Signals and Waveform Synthesis

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2.1. INTRODUCTION

A signal may be considered to be a function of time that represents a physical variable of interest associated with a system. In electrical systems, the excitation (input) and response (output) are given in terms of currents and voltages. Mostly, these currents and voltages are function of time. In general, these functions of time are called signals, *i.e.* signals are also called as the functions.

Signals play an important role in science and technology as communication, aeronautics, bio-medical engineering and speech processing etc.

2.2. CLASSIFICATION OF SIGNALS

Signals may describe a wide variety of physical phenomena, as follows:

2.2.1. Continuous-Time and Discrete-Time Signals

2.2.2. Even and Odd Signals

Another set of useful properties of signals relates to their symmetry under time reversal.

A signal $x(t)$ or $x[n]$ is referred to as an even signal if it is identical to its time-reversed counterpart. In continuous-time a signal is *even* if

$$x(-t) = x(t)$$

while a discrete-time signal is even if

$$x[-n] = x[n]$$

Examples: (*i*) t^n (where n is even) or t^{2n} (where $n \in$ integer) *i.e.* t^2, t^4, ...

(*ii*) $\cos t$, $\sin^2 t$, etc.

A signal is referred to as an odd if the signal is negative of its reflection, *i.e.*,

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

Examples: (*i*) t^n (where n is odd) or t^{2n+1} (where $n \in$ integer) *i.e.*, t, t^3, ...

(*ii*) $\sin t$, etc.

Note :

There are some functions (signals), which are neither even nor odd.

Examples: e^t, $t^2 + t$ etc.

Theorems:

(*i*) Sum of even functions = even function

(*ii*) Sum of odd functions = odd function

(*iii*) Multiplication of even and even functions = even function

(*iv*) Multiplication of odd and odd functions = even function

(*v*) Multiplication of even and odd functions = odd function

(*vi*) Sum of even and odd functions = Neither even nor odd.

2.2.3. Periodic and Unperiodic Signals

A signal $x(t)$ is *periodic* if and only if

$$x(t + T_0) = x(t), \qquad -\infty < t < \infty \qquad \text{...(2.1)}$$

where the constant T_0 is the period of $x(t)$. The smallest value of T_0 such that equation (2.1) is satisifed is referred to as the Time-period. Any signal not satisfying equation (2.1) is called unperiodic or aperiodic.

Examples: (*i*) $\sin t$, $\cos t$, $\sin {}^t/_2$, $\cos 4t$ etc. are periodic signals

(*ii*) e^t, t^2, t etc. are unperiodic signals.

2.3. STANDARD SIGNALS OR SINGULARITY FUNCTIONS

In order to simulate any signal, some standard signals which are realisable in the laboratory environment are described in this section.

Singularity function can be obtained from one another by successive differentiation or integration.

2.3.1. Step Signal

The step signal $f_s(t)$ is defined by

$$f_s(t) = \begin{cases} 0 & ; t < 0 \\ K & ; t > 0 \end{cases} \qquad \text{(where } K \text{ is the amplitude of the step signal)}$$

The signal $f_s(t)$ is graphed in figure 2.2 (*a*). Note that the function is undefined and discontinuous at $t = 0$.

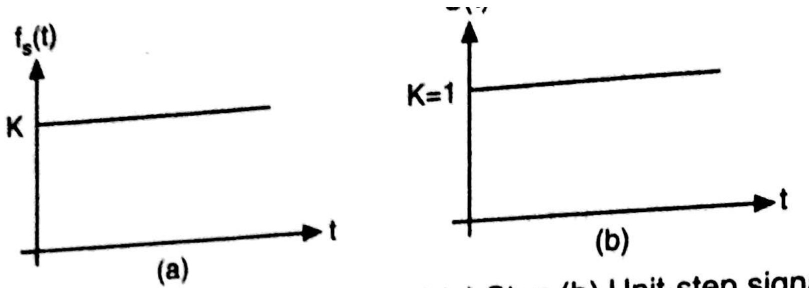

Fig. 2.2. Graphical representation of (a) Step (b) Unit-step signals.

If the value of $K = 1$ (unity), then, this step signal $f_s(t)$ is called as *unit step signal* $U(t)$, which is shown in figure 2.2 (b) and defined as

$$U(t) = \begin{cases} 0 \; ; t < 0 \\ 1 \; ; t > 0 \end{cases}$$

It can be observed from above, that the step and unit step signals are zero whenever the argument within the parentheses, namely, t is negative, and they have magnitude K and 1 respectively when the argument is greater than zero. This helps us in defining the shifted or delayed signals as

$$f_s(t-a) = \begin{cases} 0 \; ; t < a \\ K \; ; t > a \end{cases}$$

and
$$U(t-a) = \begin{cases} 0 \; ; t < a \\ 1 \; ; t > a \end{cases}$$

as shown in figure 2.3 (a) and (b) respectively.

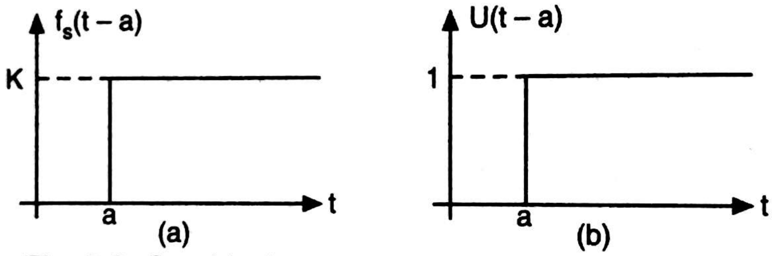

Fig. 2.3. Graphical representation of (a) Shifted step (b) Shifted unit-step signals.

EXAMPLE 2.1 **Express the given waveform as shown in figure 2.4 in terms of step signal.**

Solution:
$$f(t) = K\,U[t-(-t_1)]$$
$$f(t) = K\,U(t+t_1)$$

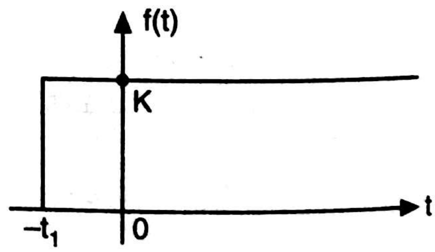

Fig. 2.4.

