EE 765

Network Information Theory

Project Report

Study on Wireless Network Information Flow Summary

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1 Introduction

Broadcast and *superposition* are the two main distinguishing features of wireless communication. Finding a maximum achievable rate of information flow in a wireless network with single source, single destination and an arbitrary number of relay nodes, has been a long-standing problem. This paper[1] attempts to solve this problem using a two-step approach.

1. A deterministic channel model has been proposed, which captures the key properties of signal strength, broadcast and superposition.

2. And this developed deterministic analysis has been used to design a new *quantize-map-and-forward* scheme for Gaussian networks.

Wireless networks operate in the interference-limited regime where noise power is small compared to signal power and most of the interference comes from other signals. This is a major motivation for the designed approach, and so the model focuses more on signal interactions than the background noise. When multiple copies of the transmitted signals are received at different relays, instead of considering them as interference, *quantize-map-and-forward* scheme forwards all the available information as, all the received signals are a function of the same transmitted message by the source.

2 Summary

2.1 The Principle Idea

The main equation of the designed model is (motivated for point-to-point link)

$$y = hx + z \tag{1}$$

where, $z \sim \mathcal{N}(0, 1)$, and $E[|x|^2] \leq 1$ at the transmitter. Transmit power and noise power are both normalized to unity and channel gain h is related to SNR by

$$|h| = \sqrt{SNR}$$

Now writing 1 in the binary expansion form, we get

$$y = 2^{\frac{1}{2}log(SNR)} \sum_{i=1}^{\infty} x(i)2^{-i} + \sum_{i=1}^{\infty} z(i)2^{-i}$$

Now, substituting $[\frac{1}{2}log(SNR)]^+$ as n, we get

$$y \approx 2^n \sum_{i=1}^{\infty} x(i)2^{-i} + \sum_{i=1}^{\infty} z(i)2^{-i}$$

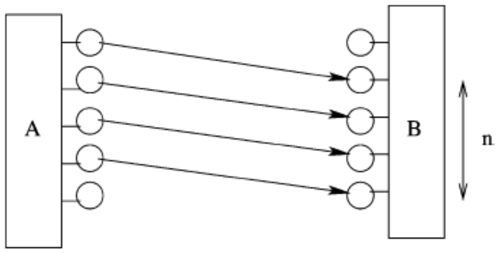

Figure 1: Pictorial representation of deterministic model for point-to-point channel (taken from [1])

We can see that 1^{st} n bits of the binary expansion of x are above the noise power level because of multiplication by 2^n, so we represent the above equation as,

$$y \approx 2^n \sum_{i=1}^{n} x(i)2^{-i} + \sum_{i=1}^{\infty}(x(i+n) + z(i))2^{-i}$$

Now, we truncate the second part, as those are the bits below noise power, and pass the signal bits above noise level. Thus, n most significant bits of x are being transmitted, where,

$$n = [\frac{1}{2} log(SNR)]^+$$

This input-output relationship can algebraically written as

$$y = \mathbf{S}^{q-n}x$$

After developing this basic framework, the paper[1] goes ahead to show how we can model broadcast, superposition and relay networks using this scheme, and also shows that the achievability rate is within one bit of the capacity region of Gaussian BC and MAC. And it also talks about cases like MIMO channels where this scheme does not work very well and in fact the gap as compared with Gaussian MIMO case shoots to infinity with k.

2.2 Linear Finite-Field Deterministic Model

In a relay network defined over a set of vertices $\mathcal{V}$, having one source, one (or multiple) destination(s) and relays. At each time t, node i receives a vector $\mathbf{y}_i[t] \in \mathbb{F}_p^q$ and transmits a vector $\mathbf{x}_i[t] \in \mathbb{F}_p^q$, where p is positive integer indicating field size and $q = max_{i,j}(n_{ij})$ (n_{ij} is the channel gain associated with every communication link from node i to node j). The received signal at each node is a linear, deterministic function of transmitted signals at all other nodes, i.e., if n is the total number of transmitting nodes,

$$\mathbf{y}_j[t] = \sum_{i=1}^{n} \mathbf{S}^{q-n_{ij}} \mathbf{x}_i[t] \tag{2}$$

