get,  $q_i^*, q_j^*$ <sup>1</sup>

$$q_i^* = \frac{1}{2} \left( \frac{2(1-c) + \nu(1+\theta)\beta}{2(1-\nu) + (1-\theta\nu)} + \frac{(1-\theta)\nu\Delta_i}{2(1-\nu) - (1-\theta\nu)} \right)$$

where,

$$\beta = \beta_1 + \beta_2$$

$$\Delta = \Delta_1 = -\Delta_2 = \beta_1 - \beta_2 \geq 0$$

<sup>1</sup>The equilibrium is stable if  $\nu < 1/2$

## Effects of increasing quality of interconnection

### ■ Demand Expansion Effect

$$q_1^* + q_2^* = \frac{2(1 - c) + \nu(1 + \theta)\beta}{2(1 - \nu) + (1 - \theta\nu)} \quad (7)$$

### ■ Quality Differentiation Effect

$$q_1^* - q_2^* = \frac{(1 - \theta)\nu\Delta}{2(1 - \nu) - (1 - \theta\nu)} \quad (8)$$

- When the connectivity is cost-less, smaller backbone prefers perfect connectivity and bigger one prefers perfect connectivity (if the superiority of installed base is small) or no connectivity
- So, if  $\Delta < \Delta^*$ : Preferred connectivity = 1  
Else: Preferred connectivity = 0

## Independent decision

Thus, this equilibrium is unfair to customers as well as backbone 2:

- Unattached customers benefit with higher interconnection (as  $\hat{p}$  decreases)
- Installed base customers prefer higher connectivity
- Backbone 1's profit increases with higher connectivity

So, this is definitely not a social-surplus maximizing equilibrium



## Preliminaries and Assumptions

- Four equal sized backbones,  $i = 1, 2, 3, 4$
- $\beta_i$ : Installed base of customers  $= \beta/4$  for all
- $\theta_{ij} = 0$  or  $1$
- In case of merger backbone 1 and 4 combine to form the new backbone 1, with installed base size  $= \beta/2$
- Backbone 2 and 3 remain unchanged in case of merger

## No Merger : Fight amongst equals

- As in duopoly case  $\pi_i = (p_i - c) * q_i$
- In absence of merger all backbone prefer higher quality of inter connectivity  $\theta_{ij} = 1$
- All backbones obtain identical profits  $\pi_i = (1 - \nu) * (q_i^*)^2$

$$q_i^* = \frac{1 - c + \nu\beta}{5(1 - \nu)} \quad (9)$$

Various strategies post merger  
(for dominant backbone)

Accommodation

Dominant backbone maintains good interconnection with other two backbones

Global Degradation

Dominant backbone doesn't interconnect at all

Targeted Degradation

The dominant backbone refuses interconnection with one of the small backbones and prevents transit of traffic through the other backbone

- Global degradation is not profitable; reduces demand and does not yield competitive advantage over rivals
- Accommodation increases total demand but eliminates any competitive advantage that dominant backbone could have from its bigger user base

Target Degradation  
(with transit prevention)

- Backbone 1 is connected to 3/4 of initial user base
- Backbone 2 is connected to all initial user base
- Backbone 3 is connected to 1/4 of initial user base
- Analysis confirms that backbone 2 would prefer not to offer transit services

## Targeted Degradation

- If  $\nu\beta/(1-c) \in (1, 4(1-2\nu)/(1+\nu))$  (the interval is not empty when the externalities are large enough) then targeted degradation strategy prevents targeted backbone from acquiring any new customers
- In this case market share of backbone 2 among date 1 new customers is higher than backbone 1.