2.3.2. Ramp Signal

The ramp signal $f_r(t)$ is defined by

$$f_r(t) = \begin{cases} 0 \; ; t < 0 \\ Kt \; ; t \ge 0 \end{cases}$$

(where K is the slope of the ramp signal)

The signal $f_r(t)$ is graphed in figure 2.5 (a). If the value of slope $K = 1$, then, this ramp signal $f_r(t)$ is called as *unit ramp signal* $r(t)$, which is shown in figure 2.5 (b) and defined as

$$r(t) = \begin{cases} 0 \; ; t < 0 \\ t \; ; t \ge 0 \end{cases}$$

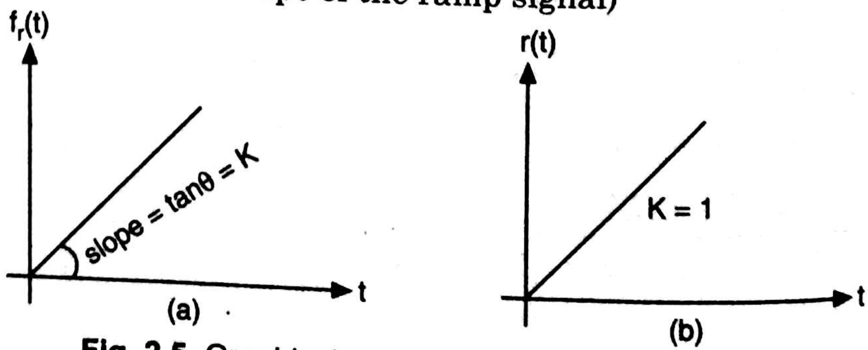

Fig. 2.5. Graphical representation of (a) Ramp (b) Unit-ramp signals

And, shifted ramp signal as shown in figure 2.6 (*a*) and is described as

$$f_r(t-a) = \begin{cases} 0 & ; t < a \\ K(t-a); & t \geq a \end{cases}$$

And, shifted unit ramp signal as shown in figure 2.6 (*b*) and is described as

$$r(t-a) = \begin{cases} 0 & ; t < a \\ (t-a); & t \geq a \end{cases}$$

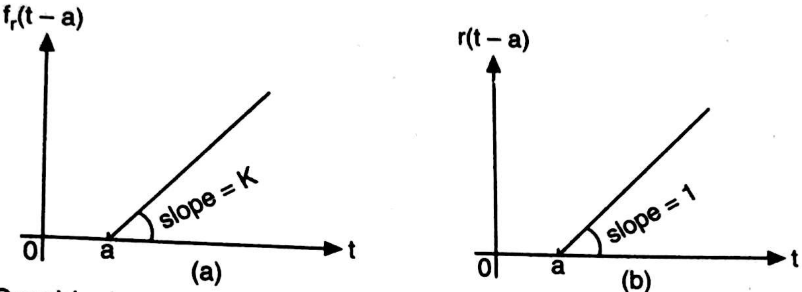

Fig. 2.6. Graphical representation of (a) Shifted ramp (b) Shifted unit-ramp signals

Note :

Unit ramp signal;

$$r(t) = t\,U(t)$$
$$r(t-a) = (t-a)\,U(t-a)$$

And ramp signal,

$$f_r(t) = K\,r(t) = K\,t\,U(t)$$
$$f_r(t-a) = K\,r(t-a) = K\,(t-a)\,U(t-a)$$

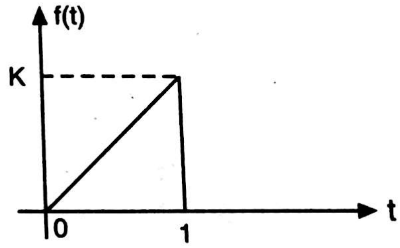

Fig. 2.7.

EXAMPLE 2.2 **Express the given triangular waveform as shown in figure 2.7, in terms of ramp and step signals.**

Solution: This triangular waveform may be generated using three signals as shown in figure 2.8 (*a*), (*b*) and (*c*).

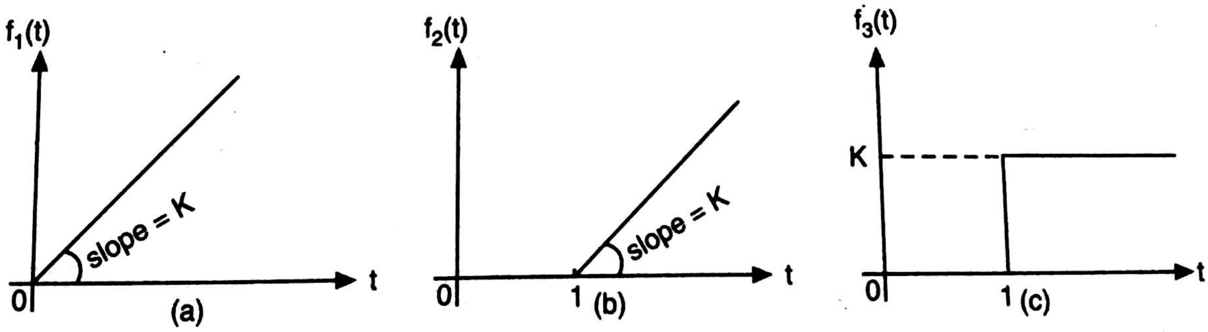

Fig. 2.8.

From the above representation,

$$\begin{aligned} f(t) &= f_1(t) - f_2(t) - f_3(t) \\ &= K\,r(t) - K\,r(t-1) - K\,U(t-1) \\ &= K\,t\,U(t) - K(t-1)\,U(t-1) - K\,U(t-1) \\ &= K[t\,U(t) - (t-1+1)\,U(t-1)] \\ &= K[t\,U(t) - t\,U(t-1)] = K\,t[U(t) - U(t-1)] \end{aligned}$$

2.3.3. Impulse Signal (It is also known as Dirac delta signal)

The impulse signal $f_\delta(t)$ is defined by

$$f_\delta(t) = \begin{cases} 0 \; ; \; t \neq 0 \\ A \; ; \; t = 0 \end{cases}$$

where A is the area of the impulse signal and some times called the strength of the impulse.

The signal $f_\delta(t)$ is graphed in figure 2.9 (a). If the value of the area $A = 1$, then, this impulse signal $f_\delta(t)$ is called as *unit impulse signal* $\delta(t)$, which is shown in figure 2.9 (b) and defined as

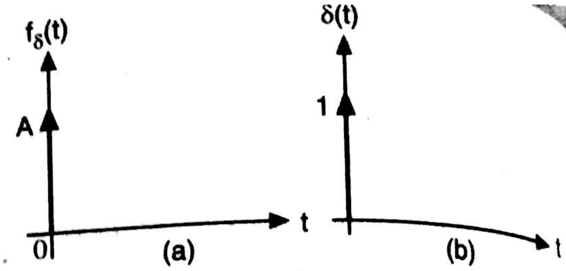

Fig. 2.9. Graphical representation of (a) Impulse (b) Unit-Impulse signals

$$\delta(t) = \begin{cases} 0 \; ; \; t \neq 0 \\ 1 \; ; \; t = 0 \end{cases}$$

The area of the unit impulse signal is defined as

$$\text{Area} = \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Therefore, unit impulse signal is also graphed as a sequence of small values $\epsilon_1, \epsilon_2, \epsilon_3$ (where $\epsilon_1 > \epsilon_2 > \epsilon_3$) as shown in figure 2.10.