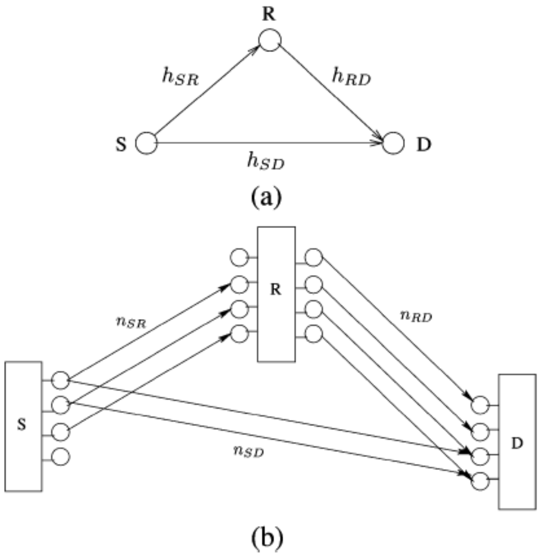

Figure 2: Example network with single relay: (a) Gaussian model; (b) linear finite-field deterministic model (taken from [1])

where, all the summations and multiplications are in $\mathbb{F}_2$ (ie $p = 2$) and $\mathbf{S}$ is a $q\ X\ q$ shift matrix.

2.3 Motivation

Whenever a network is assumed to be synchronized, i.e., all transmissions occur on a common clock, the relay is allowed to do any causal processing. And, for any such network, there is a natural information-theoretic cut-set bound, which upper bounds the reliable transmission rate R. Such a bound on relay network is proved to be

$$\bar{C} = \max_{p(\{x_j\}_{j\in\mathcal{V}})} \min_{\Omega\in\Lambda_D} I(\mathbf{y}_{\Omega^c}; \mathbf{x}_\Omega | \mathbf{x}_{\Omega^c}) \tag{3}$$

And the equivalent result for multicast network being

$$\bar{C}_{mult} = \max_{p(\{x_j\}_{j\in\mathcal{V}})} \min_{D\in\mathcal{D}} \min_{\Omega\in\Lambda_D} I(\mathbf{y}_{\Omega^c}; \mathbf{x}_\Omega | \mathbf{x}_{\Omega^c}) \tag{4}$$

where $\Lambda_D = \{\Omega : S \in \Omega, D \in \Omega^c\}$

Single relay network, diamond network and an example of a four relay network are used to motivate the approach. It is shown that the capacity as obtained using linear deterministic model is same as the cut-set upper bound. It also compares the capacities obtained by amplify-forward and decode-forward schemes against cutset bounds, and goes about to show the superiority of linear deterministic model in these examples.

Taking the example of single relay case as shown in the figure 2, we can argue that the capacity of deterministic relay channel, C^d_{relay}, should be smaller than both the maximum bits that the source can transmit and the maximum number of bits that the destination can receive.
Thus, $C^d_{relay} \leq min(max(n_{SR}, n_{SD}), max(n_{RD}, n_{SD}))$, and this also turns out to be the cutset upper bound for the linear deterministic network shown in figure 2.b.

2.4 Main Results

$\mathbf{G}_{\Omega,\Omega^c}$ is the transfer matrix associated with the cut Ω, and is induced by 2. $\mathbf{G}_{\Omega,\Omega^c}$ relates the vector of all the inputs at nodes in Ω to the vector of all outputs at nodes in Ω^c. The following theorems for linear finite-field deterministic relay networks:

1. The capacity C of a linear finite-field deterministic relay networks is given by

$$C = \min_{\Omega \in \Lambda_d} rank(\mathbf{G}_{\Omega,\Omega^c}) \tag{5}$$

2. The multicast capacity C of a linear finite-field deterministic relay network is given by

$$C = \min_{D \in \mathcal{D}} \min_{\Omega \in \Lambda_d} rank(\mathbf{G}_{\Omega,\Omega^c}) \tag{6}$$

3. The capacity of C of a linear finite-field deterministic Gaussian relay network satisfies

$$\bar{C} - \kappa \leq C \leq \bar{C} \tag{7}$$

   where $\bar{C}$ is the cutset upper bound on the capacity of $\mathcal{G}$ as described in 3, and κ is a constant which is upper bounded by a function of M_i and N_i, which are the number of transmit and receive antennas for each node.