$$q_2^* - q_1^* = \nu\beta/4(1-\nu) > 0$$

# Duopoly

- A fraction of  $\beta_i - \alpha/2$  of single homers attach to both the backbones and  $\beta_1 \geq \beta_2$
- A fraction  $\alpha$  of multihomers connect to both backbones
- Number of new customers by the Cournot equilibrium

$$q_i^*(\alpha, \theta) = q_i^*(0, \theta) + \alpha/2 \left(1 - \frac{3(1 - \nu)}{2 - \nu(2 + \theta)}\right) \quad (10)$$

- The bigger backbone prefers lower connectivity :  $\theta_1^* \leq \theta_2^*$

- If  $F(\theta) = 0$ , the equilibrium  $\theta$  decreases with the extent of installed base multihoming
- In the case of costly connectivity there exists an  $\bar{\alpha}$  between 0 and 1, such that the equilibrium quality of interconnection is equal to 1 for  $\bar{\alpha} \leq \alpha$  and is smaller than 1 for  $\alpha > \bar{\alpha}$ . Can be proved by

$$\pi_i^*(\alpha, \theta) - F(\theta) \geq \pi_i^*(\alpha, 1) - F(1) \quad (11)$$

- If only takes values 0 and 1, then under duopoly the equilibrium quality of connection is decreasing in  $\alpha$

## Target Degradation

- Assume framework of a dominant backbone 1 and smaller backbone 2 and 3
- $\beta/2 - \alpha/2$  single homers connect to backbone 1 (dominant)
- $\beta/4 - \alpha/2$  single homers connect to backbone 3 (targeted)
- $\beta/4$  single homers connect to backbone 2 (non targeted)
- Equilibrium profit of backbone 1 under targeted degradation increases with  $\alpha$ , accommodation profit is independent of  $\alpha$
- The above statement implies that backbone 1 will target the backbone with the largest  $\alpha$ ;

## Duopoly

- We only consider the cases where  $\theta = 0$  or  $1$ . In case  $\theta = 1$  the customers have no incentive to multihome
- $\eta_i$  are the number of customers in the initial base of backbone  $j \neq i$  who multihome with  $i$
- $\eta$  are the new customers who multihome
- $q_i$  are the customers who connect solely with backbone  $i$
- The possibility of multihoming in response to degradation does not perturb the equilibrium if below condition is met  $(1 - \nu)(q_2^*)^2 \geq (\nu((1 - q_1^*) + \beta_2) - c)(1 + \beta_1)$



- Add to the basic model a small proportion of customers with a higher valuation for connectivity  $(1 + \rho)\nu$  instead of  $\nu$ . There exists an interval  $(\rho_2, \rho_1)$  such that for any  $\rho$  in that interval, in the presence of degradation, these consumers
- Select the dominant backbone if unattached
- Multihome to the dominant backbone if they are a part of the smaller backbone's installed base
- Do not multihome if they are a part of the larger backbone's installed base
- If  $\rho < \rho_2$  no multihoming; if  $\rho > \rho_1$  all customers with a high valuation for connectivity multihome

## Technical Critique

- Customer is not charged as per the usage but a monthly subscription is done (they claim that this won't change the final results)
- Backbones incur a cost  $c$  from connecting each additional customer
- Cost for connectivity  $\theta_i$ , for backbone  $i$ , is  $F(\theta_i)$ . This quantity is assumed to be zero. "No transfer" case.
- Assumption that the cost of connectivity = 0
- Installed bases are assumed to be locked-in, i.e the customers will not shift in new equilibrium
- Assumed that valuation of every customer is same which is not the case in real scenario
- In the pre-existing multihoming new customers and pre-existing non multihoming customers cannot multihome

# References



Jacques Cremer, Patrick Rey, Jean Tirole

Connectivity in the Commercial Internet

*Conference on Competition and Innovation in Personal Computer Industry*, San Diego, 24 April 1999

- Wikipedia pages for *Cournot* and *Bertrand* competition

# Thank You

# Course Summary

- Introduction, Microeconomics
- Pricing
  - Without taking content into consideration
  - With CDN (co-operative analysis/double-sided market)
- Auctions