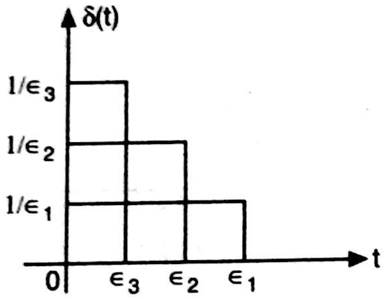

Fig. 2.10.

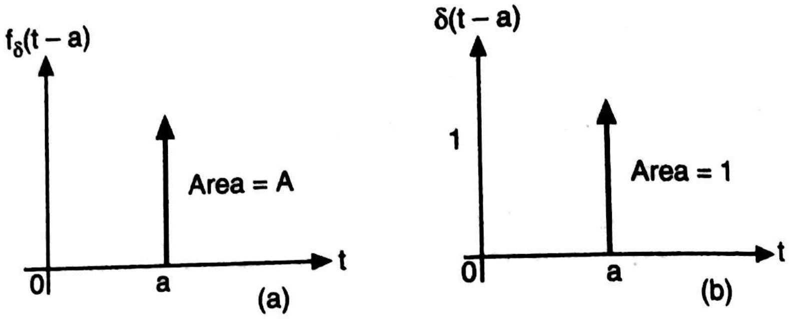

Fig. 2.11. Graphical representation of (a) shifted impulse (b) Shifted unit impulse signals.

It is clear as ϵ tends to zero, even then the area of the unit impulse function is always one, as Area

$$= \frac{1}{\epsilon_3}\cdot \epsilon_3 = \frac{1}{\epsilon_2}\cdot \epsilon_2 = \frac{1}{\epsilon_1}\cdot \epsilon_1 = 1$$

And, shifted impulse signal as shown in figure 2.11 (a) and is described as

$$f_\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ A \; ; \; t = a \end{cases}$$

And, shifted unit impulse signal as shown in figure 2.11 (b) and is described as

$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ 1 \; ; \; t = a \end{cases}$$

2.3.4. Relationship Between Standard Signals

Derivative of step signal = Impulse signal

Derivative of ramp signal = Step signal

In other words, we can say:

Integral of impulse signal = Step signal

Integral of step signal = Ramp signal

$$\frac{d}{dt}[f_s(t)] = f_\delta(t) \quad \text{or} \quad \int f_\delta(t)\, dt = f_s(t)$$

$$\frac{d}{dt}[f_r(t)] = f_s(t) \quad \text{or} \quad \int f_s(t)\, dt = f_r(t)$$

2.4. OTHER BASIC SIGNALS

2.4.1. Unit Doublet Signal

If a unit impulse signal $\delta(t)$ is differentiated with respect to t, we get

$$\delta'(t) = \frac{d}{dt}[\delta(t)] = +\infty \text{ and } -\infty;\ t = 0$$
$$= 0 \qquad ;t \neq 0$$

This signal is called unit doublet signal $\delta'(t)$ which is shown in figure 2.12 (*a*).

And shifted unit doublet signal $\delta'(t-a)$ as shown in figure 2.12 (*b*) and is described as

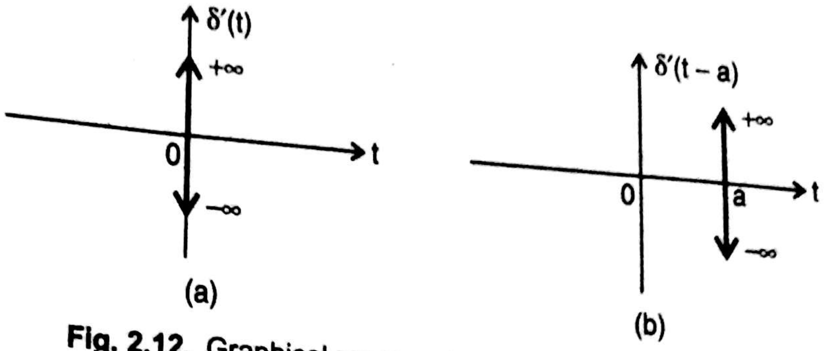

Fig. 2.12. Graphical representation of (a) Unit doublet (b) Shifted unit doublet signals

$$\delta'(t-a) = +\infty \quad \text{and} \quad -\infty;\ t = a$$
$$\delta'(t) = 0 \quad ; \quad t \neq a$$

Note :

Derivative of unit impulse signal = unit doublet signal

or

Integral of unit doublet signal = unit impulse signal

Mathematically,

$$\frac{d}{dt}[\delta(t)] = \delta'(t) \quad \text{or} \quad \int \delta'(t)\, dt = \delta(t)$$

2.4.2. Exponential Signal

Exponential signal $f(t)$ as shown in figure 2.13 and is described as

$$f(t) = \begin{cases} 0 & ; t \leq 0 \\ Ke^{-at} & ; t \geq 0 \end{cases} \qquad \text{(where } a \text{ and } K \text{ are real constants)}$$

The inverse of a has the dimension of time and is called as the *time constant*. $\tau = {}^1/_a$. This is the time taken to reach 63.2% of the total change from initial to final.

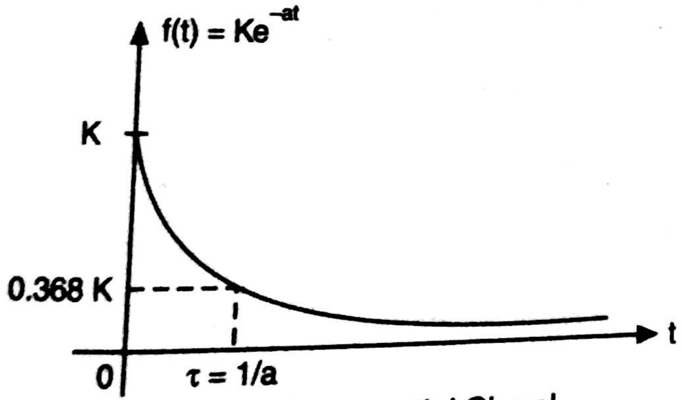

Fig. 2.13. Exponential Signal

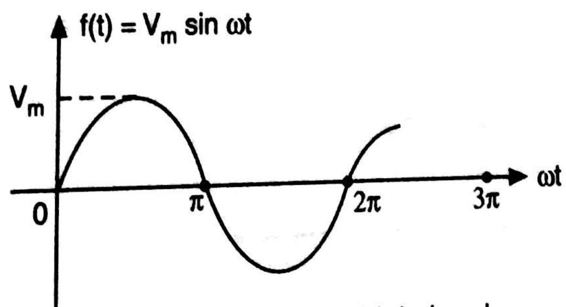

Fig. 2.14. Sinusoidal signal

2.4.3. Sinusoidal Signal

Sinusoidal signal $f(t)$ as shown in figure 2.14 and is described as

$$f(t) = \begin{cases} 0 & ; t < 0 \\ V_m \sin \omega t & ; t \geq 0 \end{cases}$$

(where V_m is peak amplitude, ω is angular frequency in rad/sec.)