4. The multicast capacity of C of a linear finite-field deterministic Gaussian relay network satisfies

$$\bar{C}_{mult} - \kappa \leq C_{mult} \leq \bar{C}_{mult} \tag{8}$$

   where $\bar{C}_{mult}$ is the cutset upper bound on the multicapt capacity of $\mathcal{G}$ as described in 4, and κ is a constant which is upper bounded by a function of M_i and N_i, which are the number of transmit and receive antennas for each node.

   It has also been shown that κ reduces further when vector quantization is used instead of scalar quantization.

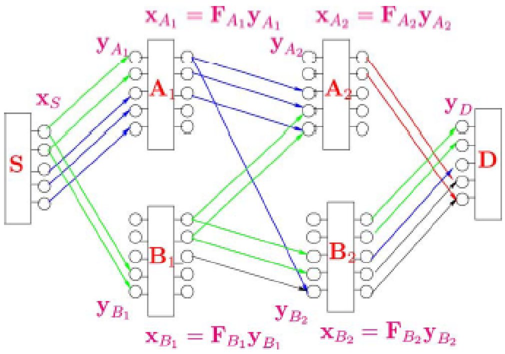

Figure 3: An example of a layered network (taken from [1])

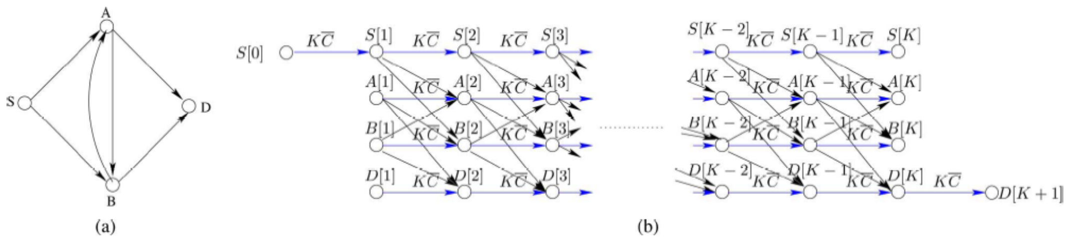

Figure 4: An example of an arbitrary network unfolded in time to form a layered network (taken from [1])

2.5 Flow of the proof

Probability of error has been analyzed in all the cases by using the notion of indistinguishability of two different messages at a set of nodes. Linear finite-field deterministic layered[1] relay networks have been focused first. An example of a layered network has been shown in figure 3. This special structure allows us to use a block of symbols for encoding sequence of messages, and these blocks do not interact as they pass through the relay nodes. This idea is then used to prove arbitrary networks (not necessarily layered), by unfolding the given network $\mathcal{G}$ to $\mathcal{G}_{unf}^{(K)}$ over time, so as to create a layered network (as shown in figure 4).

Next, the results have been proved for Gaussian relay networks, analyzing the layered networks and then extending the ideas to non-layered cases. Here, the concept of inner codebook and outer codebook has been introduced where,

- Inner codebook quantizes the received signals at each relay and then maps it independently into an i.i.d. $\mathcal{CN}(\mathbf{0}, 1)$ random vector. Also, all inner codebook

[1]Layered structure: All paths from the source to the destination have equal lengths

symbols are mapped to a Gaussian $\sim \mathcal{CN}(\mathbf{0}, 1)$ codeword.

- Outer codebook encodes the message into N inner code symbols.

And then, the average end-to-end mutual information has been lower bounded, eventually proving theorem mentioned in 7 for layered as well as non-layered cases.

References

[1] A. Salman Avestimehr, Suhas N. Diggavi, David N. C. Tse
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IEEE Transactions on Information Theory. Vol. 57, no. 4, April 2011

[2] Selected chapters from *Network Information Theory*
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