2.4.4. Gate Signal (or Gate Function)

A rectangular pulse of unit height (*i.e.* amplitude is one), starting at $t = a$ and ending at $t = b$ as shown in figure 2.15, and represented as

$$G_{a,\,b}(t) = U(t-a) - U(t-b)$$

is called as a gate function.

If this gate function starting at origin as shown in figure 2.16, then,

$$G_{0,\,a}(t) = U(t) - U(t-a)$$

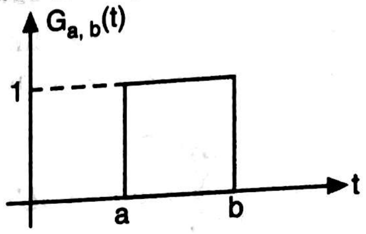

Fig. 2.15.

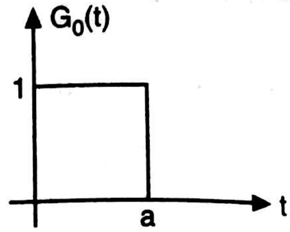

Fig. 2.16.

Application of Gate Function:

If any function (signal) multiplied by a gate function, then that function will have zero value outside the duration of the gate, and the value of the function will be unchanged within the duration of the gate.

EXAMPLE 2.3 **Synthesize the given a single half-sine waveform as shown in figure 2.17.**

Solution: Given single half-sine waveform can be constructed from the sum of two functions $v_1(t)$ and $v_2(t)$, as shown in figure 2.18 (*a*) and (*b*), as

$$v(t) = v_1(t) + v_2(t)$$

$$v_1(t) = V_m \sin \omega t \cdot U(t)$$

$$v_2(t) = V_m \sin \omega \left(t - \frac{T}{2}\right) \cdot U\left(t - \frac{T}{2}\right)$$

$$v(t) = V_m \left[\sin \omega t \cdot U(t) + \sin \omega \left(t - \frac{T}{2}\right) \cdot U\left(t - \frac{T}{2}\right)\right]$$

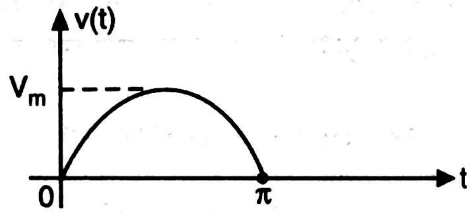

Fig. 2.17.

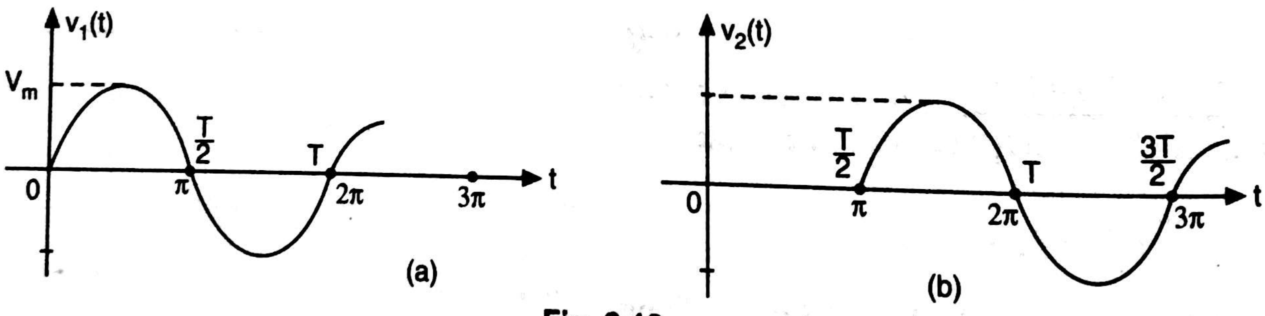

Fig. 2.18.

This same problem can be solved by using gate function as follows:

$$v(t) = V_m \sin \omega t \cdot \left[U(t) - U\left(t - \frac{T}{2}\right)\right] = V_m \left[\sin \omega t \cdot U(t) - \sin \omega t \cdot U\left(t - \frac{T}{2}\right)\right]$$

$$= V_m \left[\sin \omega t \cdot U(t) - \sin \frac{2\pi}{T}\left(t - \frac{T}{2} + \frac{T}{2}\right) \cdot U\left(t - \frac{T}{2}\right)\right]$$

Note :

Gate function is quite useful for finding out Laplace transform of periodic functions.

$$= V_m\left[\sin\omega t\cdot U(t) - \sin\left\{\frac{2\pi}{T}\left(t-\frac{T}{2}\right)+\pi\right\}\cdot U\left(t-\frac{T}{2}\right)\right]$$

$$= V_m\left[\sin\omega t\cdot U(t) + \sin\left\{\frac{2\pi}{T}\left(t-\frac{T}{2}\right)\right\}\cdot U\left(t-\frac{T}{2}\right)\right]$$

$$= V_m\left[\sin\omega t\cdot U(t) + \sin\omega\left(t-\frac{T}{2}\right)\cdot U\left(t-\frac{T}{2}\right)\right]$$

2.5. DIRECT FORMULA (OR K.M. FORMULA)

If a function is a combination of various gate functions, then we can develop a formula to represent that function directly in terms of step functions. This formula can be called as Direct formula or K.M. formula. It is given by

$$f(t) = \sum_{T=-\infty}^{\infty}(A_f - A_i)U(t-T)$$

where T is the time instant at which function $f(t)$ changes its values, A_f and A_i are the final and initial values at the corresponding time instant respectively.

If the function $f(t)$ exist (define) only for $t \geq 0$, then Direct formula reduces to

$$f(t) = \sum_{T=0}^{\infty}(A_f - A_i)U(t-T)$$

To illustrate the use of the Direct formula, let us consider several examples.

EXAMPLE 2.4 Synthesize the waveform as shown in figure 2.19 using standard signal.

(I.P. Univ., 2001)

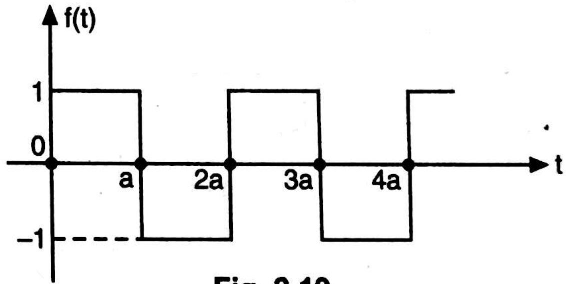

Fig. 2.19.

Solution: The function $f(t)$ can be written as the sum of the gate functions, as

$$f(t) = G_{0,a}(t) + (-1)\,G_{a,2a}(t) + G_{2a,3a}(t) + (-1)G_{3a,4a}(t) + \ldots..$$

$$f(t) = 1.[U(t) - U(t-a)] + (-1)[U(t-a) - U(t-2a)] + 1[U(t-2a) - U(t-3a)] + (-1)[U(t-3a) - U(t-4a) + \ldots..$$

$$= U(t) - 2U(t-a) + 2U(t-2a) - 2U(t-3a) + \ldots...$$

Alternative ways: (Using Direct formula)

$$f(t) = \sum_{T=0}^{\infty}(A_f - A_i)U(t-T)$$

where T is the time at which function changes its values.

$$= (1-0)\cdot U(t-0) + (-1-1)\cdot U(t-a) + [1-(-1)]\cdot U(t-2a) + (-1-1)\cdot U(t-3a) + \ldots.$$

$$= U(t) - 2U(t-a) + 2U(t-2a) - 2U(t-3a) + \ldots.$$

EXAMPLE 2.5 Synthesize the waveform as shown in figure 2.20 using step functions.

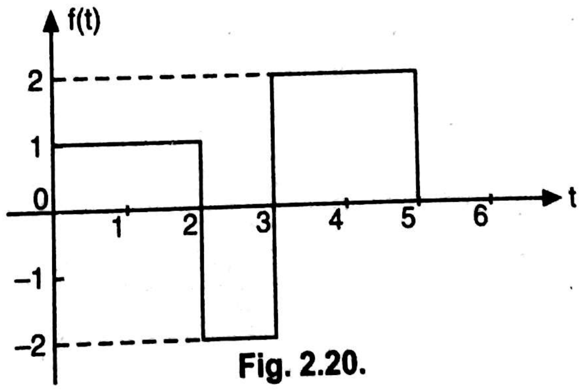

Fig. 2.20.

Solution: (*i*) Using gate functions

$$f(t) = G_{0,2}(t) + (-2)\cdot G_{2,3}(t) + 2\cdot G_{3,5}(t)$$

$$f(t) = 1\cdot[U(t) - U(t-2)] + (-2)\cdot[U(t-2) - U(t-3)] + 2\cdot[U(t-3) - U(t-5)]$$

$$= U(t) - 3U(t-2) + 4U(t-3) - 2U(t-5)$$

(*ii*) Using Direct formula:

$$f(t) = \sum_{T=0}^{\infty}(A_f - A_i)U(t-T)$$

$$f(t) = (1-0)\cdot U(t-0) + (-2-1)\cdot U(t-2) + [2-(-2)]\cdot U(t-3) + (0-2)\cdot U(t-5)$$

$$= U(t) - 3U(t-2) + 4U(t-3) - 2U(t-5)$$

EXAMPLE 2.6 **Synthesize the waveform as shown in figure 2.21.**

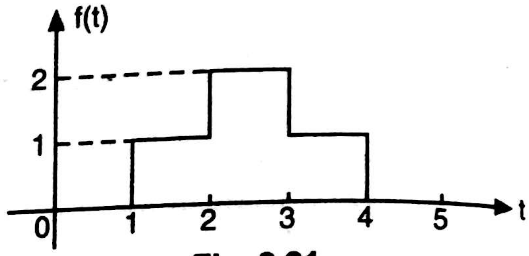

Fig. 2.21.

Solution: (*i*) Using Gate functions:

$$f(t) = G_{1,2}(t) + 2G_{2,3}(t) + G_{3,4}(t)$$

$$= 1\cdot[U(t-1) - U(t-2)] + 2\cdot[U(t-2) - U(t-3)] + 1\cdot[U(t-3) - U(t-4)]$$

$$= U(t-1) + U(t-2) - U(t-3) - U(t-4)$$

(*ii*) Using Direct formula:

$$f(t) = \sum_{T=0}^{\infty}(A_f - A_i)U(t-T)$$

$$= (1-0)\cdot U(t-1) + (2-1)\cdot U(t-2) + (1-2)\cdot U(t-3) + (0-1)\cdot U(t-4)$$

$$= U(t-1) + U(t-2) - U(t-3) - U(t-4)$$

EXAMPLE 2.7 **Synthesize the following waveforms as shown in figure 2.22. Using gate functions.**

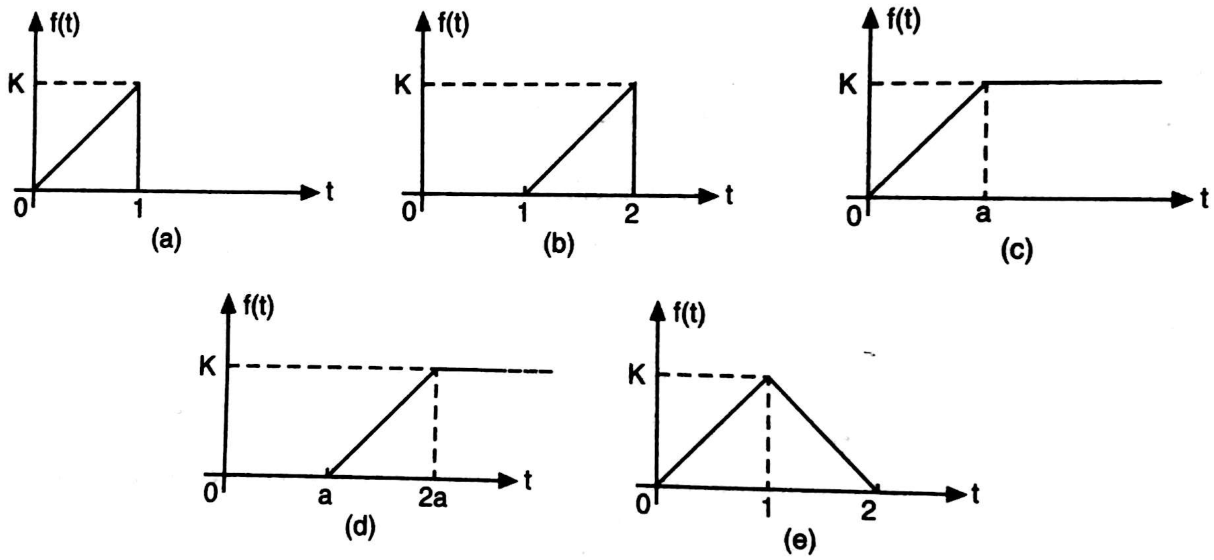

Fig. 2.22.

Solution:

(*a*) $f(t) = Kt \cdot [U(t) - U(t-1)]$

(*b*) $f(t) = K(t-1)\cdot[U(t-1) - U(t-2)]$ (since $K(t-1)$ is the expression of the line)

(*c*) This function $f(t)$ can be represented as the sum of two functions, one is from 0 to a and other is a to infinity. As,

$$f(t) = \frac{K}{a}\cdot t[U(t) - U(t-a)] + KU(t-a) = \frac{K}{a}tU(t) - \frac{K}{a}(t-a)\,U(t-a)$$

(*d*) Similar to part (*c*)

$$f(t) = \frac{K}{a}(t-a)[U(t-a) - U(t-2a)] + KU(t-2a)$$

$$= \frac{K}{a}(t-a)U(t-a) - \frac{K}{a}(t-2a)U(t-2a)$$

(e) Given waveform can break into two functions. One is from 0 to 1 and other from 1 to 2. As,

$$f(t) = K\,t\cdot[U(t) - U(t-1)] - K(t-2)\cdot[U(t-1) - U(t-2)]$$
$$= K\,t\,U(t) - 2\,Kt\,U(t-1) + 2K\,U(t-1) + Kt\,U(t-2) - 2K\,U(t-2)$$
$$= K\,t\,U(t) - 2K(t-1)\,U(t-1) + K(t-2)\,U(t-2)$$

EXAMPLE 2.8 **Synthesize the given waveform as shown in figure 2.23.**

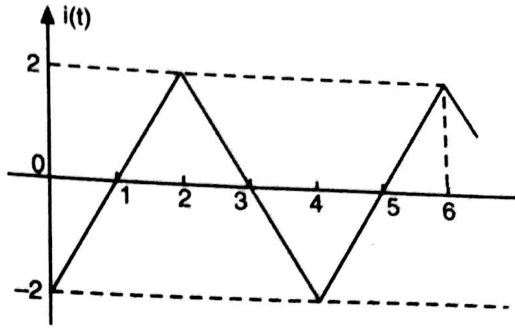

Fig. 2.23.

Solution: Using gate functions, we can represent as,

$$i(t) = (2t-2)[U(t) - U(t-2)] + (-2t+6)\,[U(t-2) - U(t-4)] + (2t-10)[U(t-4) - U(t-6) + ...$$
$$= (2t-2)\,U(t) + (-2t+2-2t+6)\,U(t-2) + (2t-6+2t-10)\,U(t-4) + ...$$
$$= (2t-2)\,U(t) + (-4t+8)\,U(t-2) + (4t-16)\,U(t-4) + ...$$
$$= 2(t-1)\,U(t) - 4(t-2)\,U(t-2) + 4(t-4)\,U(t-4) + ...$$

EXAMPLE 2.9 **Express the given waveform as shown in figure 2.24 using standard signal.**

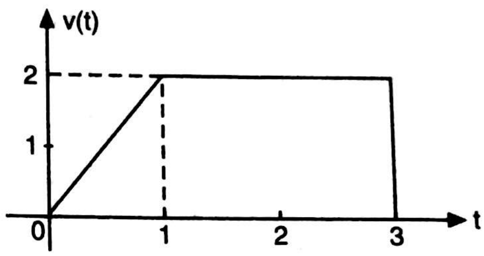

Fig. 2.24.

Solution: The function $v(t)$ can be written as the sum of a ramp and a step functions, as

$$v(t) = 2[r(t) - r(t-1)] - 2U(t-3)$$

Putting

$$r(t) = t\,U(t)$$

and $\quad r(t-1) = (t-1)\,U(t-1)$

$$v(t) = 2t\,U(t) - 2(t-1)\,U(t-1) - 2U(t-3)$$

Alternative way (using gate functions):

$$v(t) = 2t\cdot G_{0,1}(t) + 2\cdot G_{1,3}(t)$$
$$= 2t[U(t) - U(t-1)] + 2\cdot[U(t-1) - U(t-3)]$$
$$= 2t\,U(t) - 2t\,U(t-1) + 2U(t-1) - 2U(t-3)$$
$$= 2t\,U(t) - 2(t-1)\,U(t-1) - 2U(t-3)$$

EXAMPLE 2.10 **Express the waveform shown in figure 2.25 in terms of delayed functions.**

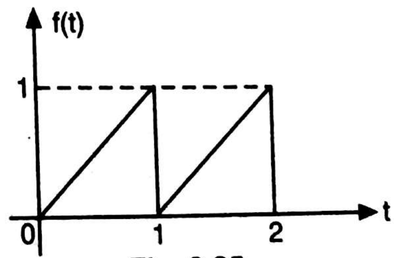

Fig. 2.25.

Solution: Using gate functions

$$f(t) = t\cdot G_{0,1}(t) + (t-1)\,G_{1,2}(t)$$
$$= t[U(t) - U(t-1)] + (t-1)\,[U(t-1) - U(t-2)]$$
$$= t\,U(t) - U(t-1) - (t-1)\,U(t-2)$$
$$= t\,U(t) - U(t-1) - (t-2)\,U(t-2) - U(t-2)$$
$$= r\,(t) - U(t-1) - r(t-2) - U(t-2)$$

EXAMPLE 2.11 **Synthesize the given waveform as shown in figure 2.26 using step and ramp signals.**

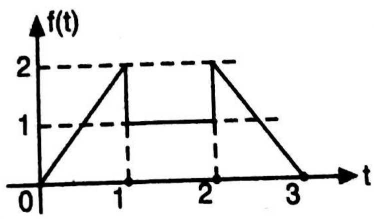

Fig. 2.26.

Solution: Using gate functions

$$f(t) = 2t\cdot G_{0,1}(t) + G_{1,2}(t) + (-2t+6)\;G_{2,3}(t)$$
$$= 2t[U(t) - U(t-1)] + [U(t-1) - U(t-2)] + (-2t+6)[U(t-2) - U(t-3)]$$
$$= 2t\cdot U(t) - (2t-1)\,U(t-1) - (2t-5)\,U(t-2) + (2t-6)\,U(t-3)$$

EXAMPLE 2.12 Sketch the waveform from the expression

$$v(t) = U(t) + \sum_{K=1}^{\infty}(-1)^K \cdot 3U(t-K)$$

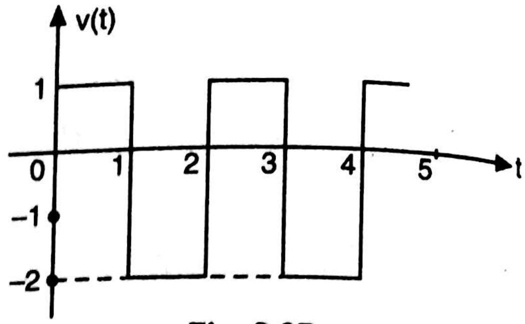

Fig. 2.27.

Solution: $v(t) = U(t) + \sum_{K=1}^{\infty}(-1)^K \cdot 3U(t-K)$

Expanding given expression, as

$$v(t) = U(t) - 3U(t-1) + 3U(t-2) - 3U(t-3) + 3U(t-4) - \ldots$$

$$= [U(t) - U(t-1)] - 2[U(t-1) - U(t-2)] + [U(t-2) - U(t-3)] - 2[U(t-3) - U(t-4)] + \ldots$$

Waveform for above expression is shown in figure 2.27.

EXAMPLE 2.13 Synthesize the given waveform as shown in figure 2.28 using gate signals.

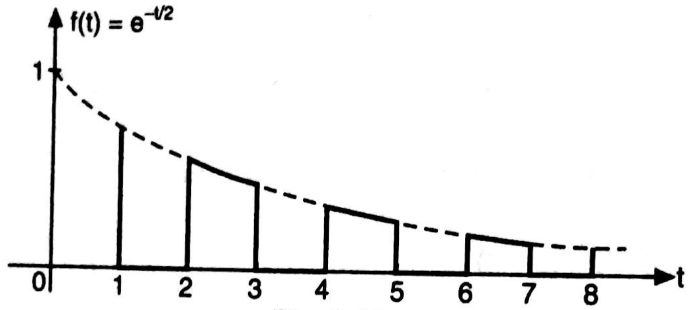

Fig. 2.28.

Solution:

$$f(t) = e^{-t/2}[U(t) - U(t-1)] + e^{-t/2}[U(t-2) - U(t-3)] + e^{-t/2}[U(t-4) - U(t-5)] + \ldots$$

$$= e^{-t/2}\,[U(t) - U(t-1) + U(t-2) - U(t-3) + U(t-4) - \ldots]$$

EXAMPLE 2.14 Express the given waveform as shown in figure 2.29 using ramp signals.

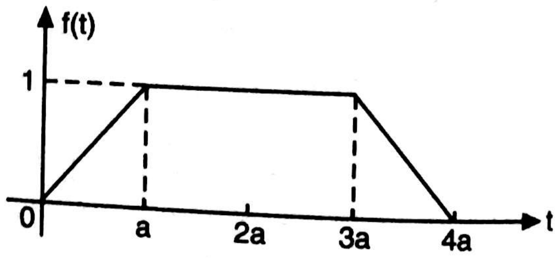

Fig. 2.29.

Solution:

$$f(t) = \frac{1}{a}\cdot t[U(t) - U(t-a)] + [U(t-a) - U(t-3a)] + \left[\left(-\frac{1}{a}\right)t + 4\right][U(t-3a) - U(t-4a)]$$

$$f(t) = \frac{1}{a}\,t\,U(t) - \frac{1}{a}\,t\,U(t-a) + U(t-a) - U(t-3a) - \frac{1}{a}\,t\,U(t-3a) + 4U(t-3a) + \frac{1}{a}\,t\,U(t-4a) - 4U(t-4a)$$

$$= \frac{1}{a}\,t\,U(t) - \frac{1}{a}(t-a)\,U(t-a) - \frac{1}{a}(t-3a)\,U(t-3a) + \frac{1}{a}(t-4a)\,U(t-4a)$$

$$= \frac{1}{a}\,r(t) - \frac{1}{a}\,r(t-a) - \frac{1}{a}\,r(t-3a) + \frac{1}{a}\,r(t-4a)$$

EXAMPLE 2.15 Sketch the waveform

$$i(t) = 1.5(1 - e^{-4t})\,U(t) - 1.5[1 - e^{-4(t-0.1)}]\,U(t-0.1)$$

Solution: If $i_1(t) = 1.5(1 - e^{-4t})\,U(t)$ as shown in figure 2.30 (*a*)

and $i_2(t) = 1.5[1 - e^{-4(t-0.1)}]\,U(t-0.1)$ as shown in figure 2.30 (*b*).

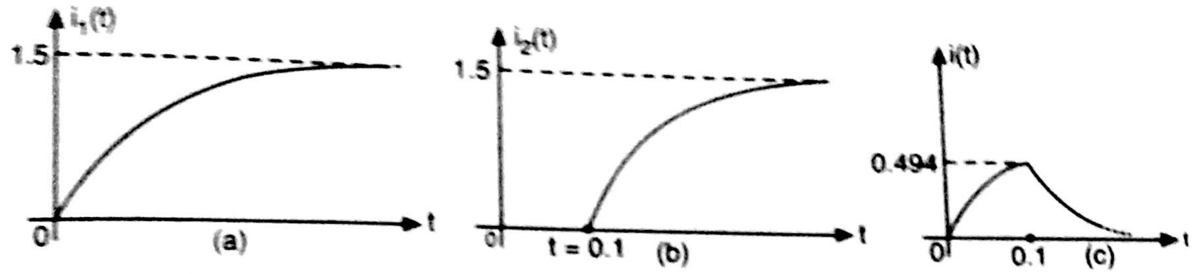

Fig. 2.30. Graphical representations of (a) $i_1(t)$, (b) $i_2(t)$ and (c) $i(t)$

Therefore, $i(t) = i_1(t) - i_2(t)$; as shown in figure 2.30 (c).

EXAMPLE 2.16 **Write an equation for the parabolic pulse $v(t)$ shown in figure 2.31 using delayed step functions.**

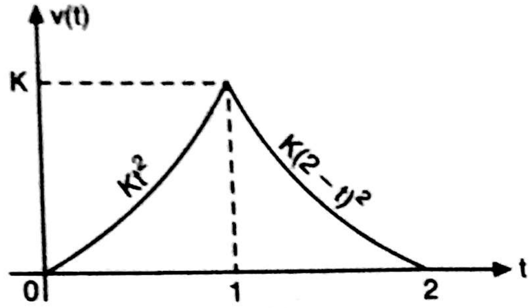

Fig. 2.31.

Solution: Using gate functions

$$v(t) = Kt^2 \cdot G_{0,1}(t) + K(2-t)^2 \cdot G_{1,2}(t)$$
$$= K[t^2 \,.\, U(t) + (-t^2 + 4 + t^2 - 4t)\; U(t-1) - (2-t)^2\, U(t-2)]$$
$$= K[t^2 \,.\, U(t) - 4(t-1)\, U(t-1) - (t-2)^2\, U(t-2)]$$

EXAMPLE 2.17 **The accompany figure 2.32 shows a waveform made up of straight line segments. For this waveform, write an equation for the $v(t)$ in terms of steps, ramps and other related function as needed.** *(I.P. Univ., 2000)*

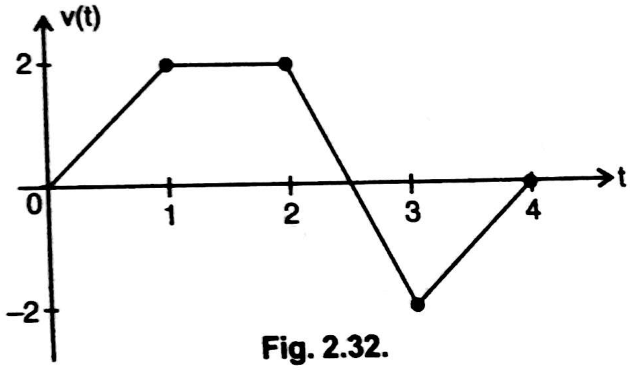

Fig. 2.32.

Solution : $v(t) = v_1(t) + v_2(t) + v_3(t)$

where, $v_1(t)$, $v_2(t)$ and $v_3(t)$ are the three parts of the given waveform from $t = 0$ to $t = 2$, $t = 2$ to $t = 3$ and $t = 3$ to $t = 4$ respectively.

$$v_1(t) = 2r(t) - 2r(t-1) - 2U(t-2)$$
$$= 2t\, U(t) - 2(t-1)\, U(t-1) - 2U(t-2)$$
$$v_2(t) = (-4t + 10)\, [U(t-2) - U(t-3)]$$
$$v_3(t) = (2t-8)\, [U(t-3) - U(t-4)]$$

Therefore,

$$v(t) = 2t\, U(t) - 2\,(t-1)\, U(t-1) + (-2 - 4t + 10)\, U(t-2) + (4t - 10 + 2t - 8)\, U(t-3) - (2t-8)\, U(t-4)$$
$$= 2t\, U(t) - 2(t-1)\, U(t-1) - 4(t-2)\, U(t-2) + 6(t-3)\, U(t-3) - 2(t-4)\, U(t-4)$$

or $$v(t) = 2r(t) - 2r(t-1) - 4\,r(t-2) + 6r(t-3) - 2r(t-4)$$

EXAMPLE 2.18 **Sketch the following signals:**

(i) $U(t-5) - U(t-8)$

(ii) $U(t-6) + U(t-4)$

(iii) $t^2\, [U(t-1) - U(t-3)]$

(iv) $(t-4)\, [U(t-1) - U(t-4)]$

Solution:

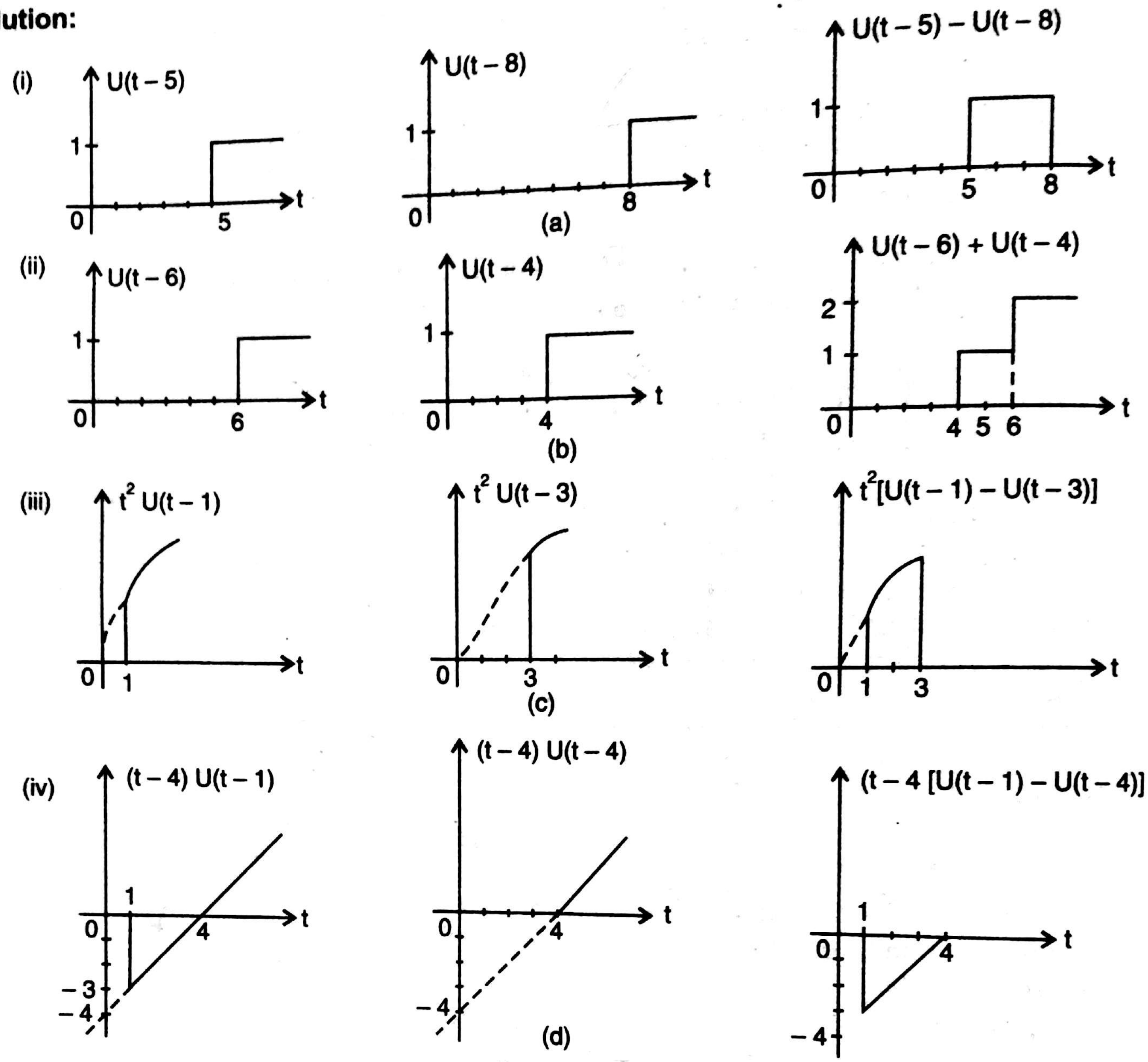

Fig. 2.33.

EXERCISES

2.1. Define a signal and also describe the different types of the signals.

2.2. Express the standard signals (or singularity functions) in mathematical and graphical forms and also write the relationship between them.

2.3. What is a gate signal and also explain the application of gate signal with the help of suitable example.

2.4. What is Direct formula (or K.M. formula). Illustrate the use of Direct formula with the help of an example.

PROBLEMS

2.1. Express $v(t)$, graphed in figure P.2.1, using the step signals.